THE ROLE OF MARKET PRICE-WEIGHT RELATIONSHIPS IN OPTIMAL BEEF CATTLE BACKGROUNDING PROGRAMS

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Approaches to the formulation of optimal beef cattle management programs have changed significantly since they were first examined in a linear programming context in the 1950s. Models were originally designed to identify feed rations which minimize the cost of providing minimum levels of energy and selected nutrients. Subsequent efforts sought to maximize overall farm or feedlot profits, including identification of optimal feeding weights, daily gains, and ration formulations (Kearl, Harris, and Fonnesbeck; Ladd and Williams). The studies varied in attention to such details as number of head fed, variety of alternative feeds, seasonality factors, animal characteristics, and the manner in which energy requirements and appetite are modeled Wilson, and others cast the California net

A significant improvement was achieved in optimal beef diet analysis when Brokken et al., Wilson, and others case the California net energy system in an economic framework. These authors, notably Wilson, emphasized that for finishing programs the California system's net energy requirements generally favor maximum voluntary intake of high-caloric-concentration rations, and thus maximization of daily weight gains. The purpose of our article is to expand the analysis of the net energy system by applying it to beef cattle winter backgrounding programs. The model developed suggests that, although Wilson's conclusions apply well to feedlots, they are not necessarily true for backgrounding situations, in which cattle are prepared for subsequent sale or use as feeders.

Just as with cattle finishing, any revenues earned from winter backgrounding are derived from weight added, from changes in the animal's condition, and/or from capital gains associated with increases in market prices while the animal is held. The value per unit weight of an animal in a winter backgrounding program is largely determined by the profits the animal would be expected to bring in a feedlot. Such expectations are related to the animal's sex, age, frame size, weight, degree of fat or conditioning, and other characteristics. For example, after correction for differences in other factors, imputed value per pound of feeder usually (but not always) declines as weight rises.[1] Thus backgrounders need to know the impact of alternative rates of gain on expected animal sales revenues as well as an expected feed costs.

RETURNS ABOVE FEED COST ON A GIVEN DAY

To demonstrate the last statement more clearly, we initially developed a framework in which a winter backgrounder formulates the quantity and quality of a ration so as to maximize the difference between an animal's value at the end of a given day and its value at the beginning of the day plus feed cost.

Net Revenue and Feed Cost Functions

Suppose for this purpose that market feeder cattle prices (P) are expressed as a function of all animal characteristics that may affect price, then all except the weight variable (W) are collapsed into the intercept term. The typically negative partial relation between weight and price may be approximated as a linear function over the 400-750-lb weight range in which most steer cattle are backgrounded; that is

$$(1) \quad P = a - bW, \qquad\qquad a, b > 0.$$

Defining W_0 and P_0 as beginning weight and price, and W_e and P_e as ending weight and price, we find that the difference or total net revenue (TNR) between the animal's value at the beginning and end of the day is[2]

[1] An animal's condition or percentage of body fat tends to increase along with weight as higher daily gains are achieved, because both conditioning and weight gain are affected by the amount of gain net energy consumed per day. All else constant, cattle with relatively little body fat are most preferred for feeding and grazing programs, because they offer the greatest promise of compensatory gains. But the preference is justified only if leanness results from low energy intake rather than poor genetic capacity for growth.

[2] "Net revenue" in this article refers to the difference between an animal's value at the end of a period and its value at the beginning of a period. Use of "net" in this sense does not imply an accounting for production costs such as feed expenses.

(2) $\text{TNR} = W_e P_e - W_o P_o$

$= W_e(a - bW_e) - W_o(a - bW_o).$

$= a(W_e - W_o) - b(W_e^2 - W_o^2)$

$= a(W_e - W_o) - b[(W_e - W_o)^2 + 2W_eW_o - 2W_o^2]$

$= a(W_e - W_o) - b(W_e - W_o)^2 - 2bW_eW_o + 2bW_o^2$

$= a(W_e - W_o) - b(W_e - W_o)^2 - 2bW_o(W_e - W_o)$

$= (a - 2bW_o)(W_e - W_o) - b(W_e - W_o)^2.$

Substituting $g = W_e - W_o$ for daily weight gain, we can represent equation 2 in the form:

(2') $\text{TNR} = (a - 2bW_o)g - bg^2.$

Net revenue at first rises, but at a continually decreasing rate, with increases in daily gain. Dividing equation 2' by g produces average net revenue, or the net revenue earned per pound of gain added. Of greater economic importance is marginal net revenue (MNR), that is, the addition to animal sale value caused by each pound added:

(3) $\text{MNR} = d\text{TNR}/dg = (a - 2bW_o) - (2b)g.$

Under the assumptions that a and b are positive, MNR is a linear and negatively sloped function of daily weight gain. The slope of this function (2b) varies directly with the slope of price-weight relationship (1) from which it is derived, whereas the intercept $(a - 2bW_o)$ increases with increases in the intercept and decreases with algebraic decreases in the slope of the price-weight function.

The relationship of daily feed cost to daily gain may be derived by use of the National Research Council (NRC) gain and maintenance net energy functions together with a specified ration and appropriate feed prices. In the NRC functions, a steer's or heifer's daily requirement of net energy for maintenance (NE_m) is a linear function of its metabolic weight, and its daily requirement of net energy for gain (NE_g) is a function of both metabolic weight and daily weight gain.[3] The price of a unit of maintenance net energy (P_{nem}) or gain net energy (P_{neg}) may be determined by dividing each feed price per unit dry matter by its NE_m or NE_g concentration per unit dry matter, multiplying by the proportion of ration dry matter accounted for by each feed, and summing these products. Total daily feed cost is then

(4) $\text{TFC} = (NE_m)(P_{nem}) + (NE_g)(P_{neg}).$

Expressing the NRC requirement functions for a steer on a pound-weight basis, and substituting into equation 4, we find

(4') $\text{TFC} = .077(W/2.2)^{.75}(P_{nem}) + (.02391g + .00141g^2)(W/2.2)^{.75}(P_{neg}).$

For any weight $W = (W_e + W_o)/2$, the corresponding average feed cost, or cost per pound of gain, is found by dividing equation 4' by g. The marginal feed cost function (MFC), that is, the feed cost of an additional pound of daily gain, is

(5) $\text{MFC} = d\text{TFC}/dg = (.02391 + .00282g)(W/2.2)^{.75}(P_{neg}),$

a linear and positively sloped function of daily weight gain.

Optimal Daily Weight Gain

Returns on a given day over feed and cattle purchase costs are now maximized by equating falling marginal net revenue (3) with rising marginal feed cost (5) and solving for the maximum-return rate of daily gain g*:

(6) $g* = \dfrac{a - 2bW_o - (.02391)(W/2.2)^{.75}(P_{neg})}{2b + (.00282)(W/2.2)^{.75}(P_{neg})}$.

Equation 6 defines a wide range of optimal daily gains, depending on levels of a, b, W, and P_{neg}. For example, as average weight W increases, optimal gain g* declines. Only some of these optimal levels are feasible in the sense of being consistent with the steer's ability to consume dry matter, an ability related to the steer's weight, to the energy concentration of the ration, and to other factors such as feed palatability (Fox and Black). Let, for example, g_m be the absolute maximum daily gain achievable by a steer at a given weight and ration energy concentration. Then if $g* \geq g_m$, a corner solution prevails at g_m. If $g* < g_m$, the marginal conditions are fulfilled and returns are optimized by operating at less than the maximum daily gain level. Less than maximum daily gains may be achieved by feeding less than the calf's voluntary intake limit or by feeding a relatively high roughage diet.

The responsiveness of optimal daily gains g* to the cattle price-weight relationship (1) is characterized by differentiating equation 6 with respect to price-weight intercept a and slope b. It is easy to show that response $\partial g*/\partial a$ is positive, meaning that increases in the cattle price-weight intercept augment opti-

[3]Where NE_m and NE_g are expressed in megacalories, W in kilograms, and g in kilograms/day, the NRC requirement functions for steers are $NE_m = .077W^{.75}$; $NE_g = W^{.75}(.05272g + .00684g^2).$

mal daily weight gains. The effect is explained by the fact that an upward shift of the price-weight intercept in equation 1 also increases the intercept of the marginal net revenue function (3). In contrast, the fact that response $\partial g*/\partial b$ is, under realistic feed price structures, negative implies that increases in the rate at which feeder prices fall with added weight serve to reduce optimal daily gains. In this case, the decline is caused by a downward shift of the intercept *and* an increase in the negative slope of the marginal net revenue function as the negative price-weight slope increases.

Hence, given a particular set of feed prices, there is some negative cattle price-weight slope b above which optimal daily gains g* fall below absolute maximum feasible daily gain g_m. This situation is depicted in Figure 1, where g_1^*

FIGURE 1.   ILLUSTRATION OF DAILY WEIGHT GAIN   OPTIMIZATION WHEN CATTLE PRICES ARE NEGATIVELY RELATED TO WEIGHT

Daily Weight Gain: g (lbs/day)
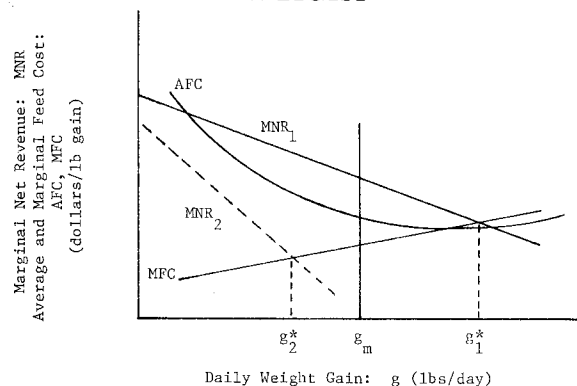
refers to an infeasible optimal daily gain given price-weight slope b_1, and g_2^* a feasible optimal daily gain given slope b_2 ($|b_2| > |b_1|$). A similar derivative of equation 6 with respect to feed ration price P_{neg} may also be shown to be negative, suggesting that increases in ration prices dampen optimal daily gain levels by shifting upward both the slope and the intercept of the marginal feed cost function.

RETURNS ABOVE FEED COST FOR ENTIRE BACKGROUNDING PERIOD

One reasonable choice criterion is that the backgrounder will seek to maximize the present value of the expected excess of net revenue over feed cost during the entire backgrounding period. Generalization of equations 1 through 6 to represent more than one day and to include a positive capital cost is not difficult. Additional complication occurs if the operator is allowed to select not only a constant rate of gain for the period but an optimal sale date as well. To permit such selection,

note that price equation 1 can be generalized to reflect changes in feeder prices during the time an animal is held for backgrounding. Specifically, both the slope and intercept of the equation can shift as time (t) passes. If a and b represent the feeder price intercept and slope, respectively, when an animal is first introduced to a backgrounding program, the price at any time during the program is:

(7) $P = a - bW + ct - dtW,$

where c and d represent shifts per time period in the intercept and slope. During most years, feeder price-weight intercepts shift upward between the fall and the spring because the supply of calves is lower in the spring and there is an added demand for spring grazers; thus $c > 0$. Moreover, premiums for lightweight over heavyweight feeders are generally greater in the spring than in the fall, so that d > 0 (see footnote 5).

The present value of net revenue (PVTNR) earned during the entire backgrounding period is specified by substituting the right side of equation 7 for P_e in equation 2 and the right side of equation 1 for P_0 in equation 2. Regrouping terms, noting that $W_e = gt + W_o$, and applying a present value operator, we have

(8) $PVTNR = \{(a - 2bW_o)(gt) - b(gt^2) +$
$(gt^2 + W_ot)[c -$
$(gtd + W_od)]\}(1+i)^{-t}$

where t is the number of days in the backgrounding program and i is a daily interest rate.

Because feed costs are incurred each day, the present value of feed cost (PVTFC) is found by substituting (gt + W_o) for W in equation 4′, multiplying 4′ by the present value operator, then summing 4′ over the t days the animal is backgrounded:

(9) $PVTFC = \sum_t \{.077\left[\dfrac{W_o+gt}{2.2}\right]^{75} P_{nem} +$
$(.02391g + .00141g^2) \cdot$
$\left[\dfrac{W_o+gt}{2.2}\right]^{75} P_{neg}\}(1 + i)^{-t}.$

Unlike equation 6, which identifies the optimal rate of gain for a single day in the backgrounding period, equations 8 and 9 may be used to determine an optimal average rate of gain g** during the entire backgrounding period and the optimal length t** of the period. Optimal values g**, t** are those values for which the first order conditions

(10) $\partial PVTNR/\partial g = \partial PVTFC/\partial g$
$\partial PVTNR/\partial t = \partial PVTFC/\partial t.$

are simultaneously satisfied.

The time dimension of the conditions in equations 10 implies that the animal is sold when the rate of increase in animal value $(\partial P_e W_e / \partial t)$, less feed cost in current period t, equals the implied return per period from investing the animal's sales proceeds at rate i. The gain dimension implies that the daily rate of gain is chosen for which increases with respect to gain in total net revenue just equal increases with respect to gain in total feed cost. Of course, optimal gain g** may exceed maximum feasible gain g_m; if it does, a corner solution occurs at g_m, t**. In general, the conditions of equations 10 are based on the assumption that the decision maker has linear (risk neutral) utility and is willing to base his decisions on expected values of c and d in equation 8 and P_{nem} and P_{neg} in equation 9.

Most important, the first order conditions represent a global optimum only if the ration employed is a least-cost ration at each rate of gain specified. This could be ensured by utilizing a trial ration for the purpose of first expressing as-fed feed prices on an NE_m and NE_g basis. Equations 10 would then be solved for a trial optimum g**, t** and a least-cost ration formulated for this level of gain. NE_m and NE_g-basis feed prices would subsequently be recalculated and the iterative process repeated with the hope that acceptable convergence would soon be achieved. Alternatively, simultaneous optimization with respect to daily gain, days in backgrounding, and ration formulation could be approached by mathematical programming methods.

LP APPLICATION

An LP model was developed to accomplish these objectives and, although it maximizes returns to owner equity rather than nonfeed costs, it serves otherwise to demonstrate the relationships outlined heretofore (Jessee and Buccola). The model spans a 1-year time horizon in which a farm operator is considered to make cattle and crop management decisions at the beginning of each quarter. Only implications for the early winter period are developed here. Potential feed constituents include corn grain, corn silage, hay, and pasture; rations are not restricted to the moderately high roughage range typical in backgrounding operations. Steers are available for purchase, or subsequent sale, at 100-lb weight increments between 500 and 1000 lbs. At each such weight, the operator has the option to feed at maintenance level (zero weight gain), at 1.1 lbs/day, and at 2.2 lbs/day. Net energy requirements are taken from the National Research Council, minimum protein requirements from

Carlson, and dry matter appetite relations from Nino and Hughes and from Fox and Black.

A baseline solution was first obtained with 1968-77 Virginia average crop yields, feed prices, and feeder cattle price-weight relations, as inflated to 1977 dollars.[4] This solution favored purchasing light steers, feeding them for maximum daily gain, then selling after one quarter. Corn prices were subsequently varied from $1.80 to $3.20/bu, corn grain yields from 70.5 to 119.5 bu/acre, and corn silage yields from 12.1 to 25.3 tons/acre. Such parameter alterations resulted in little variation in optimal cattle production activities. As a partial confirmation of Wilson's conclusions, ration caloric density changed only slightly, optimal daily gain levels remained at their maximum, and no shifts occurred in purchase or sale weights.

However, there was significant program reaction to changes in cattle price-weight functions. To render these results comparable to the given-day returns framework of equations 1-6, the steer price-weight function (1) for January 1 was set equal to that for October 1. The negative slopes of this function were then simultaneously increased (decreased algebraically) for both periods in intervals of $0.001/lb/100-lb weight increase, over the range $0.019 to $0.030. Optimal daily gains first fell below the program's maximum (2.2 lbs/day) to 1.1 lbs/day when the slope was decreased to $0.029/lb/100-lb weight increase. At this slope, the price of a 600-lb steer, for example, would be about $2.90/cwt below that of a 500-lb steer. As an indication of the frequency of such occurrence, average slopes steeper than this (1977 dollar basis) have characterized Virginia fall feeder steer sales during 5 of the past 20 years. Among Choice grade steers as a group, slopes have exceeded this during 11 of the past 20 years.

SIMULATION APPLICATION

A second approach to determining optimal average daily gain g** and optimal backgrounding period t** involves the use of a model which simulates the daily performance of a steer on a backgrounding ration. Equations 8 and 9 were evaluated at selected g and t values and those values were identified for which difference PVTNR − PVTFC was a maximum. This approach permitted consideration of smaller increments of g and t than was feasible under the linear program which, with only three alternative gain levels per season, exceeded 1,800 activities. The simulation program was used to model a winter background-

ing operation in which a medium framed, Good grade steer is started at 450 lbs and carried a maximum of 182 days. Sale is allowed at the end of any week during this period and a daily gain can be selected at any quarter-pound interval. The ration used consists of 88.6 percent corn silage (NRC #3-08-156) and 11.4 percent soybean meal (NRC #5-04-600), dry matter basis. Daily ration amounts required at each weight and daily gain level were calculated from the NRC requirements functions for maintenance and gain net energy (see footnote 3).[5] No attempt was made to develop a least-cost combination of ration nutrients; the ration cited is commonly used in backgrounding, however, and serves as a suitable basis for illustrating our conclusions.

In an actual fall decision-making context, a user would need to supply a forecast of the feeder steer price-weight relation he expects will prevail the following spring. For research purposes, we first calculated the 1968-77 mean intercept and slope of October price-weight relations for Good grade steers in Virginia (1977 dollar basis), and the corresponding means for April price-weight relations. These means were used to develop baseline estimates of coefficients a and b, and expectations of coefficients c and d, in equation 8. A set of parametric solutions were then obtained utilizing intercept and slope values one standard deviation below and above the 1968-77 mean levels. High intercept-low slope, high intercept -high slope, low intercept-high slope, and similar combinations were tried, as illustrated in Figure 2. Because these solutions always em-

FIGURE 2. SAMPLE OF LINEAR FEEDER CATTLE PRICE-WEIGHT RELATIONSHIPS USED IN WINTER BACKGROUNDING SIMULATIONS[a]
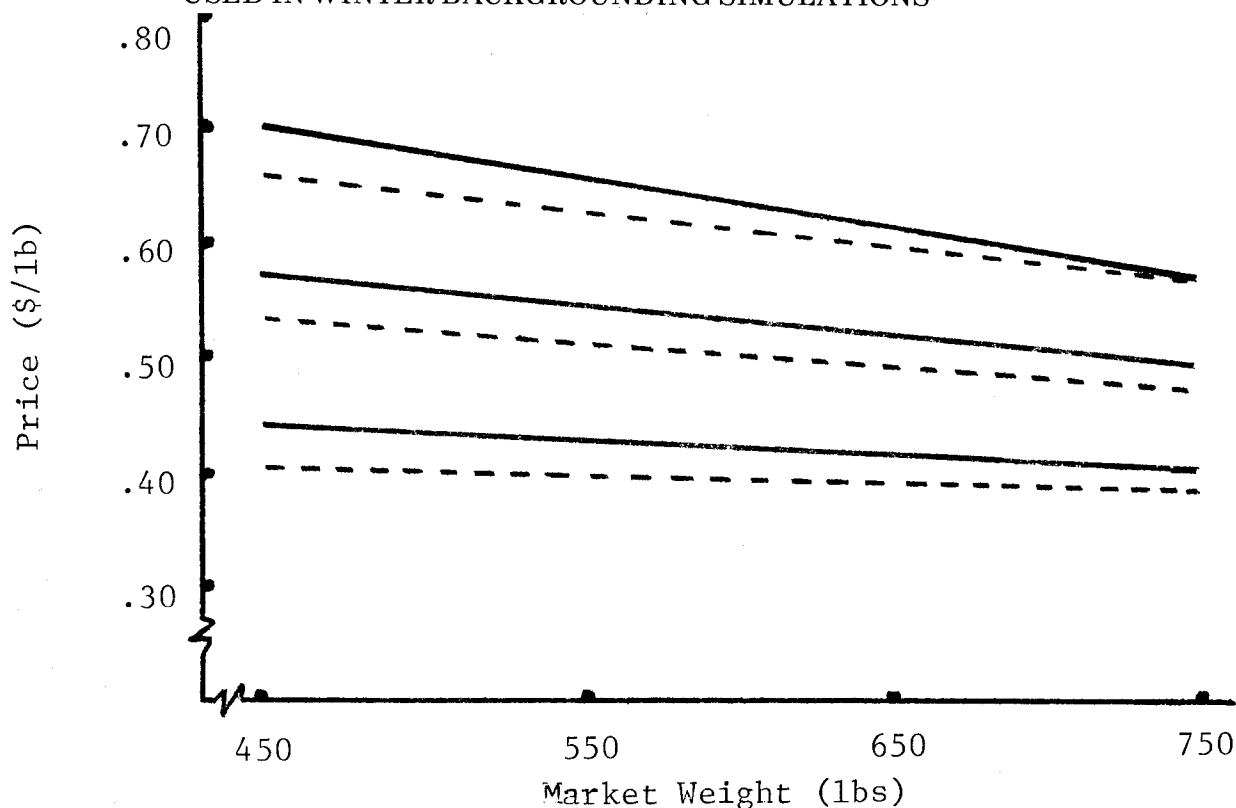
[a]Solid lines refer to spring, dotted lines to fall. The middle set illustrates the 1968-77 mean intercept and slope in Virginia state-graded feeder cattle sales (1977 dollar basis). The top set reflects intercept and slope one standard deviation above the mean, and the bottom set one standard deviation below the mean.

[5]The ration cited provides 1.617 Mcals NE_m/kg DM, or 1.033 Mcals NE_g/kg DM. The silage is assumed to contain 35 percent DM and the soybean meal 90 percent DM.

[6]The 1968-77 mean price-weight relation for Good grade steers was, in the fall, $P = .6121 - .000183W$, and in the spring, $P = .6840 - .000259W$, where P is in $/lb and W is in lbs. Standard deviations of these coefficients were, in order, .1809, .000125, .1984, .000156. The "low" feed price scenario consisted of $17.75/ton silage and $0.07/lb soybean meal ($P_{nem} = \$0.043$/Mcal, $P_{neg} = \$0.067$/Mcal). The "high" feed price scenario consisted of $27.38/ton silage and $0.10/lb soybean meal ($P_{nem} = \$0.065$/Mcal, $P_{neg} = \$.101$/Mcal).

[7]For each alternative backgrounding duration t, a maximum feasible daily gain g_m was calculated by expressing net energy requirements on a kg DM basis and comparing these with daily kg DM intake limits as estimated by Nino and Hughes and by Fox and Black. At the energy concentration (2.59 Mcals ME/kg DM) of the assumed ration, and assuming good feed palatability, a dry feedbunk, and other ideal conditions, both authors estimate daily DM intake for medium framed, growing steers to be approximately one-tenth the animal's metabolic weight ($W^{.75}$), as expressed in kilograms. Most farms do not have such ideal conditions and hence could not consistently achieve 2 lbs/day on the assumed ration. Thus the 2 lbs/day cited is used only for illustrative purposes.

ployed high fall slopes (−b) and high spring slopes (−b−dt) together, and low fall and low spring slopes together, a second set of parametric solutions was obtained in which only slopes of spring price-weight lines were permitted to decrease sequentially from their 1968-77 mean value and in which calf purchase price remained fixed. All solutions were derived for both "low" and "high" values of feed prices P_{nem} and P_{neg}, as calculated on the basis of the assumed ration and selected corn silage and soybean meal prices.[6]

Variable Backgrounding Period

A consistent feature of all the solutions is that the maximum-profit daily gain g** always exceeds the maximum feasible gain g_m of 2 lbs/day.[7] Also, where slopes of spring price-weight relations are equal to or flatter than the mean 1968-77 level and "low" feed prices are assumed, the optimal winter backgrounding duration t** is consistently 26 weeks, the maximum permitted in the model. Shorter backgrounding durations are usually preferred when intercept differences (ct) between fall and spring are less, or spring slopes (−b−dt) are steeper, than their 1968-77 mean values. Increases in feed prices also shorten optimal backgrounding durations t**. Tables 1 and 2

TABLE 1. OPTIMAL WINTER BACK-GROUNDING PROGRAMS FOR ALTERNATIVE SPRING PRICE-WEIGHT SLOPES AND LOW FEED PRICE LEVELS a b

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Price Discount For Increased Weight, Spring	Duration of Backgrounding	Sale Weight	Sale Price	Returns Over Feed Cost
($/cwt/cwt increase)	(days)	(lbs)	($/cwt)	($/head)
−2.60	182	814	47.33	59.53
−2.65	168	786	47.59	56.47
−2.74	161	772	47.30	51.45
−2.83	147	744	47.45	47.06
−2.92	133	716	47.71	46.68
−3.01	133	716	47.23	39.68

[a]Purchase weight is 450 lbs and purchase price is $53.00/cwt. The spring intercept is $68.00/cwt. In each solution, optimal daily gain exceeded 2 lbs/day. Maximum feasible daily gain is here assumed, under ideal conditions, to be 2 lbs/day, although the table could just as well be re-calculated to conform to the lower maximum feasible gains on many farms.
[b]The real daily interest rate (i) utilized in these solutions was zero, under the assumption that inflation has nearly kept pace with nominal borrowing rates. Re-solution with positive real rates reduced backgrounding durations and returns over feed cost.
[c]P_{nem} = $0.043/Mcal, P_{neg} = $.067/Mcal. For the assumed ration ingredients, this corresponds to $17.75/ton corn silage and $0.07/lb soybean meal.

TABLE 2. OPTIMAL WINTER BACK-GROUNDING PROGRAMS FOR ALTERNATIVE SPRING PRICE-WEIGHT SLOPES AND HIGH FEED PRICE LEVELS a

Price Discount For Increased Weight, Spring	Duration of Backgrounding	Sale Weight	Sale Price	Returns Over Feed Cost
($/cwt/cwt increase)	(days)	(lbs)	($/cwt)	($/head)
−2.60	112	674	50.16	25.29
−2.65	112	674	49.95	23.95
−2.74	98	646	50.14	21.65
−2.83	91	632	50.12	19.62
−2.92	91	632	49.83	17.80
−3.01	84	618	49.88	16.19

[a]P_{nem} = $0.065/Mcal, P_{neg} = $0.101/Mcal. For the assumed ration ingredients, this corresponds to $27.38 corn silage and $0.10/lb soybean meal. See also footnotes a and b under Table 1.

illustrate these results for the second set of solutions in which the fall price of a 450-lb steer is held at $0.53/lb (1977 basis) and the spring price-weight slope is allowed to steepen sequentially from its 1968-77 mean.

Examination of Tables 1 and 2 suggests that winter backgrounders should indeed react to steeper weight discounts on per-pound prices by reducing sale weights. But if they can find a market for their cattle in midwinter, it appears more profitable to reduce sale weights by reducing the period for which the cattle are held than by reducing their daily gains. Most of the solutions in Table 2 involve negative returns over total cost, a fact not surprising in view of the relatively high feed prices and low cattle sale prices assumed. In the short run we are considering, negative net returns would not discourage farmers from operating as long as revenues are sufficient to cover feed cost and unsunk nonfeed costs such as labor.

Fixed Backgrounding Period

Some backgrounders may feel that adequate midwinter markets for their cattle are not available. State-graded feeder cattle sales, for example, are often held only during the fall and early spring and many farmers prefer patronizing these markets. In general, if background-ing period t is held fixed at t_0 days, first order conditions (10) are reduced to the single equation dPVTNR/dg = dPVTFC/dg, in which PVTNR and PVTFC are evaluated at t = t_0 and in which g** is a conditional optimum subject to t = t_0. The LP model represents an example of these conditions in the sense that the holding period is restricted to at least 90 days.

Simulation results indicate that g** tends to decline as t_0 rises, although g** does not normally fall below 2 lbs/day for any t_0 less than 140 days. Table 3 shows the conditionally

TABLE 3. CONDITIONALLY OPTIMAL WINTER BACKGROUNDING PROGRAMS FOR ALTERNATIVE SPRING PRICE-WEIGHT SLOPES AND HIGH FEED PRICES, ASSUMING STEERS ARE HELD 182 DAYS a

Price Discount For Increased Weight, Spring	Average Daily Gain	Sale Weight	Sale Price	Returns Over Feed Cost
($/cwt/cwt increase)	(lbs/day)	(lbs)	($/cwt)	($/head)
-2.60	1.75	768	48.51	15.21
-2.65	1.75	768	48.14	12.31
-2.74	1.75	768	47.44	6.93
-2.83	1.75	768	46.75	1.56
-2.92	1.50	723	47.37	-3.21
-3.01	1.50	723	46.72	-7.97

$^aP_{nem}$ = $0.065/Mcal, P_{neg} = $0.101/Mcal. For the assumed ration ingredients, this corresponds to $27.38 corn silage and $0.10/lb soybean meal. See also footnotes a and b under Table 1.

optimum daily gains g**, corresponding to the same spring cattle prices and feed prices as in Table 2, where the backgrounding period is held at its maximum 182 days. It should be emphasized that in these solutions net revenues above feed cost are considerably below those attained in the variable feeding period optima shown in Table 2.

CONCLUSIONS

An important difference between designing optimal winter backgrounding programs and

designing optimal finishing programs for beef cattle is that backgrounders typically face declining per-pound prices with increases in sale weight. Wilson has shown that feedlots will maximize single-day returns if they allow maximum voluntary intake and maximize daily gains. But for winter backgrounders, returns on a given day may be greatest at some daily gain less than the feasible maximum if feeder cattle sale prices fall rapidly with increases in sale weight.

A more inclusive picture emerges when one analyzes the returns to the backgrounder over the entire holding period. Increases in the rate at which per-pound cattle sale prices fall with added weight, or increases in feed prices, induce backgrounders to sell at lighter weights; however, weight reduction may be effected by shortening the holding period as well as by reducing daily gains. Model results suggest that, when reduction in sale weight is desirable, it is generally most profitable to hold the animal for a shorter period and to continue to permit maximum voluntary intake, that is, to maximize the daily gain permitted by the moderately high roughage backgrounding ration. Optimal daily gain does tend to fall as the duration of the backgrounding program is constrained to increase. If backgrounding is required to last beyond 140 days, less than maximum feasible daily gains are often preferable.

It should be emphasized that these conclusions do not take into explicit account the influence of rate of gain on animal condition. High rates of gain may be associated with improvements in condition, and thus a reduction in the compensatory gains expected by buyers when the animal is put on feed or pasture. Explicit inclusion of the conditioning factor may very well strengthen the case for lower rates of gain.

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