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MONEY DEMAND AND SUPPLY SHOCKS, AND UNEMPLOYMENT*

R. J. SWEENEY

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This paper investigates the relative importance for U.S. unemployment of money demand and supply shocks. It develops a rational expectations model where price is fixed in the short run and disturbances to aggregate demand are met with inventory changes. Aggregate demand disturbances depend on money demand and supply shocks. Empirical evidence indicates money demand shocks are a substantial fraction of total monetary disturbances; the importance of supply shocks increases with the volatility of the period considered.

I. Introduction

In this model, money supply shocks, net of money demand shocks, cause excess demand for output which is met by running down inventories. Businesses then adjust output and employment to restore inventories to optimum levels. The cross correlation function of these net monetary shocks and unexpected changes in the rate of unemployment provides a weak-form test of the model, a test not requiring detailed specification and estimation of structural equations.

There is no time series on net monetary shocks, though series on money supply shocks have been derived (for example, Barro [1977]). Section II notes, though, that variations in real balances per unit of output (M/PY) are related to these net monetary shocks, and derives the relationship between the unanticipated variations in M/PY and money demand and supply shocks. A theoretical cross correlation function (ccf) is derived for the unanticipated changes in M/PY and unanticipated changes in the unemployment rate (Nu); the properties of this function depend crucially on the ratio of money demand to supply shocks. Hence, the model is tested by examining the sample ccf, where unanticipated changes in the series are taken as the innovations found by whitening them with Box-Jenkins [1976] techniques, to see if it corresponds adequately to the range consistent with the theory; further, the structure of the sample ccf is also evidence of the relative importance of the two sources of monetary shocks.

Section III presents empirical results. Series on real balances per unit of output are examined for base money, M1 and M2. Each series is whitened and then cross-correlated with the innovation in the U.S. civilian unemployment rate. The two periods considered are (a) 1953I - 1971III, a relatively quiet period broken by accelerating inflation and by the imposition of price controls in August, 1971; and (b) 1948I - 1976III, a longer, more turbulent period where data are marred by price control programs of the 1940's, 1950's, and 1970's. Interpretation of the relative influence of money supply and demand shocks turns on the value of the correlation coefficient at lag zero. It seems plausible that during the shorter period, money supply shocks accounted for between 21% and 57% of total monetary shocks; and during the longer, more volatile period, from 29% to 68%. From this finding, increased money supply instability might be held responsible for the greater real variability of the longer period. However, focusing solely on money supply shocks seriously understates total monetary shocks.

II. A Simple Model of Macro Adjustment

This section describes a macro model with rational expectations. Earlier models (Lucas [1972], Sargent and Wallace [1975]) did not display serial correlation in output changes or in changes in unemployment. There exists, however, notable serial correlation in, for example, unemployment rates. Some later work (e.g., Phelps and Taylor [1977], Haraf [1979]) emphasized the buffer stock role of inventories as the transmission mechanism from current demand disturbances for output to future changes in outout levels. This is the approach exploited here.

Suppose the typical firm sets price and holds it constant for a period of time (this period being endogenous over the longer run). Price is at the general equilibrium level based on best guesses of demand and costs, where "best guesses" are rational in the sense of Muth [1961]. During the period, the firm (almost surely) discovers its anticipations were wrong, with demand being higher or lower than expected. The firm reacts in two ways. At the start of the next period, it adjusts price to the level consistent with its best guess, now based on the information set including last period's experience. Furthermore, in the first period, the firm had unexpected inventory decumulation or accumulation, as demand was greater or less than expected. In the second period, it readjusts inventories; it hire more (fewer) workers, works more (less) over-time, etc. In particular, changes in inventories intermediate between demand shocks in one period, and output and unemployment adjustments in following periods.

Subsections A–C below describe (A) the behavior of M/PY due to money demand and supply shocks; (B) the behavior of Nu in response to these shocks; and (C) the relationship of innovations in M/PY and Nu.

A. Description of the Model; Behavior of M/PY.

There are four goods — money, output, bonds and labor services — in the model. The law of excess demand requires that the sum of the values of the excess demand for money, output and bonds equal zero, or

$$(1) \qquad P(Y^d - Y) + (M^d - M) + P_B(B^d - B^s) \equiv 0,$$

where P is the price level of commodities, Y equilibrium real output, Y^d real aggregate demand, M^d money demand, M money supply, P_B the price of bonds,

B^d the demand and B^s the supply of bonds. P is fixed at the start of the period, but the interest rate (and hence P_B) adjusts at all times to clear the bond market, that is, to set $B^d - B^s = 0$. Thus, taking account of how the interest rate adjusts, the law of excess demand (1) ensures

(2) $$(M^d - M) + P(Y^d - Y) = 0,$$

(3) $$(M^d/PY - M/PY) + (Y^d/Y - 1) = 0.$$

Disturbances to aggregate demand relative to output (Y^d/Y) can thus be analyzed in terms of shocks to money demand and supply, or to M^d/PY and M/PY.

To investigate disturbances, consider first money demand and then money supply shocks. To start, suppose M and Y are constant (or more generally that M and Y grow at exogenous, perhaps different, rates). At the end of period _t_, price setters set P_{t+1} to make $E(Y^d_{t+1}) = Y$, where E is the mathematical expectation operator. From (3) this requires

(4) $$E(M^d/PY)_{t+1} = (M/PY)_{t+1}.$$

With expectations formed rationally, the difference between $(M^d/PY)_{t+1}$ and $E(M^d/PY)_{t+1}$ is a white noise, ε_{t+1}. In an assumption relaxed later, assume the best guess about next period's M^d/PY is that it will equal this period's M^d/PY. This implies M^d/PY is generated by a random walk process, or

(5) $$(M^d/PY)_t = (M^d/PY)_o + \sum_{j=1}^{t} \varepsilon_j.$$

Thus, when business sets price to make $(M/PY)_{t+1} = E(M^d/PY)_{t+1}$, it sets $(M/PY)_{t+1} = (M^d/PY)_t$; hence, from (5)

$$(6) \qquad \Delta(M/PY)_{t+1} \equiv (M/PY)_{t+1} - (M/PY)_t = \varepsilon_t.$$

In this simplified case, M/PY can be represented by a random walk process;[1/] actual real balances adjust with a one-period lag to unforecasted changes in demand for them.

Suppose that instead of constant growth, the money supply has a deterministic growth component α, and a random component $e_t P_t Y_t$, with e_t a white noise.[2/] Again suppose that price-setters consider only expected value. Then, since $E(e) = 0$, price-setters ignore this element in setting price. When price-setters set $E(M^d/PY)_{t+1} = E(M/PY)_{t+1}$, from the demand side, $E(M^d/PY)_{t+1} = (M^d/PY)_t$ and hence they set $E(M/PY)_{t+1} = (M^d/PY)_t$. However, the forecast error for $(M/PY)_{t+1}$ is e_{t+1}, so $(M/PY)_{t+1} = (M^d/PY)_t + e_{t+1}$. Thus, using (5),

$$(7) \qquad \Delta(M/PY)_{t+1} \equiv (M/PY)_{t+1} - (M/PY)_t = (M^d/PY)_t + e_{t+1} - \left[(M^d/PY)_{t-1} + e_t\right]$$

$$= \varepsilon_t + e_{t+1} - e_t.$$

So far, $\Delta(M/PY)_{t+1}$ reflects permanent demand shocks, and also permanent money supply shocks that are however eliminated (with a one-period lag) by price changes. If $\varepsilon_t \equiv 0$ for all _t_, M/PY is stationary.

ε and _e_ are assumed orthogonal, since any systematic relationship could arise only if the monetary authorities could, in period t-1, forecast ε_t better than the market.

Under more general conditions, there will be cyclical variations in monetary policy and real growth, and demand shocks need not all be permanent. Nevertheless, over the longer term the evolution of (M/PY) will be governed by the sequence of terms $(\varepsilon_t + e_{t+1} - e_t)$. Short run influences can take on a wide variety of forms. Suppose, however, that all stochastic variation in (M/PY) is made up of some weighted average of current and past values of the long run forcing term, $(\varepsilon_t + e_{t+1} - e_t)$. This can be modelled, then, as

$$(8) \qquad \Delta(M/PY)_{t+1} = K(B)(\varepsilon_t + e_{t+1} - e_t),$$

where K(B) is a (possibly infinite) polynomial in the backshift operator B.[3/]

The unobservable term $(\varepsilon_t + e_{t+1} - e_t)$ can be investigated as follows. Let

$$(9) \qquad b_t \equiv \varepsilon_{t-1} + e_t - e_{t-1}.$$

b_t has an autocorrelation function (acf) of order one, and b_t can be viewed as a first order moving average transformation of some white noice processes a_t (Ansley, Spivey and Wrobleski [1977]), where

$$(10) \qquad b_t = (1 - B\theta)a_t,$$

$$(11) \qquad \frac{b_t}{1-B\theta} = (1 + \theta B + \theta^2B^2 + \ldots)b_t = a_t.$$

From (8) and (10),

$$(12) \qquad \Delta(M/PY)_t = K(B)(b_t) = K(B)[(1 - B\theta)a_t].$$

Consequently, if Δ(M/PY) is whitened using Box-Jenkins [1976] techniques, the aim is to obtain a_t, which can be used to investigate b_t.

Section III examines the whitened series on M/PY and on unemployment, both to test the model built in this section and to draw influences about the relative magnitudes of σ_ε^2 and σ_e^2, the variances of money demand and supply shocks. The next step is to consider the reaction of output and unemployment to monetary shocks.

B. The Behavior of Output and Unemployment

If aggregate demand shocks reduce inventory below its optimum, it becomes progressively more important to increase production beyond normal levels to rebuild stocks "dangerously" low relative to possible coming shocks. The higher the costs of accelerated production (AP), the more gradual such buildups and the larger the optimum stock in order to forestall such accelerations. However, the more costly is holding inventories, the more will accelerated production be used.

The effects of shocks on output are considered for convenience as deviations from a given "natural" (trend) rate of full-employment output, Yn. Let S_t^d be the stock of goods businesses, at the start of period _t_, desire to have on hand at the end of the period, and S_t the actual stock on hand. Assume that some fraction of the deviation $(S_t^d - S_t)$ is made up every period by having Y exceed Yn. AP_t per unit of "permanent" output is $ap_t \equiv AP_t/Y_n$.

It is cheaper to rebuild stocks over several periods. Let AP be distributed linearly through the next _n_ periods as a function of the current gap $(S_t^d - S_t)$, or in the absence of forther shocks,

$$(13) \qquad AP_{t+j} = a_j(S_t^d - S_t), \qquad \sum^n a_j = 1, \qquad a_j \geq 0.$$

ap is derived as follows. In period 1, $ap_1 = a_1(S_1^d - S_1)/Y_n$. In period 2,

$ap_2 = a_2(S_1^d - S_1)/Y_n + a_1(e_1 - \varepsilon_1)$, where $a_2(S_1^d - S_1)/Y_n$ represents the continuing rebuilding of stocks begun in period 1, and $a_1(e_1 - \varepsilon_1)$ represents the additional rebuilding of stocks (begun in period 2) necessitated by the shock to aggregate demand (the negative of the monetary shock) in period 1.[4/] Similarly, $ap_3 = a_3(S_1^d - S_1)/Y_n + a_2(e_1 - \varepsilon_1) + a_1(e_2 - \varepsilon_2)$ and finally $ap_n = a_n(S_1^d - S_1) + a_{n-1}(e_1 - \varepsilon_1) + \ldots + a_1(e_{n-1} - \varepsilon_{n-1})$.

These relations can be combined to give $ap_2 = a_2(S_1^d - S_1) + a_1(e_1 - \varepsilon_1) = (a_2/a_1)ap_1 + a_1(e_1 - \varepsilon_1)$, $\quad ap_3 = \left[\frac{a_3}{a_1} - \left(\frac{a_2}{a_1}\right)^2\right] ap_1 + \frac{a_3}{a_1} ap_2 + a_1(e_2 - \varepsilon_2)$ and so forth to

$$(14) \qquad ap_n = + a_1(e_{n-1} - \varepsilon_{n-1}) + \sum_{n=1}^{n-1} c_j ap_{n-j},$$

with the c_j derived as above.

The current employment rate is Nu_t; assume Nu_t and ap_t are negatively and (locally) linearly related, with factor β. The innovation in Nu, the part not predictable from past values of Nu, depends on the innovation in ap_t, i.e., on $(e_{t-1} - \varepsilon_{t-1})$. However, Nu is not stationary; but, as seen in Table I, $Nu_t - Nu_{t-1}$ is stationary. Hence the relevant forcing term for $\Delta Nu_t (\equiv Nu_t - Nu_{t-1})$ is $-\{\beta(e_{t-1} - \varepsilon_{t-1}) - \beta(e_{t-2} - \varepsilon_{t-2}) - v_t\}$, where v_t is assumed a white noise that constitutes a transitory change in ΔNu (but a permanent change in Nu); Nu_t depends on $\sum_{j=0}^{\infty} v_{t-j} - \beta(e_{t-1} - \varepsilon_{t-1})$. Thus, changes in the unemployment rate are modelled as

$$(15) \qquad \Delta Nu_t = K'(B)(b'_t),$$

$K'(1) =$ constant, where $K'(B)$ is a polynomial in B,[5/] and

$$(16) \qquad b'_t \equiv v_t - \{\beta(e_{t-1} - \varepsilon_{t-1}) - \beta(e_{t-2} - \varepsilon_{t-2})\}.$$

b'_t is a first order moving average process based on some white noise a'_t, where

(17) $\quad b'_t = (1 - B\theta')a'_t,$

(18) $\quad \frac{b'_t}{(1 - B\theta')} = (1 + \theta'B + \theta'^2B^2 + \ldots)b'_t = a'_t.$

Thus,

(19) $\quad \Delta Nu_t = K'(B)[(1 - B\theta')a'_t].$

Consequently, using Box-Jenkins techniques to whiten the observed Nu series attempts to obtain a'_t.

C. Relation of Innovations in Nu and M/PY

Consider the implications of the foregoing analysis for the intertemporal relationship of a_t and a'_t, as shown by their cross correlation function. The covariance of the innovations in ΔNu and Δ[M/PY] at lag zero is

$$E[a'_t\, a_t] = E\{[(1 + \theta'B + \theta'^2B^2 + \ldots)b'_t] \cdot [(1 + \theta B + \theta^2B^2 + \ldots)b_t]\}.$$

The covariance of b'_t with $(1 + \theta B + \ldots)b_t$ is

$$[+\beta(\sigma_e^2 + \sigma_\varepsilon^2) - \theta\beta(\sigma_e^2 + \sigma_\varepsilon^2) - \beta\theta\sigma_e^2 + \beta\theta^2\sigma_e^2];$$

the covariance of $\theta'b'_{t-1}$ with $[(1 + \theta B + \ldots)b_t]$ is

$$\theta'\theta[+\beta(\sigma_e^2 + \sigma_\varepsilon^2) - \theta\beta(\sigma_e^2 + \sigma_\varepsilon^2) - \beta\theta\sigma_e^2 + \beta\theta^2\sigma_e^2],$$

and so on. Summing,

$$E[a'_t a_t] = \frac{\beta}{1-\theta'\theta}[(\sigma_e^2 + \sigma_\varepsilon^2) - \theta(\sigma_e^2 + \sigma_\varepsilon^2) - \theta\sigma_e^2 + \theta^2\sigma_e^2] .$$

Hence, the correlation coefficient for a'_t and a_t is

$$(20)\qquad \rho(0) = \frac{\frac{\beta\sigma_e^2}{1-\theta'\theta}[(1+\sigma_\varepsilon^2/\sigma_e^2)(1-\theta) - \theta+\theta^2]}{[\sigma_a^2]^{\frac{1}{2}}\quad[\sigma_{a'}^2]^{\frac{1}{2}}} \geq 0.$$

It can be shown that (20) can be transformed to 6/

$$(21)\qquad \rho(0) = \frac{\frac{1}{1-\theta'\theta}\,[(1+\sigma_\varepsilon^2/\sigma_e^2)(1-\theta) - \theta+\theta^2]}{\left[\frac{1}{\theta}\right]^{\frac{1}{2}}\cdot\left[\frac{(1+\sigma_\varepsilon^2/\sigma_e^2)}{\theta'}\right]^{\frac{1}{2}}}.$$

θ and θ' can be found as 7/

$$(22)\qquad \theta = \frac{(\sigma_\varepsilon^2/\sigma_e^2 + 2) - [4(\sigma_\varepsilon^2/\sigma_e^2) + (\sigma_\varepsilon^2/\sigma_e^2)^2]^{\frac{1}{2}}}{2}$$

and 8/

$$(23)\qquad \theta' = \frac{2 + \frac{\sigma_v^2/\beta^2\sigma_e^2}{(\sigma_\varepsilon^2/\sigma_e^2+1)} - \left\{\left[\frac{\sigma_v^2/\beta^2\sigma_e^2}{(\sigma_\varepsilon^2/\sigma_e^2+1)}\right]^2 + 4\left[\frac{\sigma_v^2/\beta^2\sigma_e^2}{(\sigma_\varepsilon^2/\sigma_e^2+1)}\right]\right\}}{2}.$$

Thus, (22) and (23) can be solved to find θ' and θ when $\sigma_\varepsilon^2/\sigma_e^2$ and $\sigma_v^2/\beta^2\sigma_e^2$ are known, and then $\rho(0)$ can be found from (21). It is also useful to examine the correlation coefficient of a'_t and a_{t-1}; $\rho(-1) = \theta'\rho(0)$. The coefficient for a'_t and a_{t+1} is $\rho(+1) = \theta\rho(0)$. Clearly, $\rho[\pm(1+k)]$ falls off rapidly for $k \geq 1$.

III. Empirical Results

This section reports on whitened series on M/PY and Nu,9/ and on the cross correlation functions of the innovations found for these series. The pattern found is quite consistent with the model developed above.

Estimates suggest that for the period 1953II - 1971II, M1 money supply shocks were responsible for between 21% to 57% of the total variance of unanticipated net money demand and supply changes. Estimates for this period, using M2 and the monetary base (MB), give virtually the same results as the M1 concept for the relative importance of money demand and supply shocks. For the longer and more turbulent period, 1947II - 1976III, M1 money supply shocks accounted for between 29% to 68% of net monetary shocks, supporting the view that money supply shocks were responsible for the greater turbulence of the period; using MB gives very similar results.

Table I displays the results of using Box-Jenkins [1976] methods to whiten the ΔNu series. The autocorrelation function (acf) for the residuals and the chi-square test show no signs of model inadequacy.[10/] Table II shows the results for whitening the first difference of M/PY for the M1 money supply. In this case also, the acf of the residuals and the chi-square test give no indications of model inadequacy.

The ccf between the residuals of the whitened Nu and M/PY series is given in Table III. The coefficient for lag zero, $\hat{\rho}(0)$, is more than four standard errors away from zero, with the correct sign. (With Nu lagging M/PY, there are borderline significant coefficients at lags 4 and 7; the former might conceivably be due to seasonality, but the latter has no reasonable role other than sampling variability.) This is strong evidence of the fundamental relationship between net monetary shocks and unemployment disturbances. (Note that Box-Jenkins techniques are free of the tendency to "spurious regression", and indeed it is often difficult to find relationships with them even when theory strongly indicates such relationships.[11/])

A major interest is to infer $\sigma_e^2/\sigma_\varepsilon^2$ from the ccf in Table III. The smooth curve in Figure 1, based on equations (21)-(23) shows combinations of the unobservable $\sigma_\varepsilon^2/\sigma_e^2$ and $\sigma_v^2/\beta^2\sigma_\varepsilon^2$ that give $\rho(0) = 0.495$.[12/] The non-linear equations (21)-(23) give three local extrema for values of $\sigma_\varepsilon^2/\sigma_e^2$ between .1 and 5.0.[13/] The ratio $\sigma_v^2/\beta^2\sigma_e^2$ gives the relative importance of autonomous shocks to Nu versus those due to money supply disturbances (recall β is a scale factor relating aggregate demand shocks to unemployment). It seems reasonable that $\sigma_v^2/\beta^2\sigma_e^2$ may be between, say, 0.5 and 2.0. Over this range, the values of $\sigma_\varepsilon^2/\sigma_e^2$ that give $\rho(0) = .495$ range from .77 to 3.8, implying that the variance of money supply shocks relative to total monetary shocks, $\sigma_e^2/(\sigma_e^2 + \sigma_\varepsilon^2) = (\sigma_e^2/\sigma_\varepsilon^2) \div (1 + \sigma_e^2/\sigma_\varepsilon^2)$, lies between 21% to 57%.[14/]

This estimate gives, perhaps, a surprisingly small role to money supply shocks relative to demand shocks. But recall that 1953 - 1971 was not a dreadfully volatile period compared to the historical norm.[15/]

As noted above, when the whitened M/PY series for M2 and MB are cross correlated on whitened Nu, each gives results virtually identical to those for M1.[16/]

Section II showed that the theoretical values of $\rho(1)$ and $\rho(-1)$ are positive, and this is seen in Table III (the same is true for the M2 and MB concepts). While these coefficients are less than two standard errors from zero, from Section II this is to be expected with $\rho(0) = .495$.[17/]

For the longer period QII 1948 - QIII 1976, the sample ccf for the M1 concept yields $\hat{\rho}(0) = .352$, almost four standard errors away from zero, and with the correct sign. ($\hat{\rho}(-1)$ has the correct sign; $\hat{\rho}(1)$ does not, but is quite insignificant.) The results for MB are virtually identical. As with the shorter period, there are significant spikes for $\hat{\rho}(4)$. But there are also a larger number of significant spikes at rather bizarre lags of, say, -19 and +24 quarters for the M1 concept. The cross-hatched curve in Figure 1 shows that if the hypothesis $\rho(0) = .352$ is entertained, the money supply shocks contributed 29% to 68% of total variance due to monetary disturbances.[18/]

It seems plausible that the estimates of $\rho(0)$ differ because the population $\sigma_e^2/\sigma_\varepsilon^2$ was larger over the longer period. However, the two estimates are within two standard errors of each other. The range of estimates for $\sigma_\varepsilon^2/\sigma_e^2$ for the two periods imply that money supply shocks were only about 1.21 to 1.36 times as important for the longer period as compared to the shorter period.[19/]

IV. Conclusions

Monetary disturbances clearly affect the real economy. This paper examines a model where all disturbances to aggregate demand can be analyzed in terms of money demand and supply shocks. This follows from the assumption that price is set in the short run at the value expected to clear the market for output (on the basis of rational expectations) and the interest rate adjusts to set the excess demand for bonds equal to zero in each period. Shocks to money demand and supply at the given short run price level thus show up as unexpected disturbances to aggregate demand that are met by changing inventories. These inventory disturbances cause changes in production, employment and unemployment over time.

This paper does not test competing models. However, the present model implies that the whitened series on M/PY and Nu will have a ccf with the major spike at lag zero, the major spike positive, and other spikes falling off rather sharply on both sides of lag zero. Thus, (a) a statistically significant negative spike at lag zero, (b) failure to find a significant spike at lag zero or failure of the spikes to fall off, and (c) a significant cluster of spikes away from lag zero, would constitute evidence against the theory. The observed ccf supports the model.

It might be argued that a simple inverse relationship between Y and Nu might well give the observed ccf in Table III. For example, suppose innovations in Y and Nu are inversely related, but neither are influenced by money demand or supply. Of course, a major interest of the paper is to suppose that money demand and supply shocks <u>are</u> the forcing terms (along with v), and to determine their relative influences. Note, however, that the proposed alternative does not show how fluctuations in Y arise (other than from v), nor how such fluctuations in Y can arise other than from shocks that will be reflected in either money demand or supply in the context of the model used here.

The observed ccf indicates that money supply shocks were somewhere between 21% to 57% of total monetary disturbances in the period 1953QI to 1971QII. In the longer, more volatile period 1948I - 1976III, money supply shocks were somewhere between 29% and 68%. This might well be interpreted as saying that the increased volatility of the longer period was due to larger money supply disturbances. However, in both periods money demand shocks played a substantial role, and did so whether high or low estimates are taken of the relative role of money supply shocks.

FOOTNOTES

1/ This process, or the more generalized process discussed below, might display drift, that is, the mean rate of change of $\Delta(M/PY)$ may be non-zero.

Note that the text's discussion nowhere requires that M^d/PY be independent of Y, or M^d/P unit elastic with respect to Y. Indeed, let $M^d/P = kY^{\eta}$, $\eta>0$, so $M^d/PY = kY^{(\eta-1)}$. Then, in the absence of demand and supply disturbances, $M/PY = M^d/PY = kY^{(\eta-1)}$ and $(M/\dot{P}Y)/(M/PY) = (\eta-1)(\dot{Y}/Y)$. If the trend real growth rate is $\dot{Y}/Y = n$, then M/PY shows a drift of $(\eta-1)n$ per period. There would, of course, be fluctuations around this drift along lines discussed in the text. Gould and Nelson (1974) show that yearly M2 velocity data from 1867-1960 appear to be generated by a random walk process; they note, however, that quarterly data in the post-WW II period do not.

2/ Different formulations would of course alter the particular formulae given below, but the general points of this section remain valid. See below for a discussion of when there are also transitory money demand and supply shocks built on these permanent shocks.

3/ K(1) equals a constant $\gtreqless$ 0, depending on the trend rate of change in M/PY. See footnote 1 above for effects on trends in velocity if the income elasticity of demand for real balances is not equal to unity.

4/ This follows from the budget constraint in (2).

5/ K'(1) equals a constant $\gtrless$ 0, depending on the trend rate of change in Nu.

6/ From (9),

$$E[b_t b_{b+1}] = E\{(\varepsilon_{t-1} - e_{t-1} + e_t) \cdot (\varepsilon_t - e_t + e_{t+1})\} = -\sigma_e^2;$$

and from (10),

$$E[b_t b_{t+1}] = E\{(1 - \theta B)a_{t+1} \cdot (1 - \theta B)a_t\} = -\theta\sigma_a^2.$$

It follows that

$$\sigma_a^2 = \frac{\sigma_e^2}{\theta} .$$

Similarly, from (16) and (17),

$$E[b'_t b'_{t+1}] = -\beta^2(\sigma_e^2 + \sigma_\varepsilon^2) = -\theta'\sigma_{a'}^2 ,$$

and thus

$$\sigma_{a'}^2 = \frac{\beta^2(\sigma_e^2 + \sigma_\varepsilon^2)}{\theta'} .$$

Substituting these two results into (20) for σ_a^2 and $\sigma_{a'}^2$ gives

$$\rho(0) = \frac{\frac{\beta\sigma_e^2}{1-\theta'\theta}[(1 + \sigma_\varepsilon^2/\sigma_e^2)\ (1-\theta) - \theta + \theta^2]}{\left[\frac{\sigma_e^2}{\theta}\right]^{\frac{1}{2}} \cdot \left[\frac{\beta^2(\sigma_e^2 + \sigma_\varepsilon^2)}{\theta'}\right]^{\frac{1}{2}}}$$

This can be simplified to (21).

7/ From (9) and (10),

$$E[b_t]^2 = E[\varepsilon_{t-1} - e_{t-1} + e_t]^2 = E[(1 - \theta B)a_t]^2$$

and thus

$$\sigma_\varepsilon^2 + 2\sigma_e^2 = (1 + \theta^2)\sigma_a^2.$$

Substituting in the result in footnote 6, that $\sigma_a^2 = \sigma_e^2/\theta$, gives

$$\sigma_\varepsilon^2 + 2\sigma_e^2 = \frac{1 + \theta^2}{\theta}\sigma_e^2 .$$

Solving this quadratic for θ gives two roots, both positive, one greater than unity, the less than unity. To make the moving average process invertible, the smaller root, shown in the text, is selected.

This choice is not arbitrary, but is dictated by economic logic. If the larger root were chosen, past shocks would have larger and larger weight in current M/PY as time went on (and relative to current shocks). See Box and Jenkins [1976].

8/ From (16) and (17)

$$E[b'_t]^2 = 2\beta^2(\sigma_e^2 + \sigma_\varepsilon^2) + \sigma_v^2 = (1 + \theta'^2)\sigma_{a'}^2.$$

Further, as seen in footnote (6) above,

$$E[b'_{t+1}b'_t] = -\beta^2(\sigma_e^2 + \sigma_\varepsilon^2) = -\theta'\sigma_{a'}^2.$$

Combining these two results,

$$2\beta^2(\sigma_e^2 + \sigma_\varepsilon^2) + \sigma_v^2 = (1 + \theta'^2)\,\frac{\beta^2}{\theta'}\,(\sigma_e^2 + \sigma_\varepsilon^2),$$

or

$$\theta'^2 - \theta'\left[2 + \frac{\sigma_v^2}{\beta^2(\sigma_e^2 + \sigma_\varepsilon^2)}\right] + 1 = 0.$$

This quadratic can be solved for θ', giving two roots, both positive, one greater than unity, the other less than unity. To make the moving average process invertible, the smaller root, shown in the text, is selected.

9/ Data are from the NBER data base, accessed through the NY Federal Reserve Bank's computer system. The money supply series, averages of daily figures, seasonally adjusted, are

Series	Retrieval Code
Monetary Base	FMBASE
M1	FMS
M2	FMM2X

The unemployment rate (LHUR) is for the total civilian labor force.

10/ While the t-statistic is only 1.29 for the second-order autoregressive parameter and the correlation of parameter estimates is -.649 between this parameter and the first-order autoregressive parameter, the results reported below are not very sensitive to inclusion of this parameter. The chi-square statistic tests the hypothesis that the acf as a whole is not significantly different from zero. A large sample statistic argues against the null hypothesis; the "Confidence Level" indicates the percentage of experiments in which one would expect a larger statistic under the null hypothesis of zero correlation. See Box and Jenkins [1976]. Further, ΔNu showed no significant correlation with future values of the residuals in any tests (and the same is true of ΔM/PY and its residulas); such cross correlation results are ommitted from the tables.

11/ See Pierce [1977] and the comments following by Granger and Sims, as well as Feige and Pearce [1976].

12. The curve in Figure 1 was found as follows. First, a value of $\sigma_\varepsilon^2/\sigma_e^2$ was picked (between 0.1 and 5.0, in increments of 0.1). From (22), θ was then found. When the value of $\sigma_\varepsilon^2/\sigma_e^2$ and its associated θ was inserted in (21), along with $\rho(0)$ equal the estimated .495, θ' was found. $\sigma_v^2/\beta^2\sigma_e^2$ was then found on the basis of (22) for the given value of $\sigma_\varepsilon^2/\sigma_e^2$ and the associated θ'.

13/ There is a local minimum around $\sigma_\varepsilon^2/\sigma_e^2 = .35$, with $\sigma_v^2/\beta^2\sigma_e^2 \cong .0025$. There is a local maximum around $\sigma_\varepsilon^2/\sigma_e^2 = 1.9$, with $\sigma_v^2/\beta^2\sigma_e^2 \cong 6{,}000$. Around $\sigma_\varepsilon^2/\sigma_e^2 = 3.3$, there is another local minimum with $\sigma_v^2/\beta\sigma_e^2 \cong .0002$. Even for $\sigma_v^2/\beta^2\sigma_e^2 \cong .1$, $\sigma_\varepsilon^2/\sigma_e^2 \cong .58$ and money supply shocks account for 63% of total monetary shocks.

14/ If only the lowest branch of the curve in Figure 1 is considered, values of $\sigma_v^2/\beta^2\sigma_e^2$ between 0.5 and 2.0 imply a range of $\sigma_\varepsilon^2/\sigma_e^2$ from approximately .77 to 1.05. Thus, in this range that emphasizes σ_e^2 relative to σ_ε^2, the ratio of the variance of money supply shocks to the total variance due to monetary shocks is from 48.78% to 56.50% -- approximately half of all monetary shocks are money demand disturbances. See also footnote 13 above.

15/ The non-stationarity of the unemployment rate means the variations of the Nu series give an exaggerated picture of the variability of the real economy, since the sample variance of a non-stationary series tends to grow over time.

16/ The ccfs of the three whitened real balance series on each other have a correlation of approximately .9 at lag zero, and essentially zero elsewhere.

17/ A rough estimate of θ' is the ratio of the estimated $\rho(-1)/\rho(0) = .39$, and for θ, $\rho(1)/\rho(0) = .06$. For values of $\sigma_v^2/\beta^2\sigma_e^2$ and $\sigma_\varepsilon^2/\sigma_e^2$ that give $\rho(0) = .495$ in the lowest branch of the curve in Figure 1, θ' is approximately equal to .39 at $\sigma_v^2/\beta^2\sigma_e^2 = 1.0$. This would imply that $\sigma_e^2/(\sigma_e^2 + \sigma_\varepsilon^2) = .5$, or that money supply disturbances contributed approximately 50% of the variance of overall monetary disturbances. On the other hand, θ is greater than .06 for all values of $\sigma_\varepsilon^2/\sigma_e^2$ in the range 0.1 to 5.0.

Alternatively, for $\sigma_\varepsilon^2/\sigma_e^2$ in the range .77 to 1.05 in the lower part of Figure 1, $\hat{\rho}(1)$ and $\hat{\rho}(-1)$ are within two standard errors of their theoretical values.

<u>18</u>/ In the lower branch of the curve in Figure 1, $\sigma_v^2/\beta^2\sigma_e^2$ between .05 and 2.0 gives $\sigma_\varepsilon^2/\sigma_e^2$ between .47 and .75. This implies money supply schocks contributed between 57.14% to 68.03% of total monetary disturbances. Thus, in this part of the curve that emphasizes the contribution of money supply shocks, over 30% to 40% of all monetary shocks are money <u>demand</u> shocks. Even with a $\sigma_v^2/\beta^2\sigma_e^2$ as small as .1, $\sigma_\varepsilon^2/\sigma_e^2$ = .34, and money supply shocks account for about 75% of total monetary shocks.

<u>19</u>/ $\sigma_\varepsilon/\sigma_e$ for the quarters on either side of the shorter period would have to be considerably smaller in order to drag down the average. The results for both periods, including parameter estimates, autocorrelation functions of residuals and chi-square statistic are little altered if the natural logarithm of M/PY is used in estimation. Further, constraining the mean to zero has the main effect of increasing the absolute value of the sample autocorrelation at lag one for the M/PY models.

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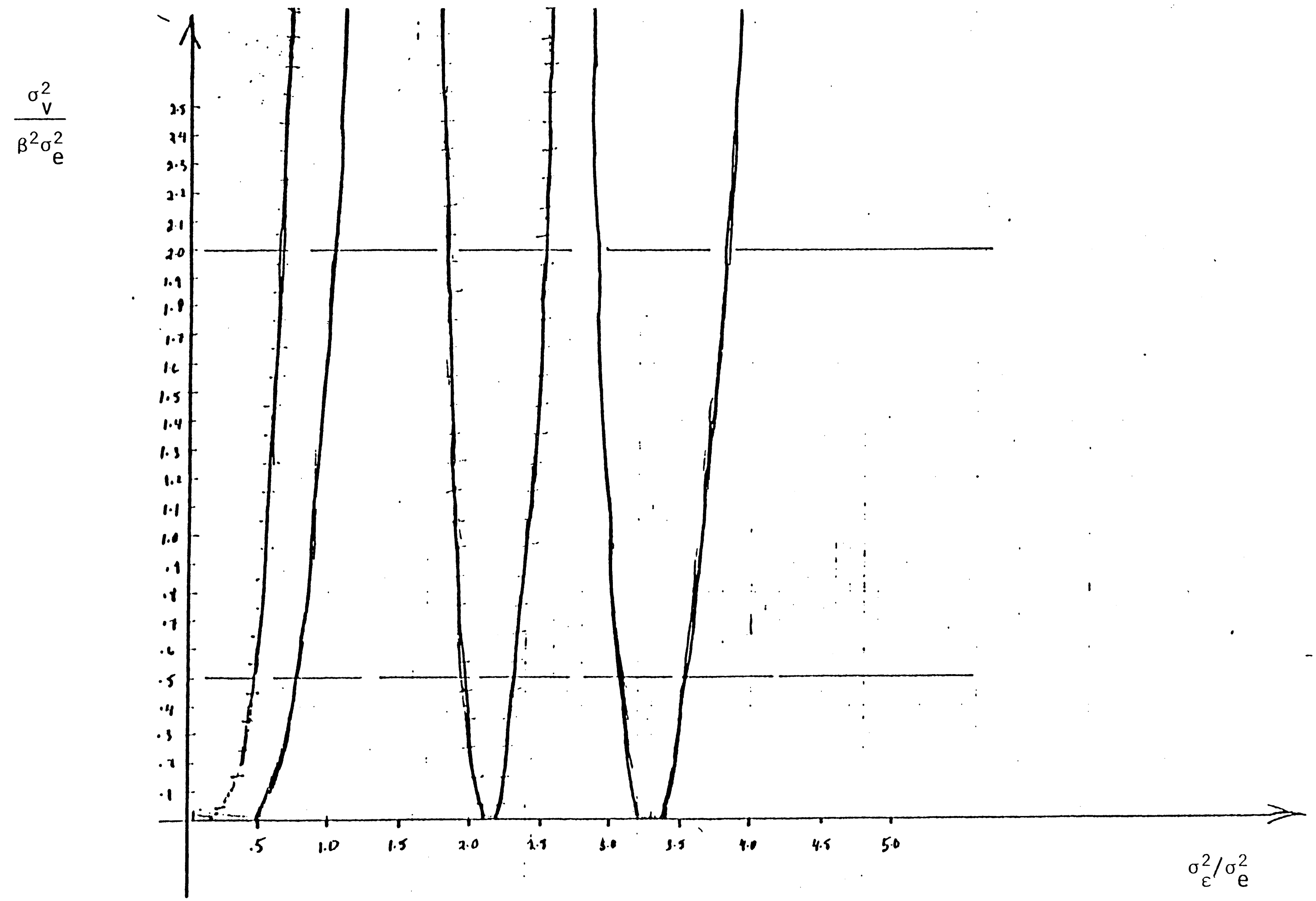

Figure 1

Table I: The Whitened Unemployment Rate, 1953II - 1971II

$$\underset{(6.39)}{(1 - .741B} + \underset{(1.29)}{.155B^2)} (1-B)\ Nu_t = (1 - \underset{(8.27)}{.684B^4}) a'_t$$

Correlation Matrix of Parameters

1.0		
-.649	1.0	
-.149	.287	1.0

Autocorrelation Function of Residuals:

Lag	1	2	3	4	5
Value	-.03	.09	.09	.12	.02
	6	7	8	9	10
	.01	.06	-.12	.08	.02
	11	12	13	14	
	.06	.01	.12	0.0	

Standard Error: .12

Degrees of Freedom	Chi-Square Statistic	Significance Level
11	4.72	.944

Table II: The Whitened M1 Real Balances Per Unit of Output, 1953II - 1971II

$$(1 - B)\ (M1/PY)_t = \underset{(4.34)}{-.00181} + (1 + \underset{(2.87)}{.332}B)\ a_t$$

Correlation Matrix of Parameters

1.0	
-.021	1.0

Autocorrelation Function of Residuals:

Lag	1	2	3	4	5
Value	.02	.02	-.06	-.12	-.07
	6	7	8	9	10
	.07	-.02	-.14	.01	.04
	11	12	13	14	
	.00	-.01	-.07	.05	

Standard Error: .12

Degrees of Freedom	Chi-Square Statistic	Significance Level
12	4.3	.977

Table III: Cross Correlation Function for the Residuals of the Whitened M1 Real Balances per unit of Output (a_t) and Unemployment Rate (a'_t) Series, 1953II - 1971II

$$\frac{\text{Cov}\,(a'_t,\ a_{t+k})}{\sigma_{a'}\ \ \sigma_a}$$

k	-15	-14	-13	-12	-11
ρ(k)	.040	.160	-.110	.108	-.044
	-10	- 9	- 8	07	06
	-.036	-.049	-.018	.049	-.089
	- 5	- 4	- 3	- 2	- 1
	.050	-.062	-.119	-.165	.195
0	1	2	3	4	5
.495**	.027	.181	-.227	-.276**	-.001
	6	7	8	9	10
	.034	-.255**	-.227	.038	-.037
	11	12	13	14	15
	.015	-.078	-.021	-.178	-.057

** greater than $2/(N)^{1/2} = .234$

Table IV: The Whitened Unemployment Rate, 1948II - 1976III

$$\underset{(10.3)}{(1 - .705B)}\ (1 - B)\ Nu_t = \underset{(16.1)}{(1 - .852B)}\ a'_t$$

Correlation Matrix of Parameters

1.0	
0.085	1.0

Autocorrelation Function of Residuals

Lag	1	2	3	4	5
Value	.05	-.11	-.13	-.16	-.02
	6	7	8	9	10
	.02	-.02	-.07	.11	.03
	11	12	13	14	15
	-.03	-.09	.13	-.06	-.10
	16	17	18	19	20
	.06	.08	.15	-.02	.01
	21	22			
	-.21	.10			

Standard Error: .09

Degrees of Freedom	Chi-Square Statistic	Significance Level
20	22.8	.30

Table V: The Whitened M1 Real Balances

Per Unit of Output, 1948II - 1976III

$$(1 - B)\,(M1/PY)_t = \underset{(4.59)}{-.00246} + (1 + \underset{(5.76)}{.356B} + \underset{(6.06)}{.487B^2} - \underset{(3.53)}{.247B^5})\,a_t$$

Correlation Matrix of Parameters

1.0			
0.313	1.0		
0.040	- .184	1.0	
0.002	0.002	0.000	1.0

Autocorrelation Function of Residuals

Lag	1	2	3	4	5
Value	-.05	-.08	.03	-.06	.03
	6	7	8	9	10
	-.09	-.03	.03	.10	.09
	11	12	13	14	15
	.04	-.08	-.08	.02	-.08
	16	17	18	19	20
	-.05	.15	.10	-.04	.07
	21	22			
	-.02	-.06			

Standard Error: .09

Degrees of Freedom	Chi-Square Statistic	Significance Level
18	12.21	.836

Table VI: Cross Correlation Function for the Residuals of the Whitened M1 Real Balances Per Unit of Output (a_t) and Unemployment Rate (a'_t) Series, 1948II - 1976III

$$\frac{\text{Cov }(a'_t,\ a_{t+k})}{\sigma_{a'}\ \sigma_a}$$

k	-25	-24	-23	-22	-21
(k)	.019	.159	.090	.180	.043
	-20	-19	-18	-17	-16
	-.105	-.193**	.081	.186	-.169
	-15	-14	-13	-12	-11
	.042	.090	-.102	-.112	.029
	-10	- 9	- 8	- 7	- 6
	.119	.000	.054	.061	.109
	- 5	- 4	- 3	- 2	- 1
	-.083	.019	-.159	.008	.238**
0	1	2	3	4	5
.352**	-.068	-.040	-.070	-.278**	-.026
	6	7	8	9	10
	-.043	-.153	-.111	.035	.070
	11	12	13	14	15
	.011	-.053	-.035	-.038	-.053
	16	17	18	19	20
	.057	-.016	.108	-.020	-.032
	21	22	23	24	25
	-.073	.110	-.025	-.260**	-.061

** greater than $2/(N)^{\frac{1}{2}} = .187$