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Estimating Input Demand for Water

I. Introduction

A common perception is that resource owners, when they consume their own holdings, see the cost of their consumption solely in terms of associated expenses. Undeniably, associated expenses are contributive to cost. But expenses alone represent an incomplete accounting of own-use cost. For instance, a corn farmer might estimate the cost of having a few ears for dinner as merely the associated expenses of picking and cooking. In situations where quantities of own-use are small, consumption expenses are an adequate approximation of economic cost. However, if the farmer were deciding whether to use half of a commercial corn crop in a joint cannery operation, the farmer's resource opportunity cost of corn, per se, would also have to be included. Opportunity cost might be indicated by corn's market value. Lost sales revenue to farming would enter into the "price" used to judge appropriate quantities of corn to use in canning.

Similarly, opportunity cost versus expense definitions of price are a keystone of derived demand models for own-water use. Below, we critique prior studies of industrial own-water users for which the derived demand price was specified in units of associated expenditures. We then report our own model and estimates of input demands for water, illustrating the effectiveness of opportunity cost definitions of price for measuring demand elasticities. Our emphasis is on showing the rational of opportunity cost concepts as a means of modeling the behavior of water owners. We show that cost minimizing owners adjust uses of own water according to an imputed price measured by opportunity costs.

Prior studies by Jacob De Rooy (1974) and Joseph A. Ziegler and Stephen E. Bell (1984) focused on important conceptual and empirical issues of input demand for own-water. Their originative studies are based on surveys of large industrial firms that owned water and engaged in "self-supply."[1] Both De Rooy and Ziegler-Bell proposed that associated expenses, of: a) own-water aquisition, b) treatment, and c) disposal, were an appropriate measure of a firm's internal water price. Opportunity cost of owned water does not constitute any part of price by their definition. We believe the third category of associated expenses (disposal costs) that Zeigler-Bell, and De Rooy include in their expense approaches is irrelevant. Disposal costs refers to getting rid of effluents and doing recovery treatment. This seems to us to be a conceptual mistake. We believe that costs of disposal enter into valuation of owned inputs only to the extent that uses of alternative sources of substitute inputs imply dissimilar costs of disposal.[2]

Our primary criticisms of De Rooy and Zeigler-Bell methods, however, are not quibbles about expense categories, but rather with the entire approach. Our criticisms are twofold: 1) They presume restricted, special situations in which, implicitly, no alternatives to own-use exist, and 2) Their econometric models and specifications overly restrict the quantity-price relations which exist in more general production frameworks. We criticize their conceptual approaches to price specification (Section II), and illustrate our specific misgivings concerning model restrictions through a detailed critique of the Zeigler-Bell study (Section III).

We present our own model and estimates of derived demands for water inputs in Sections IV - VI below. We show that opportunity cost approaches to price specification provide cost and derived demand estimates which are entirely consistent with the economic theory of cost minimizing firms. Our results are generated from a general, multiproduct, translog cost function with simultaneously estimated input demand functions. The model requires that water owners have alternatives to own-use. Our estimates confirm that self-supplying firms act as if their own uses of water vary according to external market price conditions as well as internal associated expenses. From a behavioral point of view, self-supplying firms do not implicitly price their own-water uses solely on the basis of associated expenses as postulated by prior studies. Rather, input demand for own water appears to respond to the same economic conditions as derived demands for purchased inputs.

II. Own-Use: Alternative Measures of Value

A variety of econometric techniques and variable definitions are potentially applicable for estimating input demands; Zeigler-Bell employed a two stage estimation procedure. In the first stage they estimated a total cost function for producing and disposing of own-water. Expenses associated with self-supply were made a function of the quantity of own-water used. Both marginal and average cost relations were then derived from their estimated total cost function. In stage two, the derived marginal and average costs were used as pseudo price observations for each firm's input demand for own water. Water price, by this definition, depends on expenses of water handling which, in turn, depend on the amount self-supplied. Zeigler-Bell wished to test whether marginal pseudo prices were a better measure for demand behavior than their average psuedo prices. Quantity demanded was measured by what each firm actually used. Zeigler-Bell's resulting input water demand is a single equation model, containing an expense specification of own-use price as its only price variable. Firm output, upon which input water demand is conditioned, is not included as a variable in the Zeigler-Bell demand function.

Our input water demand equations are derived from a class of generalized cost functions that have been developed from advances in duality theory.[3] We use a modified, multiple input, multiproduct, translog cost function to represent the operating behavior of firms which produce, use, and distribute water. Cost minimization behavior is assumed for the firms, this allowing application of Shepard's Lemma (Shepard (1970)). By this lemma, input demand equations for all variable inputs, including own-water, can be derived from the cost function. In addition to own-price, each input demand equation contains (cross) price variables for all other inputs and quantity variables for all outputs.

Our general-form specification of input water demand has several advantages (discussed in Section III) compared to more restrictive methods such as those employed by De Rooy and Zeigler-Bell. Of special importance to us are the conceptual differences in specifications of the own-water price variable. We justify our opportunity cost approach by assuming that water-owning firms regard water production as a SUPPLY activity. Whether firms deliver input units internally (self-supply) or externally, is not crucial.

Owners of water produce input supplies, W_s. These are amounts a firm is willing to produce, for given price conditions, from its own sources. A firm decides its (total) quantity demanded of water inputs, W_d, based on relative costs and productivities of competing inputs, whether from internal or external sources. With respect to own-water inputs, firms must realize one of three conditions:

$$W_s > W_d, \quad W_s < W_d, \quad \text{or } W_s = W_d$$

We define self-supplying firms as those for whom $W_s > 0$ and some portion of W_d is supplied from W_s (internal source). Self-supplying firms are thus a subset of water-owning firms and may have any one of the relative W_s-to-W_d relations shown above.[4] Zeigler and Bell limited their study to observations for which $W_d = W_s > 0$. De Rooy studied primarily firms for whom $W_d > W_s > 0$.

Economic theory proposes that input choices are based on assessment of value of marginal resource productivity, relative to marginal input costs. In theory, decision-relevant input costs are measured by opportunity values. From this, self-supplying firms should be expected to judge own water uses by opportunity costs. In order to do this, rational owners of water must act as if own water value depends not only on associated production expenses, but also on own water's value to other users. For firms in the real world, there are usually no observable, internally-generated accounting prices for owned resources such as water. Prices (values) of own uses must be inferred from owner behavior.

Owners who can potentially produce commercial quantities of water generally have opportunities to make external deliveries (if only to neighbors, fellow coop members, or a local water agency). Even if water sales or supplemental purchases of water are only potentially available alternatives, prices generated by others' transactions may act as appropriate measures of opportunity cost for non-transactors. That is, prevailing water prices for similar activities may provide owners with information about resource value, even when their own market opportunities are unrealized.[5]

Based on this notion, that prices are information signals, our model assumes that prices paid for purchased water are measures of own-water opportunity cost --- whether or not a firm actually avails itself of purchases or sales. If water is being consumed by its owner, we assume the owner has considered its value to others in terms of: the value that unextracted water might have to others (value of extraction rights), the cost of substitute water from another source, and sales value. Informed owners should realize approximately equivalent (marginal) values from units of own-water supplied, whether it is supplied internally or externally.

Assume that a firm has access to water deliveries from a local utility at a constant price, P_u.[6] The firm's marginal (per-unit) return on its owned water is (V):

$$V = P_u - (DELCOST + QUALCOST)$$

DELCOST is the firm's (marginal) cost of providing own-water delivery services equivalent to the availability offered by the utility company at price P_u. QUALCOST is the firm's marginal cost of equaling the water quality available at price P_u.

Marginal return V is related to the firm's water processing expenditures but also an external price. If the firm's own water were immediately available at a convenient surface location (DELCOST = 0), and if no treatment of internal water were required to bring it to the same quality as the utility's water (QUALCOST = 0), then V would be determined solely by the external water price: ($V = P_u$).

A firm could derive this marginal return by substituting own water for delivered water. Alternatively, the return (net of marketing and delivery expenses) might be obtained from supplying the water to others. From an owner's point of view, the cost (or internalized price) of using own water can, in this way, be related to a foregone (opportunity) value from potential market exchanges, P_u. However, in order to justify assigning own-water a marginal value equivalent to commercially delivered water, the acquisition, delivery, and quality control services associated with QUALCOST and DELCOST must also be supplied (or purchased) by the firm.[7]

In sum, the higher are DELCOST and QUALCOST, the higher must be the required marginal value (opportunity cost, P_u) of own water, for any given value of V, in order to make economic use of own water. The higher is V, on the other hand, the greater is own water's marginal value in own uses, for given levels of DELCOST and QUALCOST. The value of own water will change with shifts in market prices of alternative water sources. In some instances, if market prices of water increase relative to own water's production costs, own use may yield marginal rents --- a return (V) greater than normal returns. In equilibrium, and under competitive conditions, V should approach a normal level, at which firms are indifferent at the margin of use between more own-water versus purchased water inputs. Some firms would be observed using both, even when they were supplying some own-water to others. A departure from the normal V signals a possible short-term disequilibrium and, at least temporarily, a higher or lower marginal value in use than any next best input alternative. Thus, given competitive conditions and valid expectations on the part of water owners, we expect that:

$$P_u = DELCOST + QUALCOST + V$$

will hold at the margins of use and indicates two approximately equivalent methods of measuring the opportunity cost of own-water uses.

The most important difference between an expense versus an opportunity cost specification is illustrated above in the relation of V, P_u, and DELCOST.[8] For V = 0, P_u would be equal to DELCOST and the two approaches would then give similar measurements for marginal price. For non-zero V, the approaches give dissimilar measures. Further, an expense definition of price implies that as DELCOST increases so does its value; higher production costs, all else the same, will always increase own water value by a strict expense definition. In contrast, an opportunity cost approach implies that increases in production costs, relative to P_u, should decrease marginal return (V) and thus lower own-water value --- an effect consistent with intuition and opposite to that implied by an expense definition of price. Similarly, an increase in the market price of substitute water (P_u), with DELCOST unchanged, should increase the value of own water to an owner-supplier. This implies that the supply price of own-water (even for internal uses) should increase with P_u, if imputed from opportunity cost, but would remain unchanged if measured by a strict expense definition.

III. Model Restrictions: Prior Studies

The Zeigler-Bell statistical model relies heavily on De Rooy's earlier study, which is precursive for both expense definitions of water prices and functional form of input demand. De Rooy should be credited with performing what remains one of the few published, empirical studies of derived demands for water. Unfortunately, the majority of own-water use accounted for explicitly by De Rooy was recycled water. This feature alone prevents us from comparing his specific parameter estimates with our own. We discuss, instead, the limited usefulness of their results, caused by choices of variable definitions and statistical models.

For most firms in De Rooy's study, the unit costs of recycling water would have exceeded prevailing prices of fresh input water. For the minority of firms in his study which did recycle, the effect on total water demanded was accounted for by calculating an amount of (fresh) water input that would have been required had no recycling occured. This estimated, gross amount of water demanded was then functionally related to a weighted average price, composed of the price of fresh water, the proportion of gross water demand derived from recycled water, and a "price of recycled water." Recycled water prices were individually measured according to each firm's per unit recycling expenses. De Rooy also measured the price of fresh water inputs as a sum of three expense items: the price paid for purchased (delivered) water, unit costs of pumping and treating own (well) water, and unit costs of waste water disposal. Expenses of own-water production were defined to include amortized capital values.

De Rooy disaggregated each industrial plant's gross demand for water inputs into four quantities according to water uses --- cooling, processing, power production, and sanitation. The four resulting demand equations had log linear forms, and were estimated independently using OLS. This method of estimation is inefficient because the equations in his system are dependent --- their error terms and probably some of the independent variables have significant cross-equation correlations.[9] Even though De Rooy modeled water demand as a set of equations and utilized some market price information, we characterize his demand estimates as single equation models with expense-defined prices.

A feature differentiating the De Rooy and Zeigler-Bell studies turns on the range of water supply activities present in their observations. De Rooy's observations are characteristically ones for which $0 < W_s < W_d$, with own water supply coming from recycled sources. Because of gross demand aggregation and expense-defined prices, De Rooy could not obtain direct estimates of own water demand. Possible substitute relations between own water and other (external) sources were also lost. Zeigler-Bell observations are for $W_s = W_d$ firms; input water supply is exclusively from own sources and for own use. A direct estimate of own water demand is obtained, but all interactions with output level and other inputs, especially alternative water sources, are lost. We attempt, through our own data, variables, and model specifications, to comprehend a full range of self-supply activities.

As described, De Rooy and Zeigler-Bell made quantity demanded (Wd) a function of (an expense-defined) water price (Pw) and two shift variables.[10] They omitted prices of other inputs because "...(they are) not important for a short-run time framework".[11] De Rooy excluded prices of other inputs for "...simplification."[12] Zeigler-Bell also excluded output levels from their input demand equation.

These limitations cause their results to be suspect as general descriptions of own-water demand. We show the extent of Zeigler-Bell limitations by detailing their two-stage estimating methodology. They began with:

$$W_d = f(P_w, \ldots.) = f(c(W_s), \ldots) \qquad \text{(Z 1.0)}$$

for, $P_w = c(W_s)$

Quantity demanded of own water (W_d), is a function, f( ), of its own price (P_w), and two shift variables. The price of water P_w is assumed to be related to per-unit production costs, c( ) which derive from a total cost function, $C(W_s)$, for producing own-water inputs. Zeigler-Bell estimated the cost function to be:

$$C = a_o + a_1(W_s)^2 \qquad \text{(Z 2.0)}$$

$a_o = 317.04, \quad a_1 = .001$

MC = marginal cost = $.002(W_s)$

AC = average cost = $317.04(W_s)^{-1} + .001(W_s)$

Zeigler-Bell assumed output production was Cobb-Douglas (following De Rooy), so that the following derived demand function for water input could be used:

$$\ln W_d = b_o(\ln(K)) + b_1(P_w) \qquad \text{(Z 1.1)}$$

where, $\ln(K)$ = effect on $\ln W_d$ of other (shift) variables

Zeigler-Bell estimated two expense-price versions of equation (Z1.1):[13]

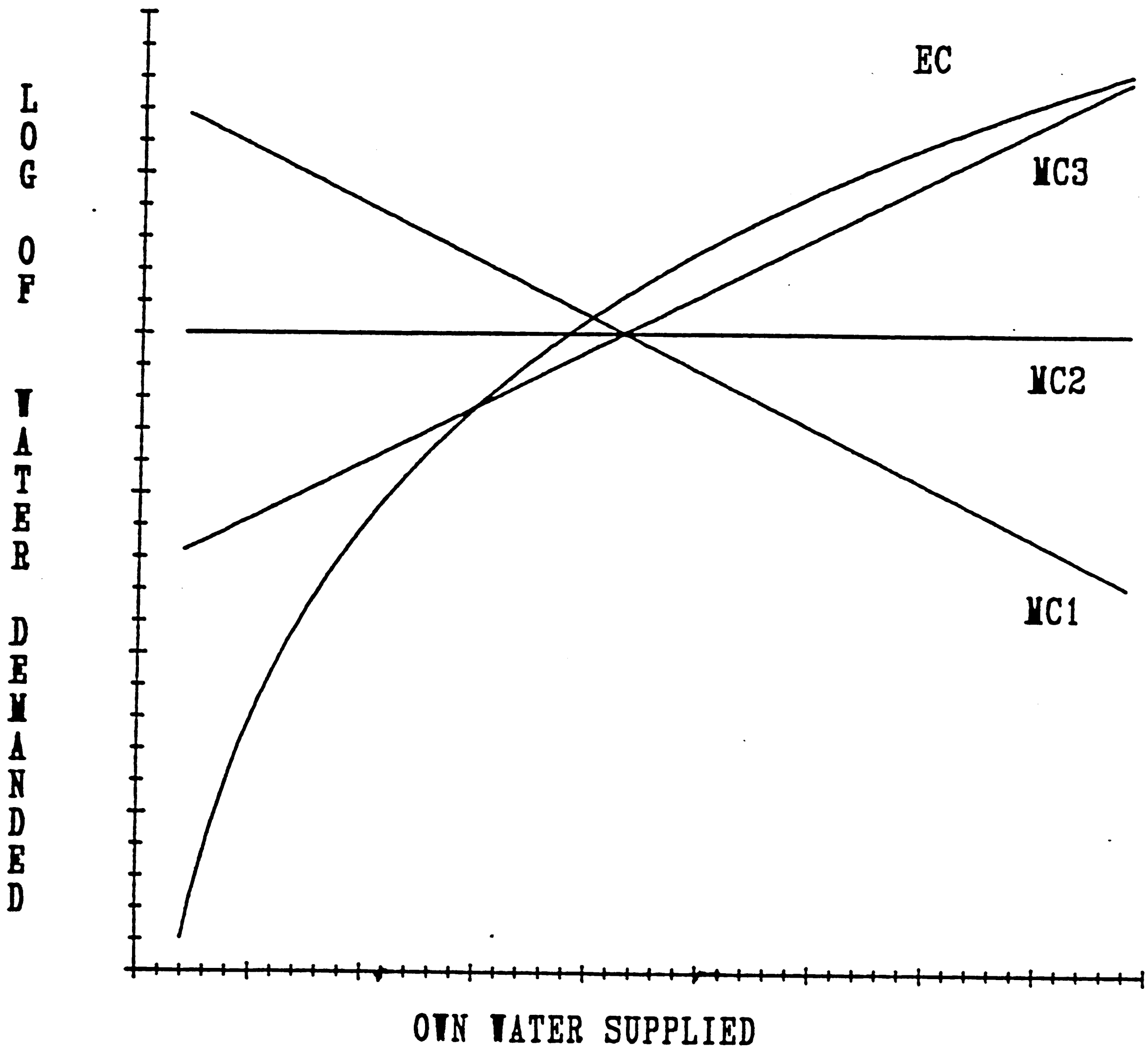
version 1: $P_w = c(W_s) = MC$

version 2: $P_w = c(W_s) = AC$

Regression estimates for both versions of equation (Z1.1) must fit a scatter of data points around an Equality Curve (shown as curve EC in

Figure I) which represents the set of (Ws, lnWd) combinations for which $W_d = W_s$ --- a condition required by their observations and statistical method.

FIGURE I



For version 1, $P_w = .002*W_s$, given ln(k) and b_o are positive, only three regression results were possible:

$b_1 < 0$ (shown as curve MC1 in Figure I)

$b_1 = 0$ (shown as curve MC2 in Figure I)

$b_1 > 0$ (shown as curve MC3 in Figure I)

In order to fit points along EC with (Z 1.1), a positive b_1 slope coefficient condition must result from the two-stage process (MC3 curve). The fewer small observations there are in the data set, the closer the MC2 ($b_1 = 0$) and MC3 fits to EC become. This explains the anomalous result reported by Zeigler-Bell that their marginal cost price specification was significant (b1 coefficient positive though near zero), but "... (that) the MC price variable did not display the expected negative sign."[14] We have shown that restrictive misspecifications forced this contradictory result. Thus, the Zeigler-Bell test fails to determine whether a marginal or average price specification is appropriate.

Version 2 of equation (Z1.1) is shown in Figure II. For $b_o > 0$, three results are possible:

$b_1 < 0$ (shown as curve AC1 in Figure II)

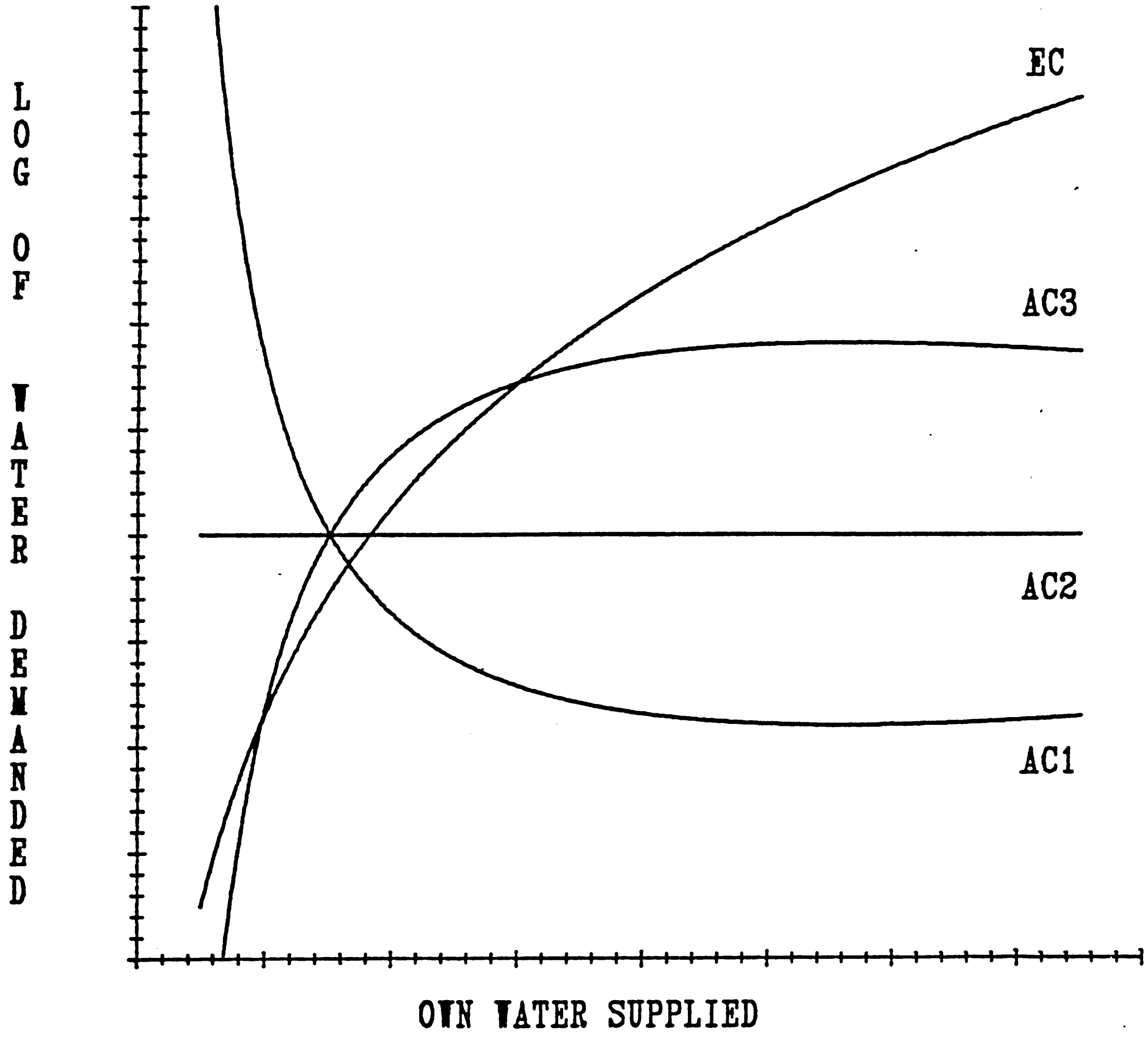
$b_1 = 0$ (shown as curve AC2 in Figure II)

$b_1 > 0$ (shown as curve AC3 in Figure II)

The best fit of data points along curve EC is acheived for $b_1 < 0$, shown as AC1 curve in figure II. The condition $b_1 < 0$ does not imply a negatively sloped demand for own water; rather, AC1 is a foreward falling demand that is negatively sloped for high prices (larger W_s) and positively sloped for low prices (smaller W_s).

Zeigler-Bell claim that their average cost specification for own-water price is superior to their marginal cost version. We feel this

FIGURE II



issue remains important but inconclusively tested. We show that their results are largely explainable as consequences of very specific model restrictions and variable definitions. Moreover, We are skeptical that strict expense approaches can correctly specify derived demand for own-water. Even for the case of $W_d = W_s$, and granting no alternatives to own use or substitutes for own water, we believe more general models of demand, incorporating opportunity cost specifications of price, would deliver superior results.

IV. A Translog Derived Demand for Own Water

Our model is general and free-form, meaning that it conforms to a neoclasical economic paradigm of cost minimizing behavior while leaving technological relations of an underlying production function virtually unrestricted. Consequently, specific production technologies appropriate to firms in our data are conceptually inferrable from resulting cost function estimates. All firms in the study are assumed to face the same production function, though they differ greatly in size and product mix. For our model, the costs that firms minimize are ECONOMIC costs. We take care, below, to reconcile definitions of economic cost with reported costs, the sum of expenditures or outlays for purchased inputs. Our model also requires input prices to be exogenous constraints on input choice. That is, individual firms are assumed to have little influence over input prices.[15]

We estimate a cost function rather than (directly) estimating a production function. Indirect cost function methods yield superior estimates of technical relations while providing an important secondary advantage --- input demand equations can be simultaneously estimated if the data set contains information about quantities of inputs used by each firm. We derive a set of input demand equations from our cost equation and jointly estimate the full system of equations.[16] This system approach improves estimation efficiency and expands degrees of freedom for a given data set.[17]

Several functional forms meeting the above criteria are available. We adapted a Translog cost function introduced by Christensen, Jorgensen and Lau (1971, 1973) and later generalized for multiproduct cost functions by Caves, Christensen, and Tretheway (1980). Total quantity demanded (W_d) of water inputs is derived from purchased water and own water sources. Reliance on own water varys in the sample from zero to one hundred percent. And, the portion of own water production (W_s) which is self-supplied varies from zero to one hundred percent.

Multiproduct Translog Cost Equation

Translog equations provide a second-order, logarithmic, Taylor's series approximation of an arbitrary function. Our application is to approximate a cost function assumed to be shared by firms in our sample. Approximation is made at a point, interpreted as the mean values of variables used in the estimation.[18] The variables of the translog are also normalized (made relative to) their respective means; this normalization convention makes it easier to calculate cost elasticities and interpret input demand properties from regression results.[19]

The cost equation is:

$$\begin{aligned} C = \; & F(P,Q) \\ = \; & A_o + \Sigma_i(A_i*P_i) + \Sigma_k(B_k*Q_k) \\ & + .5*\Sigma_i\Sigma_j(A_{ij}*P_i*P_j) + \Sigma_{ik}(D_{ik}*P_i*Q_k) \\ & + .5*\Sigma_k\Sigma_l(B_{kl}*Q_k*Q_l) + e \qquad (C\ 1.0) \end{aligned}$$

for, C = total economic cost of production (logged)
P = a vector of i logged input prices (i,j = 1,..,n; n=7)
Q = a vector of k logged outputs (k,l = 1,..,m; m=3)
e = an error term

A_i and B_k are coefficients of first-order effects. A_{ij}, B_{kl} and D_{ki} are coefficients of second-order, "interaction effects."[20]

Equation (C1.0) is a second-order polynomial defined in terms of the levels of three outputs and the prices for seven inputs. The Output categories are:

Q_1 = Metered residential-commercial-small scale governmental, including fire hydrant water delivery services (called METERED output)

Q_2 = Metered agricultural-industrial-resale water delivery services (called BULK output)

Q_3 = Unmetered water delivery services (called UNMETERED output)

Inputs are:

X_1 = Own Water

X_2 = Purchased Water

X_3 = Field Labor (pumping, treatment, distribution/transmission)

X_4 = Office Labor (billing, sales,...)

X_5 = Management and Engineering Labor

X_6 = Energy

X_7 = Capital and Materials

A problem arises for equation (C1.0) where firms do not produce any of a particular output. The logged value of zero is undefined. A solution to this problem is provided by using a more general, Box-Cox (1963) transformation on output variables which transforms q into Q, as:

$$Q = L(q,\lambda) = \{(q^{\lambda})-1\}/\lambda, \quad (\lambda \neq 0) \qquad \text{(C 2.0)}$$

such that, $(Q \rightarrow \ln(q)$ as $\lambda \rightarrow 0)$

We use the same λ value for all outputs. The value of λ is determined by estimating it jointly with all other parameters in the translog cost equation. We call the estimated value an "optimal λ."[21]

Another implementation problem for equation (C1.0) occurs if input quality variations are correlated with observed input price variations. This quality-price covariation constitutes a particularly important problem for input demand estimation because of the model's quality-constant presumption regarding input units. We attempt to control for as much variation due to quality changes as our data allows. Fortunately, we have some information about input water quality, in addition to quantity used. We use the water quality information to generate "effective" prices --- supplementing observed dollar prices for amounts purchased. Effective prices are intended to measure dollar equivalents for constant-quality water units.

Our specification of effective input price (p_i) is:

$$P_i = \ln[p_i + w_i(\ )] \qquad (C\ 3.0)$$

where,

$w_i(\ )$ = equivalency function assigning dollar weights to quality indicators

P_i = logged effective, quality-adjusted price of x_i

p_i = observed dollar amount per unit, quality varying, of x_i

Although quality adjustments would have been appropriate for nearly all input categories in our model, we were able to create effective price adjustments only for purchased (x_2) and own water (x_1) inputs.

Total economic cost (c) of equation (C1.0), is defined as the sum of prices times quantities:

$$c = \Sigma_i (p_i * x_i), \quad i = 1, 7 \qquad (C\ 1.1)$$

Unfortunately, reported accounting costs, indicated below as (acc), are a poor measure of equation (C1.1) because, although most firms use own water inputs, and therefore incur an economic or opportunity cost, they have no cash outlay for own water; $p_1 = 0$ is the reported cost. The relation of accounting cost to economic cost is:

$$acc = c - (p_1 * x_1), \quad \text{for } p_1 = \text{opportunity cost}$$

In order to correctly represent total economic cost using reported expenditures, we substitute an estimate of own-water's per unit opportunity cost for its implied zero accounting price. Equation (C1.1) becomes:

$$C = \ln[(acc) + (p_1 * x_1)]$$

Translog Factor Share Equations

Differentiating cost equation (C1.1) with respect to the input prices yields:

$$\delta c/\delta p_i = x_i + p_i*(\delta x_i/\delta p_i) \quad \text{(S 1.0)}$$

and, $\quad = x_i \quad$ (for price independence: $(\delta x_i/\delta p_i) = 0$)

Quantity demanded of each input is equal to its cost-price derivative for firms that are input price takers. This is essentially Shepard's Lemma, allowing input demand equations to be derived from a cost equation under conditions of cost minimizing behavior for any given level of output.

A similar result can be obtained by differentiating a logarithmically transformed cost equation. The share of total cost accounted for by input i is defined as $S_i = (x_i*p_i)/c$, so that:

$$\delta(\ln c)/\delta(\ln p_i) \equiv \text{cost-price elasticity}$$

$$\equiv \{\delta \ln c/\delta c\}*\{\delta c/\delta p_i\}*\{\delta p_i/\delta \ln p_i\}$$

$$= \{1/c\}*\{\delta c/\delta p_i\}*p_i$$

$$= \{x_i*p_i\}/c = S_i$$

Cost-price elasticity or share equations thus serve as surrogate (S1.0) input demand equations. Share equations yield the same information as the factor demand equations, indicating changes in relative factor uses in response to altered price or output values.

Differentiating our translog cost equation (C1.0) with respect to n input prices yields a set of n share equations, each linear in both parameters and variables. They have the simple form:

$$S_i = A_i + \Sigma_j\{A_{ij}*P_j\} + \Sigma_k\{D_{ik}*Q_k\} + u_i \quad \text{(S 2.2)}$$

$u_i = i^{th}$ error term, $\quad (i = 1, 7)$

A corresponding factor demand equation (S3.0 below) is implied by each cost share, i^{th} quantity demanded being a scalar transformation of S_i:[22]

$$x_i = (c/p_i)*S_i \qquad \text{(S 3.0)}$$

$$= (c/p_i)*(A_i + \Sigma_j(A_{ij}*P_j) + \Sigma_K(D_{Ki}*Q_K) + v_i$$

Thus, for our model, quantity demanded of each input is a function of its own price, the price of all other inputs, and level of all outputs. Equations (S2.2) and (S3.0) are unmodified for the specific water inputs, (x_1) and (x_2). But, whereas other input price variables are specified in terms of reported expenditures, purchased water price (p_2) is specified as an effective price for quality adjusted units and own water price (p_1) is an imputed value, measured by opportunity cost.

System Estimation

Since the share equations are obtained by differentiating cost with respect to input prices, they contain only coefficients present in the translog function. Additionally, two econometric requirements of our system estimation are: 1) factor prices and error terms must be independent, and 2) the error term of the cost equation must be uncorrelated with those of the share equations. We believe these conditions are reasonably well met. However, we recognize that errors in identifying water prices directly introduce errors into our measure of total economic costs.[23]

For each firm, input shares (S_i) add to one by definition. Thus, error terms (v_i) sum to zero. As a consequence, (any) one of the share equations is dependent and must be deleted from the system of shares in order to find a solution. For our seven input model, the resulting system has seven (one cost plus six share) equations. Our estimates, produced by iterative maximum likelihood criteria based on an extension of Zellner's (1962) technique, are asymptotically invarient to the deleted equation (Christiansen and Greene, 1976).[24]

The actual equations we subjected to estimation are variants of equations (C1.0) and (S2.2). This is because:

1. Outputs were subjected to a Box-Cox transformation, adding another parameter, λ. ((C 2.0) above)

2. A technological (environmental) variable called TREAT was appended to the cost equation to control for differences in water treatment that it was necessary for each firm to perform. (see Section V, below)

3. Price homogeneity conditions, required by economic implications of cost minimizing behavior, are imposed on the cost and share equations.

Interested readers will find derivations of our estimated equations in Appendix A. As for condition 3, cost minimization requires our cost equation to be homogeneous of degree one in prices and share equations to be homogeneous of degree zero in prices. The meaning of these conditions is that if all input prices change by percent γ, total cost will increase by γ percent while input shares (S_i) and quantities demanded (x_i) of inputs will be unaffected.[25] Price homogeneity conditions imply specific parameter restrictions for the Translog equations which are derived in Appendix A.

Price Elasticities for Inputs

A primary objective of our study is to compute elasticities of input substitution, particularly for own and purchased water inputs. We measured the degree of factor substitutability by Allen-Uzawa elasticity coefficients, designated below by σ_{ij}'s. These are defined as:

$$\sigma_{ij} = c*c_{ij}/c_i*c_j$$

where, $c_i = (\delta c/\delta p_i)$

$$c_{ij} = (\delta(\delta c/\delta p_i)/\delta p_j$$

For translog cost equations, the σ_{ij} are calculated from the A_{ij} coefficients and share terms (Binswanger (1974)):

$$\sigma_{ii} = (A_{ii})/(S_i)^2 - 1/S_i + 1$$

$$\sigma_{ij} = (A_{ij})/(S_i * S_j) + 1, \quad (i \neq j)$$

Alternatively, substitution elasticities may be transformed into partial (own and cross-) price elasticities, PE_{ij} which would pertain to the derived input demand equations (see Appendix B):

$$PE_{ij} = S_j(\sigma_{ij}) \quad \text{(for all i, j)}$$

Both PE_{ij} and σ_{ij} are partial elasticities. They are measured for given output quantities and do not include effects of changes in output prices (and hence in output quantities) occasioned by changing input price. The σ_{ii} should be negative, indicating input demands that are negatively sloped for own-price. Signs of σ_{ij} may be positive or negative depending upon the degree of substitutability between input pairs.

V. Data Sources and Variable Definitions

Our data come from a private survey and public information sources. Firms in the sample are water delivery operations: private utilities, districts, and cities in the Southern California region. A mail survey was conducted in 1982-83 seeking operations data for the 1980 fiscal and calander years. It was sent to nearly 50 private stock companies delivering water in more than 100 sevice areas. Complementary data came from PUC reporting documents and 1980 census tract tapes. Similar survey requests were sent to independent water districts (115) and cities (110) engaged in water delivery. Supplemental data for these firms came from California State financial reports and census tract tapes.

The unit of analysis is service area operations. Our final sample includes 52 private company service areas (operated by 25 different private companies), 36 water districts, and 31 cities, (N = 119). Ninety-eight service areas in our sample produce own water. Own water production in service areas is not much different from that of the self-supplying industrial firms used in the Zeigler-Bell and De Rooy studies. Most own water production is fresh water; in a few cases some reclaimed water is produced. Most of the firms in our sample self-supply a majority of the water they produce. The alternative to self-supply, for production of retail deliveries, is shipment to resale or wholesale connections. In nearly all cases, such connections are to customers located outside the retail customer service area of the supplying firm. The flip side of resale-wholesale deliveries, appears in our data as input water acquisitions. A majority of firms in our sample purchase water in addition to inputs that they self-supply.

Variable Definitions

1. Water Prices: p_1 and p_2

The price of purchased water (p_2) is measured as the sum of amounts paid by a firm to suppliers, divided by total units received, plus dollar adjustments for known quality variations. Several problems with this definition are recognized. First, many firms in the sample purchase water from more than one source, and have more than one purchased water price. Second, other firms purchase no water at all. Our price definition in this case is indeterminate, zero divided by zero. If a firm truely faced a zero price, the firm would view x_2 as a free resource in which case its use would be extensive rather than zero use. Likewise, an infinitely high shadow price is inappropriate, since water could conceivably be acquired by any firm in the Southern California region at some cost not greatly exceeding prevailing prices.

A shadow price was devised for non-purchasing observations. Since an average price of purchased water, calculated from what other firms in the sample paid for their water, would not correctly represent those cases for which firms do not purchase because existing sources are too high-priced (relative to their own water options), or face heavy initial capital outlays to create new input delivery systems, we elected to set p_2 slightly above the average of actual purchases; p2 = 116.43 per acre foot or 1.25 times the sample average.

This proxies potential purchase price. A higher price than this might be reasonable for many non-purchasers. However, we wanted to avoid excessive biasing of our estimated substitution elasticities. Admittedly, any individual firm may have higher or lower price alternatives. Purchasing firms buy most of their water on long term (lower-priced) contracts and less often on (higher-priced) spot market contracts. Prices actually paid for purchased water represent payment for a bundle of reliablity and other quality-of-service attributes. For the Southern California region, the distribution of input water prices is dominated by sales of the giant Metropolitan Water District (MWD) of Los Angeles.

Other problems defining p_2 concern variations in reported costs per unit that result from differences in water quality or delivery services provided. Economic theory of cost minimization presumes inputs of identical quality. Inputs of unlike quality should ideally be categorized as either different inputs or have quality corrections applied. Further, our economic paradigm presumes that marginal prices govern factor choices. We did not have sufficiently detailed contract information concerning water purchases to distinquish marginal from average prices paid. We chose to measure p_2 as an observed average cost (assumed to be near marginal price) plus price adjustments for indicated water quality variations deriving from predelivery filtration and chemical treatment. We used 1980 MWD price increments for filtering and chemical treatment. Dollar differences indicated in our data set for prefiltration and chemical treatment were very nearly the MWD charges.

Imputed own-water price (p_1) is measured by opportunity cost indicators. As outlined in Section III, there are two approaches:

p_1(OPP) = price of substitute input water

p_1(EXP) = V + DELCOST + QUALCOST

We measured opportunity cost of x_1 by both approaches in order to compare resulting input demands. Price p_1(OPP) was measured from p_2 prices and is called the OPP model. The p_1(EXP) approach was measured from production costs and foregone returns and is called the EXP model. Estimation results for both models are reported in Section VI.

The OPP model p_1 is:

$$p_1(OPP) = [1 - .4(\rho)]*(p_2)$$

for, ρ = proportion of own water inputs derived from reclaimed sources

p_2 = price of firm's purchased water inputs

Own water's OPP specification imputes opportunity cost from the value of its closest substitute. A quality adjustment is imposed which reduces the value of own water forty cents per unit of own water derived from reclaimed sources. Our OPP specification most likely underestimates the value of own water since, in Southern California, own water quality is on average greater than purchased water. A majority of purchased input water is supplied from (surface) sources outside the area: either State Project water from Northern California or Colorado River water.

For firms not producing own water, p_1(OPP) should reflect the opportunity cost of producing under high-cost conditions, possibly facing initial start-up costs. We set this value at $72 per acre foot which is at the upper quartile of 1980 average well water production costs but would nonetheless represent a negative return on own water for most non-producing firms if own-water value were set at the price of their purchased water. We varied the $72 value to assess sensitivity on input demand and found little or no impact.

Our EXP model specification of p_1 is:

$$p_1(EXP) = V + DELCOST$$

for, V = an indicator of return per unit on asset value of own water

DELCOST = an indicator of internal per unit production and delivery expenses

QUALCOST expenditures which might be required to bring own water to a quality equivalence with purchased water are dropped. Reasons are that own water is assumed to be of higher quality than purchased water. Firm-level treatment is done in a variety of ways and often involves sequential retreatment of blended water from several sources. The net effect of own water quality on treatment expenses must be evaluated on the basis of changes in total cost of a firm's treatment operations. Since own water is on average higher quality than purchased water inputs, blending reduces overall treatment expenses. We controlled for treatment expenditures by including a technical variable (TREAT below).

DELCOST is a firm's per unit cost of producing own water. Since water production expenses are not directly reported, we estimated (separable) costs of water production from labor, energy, and other acquisition expense categories. DELCOST indicates variation in own water opportunity cost through equivalence to value of purchased water. Higher DELCOST values are associated with either higher (marginal) cost of using substitute, purchased water; higher price foregone from sales to purchasers of water; or a lower acceptable return on own water asset value (V) given the price of purchased water.

Variations in the V component of p_1(EXP) are measured from a firm's net revenue, generated from service area operations, weighted by the proportion of total water inputs derived from self-supplied own water. Ideally, V would indicate a firm's ability and incentive to capture the rent potential of own water from its uses. A higher V should reflect greater foregone own-water returns, DELCOST given, but V cannot be measured directly. It is even possible that foregone return could be associated with higher operating costs if own water returns are captured in the form of local taxes, lower service charges, cash transfers to other government budgets, etc. Our return indicator was measured as $15 (the average difference between purchased water and service area production cost per unit) plus or minus variations in return assigned by the net revenue indicator.

2. Labor Prices: p_3, p_4, and p_5

Labor prices are each measured as average expenditure per unit of labor of a particular employment category. Total firm outlay for each type of labor is divided by reported full-time annual employees of the same type. While this differs from desired, marginal factor cost of labor, firms appear to have little influence on market prices for labor of any type. We assume marginal and average labor prices are identical.

Two features of this specification concern us. First, there are quality variations in labor which account for some of the observed price variations. Quality-of-labor information such as years and quality of training or experience was not available. We expected that lack of this information would lead to smaller estimates of own- and cross-price elasticities of substitution among labor categories, and larger standard errors on labor's input price estimates. These results seems to be borne out (Section IV), especially for field labor, x_3.

Job types aggregated in x_3 are less homogeneous than x_4 or x_5 categories. Our field labor category aggregates transmission, distribution, treatment, and pumping/acquisition job descriptions. A trial disaggregation proved useful, but due to software limitations, could not be implemented concurrently with the full set of output and input disaggregations contained in the maintained hypothesis. Another concern is inconsistency of labor expenditure reporting. We suspect some variation derives from compensations reported net of full employee benefits while others represent gross labor costs. No data adjustment was made to correct for suspected differences.

3. Energy Price: p_6

Price of energy is the ratio of annual electrical outlays divided by KWHrs consumed. Adjustment was made for BTU-equivalent KWHrs for a few firms that used gas for pumping. We used conversion factors for relative energy efficiencies of electrical and thermal engines. While this conversion was a rough approximation, it affected only marginally a small number of firms. Their composite price of energy was very near the average of electricity prices.

A complication arises in assessing energy inputs for each firm. Water's head pressure embodies energy and can generate electricity as a joint output of the distribution process. Electrical cogeneration and gravity powered distributions are present in our sample but not completely nor consistently reported. Our purchased energy input x_6 therefore understates the total energy inputs consumed by some firms. No adjustment was attempted to correct for these understatements.

4. Capital/Materials Price: p_7

A base 1980 water bond rate is established to represent capital investment return. Bond rating of each firm are used to assign variations around the base rate and measure relative marginal capital cost. A second differential in capital costs is applied between private and public firms to reflect the effect of income tax deductability on capital borrowing rates of public firms, which translates into relatively lower borrowing costs for given bond ratings.

Each firm's capital/material price is measured as a weighted average of its capital rate and a materials price which is the same (arbitrarily $100) for each firm. Our weighting is the proportion of capital expenses (depreciation, interest, dividends) relative to aggregate Capital/Materials expenses.[26] Reported depreciation is not an expenditure category. But as a consequence of uniform practices and longevity in write-off periods required of water agencies in the region, we believe reported depreciation is our best dollar indicator of capital use for 1980. Our assumption that all firms face an identical materials price is rationalized by the regional similarity of our observations, similarity of materials used by all types and sizes of delivery operations, and our belief that quantity discounts or other firm-level variations are insignificant.[27]

5. Outputs: q_1, q_2, and q_3

Service outputs are indexed by gallonage delivered to three customer categories. Typically, small firms serve mostly customers in the q_1 category --- metered residential, commercial, business and local government connections. The q2 category is large, bulk deliveries to metered connections including some specialty government connections such as municipal golf courses, large parks, or cemeteries. A reporting problem for government connections, especially city-operated delivery firms, is that some receive "free" water, and their gallonage may either be unreported or be included in the unmetered category.

By assigning deliveries to multiple output categories, some cost variance due to customer-type service variations are controlled. Agricultural, wholesale water, and industrial site customers are in q_2. Industrial connections are assigned to q_2 because their average delivery per connection is large, though not uniformly large as are agriculture connections. We considered assigning industrial connections to q_1 on the basis that they share, with residential and commercial connections, more extensive fire related service requirements (higher pressures, greater reserve storage). We believe however that reporting criteria primarily distinguish industrial connections on the basis of account size which is cost-reflected in rate schedules.

Unmetered deliveries are aggregated into category q3. An alternative specification might have assigned unmetered water to q_1 or q_2 according to whether average unmetered account sizes were small or large. Output hedonics on q_1 and q_2, similar to those used by Feigenbaum and Teeples (1983), controlling for the proportion of unmetered water, would have reduced OPP and EXP models to two outputs. Alternatively, an output hedonic on q_3, indicating average account size may have accomplished a similar control but could not implemented due to software limitations on total equation size.

6. Treatment level: TREAT

Our index variable for treatment level was measured as:

TREAT = T1 + T2

where, T1 = main treatment facility of system

T2 = all other treatment facilities of system

for, T1,T2 = TYPE*PUMP*AMT*LEVEL

TYPE = 1 if treatment is conducted in central, specialized plant; .67 if treatment is conducted at dispersed locations (typically at well sites).

PUMP = 1.25 if input water is pumped into central filtration (else 1.0).

AMT = Millions of gallons treated per year divided by total water distributed.

LEVEL = Weighted vector of treatment components that may be present: aeration, flocculation, coagulation, sedimentation, softening, ion exchange, filtration, chlorination, disinfection, activated carbon fileration.

Our treatment index was designed through consultation with engineers of a large water engineering firm. Some sensitivity analysis of weights used in TREAT showed minimal variation in our estimated coefficients.

VI. Empirical Results

Overall fit of our estimated equations strongly confirms their economic representations as cost and input demand functions for water delivery firms. This is especially encouraging given that the data are cross-sectional and cover a wide range of operations, both in terms of overall scale and variety of output mixes. Estimated demand for self-supplied water shows quantity demanded responds significantly to opportunity cost specifications of own price, levels of outputs, and prices of all other inputs. Both the OPP and EXP models show that own water and purchased water are close substitutes in production.

Since we estimated using an iterative maximum likelihood method, R^2 statistics are not available. Berndt (1977) created a Pseudo R^2 statistic for use in such circumstances.[28,29] Pseudo R^2 is 0.720 for the OPP system of equations and 0.763 for the EXP model.[30] For both models, the cost equations fit more accurately than the share equations. This is partly due to inherent differences in variation of dependent variables. Cost is measured in total dollar units, while shares are percentages averaging in a range from 3.5% for the Sales/Billing labor to 30% for Capital/Materials inputs.

Some key properties of our cost models are test and the results are summarized in Table I. The set of accepted properties constitutes our maintained hypothesis which is represented by the coefficient estimates reported below for the OPP and EXP models. The tested properties are:

1. Linear Homogeneity in Input Prices (LHIP). (discussed in Section IV, above)

2. Input separability (water inputs from other inputs.)

3. Presence of the technical modifier: TREAT

Table I shows test statistics for our OPP model; parallel results were achieved for our EXP model. Price homogeneity (Appendix A) cannot be rejected at any reasonable level of significance. Separability of water inputs from other input categories is strongly rejected, meaning that delivery costs are jointly determined by interacting combinations of inputs so that substitutions among water inputs is not independent of other inputs.

Omission of interactions between water and other inputs (complementarity and substitution effects in production) would lead to biases in estimated (direct) cost effects leading to significant misspecification of both the cost and derived demand equations. This is an important result for water research since, to our knowledge, the OPP and EXP models are the first water delivery models which explicitly contain water as input categories. Roughly half the models we have reviewed use total water produced as their index of output.

Our index variable for treatment is shown to act as an important indicator of technical conditions imposed on delivery. Below, we discuss specific treatment impacts in terms of interaction coefficients for TREAT, input prices, and outputs. Results shown in Table I justify our imposition of a LHIP restriction, rejection of functional separation of water/non-water inputs, and inclusion of a technical variable on treatment. These properties are maintained for all other estimates reported in this section.

Implied Production Function

To the extent duality conditions are met, the translog cost function is descriptively equivalent to a production function for representing input-output transformations. Few specific, a priori,

TABLE I

STATISTICAL TESTS OF FUNCTION PROPERTIES
OPP MODEL

	No LHIP[1]	Maintained Hypothesis[2]	Input Separability[3]	Treatment Variable[4]
Restrictions on Parameters	None	$\Sigma_i A_{ij} = 0$	$A_{ij} = 0$	A_t, A_{tt}
		$\Sigma_i D_{ki} = 0$	$i = 1,2;$	A_{ti}, $D_{tk}=0$
		$\Sigma_i A_{ti} = 0$	$j = 3$ to 7	for all i,k
Number of Constraints[5]	None	11.0	10.0	11.0
Chi Square Statistic	NA	7.2	56.1	54.1
Critical Level (99%)	NA	24.7	23.2	24.7
Log-Likelihood Function	1076.20	1073.61	1045.54	1046.56

[1] No LHIP means that linear homogeneity of input prices is not imposed.

[2] Maintained Hypothesis means: modified translog OPP model with Box-Cox transformed outputs, input price homogeneity, treatment variable and symmetry. Coefficient-constrained models examined in Table II also satisfy maintained hypothesis properties.

[3] Strong separability is tested, water inputs ($i = 1,2$) from other inputs ($j = 3,7$).

[4] The treatment variable interacts with all the other independent variables; one of the price interactions is not independent because of the maintained hypothesis of price homogeneity.

[5] This is the number of independent constraints imposed vis-a-vis the Maintained Hypothesis, except for the NO LHIP test against our maintained hypothesis.

restrictions on production technologies are imposed by the translog approximation. Thus, our OPP and EXP models are technologically generalized, allowing tests of various production function specifications; sets of coefficient-restricted equations may be estimated, allowing comparison with the unconstrained system using likelihood ratio tests.[31] Our prior beliefs are that water delivery

TABLE II

PRODUCTION FUNCTION RESTRICTIONS AND STATISTICAL TESTS
OPP MODEL

	Maintained Hypothesis[1]	Homotheticity	Homogeneity[2]	UES[3]
Restrictions on Parameters	None	all $D_{Ki} = 0$	$D_{Ki} = 0$ $B_{Kl} = 0$ all i,k,l	all $A_{ij} = 0$
Number of Constraints[4]	None	18	24	21
Chi Square Statistic	NA	88.2	331.8	103.5
Critical Level (99%)	NA	34.8	43.0	38.9
Log-Likelihood Function	1073.61	1029.50	907.69	1021.87

[1] See Table I for Maintained Hypothesis. Constrained models satisfy price homogeneity and include the Treatment variable.

[2] Output homogeneity with degree not restricted. For constant returns (linear homogeneity) an added constraint, $\Sigma_K B_K = 1$, must also be imposed. Our log-likelihood with the added constraint is 900.67; constant returns is rejected at a 99.9% significance level.

[3] Unitary Elasticity of Substitution.

[4] Number of independent constraints

technology is not homogeneous in output and that relative input proportions are not independent of the scale of delivery output. These production characteristics are strongly confirmed by tests summarized in Table II.

Three sets of parameter restrictions are tested. They correspond to homotheticity, output homogeneity, and unitary elasticities of input substitution. The homotheticity restrictions also correspond to an output separability feature of cost functions. Below, we ascribe economic meanings to output separability, and the other restrictions. Table II summarizes results for the OPP model; Appendix C shows equivalent results for the EXP model.

The OPP and EXP models more accurately represent production of water delivery services when unconstrained by the coefficient restrictions shown in Table II. Caution should thus be used when adopting empirical models of water supply or input demand embodying implicit technological constraints of the sort reject here. For example, Cobb-Douglas or CES production functions correspond to cost and derived demand equations represented by small subsets of the unrestricted set of coefficients estimated for the OPP and EXP models.

Estimated Coefficients

Coefficient estimates for both OPP and EXP specifications are presented in Table III. The price interaction coefficients, A_{ij}'s, are excluded from the table. They are transformed into elasticities of substitution (σ_{ij}) by equation (S4.4) and reported in Table IV. The untransformed A_{ij} coefficients are reported in Appendix B. Nearly all of the coefficient estmates in Table III have signs consistent with those "expected" by the underlying economic paradigm of cost minimizing behavior.[32]

Expected coefficient signs are:

A_o		near zero, owing to mean-normalization of variables.
A_i	> 0	Cost-price elasticity, monotonicity requirement.
B_K	> 0	Cost-output elasticity, index of returns to scale, also a monotonicity requirement.
B_{Kl}	> 0	(k = l) Diminishing magnitude of scale economies with output.
B_{Kl}	< 0	(k ≠ l) Economies of scope among output pairs, indicate local diseconomies if positive.
D_{Ki}		Biases of i^{th} input use with respect to k^{th} output scale; $D_{Ki} = 0$ for all K,i indicates strict homotheticity and justifies output separability.
D_{tK}		Biases of treatment with respect to k^{th} output scale.
A_{ti}		Biases of treatment with respect to i^{th} input price.
A_t	> 0	Cost-treatment partial elasticity.
A_{tt}		Bias of Treatment with respect to treatment level.
A_{ij}		Price interaction terms; represented by transformed counterparts: σ_{ij}
σ_{ij}	< 0	(i = j) own-price elasticity of substitution.
σ_{ij}		(i ≠ j) are cross-price elasticities of substitution. (positive for substitutes; negative for complements)

Input Coefficients

Effects of input prices on cost are evaluated as cost-price elasticities, $\delta C/\delta P_i$ (which is the i^{th} input's cost share, Si). Due to mean-normalization, all share equation terms, except the A_i cost-price term, are zero at the point of means, making $\delta C/\delta P_i = A_i = S_i$. It follows that all cost-price elasticities must be positive to have an economically meaningful cost function. Both models satisfy this monotonicity condition.[33]

The effects of input interactions on cost are measured by the translog A_{ij} coefficients which represent all $(\delta(\delta C/\delta P_i)/\delta P_j)$

TABLE III: COEFFICIENT ESTIMATES

COEF	VARIABLES	OPP MODEL ESTIMATE	OPP MODEL t-VALUE	EXP MODEL ESTIMATE	EXP MODEL t-VALUE
λ	Lambda	.06021	6.6030	.05650	5.4949
Ao	Intercept	.06329	.4186	.32370	2.0514
A1	P1 (OwnW)	.40133	10.5300	.37957	10.4150
A2	P2 (PurW)	.22139	5.6455	.24031	6.8471
A3	P3 (FieldL)	.05676	5.3825	.06265	7.2583
A4	P4 (SalesB)	.01702	3.7712	.01864	3.9810
A5	P5 (MangEng)	.03001	3.7183	.02979	3.7388
A6	P6 (Energy)	.08558	5.1021	.09224	5.1319
A7	P7 (CapMat)	.18793	5.3683	.17743	5.0496
B1	Q1 (Metered)	.23304	5.1946	.25168	5.0514
B2	Q2 (Bulk)	.16404	5.2335	.15112	4.7946
B3	Q3 (UnMeter)	.27754	8.3773	.26630	7.6408
B11	.5*Q1**2	.14648	15.3900	.13888	13.2390
B12	Q1*Q2	-.08749	-13.1680	-.07939	-11.9420
B13	Q1*Q3	-.07922	-10.5040	-.07628	-9.3419
B22	.5*Q2**2	.06083	9.7668	.05161	9.0522
B23	Q2*Q3	-.00456	-.9207	-.00237	-.4711
B33	.5*Q3**2	.06948	12.7250	.06082	10.1330
D11	Q1*P1	-.01256	-1.4445	-.00499	-.5491
D12	Q1*P2	.01120	1.3258	.00419	.4113
D13	Q1*P3	-.00538	-2.3516	-.00606	-2.2867
D14	Q1*P4	-.00005	-.0436	-.00056	-.4166
D15	Q1*P5	-.00143	-.8283	-.00128	-.6073
D16	Q1*P6	.00199	.4762	.00126	.2946
D17	Q1*P7	.00622	1.1246	.00746	1.2940
D21	Q2*P1	.00296	.4795	-.00498	-.8450
D22	Q2*P2	.01384	2.3083	.01962	2.9008
D23	Q2*P3	-.00309	-2.0103	-.00270	-1.6082
D24	Q2*P4	-.00206	-2.8075	-.00181	-2.2621
D25	Q2*P5	.00023	.1978	.00037	.3231
D26	Q2*P6	-.00287	-1.0320	-.00327	-1.2848
D27	Q2*P7	-.00901	-2.3589	-.00720	-1.8913
D31	Q3*P1	.03095	4.6810	.02982	4.9328
D32	Q3*P2	.00011	.0189	.00230	.4098
D33	Q3*P3	-.00483	-3.0395	-.00412	-2.7330
D34	Q3*P4	-.00199	-2.7190	-.00186	-2.3474
D35	Q3*P5	-.00382	-3.2252	-.00388	-3.1138
D36	Q3*P6	-.00354	-1.3677	-.00284	-.9772
D37	Q3*P7	-.01688	-3.4465	-.01917	-3.9521
Dt1	T*Q1	.03472	2.8215	.05339	3.9603
Dt2	T*Q2	.01381	1.8136	.00101	.1444
Dt3	T*Q3	-.03331	-3.7456	-.01901	-2.2209
At1	T*P1	.00597	.4975	.00225	.2124
At2	T*P2	-.03458	-2.6963	-.02751	-2.2713
At3	T*P3	.00887	2.5715	.00811	2.3027
At4	T*P4	.00046	.2996	.00074	.4555
At5	T*P5	.00037	.1423	-.00052	-.2045
At6	T*P6	.00316	.5710	.00127	.2268
At7	T*P7	.01574	1.8228	.01570	1.8296
At	T	-.13173	-2.5045	-.05411	-1.0341
Att	.5*T**2	-.04839	-1.7649	-.06593	-2.0296

evaluations of cross-price elasticities. The A_{ij}'s (Appendix B) measure sensitivity of A_i cost-price elasticities to changes in P_j. Corresponding elasticities of substitution (shown in Table IV) more effectively measure input substitution effects associated with changes in relative input prices.[34]

Elasticities of substitution presume constant-quality inputs. By reducing price-quality covariation, our adjustments for water input quality should have the effect of increasing measured price elastici-

TABLE IV

INPUT ELASTICITIES OF SUBSTITUTION: σ_{ij}

Inputs	OPP model σ_{ij}	OPP model Std Error*	EXP model σ_{ij}	EXP model Std Error*
11	-4.8909	.9291	-3.4124	.8476
12	4.5118	1.0516	2.2154	.8494
13	.2034	.5710	.1127	.5020
14	1.1510	.7273	1.3742	.7255
15	-.5777	.7552	.0187	.6782
16	-.6565	.7901	1.4739	.7106
17	.8270	.3997	.0317	.4243
22	-5.6420	1.6635	-3.4127	1.2530
23	-.2572	.8356	.0162	.6712
24	.1327	1.1164	.8817	.8792
25	2.9658	1.1147	2.3465	.8278
26	1.7356	1.2679	1.2686	.9396
27	-.8458	.5706	-.3071	.4786
33	-.2030	1.6506	-.6308	1.6880
34	-1.8026	2.3777	-1.2124	2.2500
35	-.8868	1.8070	-.8161	1.7844
36	-1.5802	1.6651	-.9750	1.7062
37	1.0383	.6440	.7662	.6455
44	-8.2674	5.8812	-7.5873	5.9323
45	1.3740	2.9769	.7705	2.9394
46	-.7240	3.0089	-1.9106	2.8408
47	.6634	.8722	.4362	.9522
55	-13.2818	3.1625	-13.4073	2.8437
56	3.2466	2.5175	2.1948	2.1949
57	-.0893	.9118	.3457	.8751
66	-10.6302	3.8776	-10.9261	3.3716
67	3.3960	1.2183	2.4969	1.2028
77	-1.7281	.5251	-1.1128	.6067

* Standard errors at data means.

ties for x_1 and x_2. This is indeed observed for the OPP and EXP specifications of water prices.

Strong substitutability of own and purchased water is shown by the σ_{12} terms, implying that firms substitute among water sources in response to relative price changes to achieve economic cost minimization. Purchased and own water inputs, while highly substitutable, are not equivalent in terms of their interaction with the other inputs. For example, water-energy elasticities show complementarity with own water, which on average requires more pumping, but substitution for purchased water inputs. Purchased water apparently supplants Management/Engineering expenditures that would be incurred producing own water. Conversely, since purchased water is on average lower quality than own water, it requires more Capital/Materials (treatment related) inputs --- a complementary relationship, while own water and Capital/Materials are substitutes.

The EXP model does not reveal the very plausable complementarity of own water and energy and, in general, yields lower elasticity estimates than OPP. Both models show rather strong substitution between Energy and Capita/Materials inputs. This is consistent with micro findings such as Daly and Rao (1985), though mixed results have been obtained from more aggregated, macro studies of industrial sectors or economy-wide capital-energy tradeoffs.[35]

Cross-elasticities between various labor categories show a mix of either complementary or weak substitutability. This result should serve as a caution against aggregation of labor categories on an assumption that labor groups are highly substitutable. However, standard errors for labor input elasticities are relatively large. We attribute this, in part, to quality-price covariation. Quality variation within aggregated labor categories appears to be most serious for our Field labor category, which aggregates the widest range of tasks and skills.[36]

Price-Quantity Interactions

If a proportionate increase in all outputs leaves the relative quantities demanded of inputs unchanged, the underlying production function is homothetic. Non-homotheticity implies that cost-price elasticities and shares ($\delta C/\delta P_i = S_i$) will change as output scale changes. Relative input biases are measured by $\delta(\delta C/\delta P_i)/(\delta \ln q_k)$ terms which, at the sample means, are equal to the translog D_{ki} coefficients.[37] Both the Chi Square test of homotheticity (Table II) and the significance of individual D_{ki} coefficients (Table III) indicate that production of water delivery services in Southern California is definitely not homothetic. Large firms are not simply scaled-up, larger versions of smaller ones; relative input usage changes systematically as output scale increases.[38]

Larger scale, unmetered delivery operations use relatively more own-water inputs, as indicated by the positive D_{31} coefficient. Relative demand for other inputs decreases as unmetered output expands, though the relative decline of energy inputs has only marginal significance. Firms carrying out extensive unmetered operations are essentially substituting own water for other resources. Unmetered delivery operations have an apparent economy in terms of accounting costs because, the greater the volume, the smaller are per unit amounts of treatment, pumping, and office expenses, thus reducing proportionately demands for labor, capital, and materials. But, own water inputs increase more than proportionately, reducing the apparent economies if measured in terms of economic costs.

The pattern is similar for Bulk metered output (q_2), with two important differences. First, most of the water purchased for resale in Southern California qualifies for price rebates to the purchasing firms. The rebates are proportional to deliveries made directly or indirectly (resale to retailers), to most agricultural connections. Deliveries to agriculture connections are more likely, therefore, to be produced from purchased water inputs than own water which has no equivalent rebate. The D_{22} bias coefficient is significant and positive, showing that firms produce larger Bulk deliveries by substituting toward purchased water inputs. Lack of a similar effect for own water is indicated by the D_{21} coefficient which is effectively zero. This is inferential evidence that firms do indeed assign an opportunity cost value their own water assets, differing from mere associated expenses of using it.

Secondly, unlike q_3, increased scale of q_2 output is not biased away from relative use of Management/Engineering labor. Finally, the D_{27} and D_{37} coefficients show that increased scale is biased away from capital. This probably reflects the fact that several big firms, specializing in large-scale deliveries, particularly to agriculture users, whether metered or unmetered, do so through capital-conserving open canal distribution systems. We attribute some of these water-using characteristics to distribution losses, much of which may be due to evaporation and leaks.

For Metered output q1, increasing scale biases inputs toward greater Capital/Material use at the expense of Field labor, especially acquisition, pumping and treatment components of x_3.[39] Water input bias increasingly shifts toward purchased water as q_1 output expands, though D_{11} and D_{12} are only weakly significant for the OPP model and insignificantly different from zero in the EXP specification.

Output separability assumes that an aggregator function exists, allowing multiple outputs of a joint cost function to be aggregated into a single index of composite output. The appropriateness of this specification for our cost and demand equations is closely related to significance of the scale distortion (D_{Ki}) coefficients. Feigenbaum-Teeples (1983) demonstrated that significant improvement in the specification of costs for water delivery firms could be achieved by replacing the customary, unidimensional gallonage index with a more complex hedonic index of output. Their hedonic output index controlled for several dimensions of delivery services that have significant cost impacts. Our OPP and EXP multiple output models also strongly reject the output homogeneity implied by separability.

For multiproduct translog cost functions, a sufficient condition for output separability is that all $D_{Ki} = 0$.[40] Rejection of this condition is shown in Table II. Overwhelming rejection of seperability implies that single output cost models can introduce substantial specification errors into derived demand estimates, whether imposed explicitly or implicitly.

Treatment Affects on Cost

Sensitivity of cost-output elasticities to changes in treatment level (T) are measured by D_{tK} coefficients. These terms also measure changes in cost-treatment elasticity as outputs vary. The D_{t1} and D_{t2} coefficients are positive and significant, showing that greater treatment increases cost-output elasticities, though the effect on Bulk q_2 output is smaller (and non-existant in the EXP model). Conversely, increased Unmetered output q_3 diminishes the cost-treatment elasticity, all else the same.

The large negative D_{t3} term helps explain an anomolous result from Table III --- the negative A_t coefficients. The connection is that when q_3 is small, as it is for almost all firms in the sample, the product $(D_{t3})*(Q_3)$ is positive and quite large.[41] Cost-treatment elasticity ($\delta C/\delta T$) includes the term (Dt3)*(Q3) which is the dominating term for almost all observations in the sample, outweighing the effect of the (negative) A_t coefficient.

Cost-treatment elasticity is sensitive to Capital/Materials and Field labor prices (both heavily represented in treatment expenses). Interestingly, the cost-treatment elasticity has a negative coefficient with purchased water, indicating that higher priced water reduces treatment effects on cost, all else the same. The most likely cause is that not all price-quality covariation has been purged from the price of purchased water.

Ground water in Southern California is generally higher quality than surface water. Firms seek least cost methods of providing water delivery consistent with required quality standards, blending higher quality with lower quality water is one method. Since our treatment index interacts with (partially) quality-adjusted water input prices, blending (as a low cost method of meeting output quality standards) is reflected in high numbers of gallons processed at low treatment levels. This index feature may affect the sign of cost-treatment elasticities.

Output Coefficients

It is widely believed that extensive scale economies exist in water delivery. However, scale economies for the firm may mistakenly be inferred from engineering economies associated with sizes of water facilities. Facility economies may not be coincident with scale economies or diseconomies for total operation costs. For one reason, facility economies are typically calculated without regard for the relative values of inputs. Most importantly for this study, neither do they account for opportunity costs of own water.

Scale economies are evaluated by cost-output elasticities, defined as $\delta C/\delta(\ln q_K)$. For our translog, cost-output elasticities are:

$$\delta C/\delta(\ln q_K) = (q_K)^{\lambda} * [B_K + \Sigma_i D_{Ki} * P_i + \Sigma_l B_{Kl} * Q_l + D_{tK} * T]$$

$$= 1 * [B_K + 0 + 0 + 0] \quad \text{at mean values for variables } q_K,\ P_i, \text{ and } T\ (\lambda \text{ given}).$$

Thus, B_K coefficients must be positive in order to have an economically meaningful cost function.

The sum of BK coefficients provides a measure of total cost-output elasticity as overall scale increases. Overall scale economies (SE) are inversely related to the scale economies:

$$SE = (\Sigma_K(\delta C/\delta(\ln q_K)))^{-1}$$

$$= (\Sigma_K(q_K{}^{\lambda} * [B_K + \Sigma_i(D_{Ki} * P_i) + \Sigma_l B_{Kl} * Q_l + D_{tK} * T]))^{-1}$$

$$= (\Sigma_K B_K)^{-1}, \qquad \text{(at variable means: } P_i,\ Q_K,\ T = 0)$$

The sum of estimated B_K terms show statistically significant overall scale economies, at variable means, measuring nearly 50%.[42] Away from data means, SE is a complex function of D_{tK}, T, D_{iK}, and B_{Kl} coeffincients. Table V presents SE measures for the OPP and EXP models at variable means and also for variables normalized at median values of total output. Median output levels are more typical of overall scales for firms in our sample.

The relation between individual cost-scale elasticities (B_K) and overall scale economies (SE) is shown above. Increasing one output has

TABLE V

SCALE ECONOMIES FOR OPP AND EXP MODELS

Model	OPP (mean)	OPP[a] (median)	EXP (mean)	EXP[a] (median)
	1.482	1.381	1.495	1.377

a. OPP and EXP models estimated using median for output normalization and calculation of scale economies. Represents typical-firm output level. Estimations use entire data set.

two effects. First, as q_K increases beyond its mean value, the corresponding Q_K term becomes positive. Second, since B_{KK} terms are positive for all outputs, increasing a single output causes the output-cost elasticity itself to increase (and measured scale economies, SE, to decline).

When scale of more than one output changes, the analysis is more complicated owing to cost-output interactions, called scope economies. Scope economies are diminutions of individual cost-output elasticities owing to particular patterns of multiple output changes. A local measure of scope effects is cost complementarity (CC_{Kl}), defined as cross partial derivatives of cost with respect to two outputs ($\delta(\delta c/\delta q_K)/\delta q_l$). For our modified translog model CC_{Kl} is:

$$CC_{Kl} = \{c*[(\delta C/\delta \ln q_K)*(\delta C/\delta \ln q_l) + (B_{Kl}*q_K^{\lambda}*q_l^{\lambda})]/(q_K*q_l)\}, \qquad (K \neq l)$$

$$= (B_K*B_l) + B_{Kl} \qquad \text{(at variable means)}$$

Measures of CC_{Kl} for the OPP and EXP models are evaluated for both means and medians of output levels and summarized in Table VI. Negative CC_{Kl} values denote scope economies (output complementarities),

TABLE VI

COST COMPLEMENTARITY MEASURE FOR OPP AND EXP MODELS

Outputs	Q1, Q2	Q1, Q3	Q2, Q3
Models			
OPP	-.0493	-.0145	.0410
OPPa	.0034	-.0216	.0093
EXP	-.0414	-.0093	.0379
EXPa	.0042	-.0141	.0121

a. Model estimated using median output normalization corresponding to a typical output level for calculation of cost complementarities. The entire data set is used for estimation.

and positive values are scope diseconomies. Metered and Bulk outputs show substantial scope economies at the sample means but not at their median output levels. Metered and Unmetered outputs show a smaller and more consistent cost complementarities across the output spectrum. Bulk and Unmetered output categories show diseconomies at both points of output normalization.[43]

SUMMARY

We measure derived demands for both purchased and own water inputs using identical functional forms. Their shape is dictated by our choice of a minimally restricted translog approximation to the cost function of firms assumed to be driven by cost minimizing incentives. The resulting estimates of water demand only differ from other inputs in terms of sizes and significance of interactions with the independent variables. Costs being minimized, within our model, are economic opportunity costs which differ from reported expenses because 1) own water has imputed value in terms of cost-saving substitutions for purchased water inputs, and 2) in terms of sales, own water can return revenues greater than a firm's costs of self-supply.

We impute approximate prices for own water uses, reflecting opportunity costs that producers might attach to their holdings. Two tacts are taken, each with differing empirical manifestations and shortcomings. The first method, our OPP model, bases own water prices on the prices of purchased water --- the closest substitute in production for own water. In doing so, we attempt to quality-correct reported cost variations for water inputs. Since own water usually comes from underground sources in Southern California, we specified prices as if own-water quality would be, on average, equivalent to the quality of pretreated purchased water. Own water from recycled sources is assigned a significantly lower value. While this approach can merely approximate the relative, quality-constant input values that our observed owners actually face, the quality and consistency of our empirical results indicate that we have correctly specified many aspects of water's derived demand problems.

The second method, our EXP model, bases economic valuation of own

water on implicit cost recovery, of both (internal) delivery and per unit rate-of-return costs that an owner would face. De Rooy and Ziegler-Bell limited valuation of own water to only the costs of using it. An expense approach is not necessarily incorrect. If firms are in long-run equilibrium, realizing zero economic rents (normal returns), then our expense approach would impute marginal values in a manner similar to our OPP specification. However, in actual short-run situations, nonzero rents are a problem, making expense approaches to price specification less accurate. For this reason, we feel our EXP version produced less successful demand estimates. But, realizing that own water can either be sold as an extraction right or at a wholesale price to producers elsewhere, we specify own water price, for equivalent quality water, as production costs per unit plus implied returns (and rents) on asset value that fall within broad limits of own water value to others. Thus, our two approaches are not divergent in concept, only in their empirical implementations.

In general, we find that firms in our sample respond to relative factor costs (as specified) and behave as though their own water is more valuable then just the associated expenses of using that water. We stress how well firm behavior approximates that expected by a paradigm of economic cost minimization and responds to our imputed water prices. Resulting estimates show that opportunity cost of own use is avoided in specific instances where owners can avail themselves of rebates on purchased water if it is used by a firm for favored (agricultural) categories of water delivery. Our input demand estimates show a wide range of substitutability between input pairs, changes in relative input intensities as outputs change in scale and mix, and substantial scale economies, all in line with a priori expectations. Our models' output disaggregation allows first-ever estimates of scope economies for types of water delivery and shows how these are directly related to derived demands for water inputs. Using this framework, future research can more appropriately investigate such issues as cross-subsidization and cost effects of alternative types of firm ownership --- questions we are presently investigating in another study.

FOOTNOTES

1. Ziegler and Bell describe firms that have their own water source and "intake" water produced from that source during a given period as "self-supplying."

2. It is costly for firms to dispose of used water. Costs for disposal may be particularly high when stringent waste water quality standards must be met. But, we question the relevance of these costs to input substitution decisions. Degredation of water quality takes place during production regardless of which source of input supply is used. And, disposal is the same for water whether it is supplied from within or outside the firm. The economic consequences of degredation should be accounted for as a separate production activity.

We illustrate this argument with a related example. Suppose a firm buys timber but supplements input demand by using its own trees. The firm values its own trees at the equivalent price of purchased wood. During production sawdust is created. Costs for disposing of the dust is the same regardless of which type of tree the sawdust comes from. The fact that a residual is produced does not differentially affect the values of externally versus internally supplied timber --- unless disposal of sawdust from one source is more costly than sawdust from another. We see no reason to suspect that self-supplied water systematically represents greater (or lesser) disposal costs than water from other sources.

3. Explication of the methods reviewed in recent survey chapters by Diewert (1983) and Jorgensen (1985).

4. W_d is total water demanded from all sources. W_s is the amount of own water supplied, and may exceed the amount self-supplied.

5. An owner's water rights may be separable, exchangable, real property rights associated with control over water extraction. One "markets" water in this situation by merely selling an extraction right, in which case water "expenses" would be nil even though own water is being supplied in the same way that landowners supply land to leasehold farmers.

6. We assume for simplicity that P_u is an average price equal to marginal price.

7. QUALCOST may be positive or negative depending on relative qualities of own- versus purchased water. If own ground water is higher quality (up to some production limit) than a delivered source of water, a cost minimizing strategy for firms desiring an intermediate quality for output water is to blend water sources and thereby avoid some (or all) incremental treatment costs. If delivered water must be more heavily treated (internally) than own-water, QUALCOST would be negative:

$$P_u = V - \text{QUALCOST}, \quad \text{for DELCOST} = 0$$

making V greater than P_u. A water-owning firm could be expected to capture the return (V) in part from reduced internal treatment expenses (or operations costs) and/or a price premium if own water is sold. On the other hand, if external supplies are pretreated to a quality level greater than a firm's raw own water, QUALCOST would represent the cost of bringing own water to equal quality.

8. We assume QUALCOST is zero in order to simplify quality issues. See footnote (7).

9. By his own identity equation, De Rooy assumes his water categories are exhaustive.

10. Included are non-price dummies --- one for technology of water use (date of plant construction) and another for firm type.

11. The relative factor variations that Zeigler-Bell apparently think are controlled, because their time period of analysis is short, actually remain uncontrolled because relative input prices will nonetheless vary among firms rather than over time. Exclusion of input prices means that these impacts on demand variation remain unrecognized.

12. DeRooy (1974), page 3.

13. Zeigler and Bell seem to justify the limitations of their model, in part, on the basis of few degrees of freedom resulting from their small sample of 28 firms. Appropriate F tests would have indicate whether or specific limitations are warranted.

14. Zeigler and Bell (1984), page 7.

15. This assumption seems to be satisfied. None of the firms individually has a significant impact on prices of labor (as categorized for our study), energy, or capital/material inputs. Even input water prices seem to us to be insensitive to changes in quantity demanded of individual firms. Rising marginal costs for own water production should translate into falling resource value fot own water "in situ." This implies that a fully specified own water price by the EXP approach would capture "in situ" value in the return on capital (V) term.

16. See McFadden (1978) and Kiss et. al. (1982). There are numerous advantages to estimating cost rather then production functions. For our study the advantages are: 1) Multicollinearity of input quantities and output levels across the sample make estimation of production functions difficult. Instesd, cost functions use input prices, also reducing simultaneity bias since input prices are much more likely to be exogenous than input quantities; and 2) Cost function estimates directly yield estimates of elasticities of factor substitution, and economies of scale and scope; and 3) application of Shepard's Lemma yields derived demand equations for each variable input.

17. The degrees of freedom available for our system estimation increase from 52 to 776. This increase is especially important because our estimators are asymptotic. This advantage is undermined to an extent since not all of the economic behavioral assumptions of cost minimization hold exactly.

18. We interpret the Translog function as an approximation to an underlying, "true" production structure. One implication of this interpretation is assumed symmetry --- cross partial derivatives of the trans-

log are equal regardless of the order that they are taken. Symmetry greatly reduces the number of estimated parameters.

19. For every variable, x, we rescale so that:

$$x_{new} = x \div \overline{x}, \quad \text{where } \overline{x} = \text{mean of x over all firms.}$$

The mean value of x_{new} is one.

20. The estimated equations are derived and written out in Appendix A. The number of terms is reduced somewhat because of symmetry conditions (footnote 18); $A_{ij} = A_{ji}$ and $B_{kl} = B_{lk}$.

21. Relationship of the Box-Cox transformation to our analysis is discussed in Appendix A. Thus, the notation Q_K for output means Box-Cox rather than log transformation of output q_K.

22. The factor demand equations, however, are not as simple to evaluate as share equations which are linear in all the variables. The factor demand equations are transformed by the c/p_i ratios for which c is a function of the model's independent variables. Effects on share equations of varying any one of the independent variables, is evaluated by a derivative of the share equation which results in a single coefficient. For the factor demand equation, equivalent derivatives are more complicated, except when evaluated at variable means, because c itself must be differentiated each time. See Appendix B.

23. Error term (e) in cost equation (C1.0) affects C with an approximately normal distribution. Errors (v_i) in the share equations, arising from errors in cost minimizing behavior, are assumed to have a joint normal distribution (independent of error (e) in the cost equation). A problem for the (v_i) errors is that measured shares must be contained in the interval [0,1]; this could lead to loss of efficiency in estimation. Woodward (1979) found that this was a very minor problem.

24. We deleted the Capital/Materials share equation. However, rather than reconstruct the missing coefficients and their standard errors from price homogeneity constraints and estimated variance-covariance matrix, we estimated the model deleting alternative shares. Our results were invariant to the deleted share.

25. Cost equation homogeneity (in prices) does not mean that the underlying production function is homogeneous (in outputs). Production function homogeneity is not an imposed constraint. Indeed, as we show in section VI below, it is overwhelmingly rejected. The economic conditions, referred to as linear homogeniety in input prices, imply a set of constraints on input price coefficients in the system. Homogeniety constraints reduce the order of the system by one and amount to an arbitrary normalization of prices. See Christensen and Greene (1976) and Appendix A.

26. A question arises in the application of this weighting. Should it be specific to the firm or should it be an average of firm ratios (over the whole data set)? It does not make much difference in our empirical results; we present the case where the weighting is firm-specific.

27. An alternative, variable cost function uses capital costs and materials costs (not prices) as technical variables which affect the cost environment and are treated as a third class of variables along with inputs and outputs (see Spady and Friedlaender (1981)). Below, we use this alternative for our TREAT variable, measured as an index number. There are several problems with this approach which make it difficult to implement. First, the variables are measured in dollar quantities (and all of the second-order terms incorporating them); these would be strongly colinear with cost and output variables and would reintroduce problems associated with production function estimation (footnote 16). Second, there are some delivery factors (geographic distribution of customers, topographic variation, etc.) which are positively correlated with capital and materials usage and not fully captured by other features specified for the model. This creates a saddle-point characteristic for a variable cost function's optimum: additional capital or materials should lower (other) variable costs but the correlation with unobservables impares the estimation of coefficients. This result is has been encountered by other researchers. Finally, as shown by Murray (1985), dollar measurement of technical variables would require that homogeniety in prices apply to these variables, making the model very complicated to test for homotheticity and homogeneity assumptions because constraints become non-linear and output elasticities and scale measures become more complicated to calculate and interpret.

28. UCB SHAZAM version 4.5 by Kenneth White was used for all econometric subroutines. The Nonlinear Regression subroutine was used to estimate our system. It allows a starting value to be set for each estimated coefficient. Given the model and data set sizes, a single regression typically used about 4000 CPU seconds on a VAX-11/780. The assistance of Andy Davenport and Chandra Wahjudi at the Four College Computer Center at Claremont is gratefully acknowledged.

29. Pseudo R^2 is calculated from the value of the Log Likelihood Statistic as:

$$\text{Pseudo } R^2 = 1 - e^{[(2*(LLc - LLu))/(n*t)]}$$

n = the number of equations (7)
t = the number of observations (119)
LLc, LLu = Log-likelihood statistics for constrained and unconstrained estimations of the model

Constrained estimation in this instance means that the model is evaluated at zero values for all the independent variables.

30. We estimated a linear version of each model (without the Box-Cox λ) using 3SLSQ's. This procedure yields R^2 statistics for each equation. The resulting R^2's for cost equations were nearly one but ranged from about .2 to .4 for share equations. The 3SLSQ's estimation procedure is not as efficient as the Full Information Maximum Likelihood (FIML) estimation that we used to fit the non-linear OPP and EXP models. 3SLSQ's has the advantage, however, of not being as susceptible to misspecification errors deriving from inappropriate application of cost minimization assumptions. We found minimal differences in the estimated coefficients for comparable models and this gives us additional

confidence that cost minimization is a valid approximation of behavior for our application. We also estimated a nonlinear version of the cost equation by itself using Maximum Likelihood estimation. The resulting psuedo R^2 was very high, (0.9924).

31. The tests are based on a Chi Square statistic where:

$$ChiSq(t) = 2*(LLu - LLc)$$

which, under the null hypothesis, has a Chi Square distribution with t degrees of freedom; t is the number of independent constraints imposed on the model. LLu and LLc are Log-Likelihood values computed for the unconstrained and constrained models, respectively.

32. Table III presents t-statistics rather than the (asymptotic) standard errors. Use of t-statistics implies a "null" hypothesis that the coefficients are zero. In the case of elasticities of substitution, Table IV, this is clearly an inappropriate null. It is also questionable for the BK terms, since a null hypothesis of zero would imply a null of infinite scale economies.

33. A necessary condition is that cost functions be monotonically increasing in input prices. The full requirement is that all cost-price elasticities (share equations) be positive, when evaluated at the observed input prices and output levels of all firms in the data set. This full requirement does not seem to present a problem.

34. Elasticities of substitution also provide a type of stability information. Cost function concavity in all input prices is a necessary condition for stability, corresponding to a globally estimated (rather than merely local) minimum cost production plan. Global cost minimization requires all diagonal elasticity elements (σ_{ii}) to be negative and the preponderance of cross-elasticities (σ_{ij}) to be substitute relationships. This condition holds for both the OPP and EXP models.

35. Field and Grebenstein (1980) estimate elasticities of substitution for ten two-digit manufacturing industries using state cross-section data. They disaggregate capital into physical and working capital, finding little evidence of substitution for energy and physical capital but general substitution for energy and working capital.

36. When the models are estimated without the price homogeneity constraint, own-price elasticity for Field labor is much larger, which indicates that price-quality covariation is a problem. Two additional problems affect estimation results. We suspect that reporting inconsistencies concerning total benefits and their inclusion in full labor compensation causes large standard errors, though systematic bias is not apparent. Second, aggregation of diverse labor tasks into a single Field labor category causes some problems. Disaggregation of field labor into transmission and distribution (x_{3a}) and acquistion, pumping and treatment labor (x_{3b}) would reduce variation. The size of this alternative specification precluded using Box-Cox output transformations in the same estimation.

Field labor disaggregation cleared up several problems. First, own price elasticity of x_{3a} is about -3.0 and -25.0 for x_{3b} (for both OPP and EXP models), resolving the price-quality covariation difficulty. Second, x_{3a} and x_{3b} have quite different substitution elasticities with other inputs. x_{3a} is complementary with purchased water and energy and a capital substitute. x_{3b} is a capital complement, independent of the water inputs, and an energy substitute. X3b is a strong complement with billing/sales labor and a management/engineering substitute --- the reverse of x3a.

37. Equivalently, input biases are evaluated as:

$$\delta(\delta C/\delta \ln q_K)/\delta P_i$$

showing the sensitivity of cost-output elasticity (discussed below) to changes in input prices.

38. Scale bias effects on relative input use are separate from input biases that occur with a changing output mix.

39. Disaggregation of Field labor indicates that it is these factors, not transmission-distribution labor expenses which are avoided.

40. If some D_{Ki} are non-zero, then output seperability can be satisfied only if the ratios of cost-output elasticities for all outputs are independent of all output levels and factor prices. This is refered to as weak separability and seems very unlikely to hold when strong from is rejected. These constraints are difficult to calculate and we did not go to the trouble because the sufficient condition is overwhelmingly rejected.

41. D_{tK} coefficients equivalently measure:

$$\delta(\delta C/\delta T)/\delta \ln q_K = \delta(\delta C/\delta \ln q_K)/\delta T$$

For all but four firms, q_3 is either zero or very small, making Q3 a large negative number and thus dominating in size other terms in the elasticity. While it is true that this term drops out at the mean, there really are no firms at the data means with respect to Unmetered output. Individual firms report q3's that either much smaller or larger than the sample mean. When cost-treatment elasticity is evaluated at data medians, the A_t coefficient (and hence treatment elasticity) is reduced by a factor of three and is no longer statistically significant.

42. The B_K's at means for the OPP model sum to 0.6746 or SE = 1.48. A one percent increase in all outputs leads to a 0.675 percent increase in economic costs. The standard errors of B_K terms are all relatively small (.03 to .045), leading to tight bounds on our estimates of scale economies. Application of scale and scope economy measures to generalized translog cost functions is found in Kiss, Karabadjian and Lefebvre (1983).

43. Output means are: q_1 = 534328, q_2 = 165986, q_3 = 1193918 (thousands of cubic feet). q_3 is dominated by three firms that deliver more then 98 percent of all unmetered water.

APPENDIX A

DEFINITIONS

$P_i = \ln(p_i)$ = log of price of input i

$\overline{P}i = Pi - Pn = \ln(\overline{p}_i) = \log(p_i/p_n)$

$C = \ln(c)$

q_K = output k

$Q_K = (q_K{}^{\lambda}-1)/\lambda$, Box-Cox transformation of q_K
(λ is same constant for all outputs.)

t = quantity of treatment index

$T = \ln(t)$

x_i = quantity of input i

$S_i = (x_i * p_i)/c$ = cost share of input i

$Ci = \delta C/\delta P_i = \delta(\ln c)/\delta(\ln p_i) = c_i*(p_i/c))$

$c_i = \delta c/\delta p_i = (c/p_i)*C_i$

$c_{ij} = \delta(c_i)/\delta p_j = \delta(\delta c/\delta p_i)/\delta p_j$

BOX-COX METRIC

We use an alternative to logarithmic metric transformations of variables because some firms report zero output for some output categories.[A1] The Box and Cox (1962) transformation, $f(q_K,\lambda)$, provides a more general metric, in that it includes the natural log transformation as a limiting case. Applied to output q_K, the Box-Cox transformation is:

$$Q_K = f(q_K,\lambda) = ((q_K{}^{\lambda})-1)/\lambda \quad (\lambda \neq 0) \qquad (A\ 1.0)$$

$$(Q_K \rightarrow \ln(q_K) \text{ as } \lambda \rightarrow 0)$$

Each output could have a unique λ, though, we implement our model using the same value for each output. All other variables in the model are log transformed (equivalent to $\lambda = 0$).[A2] A simplified version of our translog equation was estimated using distinct λ's for each output. Unmetered output, q3, which has the greatest preponderance of reported null values was associated with the largest estimated value for λ.

When the full model (our maintained hypothesis) was estimated, the estimated value of λ was 0.060271 for the OPP model and 0.056500 for the EXP model. Nearly identical values were obtained for all of the constrained models, differing by less than 0.005 from values obtained for our maintained hypothesis. An exception occured when output homogeneity was imposed, leading to a value of 0.12). Thus, our Box-Cox transformation for the maintained hypothesis is close to values produced by natural log transforms, though significantly larger by a standard t-test.

TREATMENT VARIABLE

Addition of a technical treatment variable yeilds the cost function: c = c(p,q,t). Log of t (T) is an exogenous, modifying parameter in the translog equation:

$$C(P,Q,T) = A_o + \Sigma_i(A_i*P_i) + \Sigma_k(B_k*Q_k) + A_t*T + .5*A_{tt}*T^2$$
$$+ .5*\Sigma_{i,j}(A_{ij}*P_i*P_j) + .5*\Sigma_{k,l}(B_{kl}*Q_k*Q_l)$$
$$+ \Sigma_{i,k}(D_{ik}*P_i*Q_k) + \Sigma_i(A_{ti}*P_i*T) + \Sigma_k(D_{tk}*Q_k*T)$$

where A_t, A_{tt}, A_{ti} and D_{tk} are the coefficients on the variables involving the treatment variable.

ZERO HOMOGENEITY IN INPUT PRICES: SHARE EQUATIONS

Parameter constraints are implied by economic cost minimization; the cost function must be homogeneous of degree one and share equations must be homogeneous of degree zero in input prices.

Function F( ) is homogeneous of degree zero in prices if:

$$\Sigma_j[(\delta F(\ )/\delta p_j)*p_j] = 0 \qquad \text{(A 2.0)}$$

Share equations (Si) of the translog cost function are homogeneous of degree zero if:

$$0 = \Sigma_j(\delta S_i/\delta p_j)*p_j$$
$$= \Sigma_j(\delta S_i/\delta P_j)*(\delta P_j/\delta p_j)*p_j$$
$$= \Sigma_j(A_{ij}*(1/p_j)*(p_j))$$
$$= \Sigma_j(A_{ij}) \quad \text{and (} = \Sigma_i A_{ij} \text{ since } A_{ij} = A_{ji}) \qquad \text{(A 2.1)}$$

DERIVATION OF THE SYSTEM EQUATIONS

Applying the $\Sigma_j(A_{ij}) = 0$ constraint (A 2.1) to the translog cost function and rewriting $\Sigma_j(A_{ij}*P_i*P_j)$ using (A 2.2) and the symmetry condition $A_{ij} = A_{ji}$ yeilds:

$$A_{11}*P_1*P_1 + A_{12}*P_1*P_2 + \ldots + A_{1n}*P_1*P_n +$$

$$A_{21}*P_1*P_2 + A_{22}*P_2*P_2 + \ldots + A_{2n}*P_2*P_n +$$
$$A_{n1}*P_1*P_n + A_{n2}*P_2*P_n + \ldots + A_{nn}*P_n*P_n =$$

$$A_{11}*P_1*P_1 + A_{12}*P_1*P_2 + \ldots + [- A_{11} - \ldots - A_{1,n-1}]*P_1*P_n +$$

$$A_{12}*P_1*P_2 + A_{22}*P_2*P_2 + \ldots + [- A_{12} - \ldots - A_{2,n-1}]*P_2*P_n +$$

$$\vdots \qquad\qquad \vdots$$

$$[- A_{11} - \ldots - A_{1,n-1}]*P_1*P_n + \ldots + A_{nn}*P_n*P_n$$

where A_{nn} is a negative sum of the n-1 preceeding terms. For illustration, this result in a 3x3 case yeilds the sums:

$$A_{11}*P_1*P_1 + A_{12}*P_1*P_2 + [- A_{11} - A_{12}]*P_1*P_3 +$$

$$A_{12}*P_1*P_2 + A_{22}*P_2*P_2 + [- A_{12} - A_{22}]*P_2*P_3 +$$

$$[- A_{11} - A_{12}]*P_1*P_3 + [- A_{12} - A_{22}]*P_2*P_3 +$$

$$\{[A_{11} + A_{12}] + [A_{12} + A_{22}]\}*P_3*P_3$$

Rearranging, completing the square and substituting $\Pi i = (Pi - Pn)$ yields:

$$= .5*A_{11}*(\overline{P}_1)^2 + A_{12}*\overline{P}_1*\overline{P}_2 + .5*A_{22}*(\overline{P}_2)^2 \qquad \text{(A2.3)}$$

In general, the function $f(x,y)$ is homogeneous of degree n, in variables x, if:

$$f(\theta x,y) = (\theta^n)*f(x,y), \quad \text{or}$$

$$F(\theta x,y) = \ln(f(\theta x,y)), \quad \theta = \text{proportion of change}$$

$$= n*\ln(\theta) + \ln(f(x,y)) = n*\ln(\theta) + F(x,y)$$

For cost function c( ), which is homogeneous of degree one (n = 1):

$$\ln[c(\theta p,t,q)] = r + \ln[c(p,z,q)]$$

where, $\ln(\theta*p_i) = \ln(\theta) + \ln(p_i) = r + Pi, \quad r = \ln(\theta)$

And, for translog cost function C( ), n = 1:

$$C(\theta p,t,q)] = A_o + \Sigma_i(A_i*[P_i + r]) + \Sigma_k(B_k*Q_k) + A_t*T + .5*A_{tt}*T^2$$

$$+ .5*\Sigma_{ij}(A_{ij}*[P_i + r]*[P_j + r]) + .5*\Sigma_{kl}(B_{kl}*Q_k*Q_l)$$

$$+ \Sigma_{ik}(D_{ik}*[P_i + r]*Q_k) + \Sigma_i(A_{ti}*[P_i + r]*T) + \Sigma_k(D_{tk}*Q_k*T)$$

$$= \{A_o + \Sigma_i(A_i*P_i) + \Sigma_k(B_k*Q_k) + A_t*T + .5*A_{tt}*T^2$$
$$+ .5*\Sigma_{ij}(A_{ij}*P_i*P_j) + \Sigma_i(A_{ti}*P_i*T) + \Sigma_k(D_{tk}*Q_k*T)$$
$$+ .5*\Sigma_{kl}(B_{kl}*Q_k*Q_l) + \Sigma_{ik}(D_{ik}*P_i*Q_k)\} + r*\{\Sigma_i A_i + \Sigma_k(\Sigma_i D_{ik}\}$$
$$+ \Sigma_i A_{ti} + .5*\Sigma_{ij}(A_{ij}*P_j) + P_i + r]\}$$

$$= C(P,H,Q) + r*\{\Sigma_i A_i + \Sigma_k(\Sigma_i D_{ik}*Q_k)$$
$$+ \Sigma_i(A_{ti}*T) + [a]\} \qquad \text{(A 3.0)}$$

[a] is the sums of Aij terms and is zero as a consequence of the share equations being homogeneous of degree zero in factor prices. Thus,

$$\Sigma_i A_i + \Sigma_k(\Sigma_i D_{ki}) + \Sigma_i A_{ti} + [a] = 1 \qquad \text{(A 3.1)}$$

Our translog cost function is homogeneous of degree one in factor prices if, 1) the sum over i of the A_{ti} and D_{ik} (for each k^{th} output) terms equals zero, and 2) the sum over i of A_i terms is unity.

IMPLEMENTATION OF HOMOGENEITY CONSTRAINTS

Applying the above restrictions on A_i, A_{ti}, and D_{ki} yeilds the functional form of cost that we estimated. Again, we illustrate, using equations (A3.2) and (A3.3), for a three input case:

$$\Sigma_i(A_i*P_i) = A_1*P_1 + A_2*P_2 + [1 - A_1 - A_2]*P_3$$
$$= A_1*[P_1 - P_3] + A_2*[P_2 - P_3] + P_3$$
$$= A_1*\overline{P}_1 + A_2*\overline{P}_2 + P_3 \qquad \text{(A 3.2)}$$

$$\Sigma_i(D_{ki}*P_i*Q_k) = D_{k1}*P_1*Q_k + D_{k2}*P_2*Q_k + D_{k3}*P_3*Q_k$$
$$= Q_k*\{D_{k1}*P_1 + D_{k2}*P_2 + [-D_{k1} - D_{k2}]*P_3\}$$
$$= Q_k*\{D_{k1}*[P_1 - P_3] + D_{k2}*[P_2 - P_3]\}$$
$$= D_{k1}*Q_k*\overline{P}_1 + D_{k2}*Q_k*\overline{P}_2 \qquad \text{(A 3.3)}$$

$$\Sigma_i(A_{ti}*P_i*T) = A_{t1}*P_1*T + A_{t2}*P_2*T + A_{t3}*P_3*T$$
$$= A_{t1}*T*P_1 + A_{t2}*T*P_2 + [-A_{t1} - A_{t2}]*T*P_3$$
$$= A_{t1}*[P_1 - P_3]*T + A_{t2}*[P_2 - P_3]*T$$
$$= A_{t1}*T*\overline{P}_1 + A_{t2}*T*\overline{P}_2 \qquad \text{(A 3.4)}$$

ESTIMATION EQUATION

The complete cost function, C = C(P,Q,T) for n inputs, can be rewritten using (A2.3), (A3.2), (A3.3), (A3.4). The n^{th} factor price variable in equation (A3.2) does not have a coefficient and must be subtracted. The left-hand-side of the cost equation becomes:

$$C - P_n = \overline{C}$$

All other input variables are then relative to the excluded n^{th} price. In like manner, price homogeneity restrictions are incorporated in the share equations which are rewritten as:

$$\begin{aligned}\overline{C} = {} & A_o + \Sigma_i(A_i*\overline{P}_i) + \Sigma_k(B_k*Q_k) + A_t*T + .5*A_{tt}*T^2 \\ & + .5*\Sigma_{ij}(A_{ij}*\overline{P}_i*\overline{P}_j) + .5*\Sigma_{kl}(B_{kl}*Q_k*Q_l) \\ & + \Sigma_{ik}(D_{ki}*\overline{P}_i*Q_k) + \Sigma_i(A_{ti}*T*\overline{P}_i) + \Sigma_k(D_{tk}*T*Q_k) \qquad \text{(A 4.0)}\end{aligned}$$

$$S_i = A_i + A_{ti}*T + \Sigma_j(A_{ij}*\overline{P}_j) + \Sigma_k(D_{ki}*Q_k) \qquad \text{(A 4.1)}$$

$$(i,j = 1 \text{ to } 6;\ k,l = 1 \text{ to } 3)$$

MEETING THE CONSTRAINTS

Cost stidies that have reported constraint tests have generally shown them to be acceptable when cross-sectional data was used, while time-series studies have strongly rejected them (Elie Applebaum (1978) and Evans and Heckman (1984)). Simpson (1984) found the constraint was binding. We draw an indirect inference that our OPP shawdow price specification p_1 and quality corrected price p_2 are appropriate because the price homogeneity constraint is so easily justified. Results for the EXP model would indicate that we are not as close to a correct specification since we are closer to rejection.

A1. Firms reporting zero deliveries in a customer category may in fact produce some small amount of that output. For reporting, firms may include these quantities in other categories. If there are no significant startup, overhead costs specific to an output category, a simple econometric expedient of using small, positive numbers in place of (reported) zero output levels would create only a minor errors-in-the-variables problem. But, there are two added complications with this technique. First, the "zero's-plus" data points are, after taking the log, quite far from the mean, and may stress the translog approximation. Since the Box-Cox function yields a relatively less extreme value for these transformed values, it is less stressful in this regard.

Second, for our data set, there is a large frequency of zero values for Bulk and Unmetered outputs. An alternative to the Box-Cox transformation would eliminate many zero values by aggregating Unmetered deliveries with Bulk or with Metered. This would cause mean output for most firms to change only slightly. Greater aggregation of output categories would necessitate inclusion of hedonic aggregators (on Metered and possibly on Bulk deliveries) to account for the greater variation in services within the measured output categories (and hence in cost). This approach was applied to output aggregation of water deliveries by Feigenbaum and Teeples (1983).

A2. Berndt and Khaled (1979) investigate translog-type models using Box-Cox transformations of all variables while imposing a constraint of equality among all λ's.

APPENDIX B

INPUT DEMAND EQUATIONS:

Input demand equations are derivable from the share equations by:

$$S_i*(c/p_i) = x_i$$

where, c = total cost

p_i = i^{th} input price

The ratio (c/pi) is a constant for each firm but a complex function and a price vector, respectively, for the data set as a whole. Thus, an input demand equation can be easily analyzed only at mean values for variables in the data set. For an i^{th} factor demand equation:

$$\delta x_i/\delta P_j = S_i*(\delta(c/p_i)/\delta P_j) + (c/p_i)*(\delta S_i/\delta P_j)$$

However, for factor share equations, the same derivative yields:

$$\delta S_i/\delta P_j = A_{ij} \quad \text{(a constant)}$$

INPUT SHARE EQUATIONS AND MEAN VALUES:

Our estimated share equations, with standard errors below each coefficient, are presented for own water (x_1) and purchased water (x_2).

S_1 AND S_2 FOR OPP MODEL

$$\begin{array}{l} S_1 = \underset{(.05011)}{.40133} - \underset{(.05124)}{.05202}*P_1 + \underset{(.05077)}{.13660}*P_2 - \underset{(.01282)}{.01628}*P_3 + \underset{(.00581)}{.00111}*P_4 - \underset{(.01831)}{.01831}*P_5 \\ \quad - \underset{(.02190)}{.03999}*P_6 - \underset{(.02570)}{.01112}*P_7 - \underset{(.00826)}{.01256}*Q_1 + \underset{(.00534)}{.00296}*Q_2 + \underset{(.00752)}{.03095}*Q_3 + \underset{(.01237)}{.00597}*T \end{array}$$

$$\begin{array}{l} S_2 = \underset{(.05011)}{.22139} + \underset{(.05077)}{.13660}*P_1 - \underset{(.06168)}{.04075}*P_2 - \underset{(.01668)}{.02234}*P_3 - \underset{(.00725)}{.00556}*P_4 + \underset{(.01119)}{.01984}*P_5 \\ \quad + \underset{(.02776)}{.01545}*P_6 - \underset{(.03191)}{.10324}*P_7 - \underset{(.00869)}{.01120}*Q_1 + \underset{(.00563)}{.01384}*Q_2 + \underset{(.00721)}{.00011}*Q_3 - \underset{(.01303)}{.03458}*T \end{array}$$

S_1 AND S_2 FOR EXP MODEL

$$\begin{array}{l} S_1 = \underset{(.04961)}{.37957} + \underset{(.03374)}{.03025}*P_1 + \underset{(.03180)}{.04454}*P_2 - \underset{(.01026)}{.01698}*P_3 + \underset{(.00554)}{.00243}*P_4 - \underset{(.00740)}{.01065}*P_5 \\ \quad + \underset{(.01787)}{.01074}*P_6 - \underset{(.02572)}{.05833}*P_7 - \underset{(.00790)}{.00499}*Q_1 - \underset{(.00536)}{.00498}*Q_2 + \underset{(.00668)}{.02982}*Q_3 + \underset{(.01219)}{.00225}*T \end{array}$$

$$S_2 = \underset{(.05882)}{.24031} + \underset{(.03180)}{.04454}*P_1 + \underset{(.04748)}{.03256}*P_2 - \underset{(.01245)}{.01872}*P_3 - \underset{(.00629)}{.00079}*P_4 + \underset{(.00918)}{.01404}*P_5$$

$$+ \underset{(.02200)}{.00558}*P_6 - \underset{(.02994)}{.07685}*P_7 - \underset{(.00842)}{.00419}*Q_1 + \underset{(.00573)}{.01962}*Q_2 + \underset{(.00739)}{.00230}*Q_3 - \underset{(.01331)}{.02751}*T$$

INPUT COST SHARES (S_i)

INPUT	OPP MODEL MEAN SHARE	OPP MODEL STD DEV	EXP MODEL MEAN SHARE	EXP MODEL STD DEV
x1 Own Water	.21149	.19533	.19280	.17627
x2 Pur Water	.18391	.19137	.18817	.19376
x3 Field Labor	.09663	.05284	.09957	.05325
x4 Bill/Sales L	.03485	.02203	.03547	.02257
x5 Mang/Eng L	.05488	.03469	.05612	.03490
x6 Energy	.11414	.08057	.11544	.07645
x7 Cap/Mat	.30410	.13377	.31243	.01319

CROSS-PRICE COEFFICIENTS:

SECOND-ORDER INPUT PRICE COEFFICIENTS (A_{ij})

COEF.	OPP MODEL ESTIMATE	OPP MODEL STD. ERROR	EXP MODEL ESTIMATE	EXP MODEL STD. ERROR
A11	-.05202	.05124	.03025	.03374
A12	.13660	.05077	.04454	.03180
A13	-.01628	.01282	-.01698	.01026
A14	.00111	.00581	.00243	.00554
A15	-.01831	.00839	-.01065	.00740
A16	-.03999	.02190	.01074	.01787
A17	-.01112	.02570	-.05833	.02572
A22	-.04075	.06168	.03256	.04748
A23	-.02234	.01668	-.01872	.01245
A24	-.00556	.00725	-.00079	.00629
A25	.01984	.01119	.01404	.00918
A26	.01545	.02776	.00558	.02200
A27	-.10324	.03191	-.07685	.02994
A33	.08539	.01601	.08372	.01978
A34	-.00944	.00812	-.00764	.00888
A35	-.01000	.00937	-.01015	.01070
A36	-.02846	.02095	-.02214	.02027
A37	.00113	.01892	-.00727	.01932
A44	.02360	.00757	.02471	.00813
A45	.00077	.00557	-.00059	.00583
A46	-.00686	.01201	-.01151	.01203
A47	-.00357	.00924	-.00625	.01062
A55	.01187	.00889	.01093	.00985
A56	.01408	.01481	.00747	.01481
A57	-.01818	.01522	-.01158	.01534
A66	-.03739	.05420	-.04234	.04699
A67	.08318	.04576	.05399	.04338
A77	.05180	.04856	.10620	.05488

PARTIAL PRICE ELASTICITIES OF INPUT DEMAND:

The Aij's, as Binswanger (1974) notes, have no natural economic interpretation. Comparisons of OPP to EXP on this basis can be misleading. A more appropriate comparison is provided by elasticities of substitution, shown in Table IV, Section VI. Partial price elasticities (E_{ij}) of input demand can also be used. They are defined:

$$E_{ii} = (A_{ii}/S_i) + S_i - 1, \qquad (i = j)$$

$$E_{ij} = (A_{ij}/S_j) + S_j \qquad (i \neq j)$$

or

$$E_{ij} = S_j * \sigma_{ij} \quad \text{for all } i, j$$

E_{ii} is the i^{th} input's own price elasticity and E_{ij} is cross-price elasticity for input x_i due to changes in j^{th} input price. E_{ii} and E_{ij} are measured for given output quantities. In this respect the measured elasticities are only partial because they do not include output effects --- which ordinarily would result from changes in output prices occasioned by changing the j^{th} input price. Input price increases increase production costs which adversely affect quantity demanded if output prices reflect higher production costs. Thus, partial elasticities underestimate input demand consequences, since an input's quantity demanded is ordinarily further reduced by feedback effects from lower output demand that follows from input price increases.

The E_{ii} estimates are expected to be negative, indicating downward sloping input demands. Estimates of Eij's ($i \neq j$) may be either negative or positive depending upon whether there is complementarity or substitutability relation between input pairs in response to relative price changes. The following tables present our estimates of partial elasticities of substitution. Standard errors (not shown) are simply equal to standard errors reported for σ_{ij} multiplied by share S_j.

OWN-PRICE PARTIAL SUBSTITUTION ELASTICITIES (Eii)

Inputs i,j	OPP E_{ij}	EXP E_{ij}
1,1	-1.0345	-.65030
2,2	-1.0377	-.63879
3,3	-.0196	-.05959
4,4	-.2882	-.26800
5,5	-.7289	-.74914
6,6	-1.2134	-1.25131
7,7	-.5256	-.33395

CROSS-PRICE PARTIAL SUBSTITUTION ELASTICITIES (E_{ij})

FOR OWN WATER (x1) AND PURCHASED WATER (x2) INPUTS

Inputs i,j	OPP E_{ij}	EXP E_{ij}
1,2	.95424	.42788
1,3	.04302	.02229
1,4	.24343	.26120
1,5	-.12216	.00296
1,6	-.13887	.28579
1,7	.17492	.00071
2,1	.82980	.41760
2,3	-.04729	.00020
2,4	.02440	.16578
2,5	.54544	.43842
2,6	.31924	.23655
2,7	-.15557	-.05801
3,1	.01966	.01151
4,1	.04012	.04805
5,1	-.03170	.00086
6,1	-.07495	.17112
7,1	.25152	.00115
3,2	-.02484	.00011
4,2	.00462	.03125
5,2	.16276	.13075
6,2	.19813	.14512
7,2	-.25725	-.09632

APPENDIX C

TABLE I_a

STATISTICAL TESTS OF FUNCTION PROPERTIES EXP MODEL

	No LHIP[1]	Maintained Hypothesis[2]	Input Separability[3]	Treatment Variable[4]
Restrictions on parameters	None	$\Sigma_i A_{ij} = 0$	$A_{ij} = 0$	At, Att, Ati
		$\Sigma_i D_{ki} = 0$	$i = 1,2;$	$Dtk = 0$
		$\Sigma_i A_{ti} = 0$	$j = 3$ to 7	for all i,k
Number of Constraints[5]	None	11.0	10.0	11.0
Chi Square Statistic	NA	18.0	41.2	48.9
Critical Level (99%)	NA	24.7	23.2	24.7
Log-Likelihood Function	1074.21	1065.23	1044.64	1041.28

[1] No LHIP means that linear homogeneity of input prices is not imposed.

[2] Maintained Hypothesis means: modified translog EXP model with Box-Cox transformed outputs, input price homogeneity, treatment variable and symmetry. Coefficient-constrained models examined in Table II also satisfy maintained hypothesis properties.

[3] Strong separability is tested, water inputs ($i = 1,2$) from other inputs ($j = 3,7$).

[4] The treatment variable interacts with all the other independent variables; one of the price interactions is not independent because of the maintained hypothesis of price homogeneity.

[5] This is the number of independent constraints imposed vis-a-vis the Maintained Hypothesis, except for the NO LHIP test against our maintained hypothesis.

TABLE II$_a$

PRODUCTION FUNCTION RESTRICTIONS AND STATISTICAL TESTS EXP MODEL

	Maintained Hypothesis[1]	Homothe-ticity	Homogen-eity[2]	UES[3]
Restrictions on Parameters	None	all $D_{ki} = 0$	$D_{ki} = 0$ $B_{kl} = 0$ all i,k,l	all $A_{ij} = 0$
Number of Constraints[4]	None	18	24	21
Chi Square Statistic	NA	94.0	300.3	84.2
Critical Level (99%)	NA	34.8	43.0	38.9
Log-Likelihood Function	1065.23	1018.19	915.08	1023.61

[1] See Table I for Maintained Hypothesis. Constrained models satisfy price homogeneity and include the Treatment variable.

[2] Output homogeneity with degree not restricted. For constant returns (linear homogeneity) an added constraint, $\Sigma_k B_k = 1$, must also be imposed. Our log-likelihood with the added constraint is 900.67; constant returns is rejected at a 99.9% significance level.

[3] Unitary Elasticity of Substitution.

4 Number of independent constraints

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