SCHEDULING INPUTS WITH PRODUCTION FUNCTIONS: OPTIMAL NITROGEN PROGRAMS FOR RICE

Ronald C. Griffin, M. Edward Rister, John M. Montgomery, and Fred T. Turner

Abstract

The problem of scheduling input applications can be examined by extending conventional production function analysis. Using appropriately designed agricultural experiments, it is possible to estimate production function parameters with alternative specifications for input timing (and amount). A study of nitrogen applications to rice is employed to illustrate scheduling via production functions. Alternative specifications and functional forms are simultaneously examined to determine the sensitivity of economic results to these factors. Sensitivity is found to be high, and this finding is hypothesized to be critical for other approaches to input scheduling as well.

Key words: scheduling, production functions, simulation, nitrogen, rice.

While agricultural research is prone to focusing on aggregate input levels (e.g., water, fertilizer), the timing or pattern of input applications can also influence yield (Dillon, p. 65). Where application timing is important, producers must resolve a management strategy for the number of applications as well as the level and timing of each application. Musser and Tew have discussed biophysical simulation as a means of studying such scheduling problems. While there are several advantages to such an approach (Musser and Tew), weaknesses include large information requirements and, in general, poorly validated biological and physical relationships. It is therefore appropriate to examine the extension of traditional static tools to such problems. Such research can investigate input scheduling as well as provide results having implications for biophysical simulation. As with other production research, the choice sets for variable specification and functional form are extensive.

This issue is addressed by examining economically optimal rates and timings of nitrogen fertilizer applications to rice. Application of nitrogen at different growth stages of the rice plant affects the yield response to that input, and higher yields may be obtained by selected multiple applications (Evatt; Matsushima; Brandon et al.). Moreover, the rate at which fertilizer is applied determines not only the cost of fertilizer but the cost of its application as well. Application costs can be quite substantial because nitrogen is often applied aerially due to flooded field conditions. Substantial agronomic research has been conducted on this topic (De Datta; Evatt and Hodges; Mikkelson and De Datta; Yoshida), but limited economic analyses have been reported.

Two conceptual models are constructed to account for multiple applications of variable fertilizer rates. Two functional forms are employed for estimating each model; thus, four production functions are developed. Profit is maximized for each of the production relationships, and the four sets of results are then compared.

PRODUCTION DATA

Experiments designed to provide information concerning the relationship between rice yields and sequencing of multiple nitrogen fertilizer applications were conducted with the Labelle variety by researchers at the Texas Agricultural Experiment Station from 1976 through 1979. Twenty-one experi-

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ments were conducted at two locations on the same soil type. Four different total nitrogen fertilization rates were investigated: 60, 90, 120, and 150 pounds per acre. Each rate was applied in five different timings, or sequences of applications, at various phenological-cultural stages. The four different cultural stages considered, in temporal order, were preplant (PP), early post-emergence (EP), post-flood (PF), and panicle differentiation (PD). The five timings tested in the experiment were the following: PP, PP-PF, PP-PD, PP-PF-PD, and EP-PF-PD.

Total nitrogen in timings comprising two applications was split between an initial application of 60 percent and a subsequent application of 40 percent. Total nitrogen in three-application timings was divided into an initial application of 40 percent and two subsequent applications of 30 percent each. Each treatment was replicated four times per experiment at site A, resulting in 80 observations per trial, and five times per experiment at site B, resulting in 100 observations per trial. There are a total of 1,920 observations over the 4-year period.

Few experiments were planted on the same date, and the weather continuum following each date was, of course, not constant. Weather data were not collected separately at the experimental plots but at a single national weather service station at site B, which is approximately 30 miles from site A.

PRODUCTION VARIABLES

Production functions are constructed to account for the effects of variable rates of nitrogen applied at different times and for the effects of variable weather. Two specifications are considered to model multiple applications of variable nitrogen rates. The purpose of investigating two alternative specifications is to explore the sensitivity of economic results to the choice of specification, since there is no *a priori* reason for preferring one over the other.

Nitrogen Variables

Both the total amount of nitrogen applied during each sequence and the amount applied at each individual time were varied in the experiment. The proportion of total nitrogen applied at any one time, however, was fixed for a given number of applications per timing. Therefore, one model, referred to as the Total-N model, specifies four fertilizer variables: (1) TN - total nitrogen applied in the sequence (measured in pounds per acre), (2) AP - number of nitrogen applications in the sequence, (3) D2 - a dummy variable differentiating between two-way splits (taking on a value of one if the sequence were PP-PD and zero otherwise), and (4) D3 - a dummy variable differentiating between three-way splits (taking on a value of one if the sequence were EP-PF-PD and zero otherwise). While the experiment's five timing sequences could have been distinguished by a suitable set of dummy variables (as in Swanson et al.), the nature of the five examined sequences makes it desirable to economize on the number of dummy variables by including a variable for the number of applications.

A second specification, the Split-N model, does not require the characteristic of fixed proportions. One variable is specified for each of the cultural stages at which nitrogen could be applied: (1) PP - preplant, (2) EP - early post-emergence, (3) PF - post-flood, and (4) PD - panicle differentiation. Measurements for these variables are the pounds of nitrogen applied per acre at each stage. Thus, both models specify yield as a function of four nitrogen variables.

Weather Variables

Previous investigations of crop response to fertilization have demonstrated the importance of considering the impacts of weather (de Janvry; Roumasset; Ryan and Perrin). The two major dimensions of weather affecting the yield of irrigated rice are temperature and solar radiation (Yoshida, p. 94). As with nitrogen, the effect of weather factors on yield varies with the stage of plant growth. Average daily temperatures (AVT) are summed for the period beginning at emergence and ending at panicle differentiation to form one temperature variable. Researchers conducting the experiments also hypothesize grain ripening is impaired by daily maximum temperatures above 95°F and by daily minimum temperatures above 72°F. Thus, two additional variables related to temperature during maturation are: MXT - the number of days for which the temperature reached a maximum above 95°F during the 21 days following first heading; and MNT - the number of days with a temperature minimum above 72°F during this same period.

Unfortunately, a complete series of local measurements on solar radiation is not available. Therefore, this factor is not taken directly into account by the models. It is possible, however, to include DTE, emergence date (in Julian days), which is generally correlated with solar radiation (and temperature). Obviously, if weather could be completely accounted for, such a date would constitute a superfluous variable. Yet, in comparing one season to the next, emergence data can act as a proxy variable for all weather variables taken together. The appeal of this variable is enhanced by the fact that emergence data can be envisioned as a decision variable (because producers control planting date) while solar radiation is clearly exogeneous.

Therefore, two groups of variables are hypothesized to be functionally related to yield. The following Split-N model is considered:

$$(1)\ y_S = f(PP,EP,PF,PD;\ AVT,MXT,MNT,DTE),$$

where PP is the pounds of nitrogen applied per acre preplant, EP is the rate applied early post-emergence, PF is the rate applied post-flood, PD is the rate applied at panicle differentiation, AVT is the aggregate of daily average temperatures from emergence to panicle differentiation, MXT is the number of days during the 21-day period following heading with maximum temperatures above 95°F, MNT is the number of days during the same period with minimum temperatures above 72°F, and DTE is the Julian date (excluding year) of emergence. Similarly, the following Total-N model is considered:

$$(2)\ y_T = g(TN,AP,D2,D3;\ AVT,MXT,MNT,DTE),$$

where TN is total pounds of nitrogen applied per acre in the sequence, AP is the number of individual applications, D2 is a dummy variable taking on a value of one if the sequence is PP-PD and zero otherwise, and D3 is a dummy taking on a value of one if the sequence is EP-PF-PD and zero otherwise.

FUNCTIONAL FORM

Several criteria should be considered when selecting functional form. Due to the character of the problem being investigated and the data obtained, it is preferable to use functional forms which allow zero input levels and do not impose a zero output level given a zero level of some input. Also, it is desirable to allow marginal products to move from a region of positive values to a region of negative values and to not impose any restrictions on concavity. Finally, concern shall be limited to functional forms which are linear-in-parameters so that inexpensive estimation by least squares regression is possible.

A number of functional forms is considered as possible representations of the unknown "true" relationship between variables in the two production process models already given. Of the functional forms considered, only the quadratic and the square root forms are judged appropriate for estimation.[1] Formally, the quadratic functional form is:

$$(3)\ y = \alpha_0 + \sum_i \alpha_i x_i + \sum_i \sum_j \beta_{ij} x_i x_j,$$

and the square root functional form is:

$$(4)\ y = \alpha_0 + \sum_i \alpha_i x_i^{1/2} + \sum_i \sum_j \beta_{ij} x_i^{1/2} x_j^{1/2},$$

where $\beta_{ij} = \beta_{ji}$ for all i, j. Of course, it is not possible to determine which of these two functions more closely approximates the true functional relationship between output and the eight input variables of the Total-N and Split-N models, since the true relationship is unknown.

RESOLUTION OF FINAL MODELS

Both of the chosen functional forms include cross products between all possible pairs of variables. Some of these cross products are always zero (e.g., PP•EP) and are omitted from the four models. Because of the 60-40 and 40-30-30 splits of total nitrogen employed in the experiments, perfect collinearity is present in the Split-N specifications and is accommodated by eliminating

[1] The following other forms were also considered: linear, cubic, logarithmic, Mitscherlich-Spillman, Cobb-Douglas, transcendental, resistance, modified resistance, CES, generalized Leontief, translog, and generalized quadratic. These forms were rejected from consideration because they did not possess one or more of the desired characteristics (Griffin et al.).

particular terms. For example, the following relationship can be derived:[2]

$$(5)\ PP \bullet PF - PP \bullet PD = 3(PF^2 - PD^2)/2.$$

Instances of very high simple correlation (> 0.9) are addressed by deleting one term.

The following *ad hoc* methodology is adopted for reducing the number of parameters in each model. Each full model is estimated using least squares, and eight auxiliary (partial) regressions are obtained by omitting all terms containing a particular variable. Resulting R^2's, are then used to calculate *F* statistics.[3] Variables which are not significantly different from zero at the 25 percent level are omitted. As a result, D3 does not appear in any of the final models. Remaining individual parameter estimates which do not provide a chosen level of significance of 15 percent are also omitted. In the case of both *F* and *t* tests, the chosen levels of significance are not arbitrary. These are the most restrictive levels possible without rejecting a large number of weather-related variables which, on the basis of prior information, are judged to be important elements of the models. The final models are presented in tables 1 and 2.[4]

The issue of pretesting implies that this procedure is less than satisfactory because the estimated standard errors of parameter estimates are unreliable (Wallace; Ziemer). Therefore, the use of *t* statistics is questionable, and, as reported by Debertin and Freund, tests of significance are likely to be "less wrong" if the degrees of freedom associated with the original, full model are used in all subsequent tests. While the large dataset used implies that the latter point is of no consequence, pretesting is still an issue. On the other hand, presentation and application of the full 44-term models is not practical, so some means must be chosen for reducing the scale of the models without great sacrifice of information. Even though *t* statistics are invalid, they do convey some information. Rather than reject this information completely, it is decided to overestimate the value of the *t* statistic information by explicitly assuming that *t* statistics are reliable.

ECONOMIC MODEL

Profit maximization is the assumed objective with profit defined as total revenue less total costs:

$$(6)\ \pi = p \bullet y - TC.$$

The price per pound of rice, p, is assumed to be constant. Per-acre yield, y, is assumed to be given by one of the four production models. Constraints are added to the profit maximization problem to force all solutions to conform to the experimental design. Thus, for the Total-N specifications, AP must equal 1, 2, or 3, and D2 is restricted to be 0 or 1. Within the Split-N formulations, the fixed percentage splits of total nitrogen are fixed at 60-40 or 40-30-30 for the appropriate sequences.

Cost factors can be usefully separated into three categories. Total cost is the sum of all non-nitrogen fixed and variable costs (K), the cost of nitrogen material (M), and nitrogen application costs (A):

$$(7)\ TC = K + M + A.$$

K is independent of any nitrogen level or timing variables and is, therefore, irrelevant to the determination of optimal nitrogen programs. This variable is contained in the total cost equation for the sake of completeness only.

In general, material and application costs are dependent upon some or all of the nitrogen decision variables. Therefore, M and A must be expressed as explicit functions of the control variables. Specification of these cost functions is somewhat complicated because there must be Split-N and Total-N formulations of material and application cost schedules.

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[2] Proof: Recall that the experiment tested five different timings. Note that either PP=3(PF + PD)/2 in the case of both 2 application timings or PF=PD for the single 1 application timing and both 3 application timings. Therefore, PP(PF − PD) = 3(PF + PD)(PF − PD)/2.

of both 2 application timings or PF=PD for the single 1 application timing and both 3 application timings. R signifies the reduced model, and F indicates the full model.

[4] While the sign of the intercepts for these models are largely irrelevant because a zero nitrogen level is outside of the sample (60-150 pounds), some readers may be disturbed by the large, negative intercepts. If average weather conditions are assumed and substituted into each of the four models, the new intercepts range from 1,802 to 3,420 pounds of rice per acre. These values are entirely reasonable.

TABLE 1. SPLIT-N PRODUCTION FUNCTION MODELS FOR RICE, CROWLEY SOIL LOCATIONS, TEXAS, 1976-79

Model Type					
Quadratic			Square root		
Variable	Parameter estimate	t value	Variable	Parameter estimate	t value
Intercept	−27,503.00	−11.43	Intercept	−3,538.64	−4.76
PP	10.01	2.22	PP	−16.76	−2.65
EP	16.72	3.10	EP	33.69	1.28
PF	13.69	1.89	PF	−101.85	−2.69
PD	17.15	2.78	PD	−11.70	−1.63
AVT	3.36	7.17	AVT	−4.03	−8.10
MXT	−122.30	−5.93	MXT	−551.31	−13.86
MNT	896.82	16.90	MNT	1,442.47	19.53
DTE	336.01	11.48	DTE	−50.55	−2.42
PP • PF	0.58	2.10	$PP^{1/2}$ $PF^{1/2}$	57.44	2.06
PP • MXT	−1.19	−6.00	$PP^{1/2}$ $MXT^{1/2}$	−40.00	−5.70
PP • DTE	0.06	1.63	$EP^{1/2}$ $MXT^{1/2}$	−48.79	−5.05
EP • MXT	−1.24	−3.66	$PF^{1/2}$ $MXT^{1/2}$	−19.87	−4.02
PF • MXT	−1.60	−4.44	$AVT^{1/2}$ $DTE^{1/2}$	69.79	11.82
PF • DTE	0.13	1.93	$MXT^{1/2}$ $MNT^{1/2}$	315.73	10.76
PD • DTE	−0.08	−1.71	$MNT^{1/2}$ $DTE^{1/2}$	−992.53	−19.31
AVT • DTE	−0.02	−4.22	$PP^{1/2}$	447.24	3.62
MNT • DTE	−8.50	−17.61	$EP^{1/2}$	454.73	3.49
PP^2	−0.04	−2.50	$PF^{1/2}$	264.03	5.08
EP^2	0.37	1.51	$PD^{1/2}$	105.56	2.04
PF^2	−1.25	−2.73			
PD^2	−0.13	−1.61			
DTE^2	−0.84	−11.03			
R^2 = 0.5231			R^2 = 0.5282		
F-ratio = 94.58			F-ratio = 111.94		
n = 1,920			n = 1,920		

TABLE 2. TOTAL-N PRODUCTION FUNCTION MODELS FOR RICE, CROWLEY SOIL LOCATIONS, TEXAS, 1976-79

Model Type					
Quadratic			Square root		
Variable	Parameter estimate	t value	Variable	Parameter estimate	t value
Intercept	−30,332.44	−12.60	Intercept	−21,037.47	−2.94
TN	36.81	8.21	TN	−19.94	−3.30
AP	188.36	2.07	D2	1,243.85	2.30
D2	−424.06	−1.59	AVT	−3.25	−2.93
AVT	3.36	7.20	MXT	−530.58	−12.67
MXT	−243.30	−17.86	MNT	−1,524.48	−19.27
MNT	1,026.14	18.02	DTE	−136.40	−3.49
DTE	352.28	12.10	$TN^{1/2}$ $AP^{1/2}$	−52.69	−1.75
TN • AP	−0.87	−1.59	$TN^{1/2}$ $MNT^{1/2}$	−113.29	−5.81
TN • MNT	−1.03	−6.98	$TN^{1/2}$ $DTE^{1/2}$	−46.86	−4.67
TN • DTE	−0.10	−4.23	$D2^{1/2}$ $MXT^{1/2}$	226.05	4.87
AP • MNT	−9.55	−1.44	$D2^{1/2}$ $DTE^{1/2}$	−154.04	−2.63
D2 • MXT	60.44	3.68	$AVT^{1/2}$ $DTE^{1/2}$	60.92	4.50
D2 • DTE	−6.15	−2.03	$MXT^{1/2}$ $MNT^{1/2}$	173.04	6.92
AVT • DTE	−0.02	−4.24	$MNT^{1/2}$ $DTE^{1/2}$	−916.37	−16.75
MNT • DTE	−8.50	−17.69	$TN^{1/2}$	1,34 2.40	7.62
TN^2	−0.06	−3.69	$AP^{1/2}$	533.14	1.73
DTE^2	−0.84	−11.08	$DTE^{1/2}$	2,504.18	1.79
R^2 = 0.5262			R^2 = 0.5345		
F-ratio = 124.24			F-ratio = 128.49		
n = 1,920			n = 1,920		

Material Costs

Material costs are determined by obtaining the total amount of nitrogen used in a given fertilization program and then multiplying by the per unit price of nitrogen. In general, this price is dependent on the chosen material. Since urea was the prevalent nitrogen source for the experiment and is also an economical and popular choice among producers, the per unit nitrogen cost (w) used in tnis study approximates typical nitrogen costs from urea. The following two functions describe the cost of nitrogen material for the Split-N and Total-N specifications, respectively:

(8) $M_S = w \bullet (PP + EP + PF + PD)$

and

(9) $M_T = w \bullet TN.$

Application Costs

Application costs for fertilization activities are sometimes neglected in economic research because these costs are assumed to be independent of fertilizer quantities. If this assumption is correct, application costs are pertinent to overall farm profitability but do not influence optimal fertilization programs. Certain conditions which are important to rice production, however, require the consideration of application costs in devising economically efficient programs of fertilization.

First, the prevalence of aerial application infers that application costs can represent a substantial portion of fertilization costs. Second, the producer can choose to apply nitrogen to rice acreage at more than one time during the cropping season, with an added expense for each application. Third, typical rate structures used by aerial applicators incorporate additional charges for heavier per acre nitrogen fertilization rates (Montgomery and Parker). These concerns are important to the proper determination of optimal nitrogen programs.

A typical aerial application rate schedule incorporates a fixed charge of \$3.00 per acre for each application and an additional charge for each unit of material exceeding 100 pounds (per acre). On the average, this additional charge amounts to \$0.025 per pound, and it should be noted that the 100-pound critical value pertains to material rather than actual nitrogen.

The Split-N statement of application costs is given immediately. Here, the rate schedule can be easily represented by a piecewise linear function:

$$(10)\ A_S(PP,EP,PF,PD) = \sum_{i=1}^{4} APC_i,$$

$$\text{where } APC_i = \begin{cases} \$0 & \text{if } z_i = 0 \\ \$3 & \text{if } 0 < z_i \leq 45 \\ \$3 + .025(z_i - 45) & \text{if } z_i \geq 45 \end{cases}$$

and z_i = PP, EP, PF, or PD. Note that A_S depends only on PP, EP, PF, and PD, as required. The critical value of 45 (pounds) is associated with the assumed use of urea, which is 45 percent nitrogen by weight.

To construct a Total-N statement of application costs, implicit use is made of the 60-40 split of total nitrogen (TN) in all two-application timings and of the 40-30-30 division in all three-application timings. The A_T function is as follows:

$$(11)\ A_T(TN,AP) = \begin{cases} \$3 & \text{if } TN \leq 45 & \text{and } AP=1 \\ \$3 + .025(TN-45) & \text{if } TN \geq 45 & \text{and } AP=1 \\ \$6 & \text{if } TN \leq 75 & \text{and } AP=2 \\ \$6 + .025(.6TN-45) & \text{if } 75 \leq TN \leq 112.5 & \text{and } AP=2 \\ \$6 + .025(TN-90) & \text{if } TN \geq 112.5 & \text{and } AP=2 \\ \$9 & \text{if } TN \leq 112.5 & \text{and } AP=3 \\ \$9 + .025(.4TN-45) & \text{if } 112.5 \leq TN \leq 150 & \text{and } AP=3 \\ \$9 + .025(TN-135) & \text{if } TN \geq 150 & \text{and } AP=3 \end{cases}$$

On the basis of these formulations, there are four separate profit functions to be optimized with respect to nitrogen inputs. These include quadratic and square root forms for the following two representations of producer profit:

$$(12)\ \pi_S = p \bullet f(PP,EP,PF,PD; AVT, MXT,MNT,DTE) - w \bullet (PP+EP+PF+PD) - A_S(PP,EP,PF,PD) - K$$

and

$$(13)\ \pi_T = p \bullet g(TN,AP,D2; AVT, MXT,MNT,DTE) - w \bullet TN - A_T(TN,AP) - K,$$

where π_S and π_T are profits associated with the Split−N and Total−N models, respectively. Following some choice of p and w, as well as some choice of the exogenous climatic and emergence variables, (i.e., AVT, MXT, MNT, and DTE), all of these functions are maximized. Because non-nitrogen costs are irrelevant to optimal scheduling, K is assumed to be zero.

OPTIMIZATION TECHNIQUE

For each Split-N formulation (quadratic and square root), the following procedure is adopted. Prices chosen for rice and nitrogen are, respectively, \$0.09 and \$0.25 per pound. Average values of the weather variables are computed for the 4-year experimental period and are substituted into the appropriate profit function along with prices. Constrained optimization by the Lagrangian method is separately conducted for each of the five sequences, because appropriate constraints vary among the timings. First-order conditions are computed and a solution to this system is determined. Each solution indicates an optimal choice of PP, EP, PF, and PD corresponding to a particular timing (e.g., PP-PF-PD). Comparison of optimal profit for the five different timings identifies the optimal application sequence for each model.

The procedure is largely the same for the two Total-N profit functions except that the Langrangian method is unnecessary. The same economic conditions and weather scenarios are employed, and the analysis is performed for each timing. But, in this case the optimization problem is further simplified because the choice of timing fixes AP and D2. Only one decision variable remains: TN. This one-dimensional optimization problem is solved for each timing by finding the single first-order equation for TN. Separate results for each timing are then compared to identify the preferred strategy.

As a computational matter, it should be observed that the systems of first-order equations for the quadratic models are linear. This greatly simplifies the simultaneous solution of the system. However, the square root models yield nonlinear systems of first-order conditions. This is primarily a concern for the seven-dimensional (four decision variables plus three constraints) square root Split-N optimization problem. A numerical algorithm using a Newton gradient improvement method[5] is employed to obtain a solution for this system.

With regard to second-order conditions, the production functions must be quasi-concave in the decision variables to guarantee that a local profit maximum has been determined. This condition is obviously satisfied for the quadratic Total-N model because the coefficient of TN^2 is negative. The quadratic Split-N model production function is quasi-concave in PP, EP, PF, and PD[6]. The second derivative of the square root Total-N production function with respect to TN is negative so this function is also quasi-concave. For the square root Split-N model, negative semidefiniteness must be verified for the bordered Hessian. This is the case in the neighborhood of the established extremals.

RESULTS

Economically optimal levels of total nitrogen for each timing, as well as associated profit, are summarized for all models in Table 3. Although the two Split-N models do not rank timings in exactly the same order of profitability nor identify identical optimal profit levels, results are quite similar. From most profitable to least, the five timings are

TABLE 3. MAXIMUM PROFIT PER NITROGEN TIMING FOR RICE PRODUCTION, BY MODEL TYPE, TEXAS, 1976-79

Model Type					
Quadratic			Square root		
Timing (ranked)	Profit[a] ($/acre)	Nitrogen (lb/acre)	Timing (ranked)	Profit[a] ($/acre)	Nitrogen (lb/acre)
Split-N					
PP-PF	413.68	89.54	PP-PF	410.98	76.89
EP-PF-PD	409.14	98.50	PP-PF-PD	406.58	84.44
PP	408.48	109.15	PP	406.40	93.88
PP-PF-PD	406.70	98.46	EP-PF-PD	406.37	85.26
PP-PD	403.06	112.86	PP-PD	404.35	106.18
Total-N					
PP	409.91	106.15	PP-PD	413.75	94.53
PP-PF	409.60	100.70	PP	408.95	100.93
PP-PF-PD or EP-PF-PD	407.45	96.38	PP-PF	408.85	94.53
PP-PD	405.78	100.70	PP-PF-PD or EP-PF-PD	406.94	88.89

[a]Total revenue less nitrogen material and application costs.

[5] This is an iterative procedure for determining local optima using the following rule: $\underline{x}^1 = \underline{x}^0 - k[(D^2(\underline{x}^0))^{-1} D(\underline{x}^0)]$, where $\underline{x}^1$ is the new trial solution, $\underline{x}^0$ is the initial guess solution or the trial solution from the previous iteration, k is the chosen step size, $D^2(\underline{x}^0)$ is the matrix of second derivatives evaluated at $\underline{x}^0$, and $D(\underline{x}^0)$ is the vector of first derivatives evaluated at $\underline{x}^0$. This equation is repeatedly solved until $\| \underline{x}^1 - \underline{x}^0 \|$ becomes arbitrarily small.

[6] The sign of the coefficient of EP^2 is initially disturbing until it is recognized that $EP > 0$ implies $.75EP = PF = PD$. Substitution of this information into the production function reveals concavity in EP.

ranked PP-PF, EP-PF-PD, PP, PP-PF-PD, and PP-PD for the economic model using the quadratic production function. Only the order of the three-application timings is reversed for the model using the square root production function. This is not surprising given the insignificance of the D3 dummy variable within both Total-N specifications. Timing-specific profit maxima fall within an \$11 per acre range for the quadratic model and a \$7 range for the square root model. Maximization of the quadratic profit model indicates that the highest level of profit (\$413.68 per acre) is achievable through application of 89.54 pounds of nitrogen per acre in a split of 53.72 pounds preplant and 35.82 pounds post-flood. The square root formulation indicates that the highest level of profit (\$410.98 per acre) is achievable through application of only 76.89 pounds of nitrogen—46.13 pounds preplant and 30.76 pounds post-flood.

The Total-N models show a much more disparate ranking of timings by profitability, but there appears to be no more differences in optimal profit levels than indicated by the Split-N models. The five timings are ranked, from most profitable to least, PP, PP-PF, PP-PF-PD or EP-PF-PD, and PP-PD for the quadratic production function, and PP-PD, PP, PP-PF, and PP-PF-PD or EP-PF-PD for the model using the square root production function. (Recall that no difference in yield was found between the three-application timings and since application costs are identical for the two, profit levels are also identical.) The most striking difference between these two orderings is that the PP-PD timing optimum is ranked last among the quadratic optima, but first among the square root optima. Compared to profit maxima of timings in the Split-N specification, those of timings in the Total-N occur across a narrower range (\$5) for the quadratic model than for the square root model (\$7). The largest optimum in the Total-N model (\$409.91) is achieved by application of 106.15 pounds of nitrogen preplant; whereas, the largest optimum in the Total-N model (\$413.75) is achieved with 94.53 pounds of nitrogen per acre—56.72 pounds preplant and 37.81 pounds at panicle differentiation.

CONCLUSIONS AND IMPLICATIONS

The results reported in this paper suggest that, while seeking to contribute to multidisciplinary programs, agricultural economists should exercise caution in recommending particular models, functional forms, etc. Specification of models, choice of functional form, and even format of the optimization procedure are not independent of the experimental design. Fixed splits of total applied nitrogen among the individual times make possible the Total-N specification. Without such fixed percentages, variable rates for individual applications cannot be identified by this specification nor can realistic application cost functions for the Total-N models be defined. Yet, it must be recognized that a limited number of splits restricts the analysis. It mandates constrained optimization of the Split-N models, increasing the complexity of solving first-order conditions.

The analysis indicates that determination of economically optimal levels of nitrogen for specific application sequences is sensitive to both model specification and functional form. The selection of an optimal timing is also sensitive to these factors. The rankings of maximum profit for each timing are of such diversity that no particular timing appears among the top two or bottom two optima in all four models. The small ranges of maximum profit levels between timings within each of the four models suggest the need to explore the statistical significance of the economic results.

The production function approach to scheduling inputs is restrictive in the sense that difficulties in obtaining statistically significant parameters and results will limit the number of alternative application timings that can be examined. Thus, this can be a very "discrete" method of scheduling when compared to the biophysical simulation alternative. Actually, the discreteness of the production function approach to scheduling is due to the model's adherence to experimental design. Because the method relies on statistical estimation using experimental data, scheduling is limited by design. The physical relationships which are typically embedded in dynamic models rarely are developed from actual estimation but are born of informal procedures. This is how fully dynamic methodologies escape the confines of experimental design—there is none—to achieve a more finely detailed model (with unknown validity).

Clearly, the loss of significant economic results in the present analysis may be due, in part, to the choice of timings for the experiment; these timings may not have been "optimal," and the production function approach to scheduling is sensitive to experimental design in this way. The lack of any definitive results using different specifications and functional forms may imply, however, that the choice of form and specification is also very crucial to all dynamic approaches to scheduling and that sensitivity in this respect should always be explored.

This paper addresses several issues pertinent to on-going agricultural research on production inputs. Agricultural economists must often develop economic decisionmaking rules on the basis of available experimental data. Economic interpretations of such data may be limited, due to either experimental design, unavailability of data regarding both controllable and uncontrollable variables (which vary during the experimental period), and/or modelling approach. While this paper does not explicitly address the first two sources of limitations, it does demonstrate the restricted type of information which agricultural economists must frequently work with when selecting a modeling approach. The reported levels of R^2 suggest one or more important relationships either are yet to be identified or that such relationships were unmonitored. As such, these results should not be viewed as recommendations to producers but rather as a building block complementary to the research programs of agronomic scientists.

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