PRODUCTION THEORY AND COMMUNITY SERVICES PLANNING: APPLICATION TO SOLID WASTE DISPOSAL*

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INTRODUCTION

A basic management responsibility in community service planning is to evaluate alternative methods of providing various public goods and services such as transportation services, sewage and solid waste disposal, and water for home and industry. These alternative methods frequently involve new or different technologies and various combinations of inputs such as capital and labor.

For example, in the disposal of solid waste, the use of different sizes and types of bulldozers, compactors and cranes may lead to significantly different combinations of capital and labor resources.[1] For accurate analysis, the quality and quantity of the service that can be provided with limited amounts of the various resources or inputs must be considered. Thus, the basic concepts that have been used in private business to allocate limited resources to obtain the desired output are equally applicable to the management and planning of community services.

The purpose of this discussion is to develop and relate the concepts of production economics to the provision of community services. The application of these concepts to the allocation of resources in solid waste disposal will provide an empirical example of the type of analysis required. These concepts should be useful to community planners and public works managers in making and justifying resource allocation decisions.

CONCEPTS OF A PRODUCTION PROCESS

In any production process where resources are combined to obtain a final product, both fixed and variable inputs are typically involved. In the disposal of solid waste, the number of men can be varied over time as can the type of equipment used in the disposal process. However, the disposal site is fixed in nature. In equation form, the theoretical production function can be expressed as:

$$(1) \quad Y = f(X_1, X_2 \mid X_i) \; i = 3, 4, \ldots n$$

where Y denotes the level of output, X_1 and X_2 denote the levels of the variable inputs, and X_i denotes the level of the fixed inputs which are held constant during the production period. From this production relationship the responsiveness of output to changes in the utilization of various inputs as measured by the elasticity of production, as well as the returns to scale as all inputs are increased simultaneously, can be determined in the conventional manner.[2]

Similar levels of production can be obtained with different combinations of the variable resources if there is substitutability between the inputs. The problem then arises as to what is the most efficient combination of inputs to use in the production of a specific level of output. Without a price for the output, as is frequently the case in the production of public services, the input combination which will

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[1] It is not the purpose of this paper to evaluate alternative public works technologies or types of capital equipment. Rather, it is our purpose to show how optimum mixes of resources, such as labor and capital, change for different costs (prices) of labor and capital services and for different sizes of public works projects.

[2] See [8, ch.8] for a discussion and derivation of the elasticity of production and returns to scale.

produce the desired output at a minimum cost is preferable. That is, we wish to minimize the cost function:

$$(2) \quad C = P_{X_1} X_1^* + P_{X_2} X_2^* + \text{fixed costs}$$

subject to the production function:

$$(3) \quad Y^* = f(X_1^*, X_2^* | X_i)$$

where P_{X_1} and P_{X_2} denote the prices or input cost of X_1 and X_2 and X_1^* and X_2^* are the quantities of the inputs that will minimize the cost of producing the desired level of output (Y^*). This minimum cost combination of inputs will occur when the marginal product (added output for each unit of input) generated for each dollar of cost outlay is equal for both variable inputs.

A UNIT OF COMMUNITY SERVICE OUTPUT

Numerous problems are encountered in the estimation of production functions for community services, but one of the most difficult is the specification and measurement of output.[3] Output is usually measured in terms of the number of basic output units of specified quality characteristics per unit of time. Presumably, it would require more resources (inputs) to produce a higher quality of a given amount of output. But few public services have basic output units with reasonable well-defined physical characteristics. Market prices frequently provide a good index of quality differences in output for the private sector; lower prices generally reflect a lower quality of output. However, similar indices of the quality of public works output are less frequently available since competitive market prices do not exist for most public services.

In the case of solid waste disposal services, the most obvious measurement of output is tons of waste disposed per unit of time (daily). Even if the discussion is limited to landfill as the method of disposal, many quality differences exist in landfill operations. Public health regulations have been used to distinguish between sanitary and nonsanitary landfills. Regulations for operation of sanitary landfills include such things as daily covering of waste disposed with a specified minimum cover, appropriate drainage, and varment and vector control. Thus, one quality difference in the disposal of solid waste is the sanitary or nonsanitary characteristic of the landfill.[4]

APPLICATION TO SOLID WASTE DISPOSAL

Data from a survey questionnaire on the disposal of solid wastes by means of landfill are used to illustrate the concepts of production economics in the management of public works.[5] (See Table 1).

Variable Inputs in Solid Waste Disposal

Variable inputs in the production process are (1) annual hours of labor and (2) annual short-term capital expenditures. The original data on labor inputs were in total annual wages paid at each site. Annual hours of labor were derived by dividing the total wages by the average hourly earnings of production and nonsupervisory workers in electric, gas, and sanitary services in California for 1968-1969.[6] This is only a rough indication of physical hours worked at each site since differences in wages of local labor markets are not considered. However, because of the relatively close proximity of the disposal sites, differences in wage rates are expected to be slight.

Variable short-term capital expenditures are defined to include equipment depreciation, maintenance and operating costs. In describing the capital input, "short-term" should be qualified since once management has purchased a durable capital item, such as a bulldozer, it is committed for the expected life of the capital unit. For the present analysis we are primarily interested in the substitutability between labor and capital, and short-term refers to the approximate life of the capital equipment. Alternatively, equipment depreciation may be considered equivalent to rental charges, and the subsequent annual equipment rental and operation costs would be consistent in measurement with the annual hours of labor input.

Other inputs in the production process of waste disposal include land and long-term capital expenditures for site development. For the present

[3] For a more detailed discussion of the problems encountered in estimating and using community service production functions, see [6, ch. 10-12].

[4] It should be noted that this measurement of service quality is not a complete accounting of all other social costs associated with landfill operations. As measurements of other quality dimensions become available, decisions based on social rather than private costs and benefits will be possible.

[5] The data are reported in [4, p. 23]. Data are complete for 29 disposal sites in southern California for the period 1968-1969 and are summarized in Table 1. Output is measured in tons of solid waste disposed yearly at each disposal site.

[6] Average hourly earnings equalled $4.42 in 1968-1969 as reported in *Employment and Earnings, States and Areas, 1939-1971.*

Table 1. SUMMARY OF SURVEY DATA ON SOLID WASTE DISPOSAL LANDFILL SITES*

Location of Disposal Site Name	County	Yearly Waste (tons)	Labor (hours)	Short Term Capital Services (1968-69 dollars)
Palos Verdes	Los Angeles	1,350,000	54,174.7	$339,083
Spadra	Los Angeles	142,500	14,691.6	72,650
Mission Canyon	Los Angeles	1,110,000	51,063.1	634,475
School Canyon	Los Angeles	450,000	33,244.8	250,918
Calabasas	Los Angeles	240,000	18,209.9	114,922
Cannery St.	Orange	139,000	3,965.4	13,361
Coyote Canyon	Orange	1,020,000	40,279.9	321,340
Olinda	Orange	524,000	34,133.7	150.403
Forster Canyon	Orange	111,000	8,313.1	48,850
Santiago Canyon	Orange	40,800	5,980.8	37,814
Bishops Canyon	Los Angeles	537,000	20,135.7	30,000
Tajigous	Santa Barbara	135,000	10,131.7	13,887
Sheldon Arleta	Los Angels	460,000	26,018.1	89,000
Toyon Canyon	Los Angeles	294,000	28,054.3	70,000
Chollas	San Diego	232,600	13,276.5	132,203
Otay	San Diego	155,000	14,162.9	57,600
Miremer	San Diego	229,100	14,716.3	145,064
Arizona	San Diego	81,991	5,008.6	56,791
Jamacha	San Diego	102,000	12,850.7	50,300
Tierra Rejada	Ventura	61,397	7,188.2	37,761
Wagon Wheel	Ventura	95,691	10,718.1	62,349
Windsor	Sonoma	63,000	3,393.7	50,340
Sonoma	Sonoma	34,100	3,393.7	19,025
North Coast	San Diego	68,500	13,122.2	43,000
Sycamore	San Diego	91,700	13,710.4	49,700
San Elijo	San Diego	36,500	9,886.9	26,300
Sonsali	San Diego	31,000	10,633.5	26,400
Soway	San Diego	14,596	7,941.2	14,700
Soblar	Sonoma	14,700	3,393.7	39,685

*Source See [4, p. 23].

analysis, these inputs are considered fixed and independent of the variable inputs of labor and short-term capital services. Information on the quality dimensions of solid waste disposal output was recorded in the questionnaire by means of classifying landfills as sanitary or nonsanitary. The empirical analysis of the impact of this quality dimension on the waste disposal process was inconclusive. Therefore, all further analysis excludes sanitary and nonsanitary quality differences.

Empirical Results of the Production Function

Least squares regression analysis was used to fit a model of the power form to the data. The resulting equation is:

(4) $Y = .32956\,L^{.943}SC^{.366}$; $R^2 = 0.74$

where Y denotes solid waste disposed yearly in tons, L denotes labor in hours per year, and SC denotes short-term capital in dollars per year. The exponent of labor is significant at the 1 percent level, while that of short-term capital is significant at the 10 percent level.

The responsiveness of the production of disposal services to changes in the utilization of the various inputs may be derived from the equation for the elasticity of production. The elasticity of production for labor (E_ℓ) is .943, and for short - term capital (E_{SC}) the elasticity of production is .366. Thus, as labor is increased by 1 percent, the quantity of solid waste that can be disposed will increase by .943 percent; for a similar change in short-term capital the result will be a .366 percentage increase in solid waste disposed.

The sum of the elasticities of production for the variable inputs indicates the nature of the returns to size. In this case

(5) $\sum_{i=1}^{n} E_i = .943 + .366 = 1.309.$

Table 2. **LABOR AND CAPITAL REQUIREMENTS FOR DIFFERENT VOLUMES OF SOLID WASTE DISPOSED BY MEANS OF LANDFILL**

Level of Output (tons per year)		Alternative Input Combinations					
		1	2	3	4	5	6
25,000	labor (hrs.)	2,808	3,628	4,404	4,933	5,346	5,988
	capital ($)	28,082	14,513	8,808	6,577	5,346	3,992
50,000	labor (hrs.)	4,769	6,161	7,479	8,377	9,078	10,168
	capital ($)	47,687	24,645	14,958	11,169	9,078	6,779
75,000	labor (hrs)	6,500	2,398	10,194	11,418	12,374	13,860
	capital ($)	65,002	33,593	20,389	15,224	12,374	9,240
100,000	labor (hrs.)	8,098	10,462	12,700	14,225	15,416	17,267
	capital ($)	80,979	41,850	25,400	18,966	15,416	11,511
250,000	labor (hrs.)	16,307	21,069	25,574	28,645	31,044	34,771
	capital ($)	163,070	84,275	51,149	38,193	31,044	23,180
500,000	labor (hrs.)	27,691	35,777	43,429	48,642	52,716	59,045
	capital ($)	276,913	143,110	86,858	64,856	52,716	39,063
750,000	labor (hrs.)	37,746	48,768	59,197	66,304	71,857	80,483
	capital ($)	377,456	195,071	118,394	88,405	71,857	53,644
1,000,000	labor (hrs.)	47,023	60,754	73,747	82,600	89,519	100,265
	capital ($)	470,233	243,018	147,495	110,134	89,519	66,844

Thus, the disposal of solid waste exhibits increasing returns to size, and a specific increase in total resource utilization will result in a larger increase in waste disposed. This suggests that for the observed landfill operations there are substantial ecnomies of size in solid waste disposal, and larger units can utilize resources more efficiently.

To obtain data on the substitution of capital for labor in the disposal of solid waste, the production function was solved for a number of points on specific isoquants. This information is summarized in Table 2. Data in each of the five columns in Table 2 represent the quantities of capital and labor required to produce successively larger quantities of output along each of five isoclines. Note, for example, that 75,000 tons of waste can be disposed of with 10,194 hours of labor (approximately five men working 2,000 hours a year) and $20,389 of capital expense. If bigger machinery and equipment is acquired, resulting in a larger short-term capital requirement, the labor input can be substantially reduced. Thus, an increase in the machinery and equipment operating budget from $20,389 to $33,593 will make it possible to reduce labor utilization by 1,796 hours (about a one-man reduction in the labor force) without any change in the quantity of solid waste disposed. For 500,000 tons of waste, a 23 percent increase in the machinery and equipment budget from $52,716 to $64,856 will enable the landfill manager to reduce the labor input by 4,074 hours. This is approximately a two-man reduction in the labor force.

Optimum Resource Combinations

The least-cost combination of the capital and labor inputs for the various levels of output can be found by minimizing the cost function:

$$(6) \quad C = P_{\ell}\, L + P_{SC}\, SC + \text{fixed costs}$$

where P_{ℓ} is the price of labor and P_{SC} is the price of short-term capital, subject to the production function (4). This can be accomplished through the use of calculus and a Lagrangian expression which enables the solution of the constrained function V:[7]

$$(7) \quad V = P_{\ell}L^* + P_{SC}SC^* + \text{fixed costs} + \lambda \, (Y^* - .32956\, L^{*.943}\, SC^{*.366}).$$

L* and SC* are the amounts of the capital and labor inputs required to minimize the cost of producing Y* level of output, and λ is the Lagrangian multiplier which ensures that L* and SC* are both the least-cost and sufficient levels of labor and short-term capital to arrive at Y*.

To solve the Lagrangian expression for the three unknowns (L*, SC*, and λ), the partial derivatives are taken and set equal to zero:

[7] For a more detailed discussion of the Langrangian multiplier see [2, pp. 350-354].

(8) $$\frac{\partial V}{\partial L} = P_\ell - \lambda(.32956)(.943) L^{*-.057} SC^{*.366} = 0,$$

(9) $$\frac{\partial V}{\partial SC} = P_{SC} - \lambda(.32956)(.366) L^{*.943} SC^{*-.634} = 0, \text{ and}$$

(10) $$\frac{\partial V}{\partial \lambda} = Y^* - .32956\, L^{*.943} SC^{*.366} = 0.$$

The ratio of equations (8) and (9) then results in the equation of an expansion path. Thus:

(11) $$\frac{P_\ell}{P_{SC}} = \frac{\lambda(.32956)(.943) L^{*-.057} SC^{*.366}}{\lambda(.32956)(.366) L^{*.943} SC^{*-.634}}$$

$$= \frac{.943\, SC^*}{.366\, L^*} = 2.58 \frac{SC^*}{L^*}$$

or

(12) $$SC^* = .388 \frac{P_\ell}{P_{SC}} L^*.$$

This expansion path is the locus of points of least-cost combinations for varying levels of Y. The minimum cost input combination for a desired level of output, Y*, is found by substituting equation (12) into equation (10) and solving for L* and SC*. For example, with a price ratio $(\frac{P_\ell}{P_{SC}})$ of 4.42 we have from equation (12):

(12a) $$SC^* = .388\,(4.42)\, L^* = 1.713\, L^*.$$

To determine the least-cost combination of SC* and L* for Y* = 100,000, we substitute (12a) into (10) and solve for L*. Thus,

$$1{,}000{,}000 = .32956\, L^{*.943} (1.713\, L^*)^{.366}$$

(11a) $$L^{*1.309} = 249{,}187$$

$$L^* = 13{,}257$$

Substituting L* into equation (12a) we can solve for SC*:

$$SC^* = \$22{,}742.$$

Table 3 summarizes the minimum cost input levels for various relative prices and levels of output.

Table 3. LEAST COST COMBINATIONS OF LABOR AND CAPITAL FOR DIFFERENT RELATIVE RESOURCE COSTS AND VOLUMES OF SOLID WASTE DISPOSED*

Level of Output (tons per year)		Ratio of Cost of Labor to Cost of Capital Services						
		2.21	3.32	3.98	4.42	4.86	5.53	6.63
25,000	labor (hrs.)	5,580	4,980	4,734	4,597	4,477	4,318	4,104
	capital ($)	4,787	6,417	7,313	7,887	8,445	9,268	10,562
50,000	labor (hrs.)	9,476	8,457	8,039	7,807	7,602	7,333	6,970
	capital ($)	8,128	10,897	12,418	13,392	14,340	15,738	17,936
75,000	labor (hrs.)	12,917	11,528	10,958	10,641	10,362	9,995	9,501
	capital ($)	11,079	14,854	16,927	18,255	19,547	21,453	24,448
100,000	labor (hrs.)	16,092	14,361	13,651	13,257	12,910	12,452	11,836
	capital ($)	13,803	18,505	21,087	22,742	24,351	26,725	30,457
250,000	labor (hrs.)	32,405	28,919	27,490	26,696	25,997	25,075	23,834
	capital ($)	27,795	37,265	42,464	45,796	49,037	53,818	61,332
500,000	labor (hrs.)	55,027	49,109	46,681	45,332	44,145	42,580	40,474
	capital ($)	47,200	63,280	72,110	77,768	83,271	91,390	104,149
750,000	labor (hrs.)	75,007	66,940	63,631	61,793	60,174	58,040	55,169
	capital ($)	64,337	86,256	98,292	106,005	113,505	124,572	141,965
1,000,000	labor (hrs.)	93,443	83,393	79,270	76,980	74,964	72,306	68,730
	capital ($)	80,151	107,458	122,452	132,060	141,404	155,191	176,859

*NOTE: See equations (7) - (12) for derivations.

Note the substantial differences in optimal (minimum cost) resource utilization as the ratio of input prices changes. For example, for a 100,000-ton disposal unit, the minimum cost labor utilization decreases by approximately 18 percent, from 16,092 to 13,257 hours, and the capital utilization increases by 65 percent from $13,803 to $22,742 as the input price ratio increases from 2.21 to 4.42. For a 750,000-ton disposal unit, an increase in the price ratio from 3.98 to 4.42 should result in the substitution of $7,713 of machinery and equipment capital for 1,839 hours of labor to maintain the most efficient (least cost) disposal unit. Thus, an 11 percent increase in the labor wage rate would be accompanied by an 8 percent increase in capital usage and a 3 percent reduction in labor utilization, or a one-man reduction in the labor force for this size unit.

CONCLUSIONS

Like the manager of any private business, community planners must be concerned with the efficient use of limited resources. For most community services, alternative methods that involve different quantities of resources such as capital and labor can be used to produce the desired output or service. Thus, community planners must evaluate the efficiency of and substitutability between various resources. Production economics concepts provide a theoretical basis for making this type of evaluation. In addition to analysis of physical efficiency problems, the concepts of production theory also can be used to determine economies of size and minimum cost levels of resource use to produce specified quantities of output.

Limitations exist when production theory is applied to public service functions at the local government level. Complementarity of various inputs among functions serves to complicate the analysis. For example, labor used in management of solid waste disposal also may be used in general government administration. Data problems also are present. Measurement of output is difficult in most cases, and differences in terms of quality may be impossible to quantify. Nevertheless, analysis of the allocation of resources in the disposal of solid waste provides empirical evidence for the usefulness of production theory. The empirical production function exhibited substantial economies of size and a high degree of substitutability between the capital and labor inputs. The analysis of least cost input combinations indicated that only slight changes in relative resource prices would result in substantial changes in minimum cost resource use. For example, a 10 percent increase in the price of labor relative to the price of capital results in 2.6 percent decrease in labor usage and a 7 percent increase in capital utilization for a 500,000-ton disposal unit.

Empirical production functions and the concepts of production theory are valuable tools for community planners. However, the accuracy of the analysis obviously depends on the quality of the data used in the estimation. As more data become available on the physical resource needs for community services and the measurements of both outputs and inputs are refined, the precision and accuracy of economic analysis of community services will be greatly enhanced.

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