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MODELING TO GENERATE ALTERNATIVES: A PHILOSOPHICAL TWIST

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Since the development of the simplex method some two score years ago, analysts have emphasized computational efficiency. Simplex solution involves moving about extreme points so as to locate the optimum basis while examining relatively few corners en route. Analysts have been proud to report having found the "best" solution, and perhaps to comment on sensitivity. They have most considerately spared the decision maker the distraction of reviewing other solutions to the model formulated and solved. The parallel in multiple objective decision making formulations is that only non-inferior solutions are presented (Willis and Perlack [1980]).

In conventional applications of mathematical programming, our historic preoccupation with finding the "best" solution has meant that we have all but overlooked the possible presence of multiple optima and have largely failed to exploit the useful information contained in such solutions. We have as often neglected the wisdom contained in nearly optimal solutions. This seems a clear consequence of the philosophy of computational efficiency embedded in the development of the simplex method.

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The view to be developed here is that the efficiency paradigm should be modified to admit that models are imperfect, that some objectives are generally left unquantified, and that we would often do better to provide a range of solutions that are quite different from one another, but which are as good or nearly so for the single objective linear program or nearly non-inferior for the multiple objective application. Provision of multiple and near optimum solutions will not only provide a richer set of information to decision makers, it may also enhance the predictive power of our prescriptive models. In brief, we should model to generate alternatives.

Multiple Optima in Economic Problems

Despite the relative silence of economists on the subject, multiple optima and nearly optimal solutions are not rare. Yet Paris'[1981] search of the empirical literature failed to find a single application that reported whether the solution was unique. Indeed, most failed to reveal even the numbers of rows and columns in the constraint matrix. Much is known about multiple and near-optima and how to find them. Until recently, however, there has been little show of interest among economists.

A couple of examples from economic theory may motivate the discussion--they are theory of the firm and consumer theory (the diet problem).

<u>Theory of the Firm</u>. Linear programming generally examines the firm from a short-run point of view in which production

facilities are assumed to be fixed. These limitations on facilities and on quantities of raw materials available, in turn place limitations on the various commodities that can be produced.

Rather than to deal with a continuous production function, linear programming begins at the more basic level of considering various processes of production. For firms operating in purely competitive markets, the linear programming problem can be expressed as:

(1) $\max z = c'x$

subject to:

(2) $Ax \leq b$

(3) $x \geq 0$.

Here x is the n dimensional column vector of activity levels (x_j), c' the vector of profit per unit of good made by activity j, A the matrix [a_j], and b the vector of resource limitations.

We know, of course, that an optimal solution to this problem need not have greater than m of the $x_j > 0$. The firm can maximize profit using only m activities! For any optimal solution, the dual variables, $z_j - c_j$, are non-negative for all j. The interpretation of $z_j - c_j$ is as the reduction in profit that would be associated with producing an additional unit of j and adjusting the levels of the other basic activities so as to maintain feasibility. These fundamental theorems are the equivalent to the classical theory of the firm admonitions to produce to the point at which price equals marginal cost and to employ resources to the point at which resource costs equal their values of marginal products.

Paris provided an interesting and useful insight: if firms are believed to be competitive and linear programming is considered to be capable of reflecting this environment, then one ought to expect multiple or near-optima as a frequent natural consequence. That is, in competitive markets long-run necessary conditions call for zero profitability, and generally goods are available in these markets in greater numbers than the number of constraints in a linear programming model of a single firm. In his words (p. 726),

> "In a competitive situation, where there are many firms using essentially the same technology, one observes that similar firms produce different product mixes. In the absence of uncertainty, all activities are equally profitable and therefore, it just happens that one firm chooses a particular combination of activities while others select a different mix. In LP terminology, this situation is characterized by zero relative loss not only for the optimal basic activities but also for those not in the basis. It causes the multiple optimal solution phenomenon. Hence, an extensive dual degeneracy may be interpreted as a validation of an LP economic model, where perfect competition and certainty prevail."

To be sure, the extreme case of perfectly competitive markets seldom, if ever, exists and linear programming models are not designed to reflect the long-run. However, the analyst should be heartened by the presence of multiple optima and, conversely, should view with concern the finding of a unique optimum. The latter could signal the existence of non-competitive markets or the failure of the linear programming formulation to reflect the assumed environment.

Diet problem. As indicated earlier, the diet problem is well-known in the linear programming literature as the first economic problem ever solved by these procedures. Jerome Cornfield independently formulated and solved approximately a diet problem, which he reported in an unpublished memorandum in 1941 (Dantzig [1963]). Several years later, George Stigler [1945] formulated a minimum cost diet model concerned with selecting among 77 foods subject to 9 nutrient constraints. He used trial and error procedures to solve approximately the constrained optimization, and the solution contained only 5 foods: wheat flour, evaporated milk, cabbage, spinach, and dry navy beans. In 1947, Dantzig and Laderman used linear programming to solve this same problem. Their solution, unpublished, contained 5 foods -- four as selected by Stigler and the fifth, beef liver, replacing evaporated milk (Dantzig [1963]). More recent works done for USDA have employed quadratic programming formulations, either to minimize weighted sums of squares of deviations of quantities of food groups from food consumption patterns (Peterkin et al. [1981]) or to maximize utility (Balintfy and Taj [1983]), both subject to minimum nutrient requirements and food budget.

But let us return to the problem formulated by Stigler. Its structure is that of (1)-(3) above, where the direction of the inequality in (2) is reversed and (1) is minimized. The 77 elements of c_j all take values of one and the x_j are defined as daily dollar expenditures on food item j, A is the 9 by 77 matrix of a_{ij} defined as quantities of nutrient i contained per portion of food item j, and b is the vector of minimum requirements of the 9 nutrients (calories, riboflavin, etc.).

Since a basic solution to a linear programming problem will contain at most m positive variables, nine food items would be the maximum variety consistent with a basic solution. However, another set of 9 or fewer food items might satisfy the nutritional requirements at the same cost, and if so, an infinite number of solutions would solve the problem equally well. If there are two or more extreme point solutions with the same (optimal) value of z, then any convex combination of these extreme point solutions is also optimal. And these convex combinations are not limited to at most m foods. Hence, far from being a "problem", the presence of multiple optima would give rise to the possibility of a more diversified, interesting and, in this case, palatable set of optimal solutions.

Certainly, multiple optima could occur if a non-basic food item had the same nutritional components (a_j) and cost (c_j) as a basic variable, or similar to a factor of proportionality. With a large number of food items n, relative to nutritional requirements m, it seems likely that near-optima, if not multiple optima, would be the rule rather than the exception. Many of the 77 food items examined by Stigler are similar in a_j and c_j.

Normative and Positive Models

Simon [1965] distinguished between positive and normative theories by the analogy to the distinction between declarative and imperative sentences. Positive theories describe, explain (addressing the question of "why?"), or predict behavior.

Normative theories in economic modeling generally prescribe--they are operational in the sense of providing goals or criteria as well as procedures for achieving them. The distinction is commonly referred to as one of answering questions of "is" or of "ought". Kadane and Larkey [1983, p.1365] have recently suggested that,

> "The development and application of theories of decision making in economics...have long been bedeviled by confusion between the 'is' and the 'ought' in theoretical statements. It is often not clear from either an author's claims for a theory (model) or from the context of its use whether a model is intended to describe how decisions have been made, to explain why they have been made,... or to prescribe how they should be made.

Their final point is that both normative and positive theories are important and that we should recognize that these theories (models) are likely to differ from each other. Unless the way decisions are currently being made cannot be improved, positive and normative statements are apt to differ.

What has this to do with the subject at hand? Simply this. Linear programming provides normative solutions to the problem of optimizing (1) subject to the conditions (2) and (3). The extreme point solution prescribed may or may not predict actual behavior well. In practice, one often sees the accuracy of prediction used to validate the model, and if the predictions are poor the model is revised. This is certainly sensible procedure for predictive models, but it is less so for purely prescriptive models.

Models of the firm are frequently used in agricultural economics to develop regional models with strong positive as well as normative motives. Predictions have generally been poor with these models -- usually suggesting substantially greater specialization of agricultural production by region than is observed in practice. The differences between predictions/prescriptions and observed behavior led Day [1961] to impose upper and lower bounds on production, Meister, Chen and Heady [1978] to suggest the use of rotational constraints to curtail the overspecialization (and to "model the agronomic nature of crop production"), McCarl [1982] to advocate a decomposition methodology to reconcile sectoral equilibria and farm level plans, and Howitt and Mean [1983] to advance a positive quadratic programming specification based on the discrepancy between the linear cost function and the cost function implied by the actions of farmers.

While the latter two approaches seem rather promising, most, such as imposition of upper and lower constraints on production and the use of rotational restraints are ad hoc contrivances which force solutions to resemble actual behavior more closely. Commenting on the former procedure, Howitt and Mean (p. 1) suggest,

> "This problem severely limits the value of linear models for policy purposes since models that are poorly calibrated and unbelievable will not be used. But models that are tightly constrained can only produce that subset of normative results that the calibration constraints dictate. The policy conclusions are thus bounded by a set of constraints that are expedient for the base year but often inappropriate under policy changes."

Another way of saying that the linear programming models predict over-specialization relative to actual behavior is to say that there are too few positive variables in the solution. Again, extreme point solutions limit to m the number of such activities, but in the presence of multiple optima there is no such limit to the number of positive variables contained in a convex combination of several optimal solutions. Indeed, Paris [1981] provides a procedure for selecting a convex combination of a set of k < n primal optimal solutions whose least squares objective function is to make the optimal solution close to the levels of the activities observed in practice.

Similar considerations apply to the diet problem example. Imagine even cost-conscious individuals choosing to subsist on a (Stigler) diet of wheat flour, evaporated milk, cabbage, spinach and dry navy beans for an extended period of time! And even if cost is replaced by utility measures, a unique optimum solution limits variety to at most m (9 in the original formulation) food items. Once again convex combinations of multiple optima remove this barrier to making the model more nearly predictive of actual behavior.

The argument for heeding the predictive power of the normative model is potent for the diet problem. A major use is in devising diets for major institutions: school lunches, penal institutions, military, and the like. If the menu lacks sufficient diversity or in other ways fails to conform with what individuals will eat, major portions of the meal will be discarded and as a consequence some or all of the nutritional constraints will be violated.

Modeling to Generate Alternatives

Modeling to generate alternatives is a developing branch of multiobjective programming which deliberately leaves some objectives (and perhaps constraints) unquantified. It accepts that the model is not reality and presumes to be a tool rather than a replacement for the decision maker. It provides a set of optimal and nearly optimal-- or non-inferior and nearly non-inferior-- solutions from which the decision maker can select. The intent is to offer the decision maker a manageably small set of decision vectors which are nearly optimal or nearly non-inferior and which are as different from one another as is possible. The unquantified or unquantifiable objectives can then lead the decision maker toward a final choice.

As indicated in the survey by Gidley and Bari [1986], MGA techniques involve two functions: generation of alternatives and selection of several for presentation.

Generation. To be general let us consider a mathematical programming model with several objectives expressed as:

(4) Minimize $z_k = g_k(x)$, $k=1,2,...,r$

(5) subject to $x \epsilon X$,

where $g_k(x)$ denote the r objectives and as usual X is the feasible solution set. One finds an initial solution by the usual means, and then typically seeks alternative solutions by solving (5) augmented by:

(6) $g_k(x) \leq (1+p_k)z_k^*$, $k=1,2,...,r$,

where z_k^* is the value of the kth objective function in the solution of (4) and (5), and p_k is the allowed tolerance from optimality of the k^{th} objective. Two groups of techniques have been used to generate and select solution vectors--one- and two-phase approaches.

<u>Single phase approaches</u>. This class of techniques is designed to produce a relatively small number of solutions which are as different from one another as possible. Variants include the Hop, Skip, and Jump (HSJ) method of Brill et al. [1982], the fuzzy HSJ method of Chang et al. [1983], Gibbs inner product minimization method proposed by Kshirsagar and Brill [1984], and the orthogonal search method developed also by Kshirsagar and Brill. The most widely used remains the HSJ approach, which seeks to minimize the sum of the activities which were nonzero in the solutions generated previously, subject to the constaints (5) and (6). Previously nonbasic activities tend to be forced into the basis and hence successive solutions tend to be quite different, although naturally this difference declines as the generations continue. Generation stops when either the maximum number of solutions desired has been found or the basic variables remain unchanged from the previous solution.

<u>Two phase approaches</u>. These techniques generate large numbers of solutions and a second stage must be used to select a subset for presentation. Again, four classes can be identified for first phase generation: neighboring extreme point generation (Paris [1981], Padmanabhan and Rogness [1985]), exhaustive extreme point generation (Paris [1981, 1983]), search with random

objective functions (Harrington and Gidley [1985]), and the branch and bound /screening method (Chang and Liaw [1984]).

For simplicity, the search with random objective functions has much to offer, and we shall return to it below. The approach has developed in several ways. One of these has been to generate random objective functions by selecting a uniform random deviate on an interval such as [-1,1] as an objective function coefficient for each decision and slack variable, drawing repeated samples, and collecting alternative solutions to the problem within the specified tolerance of optimality.

Selection of a set for presentation. Pitted against the benefits of this richer set of information is the added cost. One component is the cost of obtaining the alternative solutions. This is likely to be trivial in comparison with the costs of model development and initial solution. The other component relates to limitations associated with the information processing ability of the human mind-- it is possible to overload decision makers with solutions. The works in the human choice theoretical literature of Thurstone [1927], Luce [1964], Coombs [1964], Tversky [1972], and Keeney and Raiffa [1976] are relevant here, as are empirical pieces by Gehrlein and Fishburn [1976], Scott and Wright [1976], Troutman and Shanteau [1976], Wilkie and Weinrich [1973], and Wright [1974]. Miller [1956] provided an early and entertaining paper on this point. He begins,

> "My problem is that I have been persecuted by an integer. For seven years this number has followed me around, has intruded in my most private data, and has assaulted me from the pages of our most public journals. This number assumes a variety of disguises, being sometimes a little larger and sometimes a little

smaller than usual, but never changing so much as to be unrecognizable. The persistence with which this number plagues me is far more than a random accident."

Of course, this magical number is seven. He cites: 7 point rating scale; 7 categories for absolute judgement; 7 objects in the span of attention; 7 digits in the span of immediate memory; 7 days of the week; 7 notes of the musical scale; 7 primary colors; 7 deadly sins; 7 digits of telephone numbers among other "coincidences." He concludes that people are less accurate if they must judge more than one attribute simultaneously-- as they add attributes, they decrease the accuracy of the evaluation of any one. It is the reason for the importance of selecting for presentation a modest set of alternative solutions.

One way of selecting alternative solutions for presentation is by inspection based on the analyst's judgement. As Gidley and Bari caution, the danger here is that the presentation set becomes biased by the modeler's preferences. Formal selection procedures are generally based on some form of cluster analysis. The technique involves partitioning a group of vectors into relatively similar subsets using some measure of distance --e.g., the Euclidean metric-- between vectors. As Chang and Liaw [1984] indicate, the choice of metric is important and the measure should be applied only to the most important decision variables.

Clustering techniques can be divided into hierarchical and disjoint classes, and both have been used in selecting presentation sets. Hierarchical clustering involves starting with each solution vector representing a cluster, merging the two most similar clusters, and repeating the process until all vectors are contained in a single cluster. The output is in the form of a tree diagram (a dendogram). Chiang and Liaw used this approach to prune their solution set. In contrast, the number of desired clusters is prespecified in disjoint clustering approaches, and the algorithm is left to group alternatives into this given number of clusters. One practical advantage of disjoint clustering for present purposes is that it is easier to select a set for presentation from a given number of clusters than from a dendogram. Perlack and Willis [1985] used this approach to prune a larger set of non-inferior solutions obtained by generating techniques applied to a nonlinear programming formulation of Boston's sludge disposal problem.

An Illustration: The Diet Problem Revisited

The formulation of the Stigler diet problem as a linear program is provided in the Appendix. The 77 x_j are in the same order as Stigler's presentation. Table 1 below shows in the first column Stigler's approximate solution, and next to it the optimum solution to this minimum cost of subsistence problem. Stigler's approximation calls for a daily ration consisting of 16 ounces of wheat flour, a couple of ounces of evaporated milk, five ounces of cabbage, another of spinach and twelve ounces of dried navy beans. The total cost in 1939 dollars is 10.93 cents per day ($1.17 in 1986 values). The optimal solution substitutes a small portion of beef liver for the evaporated milk and reduces cost to 10.86 cents per day. This optimum solution is unique--

Table 1
Solutions* Within Various Tolerances of the Stigler Optimum

Food Item	Stigler Approximation		Optimum Solution		Near Optima p = .02		p = .05		p = .10		p = .20		p = .40	
1	Wheat Flour	16.18 (3.64)	Wheat Flour	13.28 (3.00)	Wheat Flour	16.44 (3.70)	Wheat Flour	18.22 (4.10)	Wheat Flour	19.55 (4.40)	Wheat Flour	15.11 (3.40)	Corn Meal	15.00 (4.00)
2	Evap. Milk	2.31 (1.07)	Beef Liver	0.12 (0.20)	Evap. Milk	3.25 (1.50)	Evap. Milk	5.84 (2.70)	Corn Meal	4.87 (1.40)	Corn Meal	5.56 (1.60)	Evap. Milk	7.60 (3.50)
3	Cabbage	4.80 (1.11)	Cabbage	4.84 (1.12)	Lard	0.36 (.22)	Lard	0.98 (.60)	Evap. Milk	9.10 (4.20)	Evap. Milk	8.66 (4.00)	Peanut Butter	0.90 (1.00)
4	Spinach	1.01 (0.51)	Spinach	1.00 (0.50)	Cabbage	4.84 (1.12)	Cabbage	4.89 (1.13)	Lard	0.33 (.20)	Potatoes	14.11 (2.00)	Potatoes	15.52 (2.20)
5	Dried Navy Beans	12.47 (4.60)	Dried Navy Beans	16.48 (6.10)	Spinach	1.00 (.50)	Spinach	0.88 (.45)	Cabbage	4.76 (1.10)	Sweet Potatoes	3.45 (1.10)	Sweet Potatoes	2.50 (.80)
6	--		--		Dried Navy Beans	10.48 (4.00)	Dried Navy Beans	5.97 (2.20)	Spinach	0.71 (.36)	Dried Lima Beans	1.26 (.70)	Dried Lima Beans	5.70 (3.20)
Total Cost (¢)	10.93		10.86		11.07		11.40		11.90		13.00		15.20	

*Quantities are in ounces per day and figures in parentheses are costs in cents (1939 values) rounded to the nearest hundredth.

all $z_j - c_j$ for non-basic variables are positive. However, it is barely so; alternative feasible bases can be found within 0.1 percent of the optimum solution. If we wished to find a diet with none of the least cost ration food items in it, we could do so with a 40 percent increase in cost.

If we are to perform MGA, we must select a value for the tolerance from optimality, p. A value of 0.001 seems needlessly stringent; we will not find much variety within that narrow tolerance of the optimum solution. A value for p of 0.40 would provide substantial variety, but 40 percent may not be an acceptable increase in cost. The remaining columns of Table 1 show the alternative solutions which minimize the cost of the food items in the original optimum solution subject to a maximum of 2, 5, 10, 20 and 40 percent increase in cost of rations above the minimum cost diet. For a 2 percent increase, lard is added to the diet; for a 10 percent increase corn meal is included; potatoes, sweet potatoes and dried lima beans are added at the 20 percent level; and at the 40 percent level peanut butter is consumed and all items are different from the optimum solution.

<u>Single Step Procedure</u>. To illustrate the HSJ procedure, two values of p will be used (one at .02 and the other a more generous .10). For each case we generate six solutions as different as we can make them from the standpoint of minimizing the cost of all previous basic variables in seeking another solution within the given tolerance of the least cost solution. Table 2 provides these solutions for the 2 percent tolerance case. The first two columns repeat the optimum solution and the

Table 2
Optimal and Nearly Optimal Solutions* Within a 2 Percent Tolerance of Least Cost (p = 0.02)

Food Item	Optimum Solution	HSJ Alternatives					
		1	2	3	4	5	6
1	Wheat Flour 13.30 (3.00)	Wheat Flour 16.40 (3.70)	Wheat Flour 15.00 (3.40)	Wheat Flour 14.80 (3.33)	Wheat Flour 14.44 (3.25)	Wheat Flour 12.22 (2.75)	Wheat Flour 12.84 (2.89)
2	Beef Liver 0.12 (0.20)	Evap. Milk 3.20 (1.50)	Cheddar Cheese 0.44 (0.67)	Wheat Cereal 0.80 (0.69)	Milk 0.93 (0.64)	Beef Liver 0.15 (0.25)	Beef Liver 0.17 (0.28)
3	Cabbage 4.80 (1.10)	Lard 0.36 (0.22)	Cabbage 4.80 (1.10)	Beef Liver 0.17 (0.28)	Beef Liver 0.06 (0.10)	Cabbage 3.63 (0.84)	Cabbage 4.32 (1.00)
4	Spinach 1.00 (0.50)	Cabbage 4.80 (1.10)	Spinach 1.00 (0.50)	Cabbage 4.80 (1.10)	Cabbage 4.80 (1.10)	Potatoes 4.30 (0.61)	Spinach 0.60 (0.30)
5	Dried Navy Beans 16.50 (6.10)	Spinach 1.00 (0.50)	Dried Lima Beans 0.80 (0.44)	Spinach 0.95 (0.48)	Spinach 1.00 (0.50)	Spinach 0.97 (0.49)	Sweet Potatoes 1.82 (0.58)
6	--	Dried Navy Beans 10.80 (4.00)	Dried Navy Beans 13.30 (4.90)	Dried Navy Beans 14.00 (5.16)	Dried Navy Beans 14.83 (5.47)	Dried Navy Beans 16.65 (6.14)	Dried Navy Beans 16.30 (6.01)
Min. Z (¢)		9.30	9.95	10.38	10.43	10.46	10.48
Total Cost (¢)	10.86	11.07	11.07	11.07	11.07	11.07	11.07

*Quantities are in ounces per day and figures in parentheses are costs in cents (1939 values) rounded to the nearest hundredth.

solution which minimizes the items from the optimum solution at a 2 percent increase in cost. Notice that this 11.07 cent diet contains 9.3 cents worth of food items from the optimum solution. The second alternative adds cheddar cheese and dried lima beans and contains 9.95 cents worth of food basic in either the Stigler optimum solution or the first alternative solution. Wheat cereal enters in the third alternative at a level of 0.69 cents and represents the only new food item not previously appearing. Milk appears in solution 4, potatoes in solution 5, and sweet potatoes in solution 6.

As shown in Table 3, a more generous tolerance of 10 percent of cost produces a greater variety. In this case the first alternative uses less than 6 cents worth of optimum solution food items. In fact, three new food items enter this alternative: corn meal, evaporated milk and lard. The second alternative added an additional two: cheddar cheese and potatoes. It used only 7 cents worth of items from the previous two solutions. The third alternative added a new item, the fourth an additional two items, and one more entered in each of the final two alternatives. With this generous tolerance, even the sixth alternative only used 9.7 cents worth of food items that appeared in any of the previous six bases, or 82 percent of the total cost of the ration. This contrasts with the similar situation for the 2 percent tolerance case in which 10.48 cents worth of food items from the previous six bases were involved, representing 95 percent of the 11.07 cent total cost of the ration.

In light of the range of choices contained in Tables 2 and

Table 3
Optimal and Nearly Optimal Solutions* Within a 10 Percent Tolerance of Least Cost (p = 0.10)

Food Item	Optimum Solution		HSJ Alternatives 1		2		3		4		5		6	
1	Wheat Flour	13.30 (3.00)	Wheat Flour	19.82 (4.46)	Wheat Flour	24.97 (5.62)	Wheat Flour	21.64 (4.87)	Wheat Flour	17.42 (3.92)	Wheat Flour	10.18 (2.29)	Wheat Flour	9.77 (2.20)
2	Beef Liver	0.12 (0.20)	Corn Meal	5.04 (1.45)	Cheddar Cheese	2.45 (3.71)	Wheat Cereal	4.05 (3.50)	Milk	3.30 (2.27)	Beef Liver	0.05 (0.08)	Rolled Oats	4.93 (2.19)
3	Cabbage	4.80 (1.10)	Evap. Milk	9.01 (4.20)	Cabbage	4.67 (1.08)	Beef Liver	0.39 (0.65)	Cabbage	3.89 (0.90)	Cabbage	4.84 (1.12)	Beef Liver	0.44 (0.74)
4	Spinach	1.00 (0.50)	Lard	0.36 (0.22)	Potatoes	1.55 (0.22)	Cabbage	4.80 (1.11)	Spinach	4.27 (2.16)	Spinach	1.00 (0.51)	Cabbage	4.80 (1.11)
5	Dried Navy Beans	16.50 (6.10)	Cabbage	4.75 (1.10)	Spinach	0.83 (0.42)	Spinach	0.81 (0.41)	Sweet Potatoes	2.76 (0.88)	Dried Peas	5.37 (2.65)	Spinach	0.80 (0.40)
6	—		Spinach	0.71 (0.36)	Dried Lima Beans	1.53 (0.85)	Dried Navy Beans	3.63 (1.34)	Dried Navy Beans	10.09 (3.72)	Dried Navy Beans	14.21 (5.24)	Dried Navy Beans	14.21 (5.24)
Min. Z (¢)			5.92		7.12		8.40		8.74		9.24		9.70	
Total Cost (¢)	10.86		11.90		11.90		11.90		11.90		11.90		11.90	

*Quantities are in ounces per day and figures in parentheses are costs in cents (1939 values) rounded to the nearest hundredth.

3, it might seem desireable to the decision maker and consumer to have access to this information. Surely, for example, a 2 percent (0.21 cents per day) increase in costs would generally be considered negligible, and in exchange the variety of diets seems substantial--at least relative to the small difference in cost. The quantity of information supplied in these two tables is not liable to overload most normal minds, and yet the provision of this additional set of near- optimum solutions could well result in their selection rather than the minimum cost of subsistence solution. And if we wished even greater variety, a convex combination of these diets could offer a menu consisting of 17 items at the same cost.

Two Phase Approach. As indicated earlier, as an alternative to selecting a small number of very different solutions in a single step by a procedure such as HSJ, we might generate a large number of solutions within a given tolerance of the optimum in one stage, and then apply a selection criterion to choose a manageable number to present to the decision maker. The random coefficients procedure has been selected to illustrate use of a two phase approach. In this, we select repeated series of random c′ vectors each consisting of 77 random variates drawn from a Uniform [0,1] Distribution. Eighteen such vectors were drawn and used to optimize that objective function subject to (5) and (6), with p set at 0.10. The number of solutions, 18, was selected arbitrarily for the illustration, and means that because we chose to form six clusters of solutions to compare alongside the optimum solution, the average number of solutions in a cluster will be three.

Table 4 shows the eighteen alternative solutions. For these solutions, 18 different food items are included in one or more of the diets. This compares with the 17 items appearing in the single phase approach in Table 3. The presentation of even this abbreviated example took three pages. A more exhaustive enumeration of near optima would surely make choice even more difficult. Thus selection of a presentation set is needed as a final step.

A form of disjoint clustering called Quick Cluster, available in the statistical package SPSSX, was used to group the 18 solutions into 6 clusters. The Euclidean distance metric is used to measure the distance between each solution and the center of each cluster (means of the variables for the solutions contained in the cluster). The solutions were considered as one cluster initially, and then split successively at each step until the desired six clusters had been formed. At this point, solutions are reallocated iteratively into the cluster whose center is closest. The 18 food items appearing in one or more of the 18 alternative solutions seems a needlessly large number of attributes to use in forming clusters, and would require finding a larger number of nearly optimal solutions to cluster in any case. Therefore, for purposes of clustering we aggregated these 18 food items into 7 food groups: vegetables(cabbage, spinach, asparagus), cereals and grains(wheat flour, corn meal, oats, wheat cereal), legumes(lima beans, navy beans, peas), starches(potatoes, sweet potatoes), sweeteners(sugar, molasses),

Table 4
Random Coefficients Generation of Nearly Optimal Solutions* Within a 10 Percent Tolerance of Least Cost (p = 0.10)

Food Item	Alternative Solutions											
	1		2		3		4		5		6	
1	Wheat Flour	1.15	Wheat Flour	2.20	Wheat Flour	5.34	Wheat Flour	5.53	Wheat Flour	4.18	Wheat Flour	6.48
2	Lard	1.32	Cabbage	0.20	Wheat Cereal	3.09	Evap. Milk	4.05	Cabbage	1.12	Evap. Milk	0.29
3	Cabbage	1.13	Spinach	2.33	Evap. Milk	1.18	Cabbage	1.14	Spinach	0.54	Cheddar Cheese	3.54
4	Spinach	0.52	Dried Navy Beans	6.55	Cabbage	1.13	Spinach	0.42	Dried Navy Beans	4.27	Cabbage	1.18
5	Dried Lima Beans	2.01	Sugar	0.54	Spinach	0.50	Dried Peas	0.76	Molasses	1.80	Spinach	0.42
6	Dried Navy Beans	5.77	--		Dried Lima Beans	0.66	--		--		--	

*Values are costs of the food items in 1939 cents, rounded to the nearest hundredth.

Table 4 (Continued)
Random Coefficients Generation of Nearly Optimal Solutions* Within a
10 Percent Tolerance of Least Cost (p = 0.10)

Food Item	Alternative Solutions											
	7		8		9		10		11		12	
1	Wheat Flour	4.46	Wheat Flour	4.53	Wheat Flour	5.64	Wheat Flour	1.95	Wheat Flour	5.62	Wheat Flour	4.59
2	Corn Meal	1.45	Evap. Milk	4.36	Cheddar Cheese	3.68	Corn Meal	0.67	Cheddar Cheese	3.71	Evap. Milk	4.33
3	Evap. Milk	4.24	Lard	1.32	Cabbage	1.14	Cabbage	0.83	Cabbage	1.08	Lard	1.22
4	Lard	0.22	Cabbage	1.14	Spinach	0.35	Sweet Potatoes	1.60	Potatoes	0.22	Cabbage	1.14
5	Cabbage	1.16	Spinach	0.41	Sweet Potatoes	0.23	Dried Lima Beans	1.39	Spinach	0.42	Spinach	0.41
6	Spinach	0.37	Dried Lima Beans	0.14	Dried Lima Beans	0.87	Dried Navy Beans	5.45	Dried Lima Beans	0.85	Asparagus	0.22

*Values are costs of the food items in 1939 cents, rounded to the nearest hundredth.

Table 4 (Continued)
Random Coefficients Generation of Nearly Optimal Solutions* Within a 10 Percent Tolerance of Least Cost (p = 0.10)

Food Item	Alternative Solutions											
	13		14		15		16		17		18	
1	Wheat Flour	4.24	Wheat Flour	2.46	Wheat Flour	5.09	Wheat Flour	5.53	Wheat Flour	5.51	Wheat Flour	5.64
2	Milk	2.76	Rolled Oats	0.96	Evap. Milk	4.32	Evap. Milk	4.27	Evap. Milk	2.02	Cheddar Cheese	3.68
3	Cabbage	1.06	Cabbage	0.30	Cabbage	1.14	Cabbage	0.65	Cheddar Cheese	2.07	Cabbage	1.18
4	Spinach	0.49	Spinach	2.13	Spinach	0.41	Potatoes	1.04	Cabbage	0.52	Spinach	0.42
5	Dried Navy Beans	3.37	Dried Navy Beans	6.05	Sugar	0.94	Spinach	0.41	Potatoes	1.37	Dried Lima Beans	0.98
6	--		--		--		--		Spinach	0.41	--	

*Values are costs of the food items in 1939 cents, rounded to the nearest hundredth.

dairy(cheese, milk, evaporated milk), and fats(lard). For each nearly optimal solution, the cost of each of these seven groups was calculated and used as the basis for clustering the 18 alternative solutions.

Table 5 shows how the 18 solutions found in Table 4 clustered into 6 groups. Four solutions formed a first cluster, clusters 2,3 and 4 contained a single solution each, cluster five was comprised of solutions 14 and 15, and the final cluster grouped the remaining nine solutions. The Euclidean distance for each solution is also provided. For solution 1, for example, the distance 0.801 represents the sum of the squares of the differences of the costs of the seven food groups in that solution from the means of those seven items in cluster 6. Naturally, the clusters containing a single solution have a zero distance. It was interesting that the solutions in the large cluster were each nearer that cluster centroid than several of the solutions in the smaller clusters.

Table 6 displays the presentation set based on this clustering. In it, the solution in each cluster which is closest to the centroid by the minimum distance criterion is selected as representative of its cluster. The argument here is that, while less information is contained in this set of solutions compared with the larger set enumerated in Table 4, it may be a more usable set because it is simpler for the human mind to comprehend. Had we opted to generate several hundred nearly optimal solutions, the point would be more dramatically made.

Table 5

Cluster Membership

Cluster	Solution No.	Euclidean Distance
1	4	1.782
1	8	1.830
1	13	1.525
1	18	1.141
2	16	0.000
3	7	0.000
4	17	0.000
5	14	1.101
5	15	0.551
6	1	0.801
6	2	1.253
6	3	0.924
6	5	0.877
6	6	1.225
6	9	1.058
6	10	1.049
6	11	1.340
6	12	1.030

Table 6
Presentation Set for Two Phase Solutions* Using Cluster Analysis

Food Item	Representative Solution From Clusters					
	1	2	3	4	5	6
1	Wheat Flour 10.13 (2.28)	Wheat Flour 18.58 (4.18)	Wheat Flour 18.84 (4.24)	Wheat Flour 28.80 (6.48)	Wheat Flour 24.58 (5.53)	Wheat Flour 19.82 (4.46)
2	Cabbage 0.86 (0.20)	Cabbage 4.84 (1.12)	Cabbage 4.58 (1.06)	Cabbage 5.10 (1.18)	Cabbage 4.93 (1.14)	Lard 0.36 (0.22)
3	Spinach 4.60 (2.33)	Spinach 1.07 (0.54)	Spinach 0.97 (0.49)	Spinach 0.83 (0.42)	Spinach 0.83 (0.42)	Cabbage 5.02 (1.16)
4	Dried Navy Beans 17.76 (6.55)	Dried Navy Beans 11.58 (4.27)	Dried Navy Beans 6.06 (3.37)	Evap. Milk 0.63 (0.29)	Wheat Cereal 4.68 (4.05)	Spinach 0.73 (0.37)
5	Sugar 1.67 (0.54)	Molasses 2.38 (1.80)	Milk 4.01 (2.76)	Cheddar Cheese 2.34 (3.54)	Dried Peas 1.54 (0.76)	Evap. Milk 9.18 (4.24)
6	--	--	--	--	--	Corn Meal 5.04 (1.45)
Solution	18	16	7	17	15	1

*Quantities are in ounces per day and figures in parentheses are costs in cents (1939 values) rounded to the nearest hundredth.

Concluding Thoughts

As appealing as efficiency is to economists, a limit to its usefulness exists. Generation of multiple optima when they exist and exploration of noninferior solution space should be augmented by the generation of nearly optimal and nearly noninferior solutions. This provides a far richer set of information to decision makers in normative models and can facilitate prediction in positive ones, provided the information does not itself become too burdensome to process in making final selections. The philosophy and accompanying procedures termed "modeling to generate alternatives" has been developing rapidly in the engineering and management science literature in the early to mid decade of the 80s, and it is important for applied economists to embrace the paradigm before the decade is out.

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Appendix
Stigler Diet Formulation

MINIMIZE
X1+X2+X3+X4+X5+X6+X7+X8+X9+X10+X11+X12+X13+X14+X15+
X16+X17+X18+X19+X20+X21+X22+X23+X24+X25+X26+X27+X28+
X29+X30+X31+X32+X33+X34+X35+X36+X37+X38+X39+X40+X41+
X42+X43+X44+X45+X46+X47+X48+X49+X50+X51+X52+X53+X54+
X55+X56+X57+X58+X59+X60+X61+X62+X63+X64+X65+X66+X67+
X68+X69+X70+X71+X72+X73+X74+X75+X76+X77

Constraints

SUBJECT TO:

1. 44.7X1+11.6X2+11.8X3+11.4X4+36X5+28.6X6+21.2X7+
25.3X8+15X9+12.2X10+12.4X11+8X12+12.5X13+6.1X14+
8.4X15+10.8X16+20.6X17+2.9X18+7.4X19+3.5X20+
15.7X21+8.6X22+20.1X23+41.7X24+2.9X25+2.2X26+3.4X27+
3.6X28+8.5X29+ 2.2X30+3.1X31+3.3X32+3.5X33+4.4X34+
10.4X35+6.7X36+18.8X37+1.8X38+1.7X39+5.8X40+5.8X41+
4.9X42+1X43+2.2X44+2.4X45+2.6X46+2.7X47+.9X48+
.4X49+5.8X50+ 14.3X51+1.1X52+9.6X53+3.7X54+3X55+
2.4X56+.4X57+1X58+7.5X59+5.2X60+2.3X61+ 1.3X62+
1.6X63+8.5X64+12.8X65+1.5X66+20X67+17.4X68+26.9X69+
8.7X72+8X73+34.9X74+14.7X75+9X76+6.4X77.GE.3
Calories (thousands)

2. 1411X1+418X2+377X3+252X4+897X5+680X6+460X7+907X8+
488X9+484X10+439X11+130X12+288X13+310X14+422X15+
9X16+17X17+238X18+448X19+49X20+661X21+18X22+166X25+
214X26+213X27+309X28+404X29+333X30+245X31+140X32+
196X33+249X34+152X35+212X36+164X37+184X38+156X39+
705X40+27X41+60X42+21X43+40X44+138X45+125X46+73X47+
51X48+27X49+166X50+336X51+106X52+138X53+20X54+
8X55+16X56+33X57+54X58+364X59+136X60+136X61+63X62+
71X63+87X64+99X65+104X66+1367X67+1055X68+1691X69+
237X72+77X73+11X77.GE.70
Protein

3. 2X1+.7X2+14.4X3+.1X4+1.7X5+.8X6+.6X7+5.1X8+2.5X9+
2.7X10+1.1X11+.4X12+.5X13+10.5X14+15.1X15+.2X16+
.6X17+1X18+16.4X19+1.7X20+1X21+.2X22+.1X25+.1X26+
.1X27+.2X28+.2X29+.2X30+.1X31+.1X32+.2X33+.3X34+.2X35+
.2X36+.1X37+.1X38+.1X39+6.8X40+.5X41+.4X42+.5X43+
1.1X44+3.7X45+4X46+2.8X47+3X48+1.1X49+3.8X50+1.8X51+
2.7X53+.4X54+.3X55+.4X56+.3X57+2X58+4X59+.2X60+.6X61+
.7X62+.6X63+1.7X64+2.5X65+2.5X66+4.2X67+3.7X68+
11.4X69+3X72+1.3X73+.5X75+10.3X76+.4X77.GE.0.8
Calcium

4. 365X1+54X2+175X3+56X4+99X5+80X6+41X7+341X8+115X9+
125X10+82X11+31X12+50X13+18X14+9X15+3X16+6X17+52X18+
19X19+3X20+48X21+8X22+34X25+32X26+33X27+46X28+62X29+
139X30+20X31+15X32+30X33+37X34+23X35+31X36+26X37+
30X38+24X39+45X40+36X41+30X42+14X43+18X44+80X45+
36X46+43X47+23X48+22X49+59X50+118X51+138X52+54X53+
10X54+8X55+8X56+12X57+65X58+134X59+16X60+45X61+38X62+
43X63+173X64+154X65+136X66+345X67+459X68+792X69+72X72+
39X73+74X75+244X76+7X77.GE.12
Iron

Constraints

5. 30.9X5+18.9X12+16.8X14+26X15+44.2X16+55.8X17+ 18.6X18+28.1X19+16.9X20+2.7X22+.2X24+.2X25+ .4X26+.4X28+169.2X30+.1X38+3.5X40+7.3X41+17.4X42+ 11.1X44+69X45+7.2X46+188.5X47+.9X48+112.4X49+16.6X50+ 6.7X51+918.4X52+290.7X53+21.5X54+.8X55+2X56+16.3X57+ 53.9X58+3.5X59+12X60+34.9X61+53.2X62+57.9X63+86.8X64+ 85.7X65+4.5X66+2.9X67+5.1X68+.2X77.GE.5 **Vitamin A**

6. 55.4X1+3.2X2+14.4X3+13.5X4+17.4X5+10.6X6+2X7+37.1X8+ 13.8X9+13.9X10+9.9X11+2.8X12+4X14+3X15+.2X17+2.8X18+ .8X19+.6X20+9.6X21+.4X22+2.1X25+2.5X26+1X28+.9X29+ 6.4X30+2.8X31+1.7X32+17.4X33+18.2X34+1.8X35+9.9X36+ 1.4X37+.9X38+1.4X39+1X40+3.6X41+2.5X42+.5X43+3.6X44+ 4.3X45+9X46+6.1X47+1.4X48+1.8X49+4.7X50+29.4X51+ 5.7X52+8.4X53+.5X54+.8X55+2.8X56+1.4X57+1.6X58+ 8.3X59+1.6X60+4.9X61+3.4X62+3.5X63+1.2X64+3.9X65+ 6.3X66+28.7X67+26.9X68+38.4X69+4X70+2X72+.9X73+ 1.9X76+.2X77.GE.1.8 **Thiamine**

7. 33.3X1+1.9X2+8.8X3+2.3X4+7.9X5+1.6X6+4.8X7+8.9X8+ 8.5X9+6.4X10+3X11+3X12+16X14+23.5X15+.2X16+6.5X18+ 10.3X19+2.5X20+8.1X21+.5X22+.5X24+2.9X25+2.4X26+ 2X27+4X28+50.8X30+3.9X31+2.7X32+2.7X33+3.6X34+1.8X35+ 3.3X36+1.8X37+1.8X38+2.4X39+4.9X40+2.7X41+3.5X42+ 1.3X44+5.8X45+4.5X46+4.3X47+1.4X48+3.4X49+3.9X50+ 7.1X51+13.8X52+5.4X53+X54+.8X55+.8X56+2.1X57+4.3X58+ 7.7X59+2.7X60+2.5X61+2.5X62+2.4X63+4.3X64+4.3X65+ 1.4X66+18.4X67+38.2X68+24.6X69+5.1X70+2.3X71+11.9X72+ 3.4X73+7.5X76+.4X77.GE.2.7 **Riboflavin**

8. 441X1+68X2+114X3+68X4+106X5+110X6+60X7+64X8+126X9+ 160X10+66X11+17X12+7X14+11X15+2X16+X18+4X19+471X21+ 5X24+69X25+87X26+120X28+316X30+86X31+54X32+60X33+ 79X34+71X35+50X36+68X38+57X39+209X40+5X41+28X42+4X43+ 10X44+37X45+26X46+89X47+9X48+11X49+21X50+198X51+33X52+ 83X53+31X54+5X55+7X56+17X57+32X58+56X59+42X60+37X61+ 36X62+67X63+55X64+65X65+24X66+162X67+93X68+217X69+ 50X70+42X71+40X72+14X73+5X75+146X76+3X77.GE.18 **Niacin**

9. 177X14+60X15+17X20+525X30+46X38+544X41+498X42+ 952X43+1998X44+862X45+5369X46+608X47+313X48+449X49+ 1184X50+2522X51+2755X52+1912X53+196X54+81X55+399X56+ 272X57+431X58+218X60+370X61+1253X62+862X63+57X64+ 257X65+136X66.GE.75 **Ascorbic Acid**