RATIONALITY, PRICE RISK, AND RESPONSE

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ABSTRACT

Risk has long been recognized as potentially important in determining agricultural supply. However, supply response models have either incorporated risk in an ad hoc manner or not at all. A rational expectations supply response model incorporating price risk is developed, an estimation procedure suggested, and an empirical example presented.

Key words: risk, supply response, rational expectations.

The role of risk in producer decision making has been recognized as a potentially important determinant of production. Sandmo has shown that competitive risk-averse firms produce a smaller output under price uncertainty than under the assumption of price certainty and that the higher the overall level of risk, ceteris paribus, the smaller the output. Batra and Ullah have shown that an increase in price risk leads to a decline in the firm's output in the case of decreasing absolute risk aversion.

If producers are assumed rational and risk averse, they should consider not only expected output prices and yields when allocating resources, but also expected variability in output prices and yields. The extent to which price and yield risks do in fact affect producer decisions is an empirical question. Given the rapid growth of literature concerned with risk in agricultural markets, it is somewhat surprising that the majority of empirical supply response models do not incorporate risk explicitly into supply or factor demand equations (e.g., Eckstein 1984, 1985; Helberger and Akinyosoye; Lee and Helberger; Shonkwiler and Emerson; Wohlgenant). After all, one of the leading arguments for agricultural price support programs is based on the assumption that by providing a guaranteed minimum price, price risk is decreased, and thus the welfare of both consumers and producers is increased.

The purpose of this paper is to develop a conceptual framework for incorporating a price risk variable into a rational expectations model of supply response. The model is an improvement over previous rational expectations models of supply response because it does not disregard the second moment (variance) of expected price. This approach represents a break with the Muthian reliance on the certainty equivalence assumption as a means for ignoring higher moments of the expectation (Sheffrin, pp. 10-11). Thus, the model allows the variability of an expectation to be reflected in supply response. When decision makers are risk adverse, this behavior will be evidenced by observed input allocations that are smaller than those implied by equating factor prices to marginal value products. Additionally, by using a rational expectations framework, expectations are no longer formed in an ad hoc manner as in earlier risk models. The measure of price risk developed in the paper is based on the variability of the expectations error, and it is closer to the theoretical concept of price risk than measures used in previous studies. An estimation procedure is illustrated with an empirical example based on sub-regional U.S. watermelon data.

REVIEW OF LITERATURE

Several researchers have incorporated price risk into supply response models, but they have generally used arbitrary, extrapolative measures of expected price risk. Traditionally, price risk has been proxied by the variance or standard deviation of output prices or returns. Behrman was the first to incorporate risk variables into econometric supply models. In Behrman's model, producers formed their price expectations adaptively. Price risk was defined as a moving standard deviation based on the past three periods for
observed prices. Ryan also specified adaptive expectations and used a definition similar to Behrman's for price risk; Bailey and Womack assumed adaptive expectations and defined price risk in terms of total price variability; while Brorsen et al. defined price risk in terms of a weighted moving-average of the absolute values of previous price changes. All these definitions preclude a direct relationship between price expectations and price risk.

Other researchers have defined price risk as a function of the difference between actual and expected prices. Just, as well as Hurt and Garcia, defined price risk as the squared deviation between actual and expected price, where expected price was based on adaptive expectations. Traill also assumed producers form expectations adaptively and defined price risk as the absolute value of the difference between expected and actual price.

Traill discussed the conceptual superiority of defining price risk in terms of the difference between actual and expected prices. It is expected price riskiness at the time production decisions are made that is important to a decision maker, not actual price variability. If a producer can forecast output prices accurately, price variability will not be associated with risk. Highly variable output prices that can be forecasted precisely will be less risky than those having less variability that cannot be forecasted with precision.

The empirical evidence is mixed as to whether increasing price risk leads to decreases in the quantity supplied. Behrman, Brorsen et al., Just, and Ryan found evidence to support this hypothesis, while the findings of Traill and Bailey and Womack were inconclusive. Studies by Traill, using aggregate data for late summer and total U.S. onion crops, and Bailey and Womack, using regional wheat data, found the estimated coefficients on risk variables had correct signs but were small relative to their standard errors. Traill suggested the results may be due to producers holding relatively stable long-run expectations about a crop's riskiness, but adjusting these upon learning new information. Thus, the long term risk effect was reflected in the intercept, and only short-run adjustments were reflected by the risk variable.

A major weakness of these earlier models was their assumption about producer expectations. Adaptive expectations are ad hoc, and since they are functions of past values of the variables being forecasted, do not allow producers to incorporate information about the system's structure or its exogenous variables into their forecasts. Rational expectations allow producers to form expectations for a subsequent period conditional upon current information contained in all exogenous variables as well as the structural relationships in the market. This approach to modeling agricultural supply has been shown by Goodwin and Sheffrin, Shonkwiler and Emerson, and Eckstein (1984, 1985) to appropriately model producer expectations and to yield results often superior to models based on adaptive expectations. Yet, the usual assumption of certainty equivalence—that only the first moments (means) of variables affect supply response—may be too restrictive since it does not allow risk to play a role in supply response.

**MODEL**

A simple rational expectations model which allows price risk to enter explicitly into supply response is developed.\(^1\) Let

\[
Q_t^e = a_1E_{t-1}(P_t) + a_2X_t + a_3E_{t-1}(R_t) + e_{1t},
\]

\[
Q_t^d = b_1P_t + b_2Z_t + e_{2t}, \text{ and}
\]

\[
Q_t^d = Q_t^e,
\]

where \(Q_t^e\) and \(Q_t^d\) are quantities supplied and demanded, respectively, at time \(t\), \(P_t\) is the price of the commodity, \(X_t\) and \(Z_t\) are exogenous variables, \(R_t\) is a measure of price risk, \(E_{t-1}\) is the expectations operator based on all information known at time \(t-1\), and the \(e_{1t}\) and \(e_{2t}\) are random error terms assumed to have zero mean. Assumptions concerning the variances of these error terms will not be made until later.

The standard assumption used in rational expectations models concerning the expectations of these exogenous variables is made (i.e., they are generated by low order autoregressive processes) as follow:

\[
X_t^* = d_1X_{t-1} \text{ and } u_{1t} = X_t - X_t^*
\]

\(^1\) Risk on yield or other competing crop prices can be incorporated into the model, but their inclusion would add little to the exposition.
and

\[(5) \quad Z^*_t = d_2Z_{t-1} \quad \text{and} \quad u_{zt} = Z_t - Z^*_t,\]

where the \(d_i \ (i=1,2)\) are parameters that may be evolving over time. The symbol * represents the expected value of exogenous variables at time \(t-1\).

Under the behavioral assumptions of rational expectations, producers know the structure of the model and solve for expected price accordingly (Wallis). Solving for \(P_t\) results in

\[(6) \quad P_t = \frac{1}{b_1}(a_1E_{t-1}(P_t) + \alpha_2X_t - b_2Z_t + a_3E_{t-1}(R_t) + e_{1t} - e_{2t}).\]

Taking the expectations of \(P_t\) at time \(t-1\), gives

\[(7) \quad E_{t-1}(P_t) = \frac{1}{b_1}(a_1E_{t-1}(R_t) + \alpha_2X^*_t - b_2Z^*_t + a_3E_{t-1}(R^*_t)).\]

To solve equation (7), the entire system of equations must be solved. Given the value of \(E_{t-1}(R_t), E_{t-1}(P_t)\) can be solved (or vice versa). However, in order to solve for a unique \(E_{t-1}(P_t)\), it is necessary to specify more about \(R_t\). One possibility is to define \(R_t\) as

\[(8) \quad R_t = (P_t - E_{t-1}(P_t))^2,\]

and when expectations at time \(t-1\) are taken, the following results:

\[(9) \quad E_{t-1}(R_t) = E_{t-1}(P_t - E_{t-1}(P_t))^2.\]

Using this definition, the risk variable in the model is the expected riskiness of price or the expected variability of the forecast error for price. This construction follows Traill by defining risk as a function of deviations between expected and actual price. However, it is actually closer to the concept of expected riskiness than the variables used by either Traill or Just because it goes a step further and defines risk in terms of the expected difference between actual and expected price. Since a producer must base his decision at time \(t-1\) on his expectations of price as well as riskiness, the appropriate variables are expected price and expected riskiness.

To solve for expected price, subtract equation (7) from equation (6), square the result and take expectations to obtain

\[(10) \quad E_{t-1}(P_t - E_{t-1}(P_t))^2 = \frac{1}{b_1}^2 [a_2^2 \sigma_{u_{1t}}^2 - b_2^2 \sigma_{u_{2t}}^2 + \sigma_{e_{1t}}^2 - \sigma_{e_{2t}}^2 - \text{cov}(e_{1t},e_{2t}) - a_2(b_2 \text{cov}(u_{1t},u_{2t}) + \text{cov}(u_{1t},e_{1t}) - \text{cov}(u_{1t},e_{2t}) + b_2(\text{cov}(u_{2t},e_{1t}) - \text{cov}(u_{2t},e_{2t}))).\]

From equation (10), the expected price risk variable as defined in equation (8) is a function of the variances and covariances of the error terms from equations (1)-(5) as well as the parameters of the structural system. As usual, the parameters and variances of the error terms of equations (1) and (2) are assumed to be constant over time. If the variances of the error terms for the exogenous variables are also assumed to be constant and if the covariances between all error terms are assumed to equal zero (or some other constant), the risk variable will also equal some constant. Under these restrictive assumptions, the risk variable will essentially be reflected in the intercept term, and the effect of risk on supply response may not be identified.

On the other hand, it seems reasonable to assume that the variances of the error terms from the forecast equations for exogenous variables are not constant but vary over time. This could be due to the fact that these stochastic processes are not stationary (Harvey). By making this assumption, the variance of expected price (the risk variable) could also vary over time. The validity of this specification can then be assessed by testing whether the structural parameter \(a_3\) in equation (1) is significantly less than zero.

### EMPIRICAL ESTIMATION

In this section, a sub-regional supply response model for U.S. watermelons is developed, and an estimation procedure is proposed and utilized to test the hypothesis that price risk has an effect on the aggregate supply response of producers. Formally, the model for watermelons is as follows.

Consider the market for watermelons grown in north-central Florida. Growers decide in the early spring how much acreage to allocate to watermelon production based on their expectations about costs and returns. These expectations may encompass the expected variability of returns as well. Melons
are harvested and marketed in June with most shipped to the northeastern U.S.

This market can be modeled in natural logarithms with the simple three equation system:

11 ln At = a0 + a1 ln \(E_{t-1}(P_t/W_t^*)\) + a2 \(E_{t-1}(R_t)\) + a3 ln At-1,

12 ln Yt = b0 + b1 ln \(P_t/W_t\) + b2 Tt, and

13 ln \(P_t/D_t\) = c0 + c1 ln \(A_tY_t/N_t\) + c2 ln DI_t + c3 TEMP_t + c4 DV_t,

where

\(P_t\) = average price of watermelons received by growers in north-central Florida in the month of June ($/cwt);

\(A_t\) = acreage planted to watermelons in north-central Florida (1,000 acres);

\(W_t\) = agricultural wage index on July 1;

\(R_t\) = price risk as defined in equation (8);

\(Y_t\) = yield of watermelons per planted acre in north-central Florida;

\(T_t\) = marketing year, 62-84;

\(N_t\) = U.S. population, millions July 1;

\(D_t\) = personal consumption expenditures price deflator, second quarter (1972 base);

\(DI_t\) = per capita disposable personal income deflated by \(D_t\), second quarter;

\(TEMP_t\) = deviation of average June temperature from mean at Central Park, New York City;

and

\(DV_t\) = dummy variable; zero until 1975, one afterwards.

The specification of the model is predicated on the following rationale. Producers allocate acreage to watermelon production based on expected returns. These are represented by the ratio of expected prices to expected costs and are denoted by \(E_{t-1}(P_t)\) and \(W_t^*\) (labor costs are the principal costs incurred in planting and harvesting watermelons), respectively. Expected variability in output price is captured by the variance in expected price. Lagged acreage is included to represent par-
the forecast variance of the shifters. Let \( \sigma^2_{uit} \) denote the forecast variance of the \( i \)th exogenous variable at time \( t \). Let the subscript \( r \) denote the seven data periods from \( s \) through \( t - 1 \) over which the exogenous variables are estimated. The forecast variance can then be written as

\[
\sigma^2_{uit} = a^2_{ui_t}(1 + X_t (X'_r X_r)^{-1} X'_t).
\]

Forecast variances are updated over the sample period to reflect the information available to market participants at any point in time. To update the variance forecast to \( \sigma^2_{uit+1} \), drop the earliest period \( s \) so that \( s + 1 \) becomes the first observation and add the observation at \( t + 1 \) to complete the seven period series. This appears apparent from equation (14) by replacing \( t \) with \( t + 1 \) and \( \tau \) with \( \tau + 1 \) to denote periods \( s + 1 \) through \( t + 1 \).

The structural equations were estimated as suggested by Wallis. That is, the forecasts of the exogenous variables were estimated separately and then were used in estimating the structural parameters of equations (11), (12), and (13). Unlike Antonovitz and Roe whose expected price and its variance were estimated by an ARIMA model, the simultaneous system was solved for expected price (as illustrated in equation (7)) and its variance (as illustrated in equation (10)) in terms of the model’s structural parameters, its expected exogenous variables and their variances. These highly nonlinear expressions then replaced \( E_{t-1}(P_t) \) and \( E_{t-1}(R_t) \) and system-wide estimation was attempted.

Because the variance of the expected price was proposed to be a function of the forecast variances of the shifters, the parameters of the model, and the variances of the error terms on the structural equations, the model is highly nonlinear. As a result, parameter estimates could not be calculated due to lack of convergence in the maximum likelihood estimation routine. Thus, the model was simplified to the extent that the parameter on the variance of the expected price was not restricted to depend on the structural variances since these are assumed constant. Also all forecast variances other than that of \( W_t^* \) were assumed to be zero, thus reducing the number of nonlinear parameters.

The acreage equation was then estimated as

\[
\ln A_t - \ln A_{t-1} = a_0 + a_1 \ln (E_{t-1}(P_t)/W_t^*) + a_2 (E_{t-1}(R_t) - E_{t-2}(R_{t-1})).
\]

Thus, the change in the variance of expected price is used in the empirical example to represent a change in price risk. There are reasons other than the desire to simplify the estimation procedure which may lead one to estimate equation (15) instead of (13). Just hypothesized that producers respond to changes in risk. It seems reasonable that the overall level of risk affects a producer’s choice of the commodity or commodities to produce, but once the commodity decision has been made, it is changes in risk that affect short-run, year-to-year production decisions as to how much of the commodities chosen should be planted and harvested. That is, the overall level of risk affects long-run choices regarding which commodities to produce, but changes in risk affect acreage allocation at the margin.

Parameter estimates and their associated standard errors are reported in column 4 of Table 1. The maximum likelihood method of estimation as employed by the Time Series Processor (TSP) statistical package (Hall) is used. The results indicate that changes in price risk significantly affect the amount of acreage allocated to the production of watermelons. In the acreage equation, the coefficient on expected returns, \( \ln (E_{t-1}(P_t)/W_t^*) \), is positive as expected and significant at the .05 level using a one-tailed test, while the coefficient on the change in the risk variable is negative and highly significant. Size of the coefficient shows that supply is quite responsive to changes in price risk as hypothesized. All estimated coefficients in the yield equation have the expected signs and are significant at the .05 level. The positive coefficient on the log of returns \( (\ln (P_t/W_t^*)) \) indicates that as the price of melons increases (decreases) relative to the wage rate, producers harvest more (less) thoroughly. The coefficient on the log of returns \( (\ln (P_t/W_t^*)) \) is but significant improvement in plant varieties and production practices over time.

The coefficients on quantity, \( c_1 \), and on temperature, \( c_2 \), from the demand equation
have signs as expected and are significant at the .05 level. $C_1$ is negative and implies a price elasticity of $-0.424$. This estimate is bounded by estimates of Suits ($-0.901$) and of Wold ($-0.206$). The coefficient on per capita disposable income is positive as expected and significant at the .10 level.

### Table 1. Maximum Likelihood Estimates of Structural Parameters for Supply Response of Watermelon Production in North-Central Florida, 1962-84

<table>
<thead>
<tr>
<th>Equation</th>
<th>Parameter</th>
<th>Variable</th>
<th>Unrestricted</th>
<th>Restricted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acreage</td>
<td>$a_0$</td>
<td>Constant</td>
<td>$-2.340$</td>
<td>$-7.857$</td>
</tr>
<tr>
<td></td>
<td>$a_1$</td>
<td>$\ln(\frac{E_{t-1}(P_t)}{W_t})$</td>
<td>$1.327$</td>
<td>$4.846$</td>
</tr>
<tr>
<td></td>
<td>$a_2$</td>
<td>$E_{t-1}(R_t)-E_{t-2}(R_{t-1})$</td>
<td>$-302.920$</td>
<td>$-302.920$</td>
</tr>
<tr>
<td>Yield</td>
<td>$b_0$</td>
<td>Constant</td>
<td>$-1.239$</td>
<td>$-1.278$</td>
</tr>
<tr>
<td></td>
<td>$b_1$</td>
<td>$\ln(\frac{P_t}{W_t})$</td>
<td>$1.109$</td>
<td>$1.188$</td>
</tr>
<tr>
<td></td>
<td>$b_2$</td>
<td>$T$</td>
<td>$0.244$</td>
<td>$0.203$</td>
</tr>
<tr>
<td>Demand</td>
<td>$c_0$</td>
<td>Constant</td>
<td>$5.854$</td>
<td>$7.344$</td>
</tr>
<tr>
<td></td>
<td>$c_1$</td>
<td>$\ln(\frac{A_tY_t}{N_t})$</td>
<td>$-2.361$</td>
<td>$-3.528$</td>
</tr>
<tr>
<td></td>
<td>$c_2$</td>
<td>$\ln(D_t)$</td>
<td>$1.173$</td>
<td>$-207$</td>
</tr>
<tr>
<td></td>
<td>$c_3$</td>
<td>$\text{Temp}_t$</td>
<td>$0.004$</td>
<td>$0.006$</td>
</tr>
<tr>
<td></td>
<td>$c_4$</td>
<td>$DV_t$</td>
<td>$840$</td>
<td>$1.757$</td>
</tr>
</tbody>
</table>

* Standard errors are in parentheses.

The finding that risk has a significant and negative effect on acreage response has important implications in estimating supply response models. It is well known that in a single equation model, omitting a relevant variable which is correlated with other included regressors leads to biased parameter estimates (Schmidt, pp. 39-40). In a system of equations, the implications are even more serious since omitting a relevant variable can potentially lead to biased parameter estimates in other equations in the model (White).

To see whether omitting expected price risk from the acreage equation significantly changes the parameter estimates, a restricted model was estimated setting $a_2 = 0$. Parameter estimates and associated standard errors are reported in column 5 of Table 1. The differences between the restricted and unrestricted models are considerable. In the restricted model, the estimated parameter $a_1$ on expected returns in the acreage equation more than triples in size but is no longer significantly different from zero. The parameter estimates in the yield equation change only slightly with yield being more responsive to returns. The changes in parameter estimates in the demand are also substantial. The coefficient on per capita disposable income has changed signs from positive to negative, but it is no longer significant; nor is the coefficient on temperature. The coefficient $c_1$ on quantity has increased absolutely and indicates a substantially more inelastic demand function. Additionally, a likelihood test between the restricted and unrestricted models yields $\chi^2(1) = 7.2$, and the hypothesis that expected price risk has no effect on acreage response (i.e., $a_2 = 0$) can be rejected at any conventional level of significance.

### CONCLUSIONS

In this paper it is argued that price risk may have an effect on supply response of producers. A model assuming rational expectations is developed which allows empirical testing of this hypothesis. An estimation procedure is developed for the model, and a supply response model for sub-regional U.S. watermelons is estimated for illustrative purposes. The results show that changes in risk have a negative effect on the annual changes in acreage allocated to watermelon production. That is, as price risk increases, quantity supplied by producers decreases.

The obvious question is whether the results from a single study of an unregulated commodity market can be generalized to other markets, particularly those regulated by governmental policy. Although this is an empirical question, unless producers in regulated markets are less risk adverse than those in the watermelon market in north-central Florida, one would expect similar response to price risk. Policy makers and agricultural economists have believed for decades that price risk affects production of agricultural commodities, and they have often implemented price stabilization programs by arguing that these programs increase benefits to both producers and consumers. If the
results found for the watermelon market do indeed hold for other crops, pricing policies that stabilize prices for producers would tend to increase production. In a period of unwanted surpluses, this type of policy may not be appropriate.

The implications of this model and the results found in this paper seem to open up a large agenda for further research. If risk is important for aggregate supply response, then models that do not incorporate risk explicitly may give biased results. Further research on the importance of assumptions concerning expectation formulation by producers would also be of importance. Conceptually, this would seem to be extremely important as it is likely that producers do use their full information set in forming expectations.

REFERENCES


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