AGRICULTURAL ECONOMICS

SP-80-8 July 1980

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AAEA paper presented at its annual meetings,

~~Urbana, Il July 27-30, 1980~~

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Presented as a Selected Paper at the American Agricultural Economics Association Annual Meetings, University of Illinois, July 28-30, 1980

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Abstract

Conditions are derived for determining the product volumes which expected utility maximizing firms would sell or buy under alternative contract arrangements. Supplies and demands under each arrangement are related to contract parameter levels. An application is made to negotiations between farmer and processor concerning terms of fixed-price, forward deliverable contracts.

SUPPLY AND DEMAND CONSIDERATIONS OF AGRIBUSINESS MARKETING PORTFOLIOS

In the past several decades, agricultural economists have made wide use of programming techniques to generate risk-efficient sets of farm plans and marketing portfolios. The most frequently employed tools have been E-V or MOTAD analysis (Ward and Fletcher; Barry and Willmann; Schurle and Erven) and stochastic dominance criteria (Anderson; Hardaker and Tanago). A principal limitation of these studies is that the portfolios identified as risk-efficient, and the probability density functions of risk-efficient portfolios, may change dramatically as parameters of alternative portfolio options are varied (Buccola and French). In the case, for example, of marketing portfolios, new risk-efficient frontiers, and hence new maximum-expected-utility solutions, must be developed each time new terms of trade are offered by a prospective buyer or seller.

The present paper examines the implications for optimal marketing portfolios of changes in the parameters of individual portfolio options. Attention is paid to both sales and purchases of agricultural commodities. Conditions are then explored under which equilibrium trade would be determined between a risk averse seller and a risk averse buyer of such commodities.

The Seller

Consider first a firm which plans to produce A units of a good and which has the choice of dividing the sale of these units between two possible sales arrangements or options. The first option may be a prior agreement under which the seller would be reimbursed for some proportion of his production costs, or be reimbursed at a stated, pre-fixed price per unit. The second option may be a similar agreement to be reimbursed at whatever market price prevails at the time of transaction, or a decision to sell on the open market itself. A wide class of marketing arrangements may be defined by expressing them in the form of a price base X and an associated price parameter k. For instance, the price received under the above cost-plus arrangement would be kX, where X is seller's per-unit cost of production (a random variable) and k is some positive factor greater than one. In the case of a fixed-price arrangement, both k and X are stated constants. Under the market price option, X is market price (a random variable) and k equals one.

Defining X_1 as the first or nonmarket price basis, X_2 as market price, k as the parameter associated with X_1, P as the proportion of A sold under the first option, X_3 as per-unit variable costs, and F as per-unit fixed costs, the seller's profit π_s is

$$(1) \qquad \pi_s = A[PkX_1 + (1 - P)X_2 - X_3 - F], \qquad 0 \leq P \leq 1.$$

Freund has shown that if the decision maker is a risk averter with negative exponential profit utility $U = -\exp(-\lambda\pi)$, and if returns are normally distributed, the decision maker's expected utility may be maximized by maximizing $Z = E(\pi) - (\lambda/2)\mathrm{var}(\pi)$. In order to incorporate these assumptions, let us denote the probability distributions of X_i as $N(m_i, s_i^2)$, $i = 1,2,3$, and the covariances of X_i, X_j pairs by s_{ij}, $i \neq j$. Assuming the variance of fixed costs F to be zero, function Z for the seller is

$$(2) \qquad Z = A[a + (-m_2 + m_1k)P] - (\lambda/2)A^2[b + (2c + 2dk)P + (s_2^2 - 2s_{12}k + s_1^2k^2)P^2)]$$

where $a = m_2 - m_3 - F \lesseqgtr 0$; $b = s_2^2 + s_3^2 - 2s_{23} \geq 0$; $c = -s_2^2 + s_{23} \gtreqless 0$; and $d = s_{12} - s_{13} \gtreqless 0$.

The portfolio proportion P* at which seller expected utility reaches an extreme value is found by setting the derivative of (2) with respect to P equal to zero and solving for P:

$$(3) \qquad P^* = \frac{(-m_2 + m_1k) - \lambda A(c + dk)}{\lambda A(s_2^2 - 2s_{12}k + s_1^2k^2)}, \qquad 0 \leq P^* \leq 1.$$

Function Z, and thus expected utility, is maximized at this point because $\partial^2Z/\partial P^2 = -\lambda A^2(s_2^2 - 2s_{12}k + s_1^2k^2)$, a nonpositive quantity regardless of the value of k. Hence, the seller would maximize expected utility if he could sell AP* of his product under the first contract arrangement and A(1 - P*) of his product at open market prices. Expressed in other words, AP* is the quantity supplied by the seller under the first contract arrangement. In general, we may define the seller's portfolio supply curve for the first contract arrangement to be the set of proportions P* that are generated as price parameter k is varied and λ, A and all other parameters are held fixed. P* is a rather complex function of k, but an insight into the behavior of the supply curve is provided by analysis of the responsiveness of P* to the risk aversion parameter λ.

It is useful for this purpose to define the "equi-mean" value of price parameter k as that value for which the expected sale price under the first contract arrangement equals the expected market price, that is, the value of k for which $m_1 k = m_2$, or $k = m_2/m_1$. The partial derivative of P*, the optimal portfolio share of the first option, with respect to λ is

$$\partial P^*/\partial\lambda = \frac{m_2 - m_1 k}{\lambda^2 A(s_2{}^2 - 2s_{12}k + s_1{}^2 k^2)} \quad (4)$$

Where $k < m_2/m_1$, expression (4) is positive (since the denominator is positive unless $r_{12} = 1$), meaning that P* rises with λ. The opposite is the case where $k > m_2/m_1$. On the other hand if $k = m_2/m_1$, $\partial P^*/\partial\lambda = 0$ and differences in risk aversion have no effect on optimal portfolios.

The result is that if k is below equi-mean (and thus the first sale option offers lower expected return than market prices), increases in the risk aversion coefficient increase relative preference for the first sale option. If k is above equi-mean, the impact of higher risk aversion is to decrease relative preference for the first sale option. At the equi-mean point, risk aversion has no impact on portfolio preference. As illustrated in figure 1, this in turn implies that increases in the risk aversion parameter cause the portfolio supply curve to rotate counter-clockwise around the price parameter's equi-mean value. Increasing risk aversion decreases the sensitivity with which optimal portfolios react to changes in a price parameter.

A special case of the seller's market portfolio problem is that in which a farmer has the choice at planting time of forward contracting a portion of his crop at a fixed and known sale price; the remainder can be sold after harvest at the prevailing market price which, at planting time, is a random variable (Barry and Willmann; Eidman, Dean, and Carter).

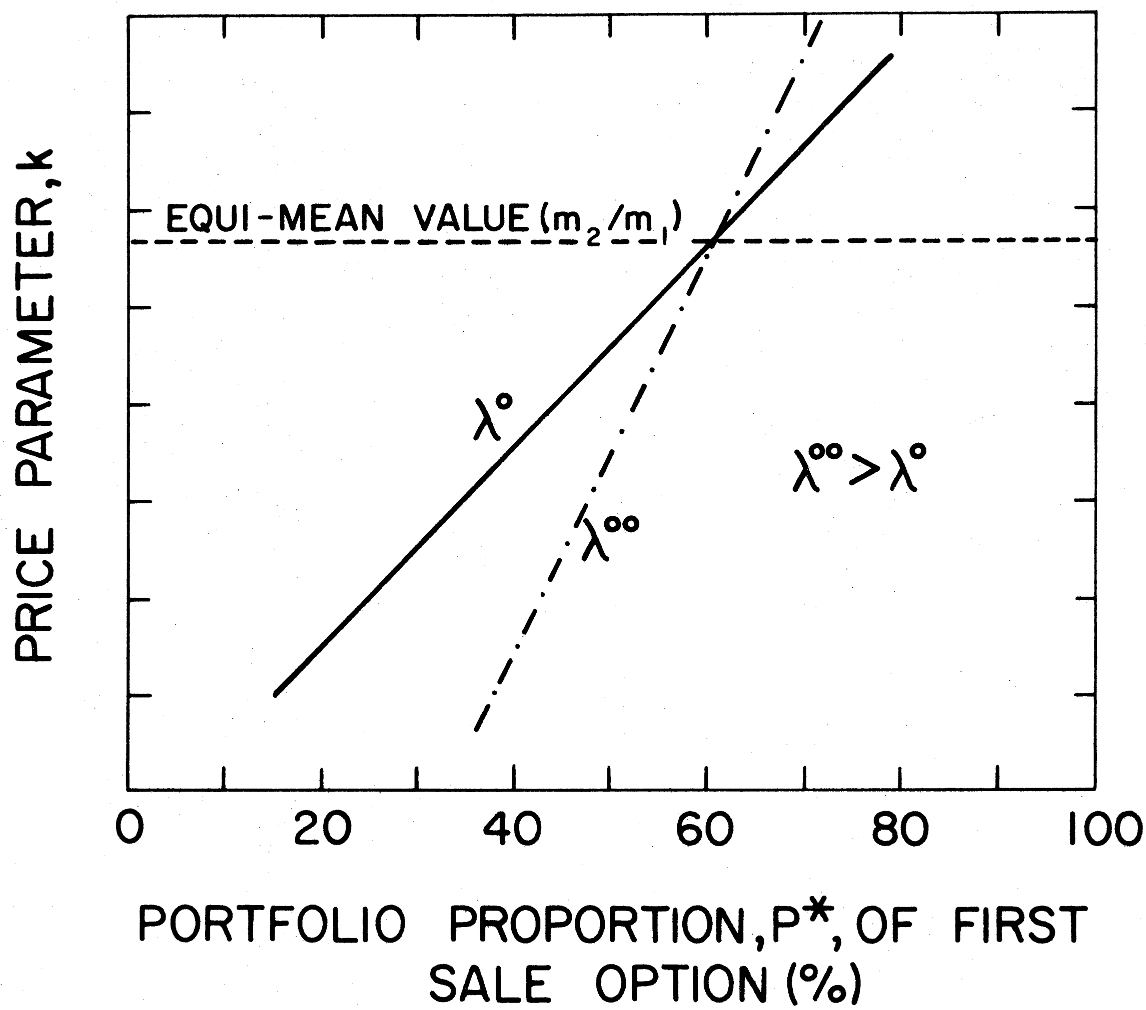

FIGURE I. ILLUSTRATION OF HYPOTHETICAL PORTFOLIO SUPPLY CURVE ROTATION AS RISK AVERSION COEFFICIENT λ RISES.

Under our terminology the promised price is m_1k. Because it is a legally or morally binding price, the variance of price basis X_1 (that is s_1^2), and all covariances involving the first price basis (s_{12} and s_{13}), are close to zero, so that (3) reduces approximately to

$$(3)' \qquad P^* = \frac{-m_2 + m_1k - \lambda Ac}{\lambda As_2^2} = \frac{-m_2 + m_1k}{\lambda As_2^2} + 1 - \frac{s_{23}}{s_2^2} .$$

Note that the impact on P*, the optimal share of sales allocated to fixed-price contracts, caused by marginal changes in the fixed price m_1k, is the positive constant $1/\lambda As_2^2$. This has several implications. First, the supply relation between the fixed price level and the proportion of sales optimally offered at a fixed price is positively-sloped and linear. Second, this supply relation is relatively inelastic for decision makers who are strongly risk averse (high λ) and also for those who perceive market prices to be very unpredictable (high s_2^2). Interestingly, extreme risk aversion does not by itself ensure that all sales will be optimally allocated to the fixed-price option. As risk aversion coefficient λ in (3)' becomes large, P* approaches $1 - s_{23}/s_2^2$, $0 \leq P^* \leq 1$, a proportion less than one as long as market price variance s_2^2 is not infinite and covariance s_{23} is positive.

The Buyer and Market Equilibrium

The buyer situation comparable to the above analysis involves a firm that wishes to purchase A units of a good and has the option of dividing its purchases between two alternative purchase options. It is assumed here that the buyer transforms the purchased good in some way and resells it. Defining, on a raw-product-equivalent basis, X_4 as the per-unit resale price, X_5 as the per-unit variable cost of transformation, G as the per-unit fixed cost of transformation, R as the proportion of goods purchased under the first purchase option, and all other symbols as previously, the buyer's profit π_b may be represented as

$$(5) \qquad \pi_b = A[X_4 - RkX_1 - (1 - R)X_2 - X_5 - G], \qquad 0 \leq R \leq 1.$$

If returns are normally distributed and the buyer has negative exponential profit utility with risk aversion parameter $z > 0$, it may be shown analogously to the above that the optimal portfolio proportion R* allocated to the first purchase option is

$$(6) \qquad R^* = \frac{(m_2 - m_1k) - zA(g + hk)}{zA(s_2^2 - 2s_{12}k + s_1^2k^2)}$$

where $g = -s_2^2 + s_{24} - s_{25} \gtreqless 0$ and $h = -s_{14} + s_{12} + s_{15} \lesseqgtr 0$. Equation (6) is the buyer's demand function for the first purchase option and can be analyzed in a similar way to the seller's supply curve. When the first option is a fixed price option, for example, (6) reduces approximately to $R^* = (m_2 - m_1k)/zAs_2^2 + 1 - (s_{24} - s_{25})/s_2^2$.

It is instructive to develop conditions under which seller and buyer would agree on the portfolio proportions to be allocated to each marketing option. Assuming the two traders have equal bargaining power and identical subjective probability distributions of revenue and cost terms, the price parameter k_e for which trade agreement is reached is that value for which P* in (3) equals R* in (6). Defining $\alpha = z/\lambda$, this is equivalent to the value of k for which

$$\frac{(-m_2 + m_1 k) - \lambda A(c + dk)}{\alpha\lambda A(s_2^2 - 2s_{12}k + s_1^2 k^2)} = \frac{(m_2 - m_1 k) - zA(g + hk)}{\alpha z A(s_2^2 - 2s_{12}k + s_1^2 k^2)} .$$

Cross-multiplying, cancelling terms $(s_2^2 - 2s_{12}k + s_1^2 k^2)$ on each side of the equality, and solving for k_e gives

$$(7) \quad k_e = \frac{m_2(1 + \alpha) - zA(g - c)}{m_1(1 + \alpha) - zA(h + d)}$$

$$= \frac{m_2(1 + \alpha) - zA(s_{24} - s_{25} - s_{23})}{m_1(1 + \alpha) - zA(s_{14} - s_{15} - s_{13})} .$$

If the revenue and cost variables in the seller's and buyer's profit functions are zero correlated, covariance terms in (7) drop to zero and the equilibrium price parameter level k_e becomes m_2/m_1, that for which the two market options have equal expected value. In general, however, the magnitude and sign of the ratio $(s_{24} - s_{25} - s_{23})/(s_{14} - s_{15} - S_{13})$ determines whether equilibrium price parameter k_e exceeds, equals, or falls short of this equi-mean value. When the first option is a fixed-price forward contract, (7) reduces approximately to

$$(7)' \quad k_e = \frac{m_2(1 + \alpha) - zA(s_{24} - s_{25} - s_{23})}{m_1(1 + \alpha)} .$$

The equilibrium fixed price $m_1 k_e$ then exceeds, equals, or falls short of expected market price m_2 according as covariance s_{23} algebraically exceeds, equals, or falls short of covariance difference $(s_{24} - s_{25})$.

An Application

Contract negotiations at planting time between California producers of processing tomatoes and a tomato processor were used to illustrate some of the relationships developed above. In the application shown here, a representative producer is considered to optimally allocate a proportion P* of his tomato acreage to fixed-price-per-ton forward deliverable contracts and the remaining proportion (1-P*) to market price contracts. The sole processor is considered to optimally allocate a proportion R* of the tomato acreage he contracts to fixed-price terms and the remainder (1-R*) to market price terms. Note that since acres rather than tons are contracted in this instance, some yield risk remains in the fixed-price contract and it is not strictly true that $s_1^2 = s_{12} = s_{13} = 0$. However, the estimated coefficient of variation of tomato yields was very small (less than 3%) and considered negligible for our purposes.

The processor was assumed to negotiate identical contract arrangements with 22 producers who cultivated an average of 1,333 acres of tomatoes each. Estimated risk aversion parameters of processor and representative producer were $z = .000045$ and $\lambda = .0012$, respectively, where income is expressed in $1,000 units. In order to make the exposition most meaningful, parameter m_1 was set equal to the expected per-acre variable cost of tomato production, and per-acre fixed price m_1k was varied by varying k. Since m_2, the expected per-acre market value of tomatoes, was greater than m_1 by a factor of 1.422, a forward contract's fixed price per ton equalled the expected tomato market price per ton when k equalled 1.422.

Supply and demand relations for fixed-price contracts under these circumstances are shown in figure 2. In the first situation, depicted by solid lines, the covariance used between market prices of raw tomatoes and market prices of processed tomato products (s_{24}) was as empirically estimated. Since this covariance was positive (r = .514) and considerably greater than the sum of covariances between tomato market prices and processor and producer variable production costs ($s_{25} + s_{23}$), the equilibrium fixed price level $m_1 k_e$ was, by equation (7)', less than expected market value m_2. In the second situation, covariance s_{24} was set at zero with the result that the processor-buyer's demand curve shifted to the right. Since quantity $zA(s_{25} + s_{23})$ is a relatively small number, the new equilibrium solution was, as verified in (7)', such that the fixed tomato price approximately equalled the expected market value of tomatoes. For the third situation, covariance s_{24} was assigned an arbitrary negative value (r = -.179); the buyer demand curve shifted further to the right and no equilibrium solution was achieved. Further buyer-seller negotiations would, if conducted on an equal-market-power basis, result in a fixed price level somewhere above the expected market value.

Conclusions

Explicit representation of decision makers' expected utility functions provides us with important insights into rational portfolio selection behavior. A decision maker's demand for each market portfolio option considered is a function of the price parameter associated with that option.

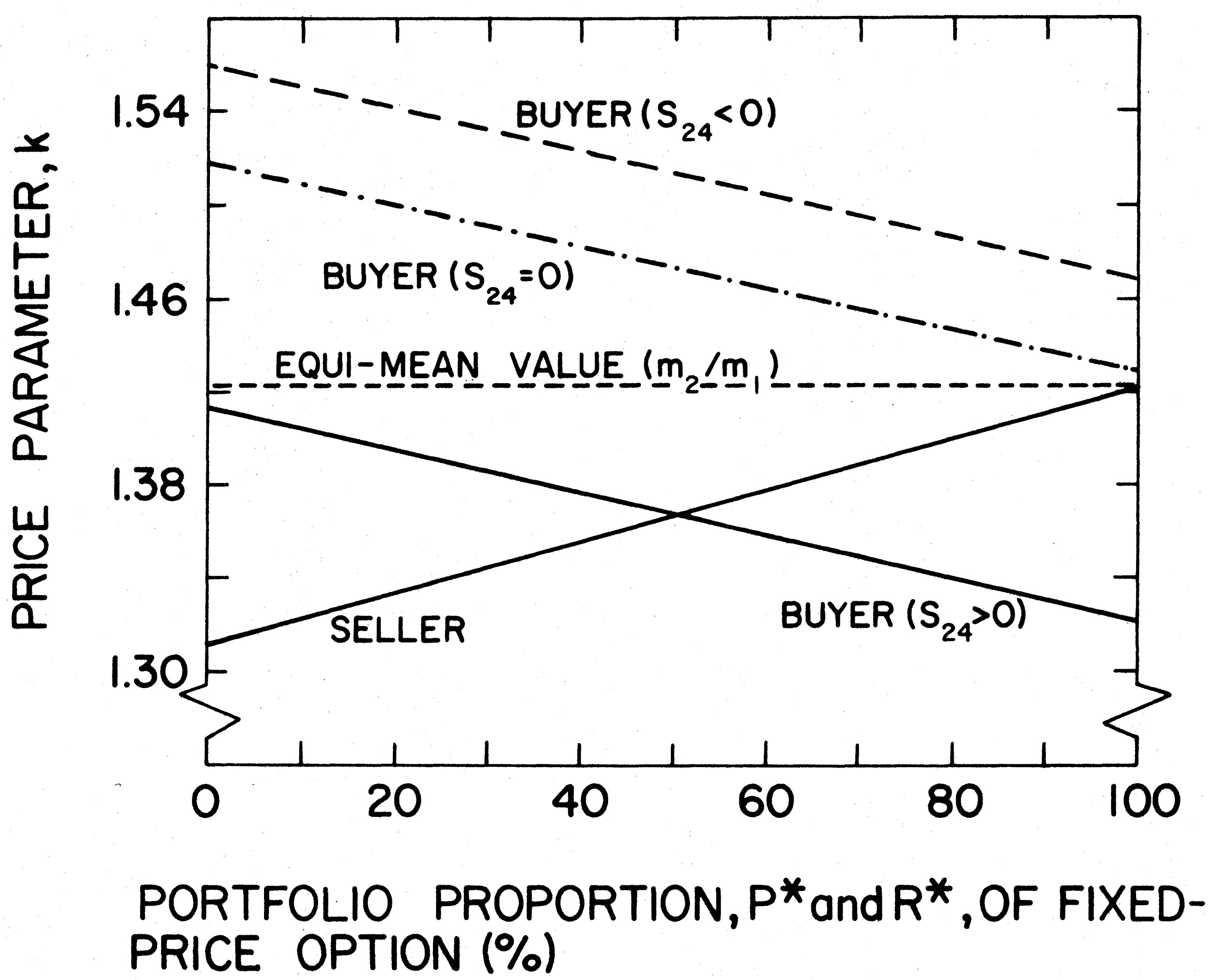

FIGURE 2. PORTFOLIO SUPPLY AND DEMAND CURVES, AND EQUILIBRIUM PORTFOLIO AGREEMENT, WHEN THE ALTERNATIVE MARKETING OPTIONS ARE FIXED PRICE AND MARKET PRICE

Quantities that sellers demand under each option may be expressed in the form of positively-sloped portfolio supply curves, and quantities that buyers demand may be expressed as negatively-sloped portfolio demand curves. Mutually agreeable marketing portfolios can then be determined using standard equilibration methods.

An important application of this analysis to present-day farmer market contracting is that in which farmers are faced with the choice at planting time between sales at fixed-price, forward deliverable contracts and sales at eventual open market prices. The example cited here suggests that mutually agreeable forward deliverable contract prices are highly sensitive to the covariance the buyer perceives between raw product and finished product market prices, and to the covariance the seller perceives between raw product market prices and farm production costs. Other than by reference to these covariances, and to the risk aversion of the buyer relative to the seller, it is generally impossible to determine whether fixed prices negotiated on an equal-market-power basis will be above, equal to, or below expected market prices.

This analysis could usefully be extended in a number of different directions. Consideration might be given to a multitude of sellers and buyers, to alternative utility functional forms, or to risk seeking behavior. The assumption of equal bargaining power might also be relaxed while retaining the above risk-theoretic elements.

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