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JUN 11 1980



## A PROFIT-MAXIMUM MANAGEMENT MODEL FOR BEEF CATTLE BACKGROUNDING AND GRAZING OPERATIONS

by

Warren B. Jessee and Steven T. Buccola

Virginia Polytechnic Institute and State University  
Blacksburg, Virginia

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Research Division Bulletin 154

December 1979

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#### ACKNOWLEDGMENTS

We wish to thank Ralph Kline, David Kenyon, and Ernest Bentley of the Department of Agricultural Economics, Virginia Polytechnic Institute and State University, for their guidance in the preparation of this manuscript. Any shortcomings in the paper remain our own responsibility.

## I. INTRODUCTION

After weaning, most cattle destined for the feedlot are retained on a farm for a certain period and, depending upon the season, either grazed out or backgrounded on a ration of roughages and concentrates. Many states which have not historically enjoyed comparative advantages in cattle finishing can profitably specialize in beef cattle grazing and backgrounding as well as cow-calf production. The backgrounding process requires careful planning because profit-maximum diets may vary greatly according to animal condition and age and according to feed prices and time of the year.

A model of a beef backgrounding and grazing operation, developed and utilized to identify profit-maximum farm plans under a variety of feed and cattle price conditions, is the subject of this report. The model was designed to represent crop and livestock production decisions made at the beginning of each of six consecutive quarters. During the first two quarters, decisions are made regarding crops grown, acres devoted to each crop, and crop fertilization and other input levels. During the last four quarters, decisions are made regarding cattle purchase and sale weights, rates of cattle gain, cattle rations, and forage production levels. The model allows the operator to carry steers or heifers and to hire labor and borrow capital in addition to that available initially on the farm. Cattle feed requirements were based on the California net energy system, and cattle purchases and sale prices per pound were permitted to vary with cattle weight.

In the specific applications of the model reported here, Virginia crop and livestock production cost data were utilized to develop:

- (1) profit-maximizing resource mixes for production of feed inputs;
- (2) profit-maximizing feed production and cattle feeding responses to alternative feed production costs and feed prices;
- (3) profit-maximizing cattle production response to alternative feeder cattle prices.

Thus, under the price structures employed, the model applications can be used to suggest successful farm plans for selected areas of Virginia.

And more generally, they can be used to gain insight into the sensitivities of such plans to changes in crop and livestock prices.

## II. MODEL FORMULATION

### Considerations in Developing an Analytical Model

To be able to make economically sound decisions, cattle backgrounding operators need to develop guidelines for the efficient utilization of farm resources such as labor, capital, and feed. Workers may be hired to supplement the operator's own labor, and capital requirements may be met either through borrowing activities or through utilization of the operator's own funds. Feed ration formulation is a crucial element of beef farm management, because rations affect rate of gain and degree of finish. Constituents of the feed ration can either be purchased or produced on the farm; if the latter, the operator must decide what and how much of each crop to produce. Two of the important controllable factors influencing yields are rate of fertilization and selection of soil type. Optimal levels of fertilization need to be simultaneously determined along with other management decisions.

More concisely, a realistic model of cattle production on the farm should include the following: (1) livestock purchase, sale, and production activities with variable rates of gain and variable purchase and sale weights; (2) feed purchase, sale, and production activities at variable levels of fertilization and on alternative soil classes; (3) capital borrowing activities; and (4) the option of hiring additional labor. The representation of these four types of activities in our model, and associated constraint specifications, are discussed below.

Livestock Purchase, Sale, and Production Activities. The model specifies separate purchase activities for each of several different animal weight categories and for each sex (steers and heifers). Once purchased, the feeder animal is transferred to livestock production activities where ration components and average daily gain for the backgrounding period are determined. The animal is then transferred either to the next backgrounding period's livestock production activities or to the current period's livestock sale activities.

Feed Purchase, Sale, and Production Activities. Purchased feeds are transferred directly to the livestock production activities. Feeds grown on the farm may either be sold or consumed by the operator's livestock. Hence, dry matter produced in the feed production activities is transferred either to the livestock production activities or to the feed sale activities.

The model also determines optimal rates of fertilization for each of the feed production activities. It is assumed that rates of fertilizer application are positively and linearly related to yields. For each of the feed production activities, there is a fertilizer use activity that, if selected, increases the crop's dry matter production level per acre.

Capital Borrowing and Labor Hiring Activities. The operator is assumed to have a certain amount of labor and capital at his disposal. Additional capital and labor are provided through specification of borrowing and hiring activities. These activities, respectively, supply capital and labor to feed and cattle purchase and production activities in each time period.

Constraint Specifications. Various constraints or restrictions must be satisfied in the model solution. The constraints utilized ensure that: (a) land, labor, and capital usage do not exceed amounts available to the farmer; (b) each beef animal's nutritional requirements are satisfied without exceeding its stomach capacity; (c) feed production or purchase is at least as great as the amount of feed consumed by all beef animals; and (d) productive resources not sold or utilized in one period are available for use in subsequent periods.

### The Analytical Model

#### Definition of Terms

The model used in this study is presented here in mathematical form. Although activities for both steer and heifer production are included, in the terms presented below cattle sex distinction is avoided for simplicity.

Indices. The indices used in the model are defined as:

- j: quarterly periods in the model,  $j=1, 2, \dots, n_j$ ;
- m: potential cropping activities,  $m=1, 2, \dots, n_m$ <sup>1/</sup>;
- k: weight classes of steers or heifers that can be purchased, produced, or sold,  $k=1, 2, \dots, n_k$ ;
- r: quarterly rates of gain that may be achieved in steer or heifer production activities,  $r=1, 2, \dots, n_r$ ;
- q: feeds that can be purchased,  $q=1, 2, \dots, n_q$ <sup>1/</sup>;
- $\ell$ : feeds that can be sold,  $\ell=1, 2, \dots, n_\ell$ <sup>1/</sup>;
- i: farm land classifications,  $i=1, 2, \dots, n_i$ .

Activities. The activities in the model are defined as:

- $X_{krj}$ : number of steers or heifers in the kth weight class gaining weight at the rth level of weight gain during the jth period, in number of head;<sup>2/</sup>
- $Y_{mij}$ : amount of land devoted to the mth crop on the ith land class during the jth period, in acres;
- $Y_{P_{qj}}$ : amount of the qth feed purchased during the jth period, in pounds dry matter;
- $Y_{S_{\ell j}}$ : amount of the  $\ell$ th feed sold during the jth period, in pounds dry matter;
- $X_{P_{kj}}$ : number of steers or heifers in the kth weight class purchased during the jth period, in number of head;
- $X_{S_{kj}}$ : number of steers or heifers in the kth weight class sold during the jth period, in number of head;
- $L_j$ : amount of labor hired during the jth period, in hours;
- $CAP_j$ : operating capital borrowed during the jth period, in dollars;
- $F_{mij}$ : amount of fertilizer purchased and applied to the mth crop produced on the ith land class during the jth period, in pounds;

---

<sup>1/</sup>When  $\ell=q=m$ , each refers to the same feed.

<sup>2/</sup>The various levels of daily gain and the various cattle weight classes must be defined such that, if an animal of the kth weight class is produced at the rth rate of gain, the animal belongs to the (k + r)th weight class at the end of the period.

- $T_{mj}$  : dry matter of the  $m$ th feed transferred from the  $j$ th period to the  $(j + 1)$ th period, in pounds;
- $NEM_{mj}$  : dry matter of the  $m$ th feed used for cattle weight maintenance during the  $j$ th period, in pounds;
- $NEG_{mj}$  : dry matter of the  $m$ th feed used for cattle weight gain during the  $j$ th period, in pounds;
- $TRC_j$  : operating capital transferred from the  $(j - 1)$ th period to the  $j$ th period, in dollars.

Price and Cost Coefficients. The price and cost coefficients used in the objective function are defined as:

- $PC_{kj}$  : steer or heifer sale price (less sale costs) in the  $k$ th weight class during the  $j$ th period, in dollars per head;
- $BC_{kj}$  : steer or heifer purchase price (plus purchasing expenses) in the  $k$ th weight class during the  $j$ th period, in dollars per head;
- $CAT_{krj}$  : operating cost, exclusive of feed, capital, and labor, of producing steers or heifers in the  $k$ th weight class at the  $r$ th level of gain during the  $j$ th period, in dollars per head;<sup>1/</sup>
- $CY_{mij}$  : operating cost, exclusive of fertilizer, capital, and labor, of producing the  $m$ th crop on the  $i$ th land class during the  $j$ th period, in dollars per acre;<sup>2/</sup>
- $CYP_{qj}$  : purchase price of the  $q$ th feed during the  $j$ th period, in dollars per pound dry matter;
- $CYS_{lj}$  : sale price of the  $l$ th feed during the  $j$ th period, in dollars per pound dry matter;
- $W_j$  : wage rate for hired labor during the  $j$ th period, in dollars;
- $R_j$  : price (interest rate) of borrowed capital during the  $j$ th period, in cents per dollar borrowed;

---

<sup>1/</sup> Cattle production operating costs include such expenses as veterinary fees, medicine, and machinery and equipment use.

<sup>2/</sup> Crop production operating costs include machinery and equipment, seed, herbicide and pesticide costs.

$CF_{mij}$  : price of fertilizer purchased and applied to the  $m$ th crop on the  $i$ th land class during the  $j$ th period, in dollars per pound;

$CT_{mj}$  : cost of storing the  $m$ th feed from the  $j$ th period until the  $(j + 1)$ th period, in dollars per pound dry matter.

Resource Availabilities. The operator's resources available for use in beef production are defined as:

CAPITAL: amount of operator's operating capital available, in dollars;

LABOR<sub>j</sub> : amount of operator's labor available during the  $j$ th period, in hours;

LAND<sub>ij</sub> : amount of land available in the  $i$ th farm land class during the  $j$ th period, in acres.

Capital Technical Coefficients. Capital utilization per unit of the various activities is represented by:

CAPX<sub>krj</sub> : operating capital required to produce steers or heifers in the  $k$ th weight class at the  $r$ th level of gain during the  $j$ th period, in dollars per head;

CAPY<sub>mij</sub> : operating capital required to produce the  $m$ th crop on the  $i$ th land class during the  $j$ th period, in dollars per acre;

CAPYP<sub>qj</sub> : operating capital required to purchase the  $q$ th feed during the  $j$ th period, in dollars per pound dry matter;

CAPXP<sub>kj</sub> : operating capital required to purchase steers or heifers of the  $k$ th weight class during the  $j$ th period, in dollars per head;

CAPL<sub>j</sub> : operating capital required to hire labor during the  $j$ th period, in dollars per hour;

CAPF<sub>mij</sub> : operating capital required to purchase fertilizer for application on the  $m$ th crop on the  $i$ th land class during the  $j$ th period, in dollars per pound;

CAPXS<sub>kj</sub> : cash receipts acquired from sale of steers or heifers of the  $k$ th weight class during the  $j$ th period, in dollars per head;

CAPYS<sub>lj</sub> : cash receipts acquired from sale of the  $l$ th feed during the  $j$ th period, in dollars per pound dry matter.

Labor Technical Coefficients. Utilization of labor by the model's activities is represented by:

- $LAX_{krj}$  : labor required to produce steers or heifers in the  $k$ th weight class at the  $r$ th level of gain during the  $j$ th period, in hours per head;
- $LAY_{mij}$  : labor required to produce the  $m$ th crop on the  $i$ th land class during the  $j$ th period, in hours per acre;
- $LAXP_{kj}$  : labor required to purchase steers or heifers in the  $k$ th weight class during the  $j$ th period, in hours per head;
- $LAXS_{kj}$  : labor required to sell steers or heifers in the  $k$ th weight class during the  $j$ th period, in hours per head;
- $LAF_{mij}$  : labor required to apply fertilizer to the  $m$ th crop on the  $i$ th land class during the  $j$ th period, in hours per pound.

Feed Nutrient Content Coefficients. Feeds, whether produced through the crop production activities or purchased in surrounding markets, provide dry matter, net energy for weight maintenance, net energy for gain, and protein to the cattle production activities. Because the multiple net energy system is used, it is necessary to distinguish in each case whether a unit of feed is used for gain or for maintenance. A single unit of feed can be used for either animal maintenance or animal weight gain, but not for both. Thus, we must specify two production activities in each period for each feed--one producing dry matter useful for cattle weight maintenance, and another producing dry matter useful for adding weight to the beef animal. The related coefficients are defined as:

- $YNEM_m$  : net energy available for maintenance from the  $m$ th feed, in megacalories per pound dry matter;
- $YNEG_m$  : net energy available for gain from the  $m$ th feed, in megacalories per pound dry matter;
- $YCP_m$  : crude protein content of the  $m$ th feed, in pounds per pound dry matter;
- $YDM_{mij}$  : dry matter yield of the  $m$ th crop (unfertilized) on the  $i$ th land class during the  $j$ th period, in pounds per acre;
- $D_{mij}$  : increase in dry matter resulting from application of fertilizer to the  $m$ th crop on the  $i$ th land class during the  $j$ th period, in pounds dry matter per pound fertilizer.

Livestock Nutritional Coefficients. Animal utilization of the various feed nutrients is represented by:

- ENM<sub>kr</sub> : net energy required to maintain weight of a steer or heifer in the kth weight class at the rth level of weight gain, in megacalories per head per period;<sup>1/</sup>
- ENG<sub>kr</sub> : net energy required to add weight to a steer or heifer in the kth weight class at the rth level of weight gain, in megacalories per head per period;
- CP<sub>kr</sub> : crude protein required to produce a steer or heifer in the kth weight class at the rth level of weight gain, in pounds per head per period;
- SC<sub>kr</sub> : stomach capacity of a steer or heifer in the kth weight class at the rth level of weight gain, in pounds dry matter per head per period.<sup>1/</sup>

#### Objective Function and Constraints

Using the above nomenclature, the objective function to be maximized can be represented as

$$\begin{aligned}
 (2-1) \text{ Profit} = & \sum_{kj} (PC_{kj})(XS_{kj}) + \sum_{lj} (CYS_{lj})(YS_{lj}) \\
 & \text{(cattle sale revenue)} \quad \text{(feed sale revenue)} \\
 & - \sum_{kj} (BC_{kj})(XP_{kj}) - \sum_{qj} (CYP_{qj})(YP_{qj}) \\
 & \text{(cattle purchase cost)} \quad \text{(feed purchase cost)} \\
 & - \sum_{mj} (CT_{mj})(T_{mj}) - \sum_{mij} (CF_{mij})(F_{mij}) \\
 & \text{(feed storage cost)} \quad \text{(fertilizer purchase cost)}
 \end{aligned}$$

---

<sup>1/</sup> Although rate of gain  $r$  is not normally specified as a determinant of net energy for maintenance or crude protein requirements, it must be specified as such in the present model; because, as rate of gain increases, so does the quarterly average weight of the animal. For further explanation see Chapter III.

$$\begin{array}{rcl}
- & \sum_j (W_j)(L_j) & - \sum_j (R_j)(CAP_j) \\
& \text{(labor purchase cost)} & \text{(operating capital borrowing cost)}
\end{array}$$

$$\begin{array}{rcl}
- & \sum_{krj} \sum \sum (CAT_{krj})(X_{krj}) & \\
& \text{(other cattle production costs)} &
\end{array}$$

$$\begin{array}{rcl}
- & \sum_{mij} \sum \sum (CY_{mij})(Y_{mij}), & \\
& \text{(other crop production costs)} &
\end{array}$$

where "profit" is a shorthand expression representing returns to fixed resources.

Land, Labor, and Capital Restrictions. The objective function is maximized subject to several restrictions. The first four listed ensure that land, labor, and operating capital requirements are met.

$$(2-2) \quad LAND_{ij} \geq \sum_m Y_{mij}, \text{ for each land class } i \text{ and period } j.$$

Land requirements of all production activities do not exceed land availability in each of the farm land classifications in each period.

$$\begin{array}{rclclcl}
(2-3) & LABOR_j & + & L_j & \geq & \sum_{kr} (LAX_{krj})(X_{krj}) \\
& \text{(operator labor)} & & \text{(hired labor)} & & \text{(labor for cattle production)}
\end{array}$$

$$\begin{array}{rclclcl}
+ & \sum_{mi} \sum (LAY_{mij})(Y_{mij}) & + & \sum_k (LAXS_{kj})(XS_{kj}) & & \\
& \text{(labor for crop production)} & & \text{(labor for cattle sale)} & &
\end{array}$$

$$+ \sum_k (LAXP_{kj}) (XP_{kj}) + \sum_{mi} (\sum (LAF_{mij}) (F_{mij}))$$

(labor for cattle purchase) (labor for fertilizer application)  
in each period j.

All labor needs are met by operator or hired labor in each period.

$$(2-4) \quad \begin{aligned} & \text{TRC}_j + \text{CAP}_j \\ & \text{(operating capital transfer from the previous period)} \quad \text{(borrowed capital)} \\ \\ & \geq \sum_{kr} (\sum (CAPX_{krj}) (X_{krj})) + \sum_{mi} (\sum (CAPY_{mij}) (Y_{mij})) \\ & \quad \text{(operating capital for cattle production)} \quad \text{(operating capital for crop production)} \\ \\ & + \sum_q (\sum (CAPYP_{qj}) (YP_{qj})) + \sum_k (\sum (CAPXP_{kj}) (XP_{kj})) \\ & \quad \text{(operating capital for feed purchase)} \quad \text{(operating capital for cattle purchase)} \\ \\ & + (\text{CAPL}_j) (L_j) + \sum_{mi} (\sum (CAPF_{mij}) (F_{mij})) \\ & \quad \text{(operating capital for labor hiring)} \quad \text{(operating capital for fertilizer purchase)} \\ \\ & - \sum_k (\sum (CAPXS_{kj}) (XS_{kj})) - \sum_{\ell} (\sum (CAPYS_{\ell j}) (YS_{\ell j})) \\ & \quad \text{(operating capital from cattle sale)} \quad \text{(operating capital from feed sale)} \end{aligned}$$

+  $TRC_{(j+1)}$   
 (operating capital transferred to the next period)  
 in each period j.

All operating capital needs are met by operator or borrowed capital or acquired through cattle and crop sale activities.

$$(2-5) \quad TRC_1 \leq \text{CAPITAL}$$

This allows operator capital to enter the model in the first period.

Livestock Nutritional Restrictions. The next four constraints are necessary to ensure that cattle nutritional needs are met without exceeding the stomach capacity of each animal.

$$(2-6) \quad \sum_m (YNEM_m) (NEM_{mj}) \geq \sum_{kr} (ENM_{kr}) (X_{krj})$$

(feed energy available for maintenance) (feed energy used for cattle maintenance)  
 in each period j.

This ensures that the net energy needs for cattle weight maintenance are met for each weight grouping in each period.

$$(2-7) \quad \sum_m (YNEG_m) (NEG_{mj}) \geq \sum_{kr} (ENG_{kr}) (X_{krj})$$

(feed energy available for gain) (feed energy used for gain)  
 in each period j.

This constraint ensures that the net energy needs for cattle weight gain are met for each weight class and for each level of weight gain in each period.

$$(2-8) \quad \sum_m (YCP_m) (NEM_{mj} + NEG_{mj}) \geq \sum_{kr} (LP_{kr}) (X_{krj})$$

(crude protein available) (cattle crude protein requirements)  
 in each period j.

This constraint ensures that the crude protein requirements of cattle are met in each weight class and for each level of weight gain in each period.

$$(2-9) \quad \sum_m (NEM_{mj} + NEG_{mj}) \leq \sum_{kr} (SC_{kr})(X_{krj})$$

(feed dry matter fed)      (stomach capacity of all animals fed)  
in each period j.

This ensures that the dry matter stomach capacity of each animal is not exceeded in any period.

Inter-Period Transfers. Finally, constraints allowing for inter-period transfer of cattle and feeds are defined.

$$(2-10) \quad \sum_r X_{krj}$$

(cattle produced in the kth weight class in period j)

$$\leq (X_{k,r=0,j-1} + X_{k-1,r=1,j-1} + X_{k-2,r=2,j-1})$$

(cattle transferred at weight k from period (j-1))

$$+ X_{kj}^P$$

(cattle purchased at weight k at beginning of period j)

$$- X_{k,(j-1)}^S$$

(cattle sold at weight k at end of period (j-1))

for each weight class k and period j;

where k-1 represents the weight difference between any two adjacent weight classes, k-2 is defined mutatis mutandis, r = 0 implies zero quarterly weight gain, r = 1 and r = 2 refer to successively higher quarterly weight gains, and

$$k-(k-1) = r_1$$

$$k-(k-2) = r_2.$$

This constraint ensures that cattle produced in any period were either purchased in that period or were cattle produced and not sold in the previous period.

$$\begin{aligned}
 (2-11) \quad & \sum_i [(YDM_{mij})(Y_{mij}) + (D_{mij})(F_{mij})] \\
 & \text{(feed produced without} \quad \text{(additional feed from ferti-} \\
 & \text{fertilizer in current} \quad \text{lizer use in current} \\
 & \text{period)} \quad \text{period)} \\
 & - YS_{\ell j} + YP_{qj} \\
 & \text{(feed sold in current period)} \quad \text{(feed purchased in current} \\
 & \quad \text{period)} \\
 & + T_{m(j-1)} \geq T_{mj} \\
 & \text{(feed transferred from} \quad \text{(feed transferred to next period)} \\
 & \text{previous period)} \\
 & + NEM_{mj} + NEG_{mj} \\
 & \text{(feed fed to cattle in current period)} \\
 & \text{for each feed } m(\ell, q) \text{ and period } j.
 \end{aligned}$$

This constraint ensures that feeds transferred to the next period or fed to cattle in the current period must either be purchased, produced on the farm and not sold, or be transferred from the previous period.

Use of the mathematical model set forth above generates, by period, the profit maximizing crop mix and rates of fertilization, optimal buying and selling weights of cattle, the optimal ration, and the optimal average daily gains for cattle, given a set of crop and livestock prices and costs.

### III. DATA PREPARATION

The present section consists of a discussion of the procedures used in collecting beef farm production and marketing data for the Appalachian and Shenandoah-Piedmont zones and interpreting these data for usage in a linear program.

Virginia has highly variable topography, soil, and climatic conditions. In order to provide representation of typical farm types, the state was divided into three relatively homogeneous areas: (1) Western or Appalachian, (2) Piedmont and Shenandoah, and (3) Eastern.<sup>1/</sup> The Appalachian and Shenandoah-Piedmont areas were then selected for analysis since these account for about 92% of cattle and calf sales in the state. The rolling hills and mountains in these parts of the state have, for the most part, encouraged production of pastures and forages at the expense of grains, but a considerable amount of corn for silage is also produced there.

#### Resource Considerations

##### Land

The Soil Conservation Service, USDA, classifies soils into eight productivity groups. The first four groups (Class I through Class IV soils) are suitable for row crop or forage production, whereas the second four groups (Class V through Class VIII soils) are generally restricted to pasture production. For the purpose of this study, Classes V through VIII are aggregated into one class suitable only for pasture production. County-level soil surveys were used to obtain, for both the Shenandoah-Piedmont and Appalachian areas, estimates of the relative amounts of land in each of the five land classes (USDA, Soil Conservation Service). Results of a 1968 survey were used to estimate the total number of acres employed on the average-sized beef farm in each area (Walker and Jobes, 1975). Using these data, a representative farm was formulated for each of the zones studied (Appendix Table A.1).

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<sup>1/</sup> These zones were defined by the Southern Regional Research Project S-116, "Supply, Pricing, and Marketing Alternatives for Cattle, Beef Systems in the South."

### Labor

It is assumed in these models that the farm operator supplies his own labor to the beef operation. Realistically, an operator may be able to supply 2400 hours per year of his own time; that is 300 eight-hour days. This figure may be divided into approximately equal fourths for each of the four quarters of the year.

### Capital

Operating capital available to the representative farms was calculated by taking the average net worth of a typical beef operator and deducting the value of land, buildings, machinery, and equipment. These dollar amounts, from Kline and Cameron (1975), are inflated at 8.5% per year to represent 1977 dollars. The available operating capital used in each model is \$18,530.92.

## Crop and Livestock Production Activities: Costs and Resource Utilization

The crop and livestock budgets used in this study are based on budgets developed by Moore, Brant, and Folmar (1976-1978). The budgets were slightly modified in incorporating them in our model.

### Crop and Fertilizer Activities

Crop production activities included are: (1) corn for grain, (2) corn for silage, (3) alfalfa hay, (4) stockpiled fescue, and (5) orchard-grass-clover pasture. Fertilizer response functions employed for each of these crops are summarized in Appendix Table B.1. The first two of these, for corn grain and corn silage, are linearized versions of quadratic functions estimated by Lutz, Jones, Moody, and Hoepner (1964). The last three functions were fitted to original data points observed by Blaser; Hallock, Brown, and Blaser (1965); and Blaser and Brady (1950).

To account for yield variations by soil class in each crop, the intercepts of each equation in Appendix Table B.1 were adjusted such that, for each soil class, the equation-predicted yield would coincide with the "long-term average yield" associated with some proportion of that soil's "fertilizer recommendation." Fertilizer recommendations and

long-term average yields used were those quoted for each soil class by Agricultural Experiment Station personnel at Virginia Tech. Readers are referred to Jessee, pp. 33-38, for further details on response function construction. Intercept adjustments for each soil class and crop are shown in Appendix Tables B.2 through B.6.

Budgets prepared by Moore, Brant, and Folmar (1976-1978) provided estimates of operating costs and labor and capital utilization coefficients for each of the above cropping activities and for the cattle purchase, production, and sale activities (see Jessee, pp. 100-105, his Appendix B).

#### Nutritional Data for Feeds

Estimates of the conversion factors expressing net energy for gain, net energy for maintenance, and crude protein per unit dry matter of feed were obtained from data published by the Nation Research Council (1976). These are reported in Appendix Table C.1. Use of the data assumes constant energy and protein content per unit of feed dry matter, regardless of the time of the year in which the feed is produced or consumed. In reality, forage energy and protein concentration vary by season.

#### Livestock Growth Functions

##### Energy Requirements

In the mathematical model presented in the previous chapter, beef cattle nutritional needs were measured by energy and protein requirements. According to the system proposed by Lofgreen and Garrett (1968), functions expressing net energy for steer or heifer maintenance and growth are specified as:

$$(3-1) \quad NE_m = .077W^{.75}$$

$$(3-2) \quad SNE_g = (.05272G + .00684G^2)W^{.75}$$

$$(3-3) \quad HNE_g = (.05603G + .01265G^2)W^{.75}.$$

where

$NE_m$  denotes net energy required for steer or heifer weight maintenance, in megacalories per day;

$SNE_g$  denotes net energy required for steer weight gain, in megacalories per day;

$HNE_g$  denotes net energy required for heifer weight gain, in megacalories per day;

$W$  denotes weight of the animal, in kilograms; and

$G$  denotes average daily gain, in kilograms.

For purposes of this study's model, functions (3-1) and (3-2) were evaluated for steers at 100-pound weight increments from 500 to 1000 pounds and at 0, 1.1, and 2.2 pound average daily gain levels over a 90-day period. Functions (3-1) and (3-3) were evaluated for heifers at 75-pound weight increments from 450 to 900 pounds and at 0, .83, and 1.67 pound average daily gain levels over a 90-day period.<sup>1/</sup>

#### Crude Protein Requirements

The assumed crude protein requirements of steers and heifers modeled in this study are based on relationships developed by Carlson (1976, p. 645). In his study, five percent of the crude protein ingested by beef cattle represents metabolic loss and is therefore not available to meet the animal's protein requirements. The relationships formulated by Carlson are, for steers,

$$(3-4) \quad CP_s = .002W_s^{.75} + (.365 - .00033W_s)G_s,$$

and for heifers,

$$(3-5) \quad CP_h = .002W_h^{.75} + (.374 - .00044W_h)G_h$$

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<sup>1/</sup>These functions were evaluated, for each rate of gain, at the average animal weight achieved during each period. Results were then multiplied by 90 to arrive at an estimate of the energy requirements for the entire 90-day period. This latter aggregation process produced slight errors in feed requirement estimates, although across all weight ranges and gain levels the errors averaged less than 0.05%.

where

$CP_{s,h}$  denotes crude protein requirements for steer or heifer, in kilograms per day;

$W_{s,h}$  denotes weight of steer or heifer, in kilograms; and

$G_{s,h}$  denotes average weight gain of steer or heifer, in kilograms per day.

Functions (3-4) and (3-5) were evaluated over the same weight groups and daily gain levels as were used in evaluation of steer and heifer energy requirements.<sup>1/</sup>

#### Dry Matter Maxima

Rations fed to meet the above nutritional requirements must not exceed the stomach capacity of the animal. Nino and Hughes (1976, p. 626) estimate the maximum dry matter intake function for steers and heifers as,

$$(3-6) \quad SC = .1W^{.75},$$

where

SC denotes maximum dry matter intake, in kilograms per day,  
and

W denotes weight of the animal, in kilograms.

Like the crude protein relations, function (3-6) was evaluated at the various weight groups and levels of gain specified for steers and heifers in the energy requirements relations.<sup>2/</sup>

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<sup>1/</sup> The evaluation and aggregation process used was the same as that employed for the energy requirement relations. Average aggregation error was approximately 1.00%.

<sup>2/</sup> This relation was also evaluated and aggregated in the same manner as for the energy relations, resulting in an average error of less than 0.03%.

## Prices

### Crop Prices

The model allows only corn and alfalfa hay to be purchased or sold. Prices paid for corn meal and prices received for corn grain are reported in Virginia Agricultural Statistics, Virginia Department of Agriculture and Commerce, for years 1968 to 1977 (see Appendix Table D.1). Alfalfa prices utilized were quarterly prices paid or received in Virginia for the years 1968 to 1977, as reported in the same source.

### Cattle Prices

Steer and heifer prices are expressed here as functions of cattle weight. The price-weight functions employed for the spring and fall months of the years studied (1973, 1974, 1976, 1977, and a 1968-77 average) were taken from an unpublished study of Virginia feeder cattle sales by Buccola and Carmichael (1978). These functions, representing Good grade cattle at special state-graded sales, are reported in Appendix Table D.2. Representative prices, calculated by evaluating these functions at selected weight levels, are shown in Appendix Table D.3.

Since special feeder cattle sales are not conducted in the winter months in Virginia, winter intercepts were estimated by adding to the previous fall intercepts the mean difference between average 500-750 lb winter feeder prices and average 500-750 lb fall feeder prices. Data for this purpose were drawn from regular weekly auction sales as reported by the Virginia Department of Agriculture and Commerce, Federal-State Market News Service (1969-1977) (Appendix Table D.4).

The prices used for steers and heifers for the summer quarter of 1977 were prices for various weights of steers and heifers as reported by Virginia Tech's Extension Division, Department of Animal Science, in Results, Virginia Feeder Calf Sales (1968-1977) (Appendix Table D.5).

### Storage, Capital, and Labor Prices

Per bushel cost of corn storage and drying used in the present model refers to a storage-drying system having a capacity of 13,440 bushels (Bauer, Donald, and Smith, 1977, p. 10). These costs, exclusive of labor and capital expenses, are 24.74¢ per bushel and 0.65¢ per

bushel per 90-day period, respectively, for storage and drying. (See Appendix Table C.2.)

The rate of interest charged on operating capital used in the model is 9.5 per cent per year. Average hourly wages paid to hired labor on Virginia farms are those reported by the U.S. Department of Agriculture, Statistical Reporting Service, Virginia Crops and Livestock, for the years 1976 to 1978 (see Appendix Table D.1).

#### IV. SOLUTION RESULTS UNDER 1976-77 AND 1973-74 PRICE STRUCTURES

Profit-maximizing solution results are reported in this section for both the Appalachian and Shenandoah-Piedmont areas corresponding to two scenarios: one utilizes actual 1976-1977 feed and cattle price structures in Virginia, the other utilizes actual 1973-1974 feed and cattle price structures.

##### Description of the 1976-1977 and 1973-74 Scenarios

In the 1976-1977 scenario, crops are considered to be produced in 1976 and available as livestock feed or for sale from October 1, 1976 onwards. Pasture is produced continuously, and is available from October 1, 1976 to September 30, 1977. Livestock may be produced in the same time interval (October 1, 1976 to September 30, 1977) with average daily gains of 0, 1.1, or 2.2 pounds per day for steers and 0, 0.867, or 1.667 pounds per day for heifers.

As a historical record, Virginia feeder cattle prices increased continuously but moderately during the October 1, 1976-to-September 30, 1977 period. For 600-pound Good-grade steers, for example, the price increased from \$36.45 to \$40.03 per hundredweight. Corn prices decreased from \$2.52 to \$1.88 per bushel.

The models used in the 1973-1974 scenario were similar to the 1976-1977 models in that nonpasture crop production was considered to occur during the spring and summer of the first year (1973) and to become available for sale or for use as a livestock feed from October 1 onward. Pasture was available for continuous grazing from October 1, 1973 through September 30, 1974. Cattle production was considered to occur over the same period. Potential average daily gains for steers and heifers were the same as those used in the 1976-1977 scenario.

During the October 1, 1973-to-September 30, 1974 period, Virginia cattlemen experienced a rapid decline in all cattle prices. Prices for 600-pound Good-grade steers, for example, dropped from \$54.59 to \$32.38

per hundredweight. At the same time, Virginia farmers faced a corn price increase from \$2.26 to \$3.46 per bushel. The price of alfalfa hay increased slightly, from \$49.50 to \$53.00 per ton.

### Results for the 1976-1977 Scenario

#### Appalachian Area

The optimal farm plan for the Appalachian Area in the 1976-1977 scenario generated \$23,670 in returns to the operator's land, labor, and capital. Livestock production activities, along with the amounts of feed offered to livestock in the optimal farm organization, are presented in Table 4.1. Associated crop production activities are presented in Table 4.2. The optimal farm plan required borrowing \$58,233 on January 1, 1977. In the Appalachian model, the largest number of cattle was stocked in the January-March period, with some of these cattle being held for sale in June at heavier weights.

#### Shenandoah-Piedmont Area

A summary of the livestock purchase, sale, and production activities for this scenario is presented in Table 4.3; the associated crop production activities are presented in Table 4.4. Capital borrowing and labor hiring activities are shown in Table 4.5. Net returns to operator's land, labor, and capital in this model amounted to \$70,442.

The land available to the typical beef producer in the Shenandoah-Piedmont Area is larger than that available to the typical beef producer in the Appalachian area. Hence, the optimal Shenandoah-Piedmont farm plan reported here represents a larger beef operation than that of the Appalachian area. As in the Appalachian model, the largest number of cattle in the Shenandoah-Piedmont model was produced during the January-March period, with some of these cattle being held until June for sale at heavier weights. And, as in the previous model, all cattle were produced at the maximum allotted gain per day.

Table 4.1. Livestock Purchase, Sale, and Production in the Optimal Farm Plan; Appalachian Area, 1976-1977 Scenario

	October 1, 1976	October 1- January 1, 1977	January 1 1977	January 1- April 1, 1977	April 1, 1977	April 1- July 1, 1977	July 1, 1977	July 1- October 1, 1977	October 1, 1977
Purchase or Sale	Purchase 47 heifers at 450 pounds		Sell 47 heifers at 600 pounds		Sell 231 steers at 700 pounds		Sell 123 steers at 900 pounds		Sell 149 steers at 700 pounds
			Purchase 354 steers at 500 pounds				Purchase 149 steers at 500 pounds		
Gain		47 heifers at 1.67 pounds per day: 450-600 pounds		354 steers at 2.2 pounds per day: 500-700 pounds		123 steers at 2.2 pounds per day: 700-900 pounds		149 steers at 2.2 pounds per day: 500-700 pounds	
Ration per Head per Day (dry matter basis)		.9 pounds corn grain  12.5 pounds pasture		2.9 pounds corn sil- age  5.9 pounds corn grain  5.6 pounds corn stover		3.8 pounds corn silage  5.4 pounds corn grain  8.4 pounds pasture		7.8 pounds corn silage  3.1 pounds corn grain  4.0 pounds pasture	

Table 4.2. Crop Production Activities, Yields, and Crop Fertilization Activities in the Optimal Farm Plan, Appalachian Area, 1976-1977 Scenario

Crop	Land Class	Acres	Yield Per Acre	Fertilization		
				N (lbs/ac)	P <sub>2</sub> O <sub>5</sub> (lbs/ac)	K <sub>2</sub> O (lbs/ac)
Corn Grain	I	11.89	129 bu. <sup>b/</sup>	162.5	<u>a/</u>	<u>a/</u>
Corn Grain	III	13.84	95 bu. <sup>b/</sup>	112.5	<u>a/</u>	<u>a/</u>
Corn Grain	IV	49.28	73 bu. <sup>b/</sup>	87.5	<u>a/</u>	<u>a/</u>
Corn Silage	I	9.84	22 tons <sup>c/</sup>	162.5	<u>a/</u>	<u>a/</u>
Corn Silage	II	10.34	20 tons <sup>c/</sup>	137.5	<u>a/</u>	<u>a/</u>
Orchard Grass-Ladino Clover Pasture	V	110.8	.778 ton <sup>d/</sup>	<u>a/</u>	<u>a/</u>	25

<sup>a/</sup> Held constant at the recommended rate (see Appendix B).

<sup>b/</sup> 84.5% dry matter.

<sup>c/</sup> 27.9% dry matter.

<sup>d/</sup> Dry matter basis, produced from October 1976 to October 1977.

Table 4.3. Livestock Purchase, Sale, and Production in the Optimal Farm Plan, Shenandoah-Piedmont Area, 1976-77 Scenario

	October 1, 1976	October 1- January 1, 1977	January 1 1977	January 1- April 1, 1977	April 1, 1977	April 1- July 1, 1977	July 1, 1977	July 1- October 1, 1977	October 1, 1977
Purchase or Sale	Purchase 60 heifers at 450 pounds		Sell 60 heifers at 600 pounds  Purchase 1449 steers at 500 pounds		Sell 1130 steers at 700 pounds		Sell 319 steers at 900 pounds  Purchase 402 steers at 500 pounds		
Gain		60 heifers at 1.67 pounds per day: 450-600 pounds		1449 steers at 2.2 pounds per day: 500- 700 pounds		319 steers at 2.2 pounds per day: 700-900 pounds		402 steers at 2.2 pounds per day: 500-700 pounds	
Ration per Head per Day (dry matter basis)		.9 pounds corn grain  12.5 pounds pasture		7.5 pounds corn grain  6.4 pounds corn sto- ver		0.9 pounds corn silage  10.2 pounds corn grain  4.2 pounds pasture		6.1 pounds corn silage  5.5 pounds corn grain  1.9 pounds pasture	

Table 4.4. Crop Production Activities, Yields, and Crop Fertilization Activities in the Optimal Farm Plan, Shenandoah-Piedmont Area, 1976-1977 Scenario

Crop	Land Class	Acres	Yield Per Acre	Fertilization		
				N (lbs/ac)	P <sub>2</sub> O <sub>5</sub> (lbs/ac)	K <sub>2</sub> O (lbs/ac)
Corn Grain	I	23.59	129 bu. <sup>b/</sup>	162.5	<u>a/</u>	<u>a/</u>
Corn Grain	II	33.13	116 bu. <sup>b/</sup>	137.5	<u>a/</u>	<u>a/</u>
Corn Grain	III	185.62	95 bu. <sup>b/</sup>	112.5	<u>a/</u>	<u>a/</u>
Corn Grain	IV	108.91	73 bu. <sup>b/</sup>	87.5	<u>a/</u>	<u>a/</u>
Corn Silage	II	21.75	20 tons <sup>c/</sup>	137.5	<u>a/</u>	<u>a/</u>
Orchard Grass-Ladino Clover Pasture	V	141.5	.778 tons <sup>d/</sup>	<u>a/</u>	<u>a/</u>	25

<sup>a/</sup> Held constant at the recommended rate (see Appendix B).

<sup>b/</sup> 84.5% dry matter.

<sup>c/</sup> 27.9% dry matter.

<sup>d/</sup> Dry matter basis, produced from October 1976 to October 1977.

Table 4.5. Operator Labor Use, Labor Hiring, and Capital Borrowing Activities in the Optimal Farm Plan, Shenandoah-Piedmont Area, 1976-1977 Scenario

	January- April 1976	April- July 1976	July- October 1976	October- January 1977	January- April 1977	April- July 1977	July- October 1977
Operator Labor Use (man days)	3.7	75.6	75.6	75.6	75.6	75.6	75.6
Labor Hired (man days)		4.8		77.6	230.1	3.6	
Borrowed Capital Outstanding (\$)		10,694	13,585	23,736	284,809		

### Results For the 1973-1974 Scenario

#### Appalachian Area

With the rapidly falling cattle prices and increasing grain prices of the 1973-74 scenario, the Appalachian model indicated it was best to produce cattle only during the October 1-December 31, 1973 period, when cattle prices were highest. If cattle had been held longer than this, they would have been sold at significant capital losses. On October 1, the solution suggested purchasing 289 heifers at 450 pounds, feeding them to gain 1.67 pounds per day, then selling them at 600 pounds on December 31. The ration fed these cattle consisted, on a dry matter basis, of 1.2 pounds corn silage, 3.7 pounds corn grain, and 8.5 pounds corn stover per head per day. Corn grain produced but not fed to the cattle (5,919 bushels) was stored at the October 1, 1973 harvest and sold on July 1, 1974.

The associated crop production activities are presented in Table 4.6. All of Class I through Class IV land was utilized, whereas Class V land remained idle. To facilitate cattle purchase on October 1, 1973, it was necessary to borrow capital on this date in the amount of \$54,143. This farm plan produced a net return of \$18,173 to the operator's resources.

#### Shenandoah-Piedmont Area

The optimal farm plan for the Shenandoah-Piedmont area in the 1973-1974 scenario was similar to that for the Appalachian area. Eleven hundred and thirty-three 450-pound heifers were purchased on October 1, 1973 and sold as 600-pound feeders on December 31, 1973. These heifers were fed to gain 1.67 pounds per day on a ration consisting, on a dry matter basis, of 1.2 pounds corn silage, 3.7 pounds corn grain, and 8.5 pounds corn stover per head per day. Associated crop production activities are presented in Table 4.7. As in the Appalachian area optimal farm plan, Class V land remained idle. On July 1, 1974 corn grain produced but not fed to cattle (23,184 bushels) was sold on the market. Capital borrowing and labor hiring activities are presented in Table 4.8. Returns to the operator's resources under this farm plan were \$65,668.

Table 4.6. Crop Production Activities, Yields, and Crop Fertilization Activities in the Optimal Farm Plan, Appalachian Area, 1973-1974 Scenario

Crop	Land Class	Acres	Yield per Acre	Fertilization		
				N (lbs/ac)	P <sub>2</sub> O <sub>5</sub> (lbs/ac)	K <sub>2</sub> O (lbs/ac)
Corn Grain	I	21.73	129 bu. <sup>b/</sup>	162.5	<u>a/</u>	<u>a/</u>
Corn Grain	II	7.68	116 bu. <sup>b/</sup>	137.5	<u>a/</u>	<u>a/</u>
Corn Grain	III	13.84	95 bu. <sup>b/</sup>	112.5	<u>a/</u>	<u>a/</u>
Corn Grain	IV	49.28	73 bu. <sup>b/</sup>	87.5	<u>a/</u>	<u>a/</u>
Corn Silage	II	2.66	20 tons <sup>c/</sup>	137.5	<u>a/</u>	<u>a/</u>

<sup>a/</sup> Held constant at the recommended rate (see Appendix B).

<sup>b/</sup> 84.5% dry matter.

<sup>c/</sup> 27.9% dry matter.

Table 4.7. Crop Production Activities, Yields, and Crop Fertilization Activities in the Optimal Farm Plan, Shenandoah-Piedmont Area, 1973-1974 Scenario

Crop	Land Class	Acres	Yield Per Acre	Fertilization		
				N (lbs/ac)	P <sub>2</sub> O <sub>5</sub> (lbs/ac)	K <sub>2</sub> O (lbs/ac)
Corn Grain	I	23.59	129 bu. <sup>b/</sup>	162.5	<u>a/</u>	<u>a/</u>
Corn Grain	II	44.46	116 bu. <sup>b/</sup>	137.5	<u>a/</u>	<u>a/</u>
Corn Grain	III	185.62	95 bu. <sup>b/</sup>	112.5	<u>a/</u>	<u>a/</u>
Corn Grain	IV	108.91	73 bu. <sup>b/</sup>	87.5	<u>a/</u>	<u>a/</u>
Corn Silage	II	10.42	20 tons <sup>c/</sup>	137.5	<u>a/</u>	<u>a/</u>

<sup>a/</sup> Held constant at the recommended rate (see Appendix B).

<sup>b/</sup> 84.5% dry matter.

<sup>c/</sup> 27.9% dry matter.

Table 4.8. Operator Labor Use, Labor Hiring, and Capital Borrowing Activities in the Optimal Farm Plan, Shenandoah-Piedmont Area, 1973-1974 Scenario

	January- April 1973	April- July 1973	July- October 1973	October- January 1974	January- April 1974	April- July 1974	July- October 1974
Operator Labor Use (man days)	.9	75.6	36.2	75.6	25.5	-	-
Labor Hired (man days)	5.4		299.4				
Borrowed Capital Outstanding (\$)	7,976	9,318	266,105				

Summary

Solutions under both 1976-77 and 1973-74 scenarios suggest that: (a) higher rates of gain are preferred to lower rates of gain; (b) lighter weight cattle are preferred to heavier weight cattle, although heavy cattle become more attractive as the negative effect of weight on price diminishes; (c) farm profitability and number of cattle stocked diminish as the feeder price-weight function shifts downward or becomes more steeply negative in slope (each producing lower feeder prices). These phenomena are explored in more detail in the following chapter.

## V. SOLUTION RESULTS UNDER AVERAGE-YEAR PRICES AND UNDER ALTERNATIVE PRICES AND COSTS

In order to provide a more complete analysis of the effects of cattle and feed prices and costs on profit-maximum farm plans, the Appalachian and Shenandoah-Piedmont models were re-programmed to reflect average cattle and feed prices observed during the 1968-1977 period. Selected price and cost changes were then performed on the Appalachian model. The parametric changes were divided into two parts: those changes affecting feed costs and crop yields and those changes affecting cattle prices.

### Results for the 1968-1977 "Average Year" Scenario

#### Appalachian Area

Under the assumption of average 1968-1977 cattle and feed prices, the optimal farm plan for the Appalachian area generated \$36,321 in returns to the operator's land, labor, and capital. The livestock purchase, sale, and production activities in this farm plan are presented in Table 5.1. Associated crop production and fertilization activities are given in Table 5.2; and utilization of the operator's labor, along with the capital borrowing and labor hiring activities, are presented in Table 5.3.

All cattle are, in this optimal plan, raised at the maximum allowable gain per day; that is, 2.2 pounds per day for steers and 1.67 pounds per day for heifers. No cattle are produced in the January-March period, and heifers are produced only in the April-July interval. Absence of steer production during the latter two periods is most likely due to the fact that steer price-weight functions on marketing dates of April 1 and July 1 had steeper negative slopes than on other sale dates. They were also more steeply-sloped than heifer price-weight lines on any sale date. Lighter-weight cattle are also preferred to heavier cattle in this solution; this is attributable to the decrease in efficiency of the conversion of feed-to-weight-gain associated with weight increase.

Table 5.1. Livestock Purchase, Sale, and Production in the Optimal Farm Plan, Appalachian Area, Assuming 1968-1977 Average Feed and Livestock Prices

	October 1 Year 1	October 1- January 1, Year 2	January 1 Year 2	April 1 Year 2	April 1-July 1, Year 2	July 1 Year 2	July 1- October 1, Year 2	October 1 Year 2
Purchase or Sale	Purchase 583 steers at 500 pounds		Sell 583 steers at 700 pounds	Purchase 83 heifers at 450 pounds		Sell 83 heifers at 600 pounds  Purchase 53 steers at 500 pounds		Sell 53 steers at 700 pounds
Gain		583 steers at 2.2 pounds per day: 500-700 pounds			83 heifers at 1.67 pounds per day: 450-600 pounds		53 steers at 2.2 pounds per day: 500-700 pounds	
Ration per Head per Day (dry matter basis)		6.4 pounds corn silage  4.2 pounds corn grain  1.0 pounds pasture  3.1 pounds corn stover			.9 pounds corn grain  12.5 pounds pasture		3.5 pounds corn grain  11.1 pounds pasture	

Table 5.2. Crop Production Activities, Yields, and Crop Fertilization Activities in the Optimal Farm Plan, Appalachian Area, Assuming 1968-1977 Average Feed and Livestock Prices

Crop	Land Class	Acres	Yield Per Acre	Fertilization <sup>a/</sup> (lbs/ac)
Corn grain	I	3.98	125 bu. <sup>b/</sup>	162.5 lbs. nitrogen
Corn grain	III	13.84	95 bu. <sup>b/</sup>	112.5 lbs. nitrogen
Corn grain	IV	49.28	73 bu. <sup>b/</sup>	87.5 lbs. nitrogen
Corn silage	I	17.75	22 tons <sup>c/</sup>	162.5 lbs. nitrogen
Corn silage	II	10.34	20 tons <sup>c/</sup>	137.5 lbs. nitrogen
Orchard grass-ladino clover pasture	IV	110.8	.778 tons <sup>d/</sup>	25 lbs. potash

<sup>a/</sup> Other fertilization held constant at the recommended rate (see Appendix B).

<sup>b/</sup> 84.5 percent dry matter.

<sup>c/</sup> 27.9 percent dry matter.

<sup>d/</sup> Dry matter basis, produced from October Year 1 to October Year 2.

Table 5.3. Operator Labor Use, Labor Hiring, and Capital Borrowing Activities in the Optimal Farm Plan, Appalachian Area, Assuming 1968-1977 Average Feed and Livestock Prices

	January- April Year 1	April- July Year 1	July- October Year 1	October- January Year 2	January- April Year 2	April- July Year 2	July- October Year 2
Operator labor use (man days)	3.9	18.5	75.6	75.6	15.3	3.0	5.7
Labor hired (man days)	--	--	31.9	62.75	--	--	--
Borrowed capital outstanding (\$)	--	--	--	151,934	--	--	--

### Shenandoah-Piedmont Area

The optimal farm plan for the Shenandoah-Piedmont area, under average 1968-1977 feed and cattle prices, produced returns of \$119,947 for the operator's land, labor, and capital.<sup>1/</sup> The livestock purchase, sale, and production activities for this solution are presented in Table 5.4. Associated crop production activities are presented in Table 5.5. Operator labor use, capital borrowing, and labor hiring activities in the plan are shown in Table 5.6.

As in the Appalachian solution described above, the optimal farm plan for the Shenandoah-Piedmont area consists of lighter weight cattle produced at the maximum gain per day. No cattle are produced in the January-March period, and heifers are produced only in the April-July production period. Although the number of cattle produced in the Shenandoah-Piedmont solution is somewhat larger than that for the Appalachian area, rations fed per head of cattle are, with the exception of the October-December production period, the same as in the Appalachian model. In the latter fall season, the Shenandoah-Piedmont model solution feeds, on a per head basis, more corn grain and corn stover but less corn silage than does the Appalachian solution.

### Results of Parametric Changes on Crop Yields and Feed Input Prices

In this section, the effects on model outcomes of parametric changes in costs per unit of feed are examined. These cost changes are accomplished by altering the assumed yields of corn grain, corn silage, and pasture produced on the farm, and by changing corn purchase and sale prices. Parametric changes are only carried out on the Appalachian model, in which the 1968-1977 "average year" solution described above serves as the base solution.

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<sup>1/</sup> It is likely that this figure is inflated by the absence of a realistic limit on the amount of borrowable operating capital. (The program solution involved the borrowing of approximately \$600,000.)

Table 5.4. Livestock Purchase, Sale, and Production in the Optimal Farm Plan, Shenandoah-Piedmont Area, Assuming 1968-1977 Average Feed and Livestock Prices

	October 1 Year 1	October 1- January 1, Year 2	January 1 Year 2	April 1 Year 2	April 1-July 1, Year 2	July 1 Year 2	July 1- October 1, Year 2	October 1 Year 2
Purchase or Sale	Purchase 2207 steers at 500 pounds		Sell 2207 steers at 700 pounds	Purchase 106 heifers at 450 pounds		Sell 106 heifers at 600 pounds  Purchase 67 steers at 500 pounds		Sell 67 steers at 700 pounds
Gain		2207 steers at 2.2 pounds per day: 550-700 pounds			106 heifers at 1.67 pounds per day: 450- 600 pounds		67 steers at 2.2 pounds per day: 500-700 pounds	
Ration per Head per Day (dry matter basis)		4.4 pounds corn silage  5.8 pounds corn grain  .3 pounds pasture  3.6 pounds corn stover			.9 pounds corn grain  12.5 pounds pasture		3.5 pounds corn grain  11.2 pounds pasture	

Table 5.5. Crop Production Activities, Yields, and Crop Fertilization Activities in the Optimal Farm Plan, Shenandoah-Piedmont Area, Assuming 1968-1977 Average Feed and Livestock Prices

Crop	Land Class	Acres	Yield Per Acre	Fertilization (lbs/ac)
Corn grain	I	3.36	125 bu. <sup>b/</sup>	162.5 lbs. nitrogen
Corn grain	III	185.62	95 bu. <sup>b/</sup>	112.5 lbs. nitrogen
Corn grain	IV	109.91	73 bu. <sup>b/</sup>	87.5 lbs. nitrogen
Corn silage	I	20.23	22 tons <sup>c/</sup>	162.5 lbs. nitrogen
Corn silage	II	54.88	20 tons <sup>c/</sup>	137.5 lbs. nitrogen
Orchard grass-ladino clover pasture	V	141.5	.778 tons <sup>d/</sup>	25 lbs. potash

<sup>a/</sup> Other fertilization held constant at the recommended rate (see Appendix B).

<sup>b/</sup> 84.5 percent dry matter.

<sup>c/</sup> 27.9 percent dry matter.

<sup>d/</sup> Dry matter basis, produced from October Year 1 to October Year 2.

Table 5.6. Operator Labor Use, Labor Hiring, and Capital Borrowing Activities in the Optimal Farm Plan, Shenandoah-Piedmont Area, Assuming 1968-1977 Average Feed and Livestock Prices

	January- April Year 1	April- July Year 1	July- October Year 1	October- January Year 2	January- April Year 2	April- July Year 2	July- October Year 2
Operator labor use (man days)	8.4	75.6	75.6	75.6	57.9	3.9	7.3
Labor hired (man days)	--	--	196.6	478.9	--	--	--
Borrowed capital outstanding (\$)	--	10,436	20,458	618,218	--	--	--

### Pasture Yield Changes

The intercepts on the orchard grass-ladino clover yield relations were first increased and then decreased in 10% increments for each soil class to 50% of their initial values. Taking Soil Class V as an example, this resulted in a range of unfertilized pasture yields of .380 to 1.149 tons, evaluated in increments of .076 tons.

The model allows pasture to be produced continuously from October of the first model year through September of the following year with no provision for storage from one period to the next. As pasture yields were increased in this application of the model, the number of cattle produced in each production period in the optimal farm plan rose, except for the period January through March. The latter period evidenced no change in cattle numbers, since pasture was scarcely made available for grazing during the winter months. The increases in cattle numbers associated with pasture yield increases in other production periods is demonstrated in Figure 5.1.

Rations fed to heifers during the April-June production period and to steers during the July-September production period remained constant as pasture yields changed; but in the October-December period, some pasture was substituted for corn silage, suggesting that corn silage and pasture act as substitutes under the assumed price structure. The amount of corn grain and corn stover fed per head also increased slightly, compensating for the lower energy concentration of pasture (see Figure 5.2).

### Corn Yield Changes

Parametric changes on corn yields were performed simultaneously for corn silage and corn grain functions. The intercepts of the fertilizer-yield relations for these crops were increased and decreased in each soil class by 50% of their base values in 10% increments. In Class I soils, for example, this produced a range of corn grain yields (unfertilized basis) of from 26.93 to 80.79 bushels per acre and a range of corn silage yields (unfertilized basis) of from 6.61 to 19.83 tons per acre. Yields were altered incrementally in units of 5.39 bushels per acre for corn grain and 1.32 tons per acre for corn silage. The ranges and incremental changes in yields of unfertilized and fertilized corn on

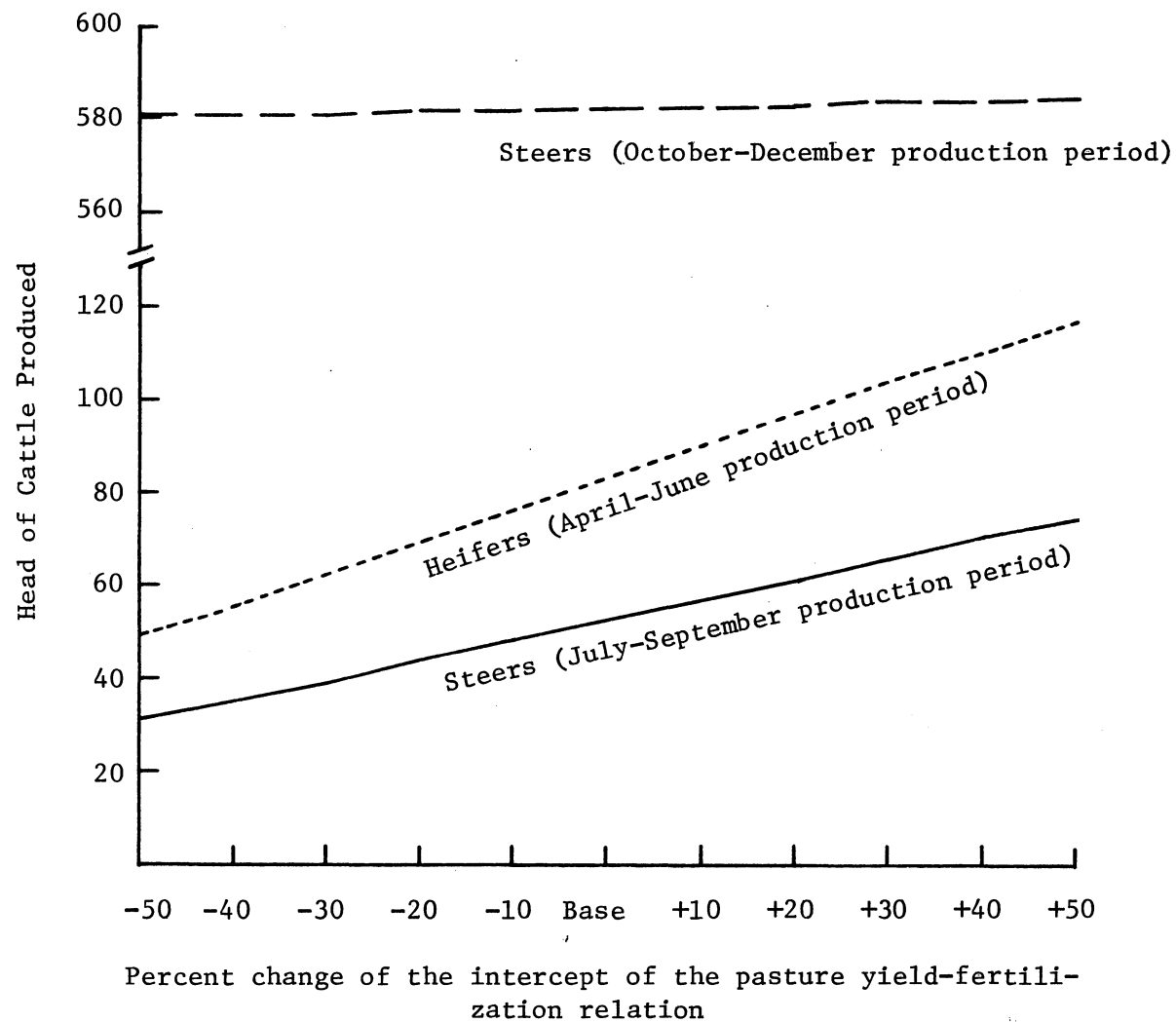


Figure 5.1. Optimal Steer and Heifer Numbers by Selected Pasture Yields, Appalachian Area, Assuming 1968-1977 Average Feed and Livestock Prices

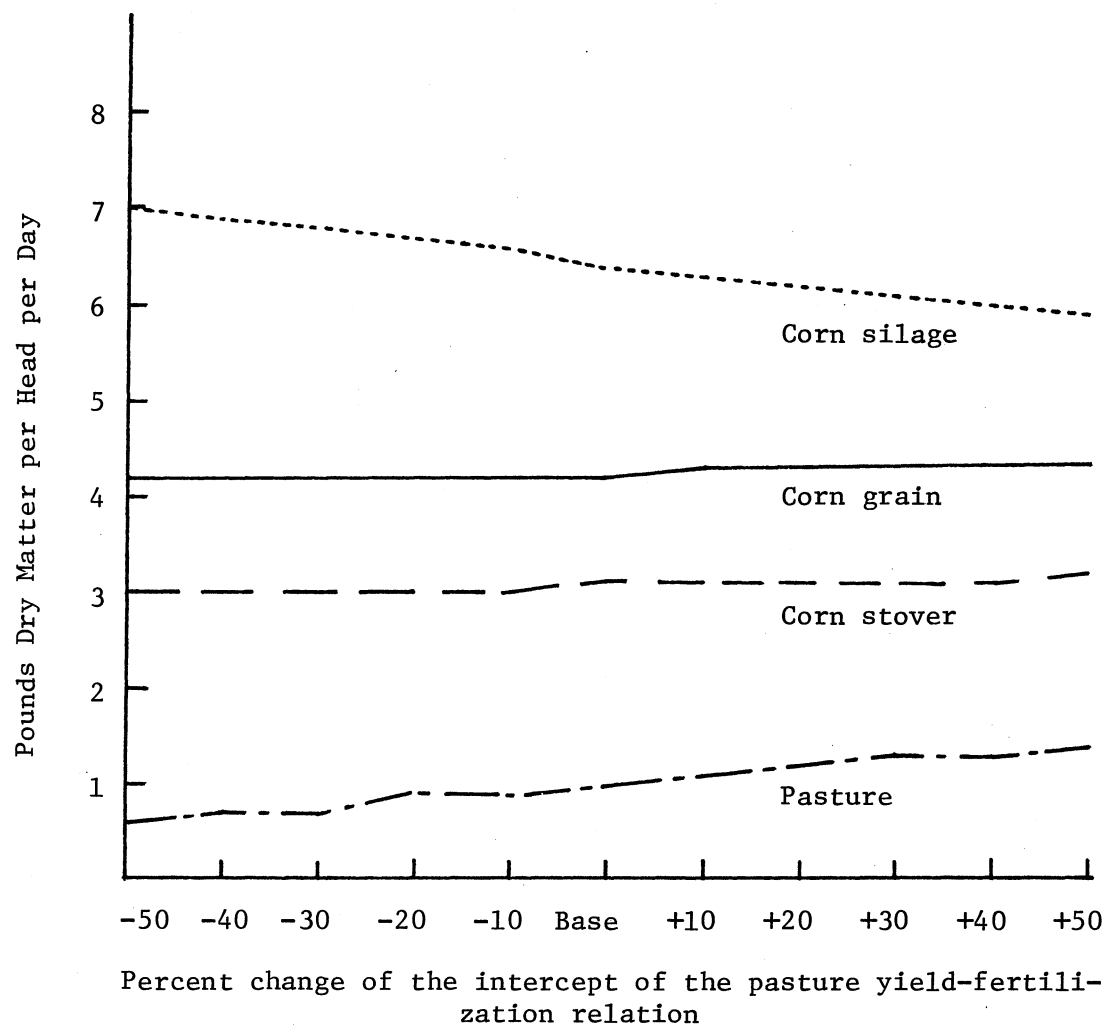


Figure 5.2. Optimal Ration for Steers Produced in the October-December Period, by Selected Pasture Yields, Appalachian Area, 1968-1977 "Average Year" Scenario

other soil classes can be calculated from the respective intercepts and slopes of the yield-fertilization relations (see Appendix B tables).

Changing corn yields had no effect on the number of cattle produced nor on the rations fed in the April-June or July-September production periods. However, the changes did have a substantial effect on the number of cattle produced and ration fed during the October-December production period, implying that the increased feed volume resulting from increased corn yields is only made available to cattle in the fall season immediately following harvest. Although this may be due in part to the presence of storage costs, it is more likely that spring pastures represent a cheaper source of net energy than corn grain or silage does at the yield levels observed.

Corn silage did tend to replace corn grain in the fall ration as both grain and silage yield intercepts shifted upward (see Figure 5.3). The probable reason for the substitution is that proportionate increases in both yield relations result in larger increases in net energy production per acre from corn silage than from corn grain. Changes in corn silage production associated with shifts in intercepts of the yield relations are depicted in Figure 5.4.

#### Corn Price Changes

The 1968-1977 "average year" base solution for the Appalachian area was next changed by parametrically increasing corn prices paid and received in each quarter by \$0.60 per bushel in \$0.20 per bushel increments. On October 1 of the first model year, the price of corn was increased from \$2.57 to \$3.17 per bushel, with no resulting change in the profit-maximum farm plan. This insensitivity indicated the superior profitability of feeding corn to cattle rather than selling the corn in surrounding markets. Presumably, if corn prices had been increased further, cash sale of corn would have entered the solution.

Corn prices paid and received were then decreased \$0.80 per bushel in each quarter in \$0.20 per bushel increments. No change in the optimal farm plan occurred until corn prices were decreased \$0.60 per bushel (to equal a cost of \$1.97 per bushel). At this point, the optimal plan responded to the lower feed costs by purchasing corn meal and by carrying

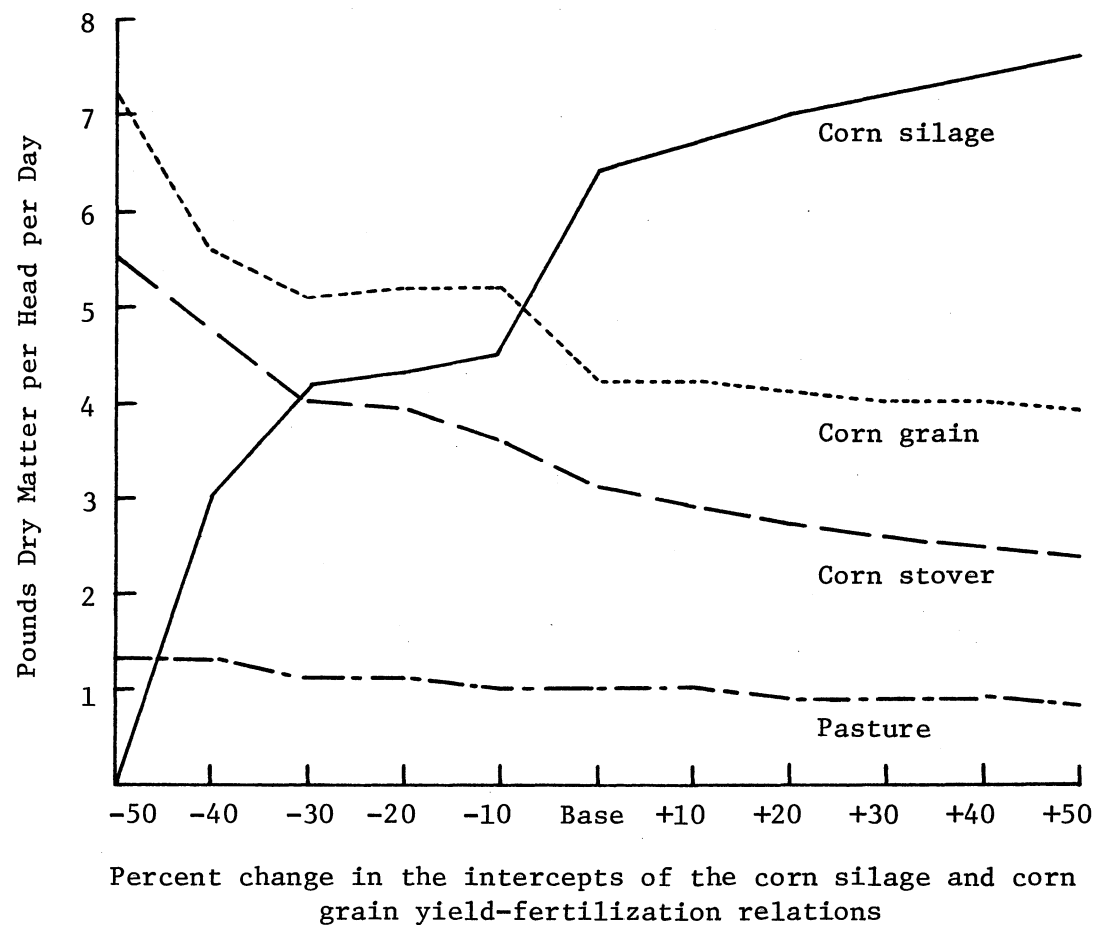


Figure 5.3. Optimal Ration for Steers Produced in the October-December Period, by Selected Corn Yields

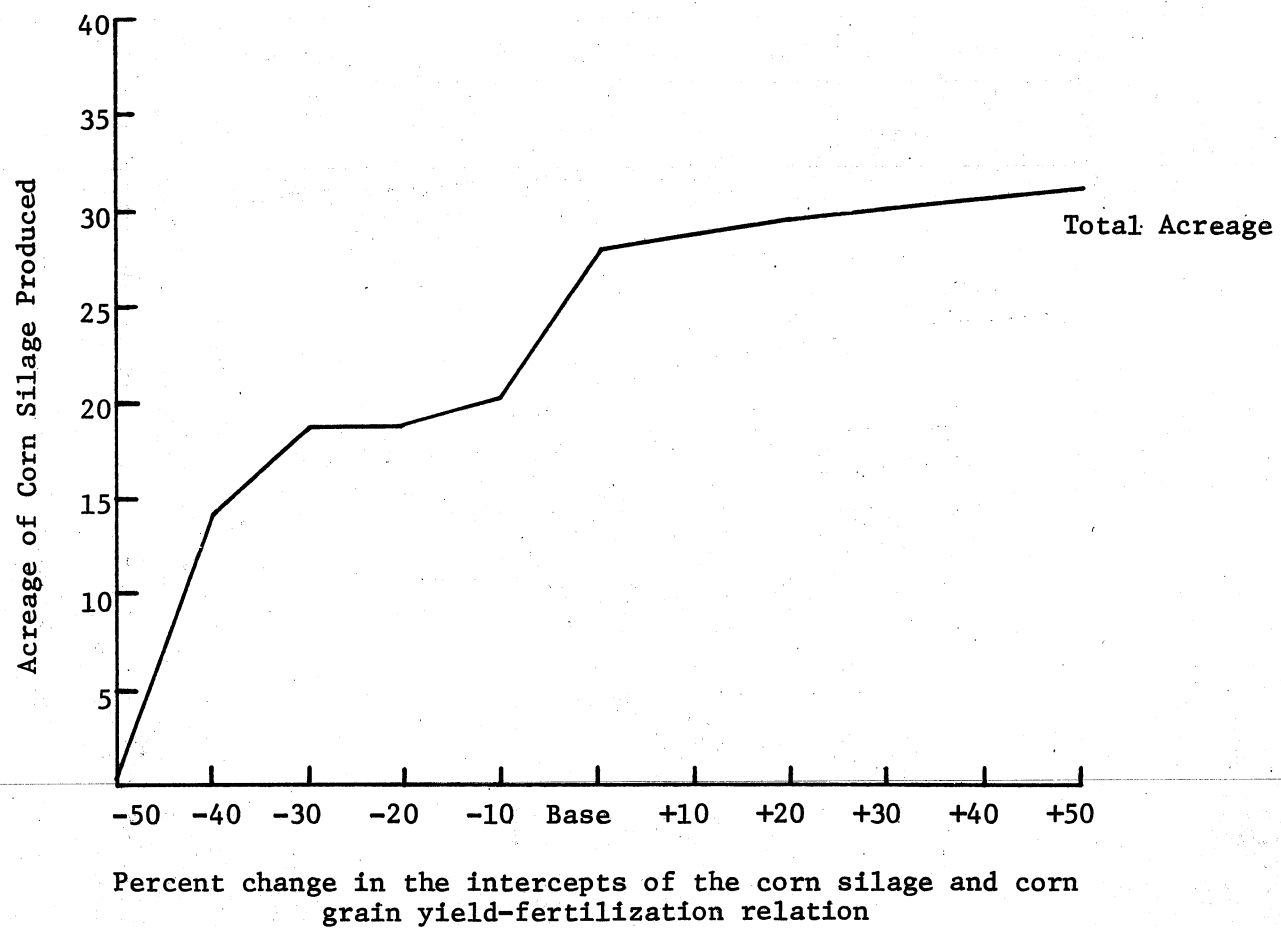


Figure 5.4. Optimal Corn Silage Acreage, by Selected Corn Silage and Grain Yield Levels

additional steers. A summary of the optimal rations in the October-December and July-September production periods when fall corn prices are \$1.97 per bushel are provided in Table 5.7. The ration fed in the other periods did not change. Twenty additional steers were produced in the October-December production period and eight additional steers were produced in the July-September production period. Corn meal, in the amount of 301 hundredweight, was purchased on January 1 of year 2, stored until July, then fed to all 61 steers produced in the July-September production period. At the same time, the amount of corn grain produced on the farm and fed to cattle declined somewhat.

This decrease in farm-produced corn grain in the optimal ration served to demonstrate the substitutability of purchased corn meal for farm-produced corn grain as a concentrate in the beef cattle diet. It is noteworthy that, although the decline in farm grain production was offset by increased roughage production in the form of corn silage, sufficient amounts of purchased corn meal were fed at low corn prices to slightly increase the steer ration energy concentration from 2.54 to 2.63 megacalories per kilogram in the July-September period.

#### Results of Parametric Changes on Cattle Prices

Cattle prices used in base and parametric solutions reported here are expressed as linear price-weight relations (see Chapter III). Hence, parametric changes on cattle prices involved intercept changes and/or slope changes. Intercept changes were sequentially carried out in three ways: (1) increasing intercepts of heifer price-weight relations in each quarter, (2) increasing and decreasing intercepts of steer and heifer price-weight relations in each quarter, and (3) increasing the July 1 intercept for steers. Slope changes were then performed on the steer price-weight relation of October 1 of year one and January 1 of year two, while holding all other cattle prices at zero level.

#### Changing the Intercepts of the Price-Weight Relations

Increasing Heifer Prices. The intercepts of the heifer price-weight relations in the Appalachian base solution were first increased in each

Table 5.7. Number of Steers Produced and Optimal Rations Fed With Fall Corn Prices Received at \$1.97 per Bushel, Appalachian Area, 1968-1977 Average Year Scenario<sup>a/</sup>

	October-December Year 1	July-September Year 2
Number steers produced <sup>b/</sup> (head)	603	61
Ration per head per day (pounds dry matter basis)		
Corn silage	6.8	--
Corn grain	4.1	1.0
Pasture	1.0	8.7
Corn stover	2.9	--
Purchased corn meal	--	5.0

<sup>a/</sup> Corn prices received per bushel in other periods are: Winter - \$1.97, Spring - \$1.94, and Summer - \$2.09.

<sup>b/</sup> The ration fed to heifers did not change from the base solution.

quarter by \$1-per-hundredweight increments up to a total increase of \$10 per hundredweight over the base solution intercepts. No change in the optimal farm plan occurred until the heifer intercepts had risen by \$4 per hundredweight. At this point, steer production dropped from the October-December production period and was replaced with 713 heifers. Crop production activities exhibited a slight increase in corn silage production, reflecting the preference of a somewhat lower energy ration for heifers at 1.67 pounds per day gain.

It is noteworthy that a shift in a heifer price-weight function intercept is sufficient to cause substitution of heifers for steers. Equal intercept shifts on both sale and purchase date cause no changes in the per pound price differentials between sale and purchase date. Thus, increased desirability of heifer production under an intercept increase is due to the increase in marginal net revenue of weight gain caused by overall increases in per hundredweight prices--and is not due to a relative change between steers and heifers in the difference between sale and purchase prices.

Changing Steer and Heifer Prices. In a second approach, the intercepts of both steer and heifer price-weight functions were decreased \$10 per hundredweight, then increased \$8 per hundredweight, in \$2 per hundredweight increments. When all cattle price-weight intercepts had been decreased \$2 per hundredweight from base solution levels, the number of steers produced during the October-December production period decreased from 583 to 573 head per day. Optimal solutions for the remaining production periods did not deviate from the base model. Indeed, no further changes in the optimal solutions in any quarter were observed as intercepts of price-weight relations were decreased to \$10 per hundredweight below base levels. The reason for this insensitivity in the optimal solution is that, although reduction of intercepts of price-weight relations decreases the marginal revenue of weight gain, marginal revenue reductions effected here were roughly the same for steers as for heifers. If price-weight intercepts and marginal revenues of gain had been reduced further, it is likely that cattle numbers and average daily gains would eventually have declined.

Increasing the intercepts of the price-weight relations had the same general effect on the optimal solution as did decreasing corn prices. Decreases in corn prices reduced the marginal cost of gain, whereas increases in intercepts of price-weight relations increased the marginal revenue of gain. Hence, each increased the marginal profit of gain. In the situation presented here, each \$0.10 per bushel decrease in corn prices was equivalent in its effect on cattle numbers to a \$1.07-per-hundredweight increase in intercepts of the cattle price-weight relations.

#### Changing the Slope of the Price-Weight Relation for Fall Steers Only

In order to focus attention on slope changes only, the intercept of the January 1 price-weight relation was lowered from that of the base solution to equal the intercept of the October 1 function; cattle prices in all other periods were set at zero level. The slopes of the January 1 and October 1 price-weight relations were then decreased algebraically (increasing the negative impact of weight on cattle price) in increments of  $.1\frac{1}{2}$ . This was equivalent to incrementally increasing the rate at which prices decreased with weight by \$0.001 per hundredweight per pound of weight increase.

When the slopes were decreased by .2, cattle production fell from base solution levels and corn silage was substituted for corn grain in the optimal ration. At this point, it also became relatively more profitable to sell part of the corn grain production than to use it for feed. Only when the slope of the price-weight relation was decreased by 1.0 (to 2.9067) did the optimal rate of gain decline to 1.1 pounds per day. It is not surprising to observe decreases in optimal daily gains with algebraic decreases in price-weight slopes because, as heavier cattle decline in value relative to lighter cattle, the marginal value of a unit of weight gain also declines.

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<sup>1/</sup> See Chapter III for the base solution price-weight relations.

## VI. SUMMARY AND CONCLUSIONS

Summary

The purpose of this study was to develop a normative model of a mixed farming operation that included beef cattle backgrounding, feed crops, cash grain, and pasture. This involved developing a framework for finding, under specified farm situations, the profit-maximizing buying and selling weights of cattle, optimal rations fed, and optimal resource mixes for production of feed inputs. It also involved determining the response of the optimal farm plan to alternative feed production costs and feed prices, and to alternative feeder cattle prices.

The model employed was a linear program with variable fertilization rates on the crop production activities, alternative cattle purchase and sale weights, and alternative rates of cattle weight gain. Description of ration constituents was made on the basis of protein and net energy content. Net energy content of feeds was in turn divided into two components: net energy for cattle weight maintenance and net energy for cattle weight gain.

The model was utilized to find profit-maximizing farm plans for typical Virginia beef farms in the Shenandoah-Piedmont and Appalachian areas of Virginia under three price scenarios: 1973-1974 prices, 1976-1977 prices, and 1968-1977 average prices. Results of the analyses indicated, in each of these cases, that profits would be maximized by production of lighter weight cattle at the maximum rate of gain considered. The optimal ration generally consisted of a mixture of corn grain and stover, corn silage, and pasture. Results substantiated the findings of Kline and Cameron (1975) and Chiang, Jensen, Kenyon, and Kline (1974) that, in Virginia, production of lightweight feeder cattle is more profitable than production of heavyweight feeder cattle.

Various parametric changes were applied to the Appalachian model under the 1968-1977 average-price scenario to evaluate the effects of such changes on the optimal farm plan. These changes entailed altering yields of corn grain, corn silage, and pasture; altering purchase and sale prices of corn; and finally, altering parameters of the feeder-cattle price-weight relations.

Proportionate increases in corn silage and corn grain yields resulted in increased cattle production in the October-December production period, increases in corn silage acreage, and decreases in corn grain acreage. Increases in pasture yields resulted in substitution of corn grain for corn silage and in slight increases in cattle numbers. In general, however, changes in intercepts of fertilization-yield relations did not significantly alter profit-maximum cattle production patterns. Although cattle numbers tended to respond positively to increases in yields, cattle continued to be produced at maximum daily gains and at lighter weights.

Essentially the same patterns as described above emerged with parametric alteration of corn prices. When corn prices declined, more cattle were backgrounded; but the optimal rate of gain and optimal cattle purchase and sale weights remained the same.

Parametric increases and decreases in intercepts of cattle price-weight functions also resulted in an increase in the number of cattle backgrounded, but usually had little effect on the optimal rate of gain. This appeared to support the contention of Wilson that, under a wide range of feed input costs, the optimal rate of gain is the maximum rate that can be obtained.<sup>1/</sup> However, as the slopes of price-weight functions were decreased algebraically--that is, as the negative impacts of cattle weight on per hundredweight price were increased--optimal rates of gain began, after some point, to decline from the maximum rate allowed in the model.

More specifically, the chief conclusions reached in this study may be summarized as follows:

- (1) Under the three scenarios employed, changes in feed production costs and feed input prices had little effect on the optimal sex, optimal buying and selling weights, or optimal rates of gain for cattle.

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<sup>1/</sup>The maximum rates of gain allowed in the model were somewhat less than those achievable by Good- and Choice-grade feeders on high concentrate diets.

- (2) Changes in parameters of the cattle price-weight relations had a greater effect than did changes in feed costs on the rate of gain, sex, or weight range of cattle produced in the optimal plan. Specifically, the change in an animal's market value from one period or weight to the next was the primary determinant of optimal purchase and sale weights and of optimal rates of gain.

### Limitations of Results

Because of the nature of the model and because of data availability, some reservations about the results should be noted. First, limited data were available for estimating fertilizer response functions of crops grown under Virginia conditions. These data were developed from research plots rather than from actual or average farm situations and tended to overstate yields that could be achieved on most farms, although some effort was made in our study to compensate for this overstatement.

Second, the estimates employed of a "typical" beef operator's resources may not completely represent current (1979) conditions. Farm resource data employed in this study were based on a survey conducted in 1968, and many changes in the typical beef operation may have occurred since that time.

Third, the functions used to estimate beef cattle nutritional requirements do not account for stress from environmental factors such as excessive heat or cold, and do not account for compensatory gains.<sup>1/</sup> There may be some inaccuracy in the net energy requirement relationships themselves under conditions of high roughage intake, since the relationships were mainly developed for cattle fed high-concentrate diets.

Fourth, there are some limitations associated with the LP model construction. The model currently allows capital to be borrowed in unlimited amounts at a constant rate of interest. This probably

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<sup>1/</sup> Compensatory gains are additional weight gains encountered when the cattle ration changes suddenly from low to high energy content.

resulted in too many cattle being produced in the fall quarter for the Shenandoah-Piedmont solutions.

Finally, some inaccuracy in optimal feed ration determination may have been caused by allowing particular feedstuffs to be devoted entirely to maintenance or entirely to gain net energy in some periods. The degree of solution bias caused by this model property is not as yet known.

APPENDIX

TABLES

Appendix Table A.1. Land Available to a Representative Beef Operation<sup>a/</sup>

Soil Class	Land Area (acres)	Proportion of Total Area (%)
<u>Appalachian Area of Virginia</u>		
Class I	21.73	8.1
Class II	10.34	3.9
Class III	13.84	5.2
Class IV	49.28	18.5
Class V	110.80	41.5
Miscellaneous	61.01	22.9
TOTAL	267.0	100.0
<u>Shenandoah-Piedmont Area of Virginia</u>		
Class I	23.59	3.8
Class II	54.88	8.8
Class III	185.62	29.8
Class IV	108.91	17.5
Class V	141.50	22.7
Miscellaneous	108.99	17.5
TOTAL	623.50	100.0

<sup>a/</sup> Includes wooded land, pits, dumps, and other land not utilized in a beef operation.

SOURCES: USDA, Soil Conservation Service; and Walker and Jobes (1975).

Appendix Table B.1. Linear Fertilizer Response Functions for Grain Corn, Silage Corn, Alfalfa Hay, Fescue Hay, and Orchardgrass-Clover Pasture<sup>a/</sup>,<sup>b/</sup>

Crop	Response Function	Researchers
Grain Corn	$G = 66.3491 + .4606N$	Lutz, Jones, Moody, and Hoepner
Silage Corn	$S = 12.2211 + .05565N$	Lutz, Jones, Moody, and Hoepner
Alfalfa Hay	$A = 4692.98 + 8.5024K + 13.5611P$ (632.08) (3.54) (6.95)	Blaser
Fescue Hay	$F = 1.100 + .006N$ (.4583) (.00173)	Hallock, Brown, and Blaser
Orchardgrass-Clover Pasture	$O = 2588.50 + 12.45K$ (37.00) (.0053)	Blaser and Brady

<sup>a/</sup> Definitions of terms: G = grain yield in bushels/acre; S = silage yield in tons/acre; A = alfalfa yield in pounds/acre; F = fescue yield in tons/acre; O = orchardgrass-ladino clover yield in pounds/acre; and N, K, and P are nitrogen, potash, and phosphoric acid, respectively, in pounds/acre.

<sup>b/</sup> Numbers in parentheses are standard errors.  $R^2$ 's for the alfalfa, fescue and orchardgrass functions were .94, .54, and .99, respectively. Sample sizes were 9, 3, and 3, respectively. The cash grain and silage functions were linearizations of quadratic functions estimated by the indicated researchers. Thus the parameter estimates reported here for these two crops do not have standard errors with the usual statistical interpretation.

Appendix Table B.2. Corn Grain: Fertilizer Recommendations, Adjusted Long Term Average Yields, and Intercepts for Corn Grain-Nitrogen Response Functions, by Soil Class

Soil Class	Fertilizer Recommendation			Adjusted Long Term Average Yield (bu/acre)	Revised Intercept
	N (lbs/acre)	P <sub>2</sub> O <sub>5</sub> (lbs/acre)	K <sub>2</sub> O (lbs/acre)		
Class I	162.5	60	60	110	53.86
Class II	137.5	50	50	100	52.50
Class III	112.5	40	40	82	43.14
Class IV	87.5	40	40	63	32.77

Source: Epperson and Hawkins (1966) and Epperson, Hawkins, and McCart (1971).

Appendix Table B.3. Corn Silage: Fertilizer Recommendations, Adjusted Long Term Average Yields, and Intercepts for Corn Silage-Nitrogen Response Functions, by Soil Class

Soil Class	Fertilizer Recommendations			Adjusted Long Term Average Yield (tons/acre)	Revised Intercept
	N (lbs/acre)	P <sub>2</sub> O <sub>5</sub> (lbs/acre)	K <sub>2</sub> O (lbs/acre)		
Class I	162.5	80	125	20	13.22
Class II	137.5	65	100	18	12.26
Class III	112.5	50	60	14	9.30
Class IV	87.5	50	60	11	7.35

Source: Epperson and Hawkins (1966) and Epperson, Hawkins, and McCart (1971).

Appendix Table B.4. Alfalfa Hay: Fertilizer Recommendations, Adjusted Long Term Average Yields, and Intercepts for Alfalfa Hay-Fertilizer Response Functions, by Soil Class

Soil Class	Fertilizer Recommendations			Adjusted Long Term Average Yield (lbs/acre)	Revised Intercept
	N (lbs/acre)	P <sub>2</sub> O <sub>5</sub> (lbs/acre)	K <sub>2</sub> O (lbs/acre)		
Class I	0	70	200	6801	5962
Class II	0	65	162.5	6234	5469
Class III	0	50	142.5	5100	4501
Class IV	0	40	112.5	3967	3488

Source: Epperson and Hawkins (1966) and Epperson, Hawkins, and McCart (1971).

Appendix Table B.5. Fescue: Fertilizer Recommendations, Long Term Average Yields, and Intercepts for Fescue-Nitrogen Response Functions, by Soil Class

Soil Class	Fertilizer Recommendations			Long Term Average Yield (tons/acre)	Revised Intercept
	N (lbs/acre)	P <sub>2</sub> O <sub>5</sub> (lbs/acre)	K <sub>2</sub> O (lbs/acre)		
Class I	180	60	100	4.0	2.92
Class II	140	55	65	3.75	2.91
Class III	100	40	40	3.25	2.65
Class IV	60	40	40	2.625	2.265

Source: Epperson and Hawkins (1966) and Epperson, Hawkins, and McCart (1971).

Appendix Table B.6. Orchardgrass-Clover Pasture: Fertilizer Recommendations, Long Term Average Yields, and Intercepts for Pasture-Fertilizer Response Functions, by Soil Class

Soil Class	Fertilizer Recommendations			Long Term Average Yield (lbs/acre)	Revised Intercept
	N (lbs/acre)	P <sub>2</sub> O <sub>5</sub> (lbs/acre)	K <sub>2</sub> O (lbs/acre)		
Class I	0	55	75	4741	3807.26
Class II	0	35	45	4042	3481.75
Class III	0	30	30	3147	2773.50
Class IV	0	25	25	2574	2262.75
Class V	0	25	25	1830	1518.75

Source: Epperson and Hawkins (1966) and Epperson, Hawkins, and McCart (1971).

Appendix Table C.1. Dry Matter Content, Net Energy for Maintenance, Net Energy for Gain, and Crude Protein Content, by Feed

Feed	Dry Matter (percent)	Net Energy for Maintenance (Mcal/kg dry matter)	Net Energy for Gain (Mcal/kg dry matter)	Crude Protein (percent)
Corn grain	84.5	2.28	1.48	10.0
Corn silage	27.9	1.56	.99	8.1
Corn stover	87.2	1.21	.55	5.9
Alfalfa hay	91.2	1.07	.28	13.6
Fescue hay	88.5	1.15	.44	11.1
Ladino clover pasture	18.0	1.69	1.11	24.7
Orchardgrass pasture	23.8	1.41	.82	18.4

Source: National Research Council (1976).

Appendix Table C.2. Costs of a Corn Storage-Drying System With Capacity of 13,440 Bushels

	Total Cost \$	Cost Per Bu ¢
<u>Variable Costs<sup>a/</sup></u>		
Energy for drying	1,269	9.44
Electricity	269	2.00
Insurance <sup>b/</sup>		
Repairs	<u>340</u>	<u>2.53</u>
Total Variable	1,878	13.97
<u>Fixed Costs</u>		
Depreciation	774	5.76
Interest	567	4.22
Taxes	26	.19
Insurance	<u>81</u>	<u>.60</u>
Total Fixed	1,448	10.77
Total Cost	3,326	24.74

<sup>a/</sup> Excludes labor and capital costs.

<sup>b/</sup> Insurance costs amount to \$88 per 3-month period, or 0.65¢ per bushel per quarter.

Source: Bauer, Donald, and Smith (1977).

Appendix Table D.1. Virginia Seasonal Average Corn and Alfalfa Prices Paid and Received By Farmers, 1973-74, 1976-77, and 1968-77 Averages<sup>a/</sup>

Date	Corn Grain Received (\$ per bu)	Alfalfa Hay Received (\$ per ton)	Corn Meal Paid (\$ per cwt)	Alfalfa Hay Paid (\$ per ton)
October 15, 1973	2.26	49.50	6.30	63.00
January 15, 1974	2.69	47.50	6.60	65.00
April 15, 1974	2.62	51.50	6.80	68.00
July 15, 1974	2.94	48.00	7.10	55.00
October 15, 1974	3.46	53.00	8.60	58.00
October 15, 1976	2.52	66.00	6.90	101.00
January 15, 1977	2.54	69.50	6.70	108.00
April 15, 1977	2.60	79.00	6.60	112.00
July 15, 1977	2.21	64.00	6.30	90.00
October 15, 1977	1.88	80.50	5.60	113.00
Fall Average <sup>b/</sup>	2.57	70.59	7.46	86.00
Winter Average <sup>b/</sup>	2.57	63.97	6.85	93.00
Spring Average <sup>b/</sup>	2.54	65.85	7.67	86.00
Summer Average <sup>b/</sup>	2.69	62.72	7.46	81.00

<sup>a/</sup> Wages paid to hired labor were assumed to be \$2.55, 2.65, 2.63, 2.08, and 2.72 per hour in October, January, April, July, and October, respectively.

<sup>b/</sup> Averages for 1968 through 1977, expressed in constant 1977 dollars.

Sources: Virginia Department of Agriculture and Commerce, Virginia Agricultural Statistics, 1974, 1975, 1976, 1977, 1978; and USDA, Agricultural Prices, Annual Summary, 1969.

Appendix Table D.2. Feeder Steer and Heifer Price-Weight Relations,  
Virginia Special Feeder Cattle Sales, 1973-74,  
1976-77, and 1968-77 Averages<sup>a</sup>

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<u>Fall, 1973</u>		
(1)	PS = 7820 - 3.75655W + .0005204W <sup>2</sup> (.041456) (.00001206)	R <sup>2</sup> = .7584 n = 5088
(2)	PH = 5193 + .1297726W - .00132199W <sup>2</sup> (.427628) (.0004108)	R <sup>2</sup> = .4667 n = 1916
(3)	<u>Spring, 1974</u>	
(3)	PS = 7346 - 4.6430414W + .00128655W <sup>2</sup> (.340384) (.0002626)	R <sup>2</sup> = .8120 n = 1062
(4)	PH = 5339 - 2.91871389W - .00136205W <sup>2</sup> (.355598) (.0002699)	R <sup>2</sup> = .5464 n = 869
<u>Fall, 1974</u>		
(5)	PS = 3428 + 1.0456738W - .00115912W <sup>2</sup> (.19002) (.000138)	R <sup>2</sup> = .4053 n = 5334
(6)	PH = 2156 + 2.0309464W - .00165471W <sup>2</sup> (.29184) (.000271)	R <sup>2</sup> = .4754 n = 1926
<u>Fall, 1976</u>		
(7)	PS = 4101 + .00813528W - .00069869W <sup>2</sup> (.2347) (.000089)	R <sup>2</sup> = .5295 n = 6579
(8)	PH = 2061 + 2.358963W - .001692W <sup>2</sup> (.24673) (.000222)	R <sup>2</sup> = .4582 n = 2652
<u>Spring, 1977</u>		
(9)	PS = 4913 - 2.02529W + .000602W <sup>2</sup> (.19563) (.000147)	R <sup>2</sup> = .7206 n = 1937
(10)	PH = 2729 + 1.53843W - .001401W <sup>2</sup> (.28379) (.000250)	R <sup>2</sup> = .6073 n = 1085

Appendix Table D.2. Continued

Fall, 1977

$$\begin{array}{ll}
 (11) \quad PS = 4353 - .140204W - .000572W^2 & R^2 = .5420 \\
 \quad \quad \quad (.11614) \quad (.0000845) & n = 6397 \\
 (12) \quad PH = 1995 + 4.15953W - .00363W^2 & R^2 = .5891 \\
 \quad \quad \quad (.27112) \quad (.000249) & n = 2847
 \end{array}$$

where

PS denotes the respective quarterly average Fancy and Choice steer price at Virginia state-graded sales in cents per hundredweight;

PH denotes the respective quarterly average Fancy and Choice heifer price at Virginia state graded sales, in cents per hundredweight;

W denotes weight of steer or heifer, in pounds.

Spring Average, 1968-77<sup>b/</sup>

$$\begin{array}{ll}
 (13) \quad PS = 6634 - 2.0269W \\
 (14) \quad PH = 5260 - 1.2823W
 \end{array}$$

Fall Average, 1968-77<sup>b/</sup>

$$\begin{array}{ll}
 (15) \quad PS = 6349 - 1.9067W \\
 (16) \quad PH = 4618 - .6331W
 \end{array}$$

where:

PS denotes the respective period average Good grade steer price at Virginia state graded sales, in cents per hundredweight;

PH denotes the respective period average Good grade heifer price at Virginia state graded sales, in cents per hundredweight; and

W denotes the weight of the steer or heifer in pounds.

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<sup>a/</sup> Standard errors are in parentheses and n refers to sample size. No standard errors are generated for equations (13)-(16).

<sup>b/</sup> These averages are expressed in constant 1977 dollars.

Appendix Table D.3. Representative Fall Steer and Heifer Prices by Weight<sup>a/</sup>

Steers		Heifers	
Weight (pounds)	Price (\$ per cwt)	Weight (pounds)	Price (\$ per cwt)
<u>Fall 1973</u>			
500	60.72	450	49.84
600	57.53	525	48.97
700	54.45	600	47.95
800	51.48	675	46.78
900	48.61	750	45.47
1000	45.84	825	44.00
1100	43.18	900	42.35
		975	40.63
<u>Fall 1976</u>			
500	39.30	450	27.80
600	38.54	525	28.33
700	37.64	600	28.67
800	36.60	675	28.82
900	35.42	750	28.78
1000	34.10	825	28.56
1100	32.65	900	28.14
		975	27.53
<u>Fall Average, 1968-1977</u>			
500	56.21	450	46.83
600	54.18	525	45.87
700	52.15	600	44.91
800	50.12	675	43.94
900	48.10	750	42.98
1000	46.07	825	42.02
1100	44.04	900	41.06
		975	40.10

<sup>a/</sup>Prices are generated from equations in Appendix Table D.2.

Appendix Table D.4. Average Seasonal Price Differentials for Feeder Steers and Heifers, Virginia Weekly Sales: Periods 1973-74, 1976-77, and 1968-77

Model	Steers		Heifers	
	Summer-Spring <sup>a/</sup> (\$ per cwt)	Winter-Fall <sup>b/</sup> (\$ per cwt)	Summer-Spring <sup>a/</sup> (\$ per cwt)	Winter-Fall <sup>b/</sup> (\$ per cwt)
1976-77	-0.57	2.64	0.22	3.14
1973-74	-4.01	-1.36	-6.73	-0.85
1968-77 <sup>c/</sup>	-3.65	3.30	-1.58	3.77

<sup>a/</sup> Average summer price minus average spring price.

<sup>b/</sup> Average winter price minus average fall price.

<sup>c/</sup> Expressed in 1977 dollars.

Appendix Table D.5. Average Steer and Heifer Prices by Weight Class, Virginia State-Graded Feeder Cattle Sales, Summer 1977

Steers		Heifers	
Weight (pounds)	Price (\$ per cwt)	Weight (pounds)	Price (\$ per cwt)
450-550	40.77	425-500	32.73
550-650	40.33	500-550	32.41
650-750	39.42	550-650	32.74
750-850	38.43	650-700	32.43
850-950	37.58	700-750	32.32
950-1000	36.52	750-850	31.20
1000+	35.80	850-900	30.80
		900+	30.97

Source: Virginia Polytechnic Institute and State University. Results, Virginia Feeder Calf Sales. Extension Division, Department of Animal Science.

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