Population

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POPULATION FORECASTING ON CITY LEVEL
AN ECONOMETRIC APPROACH

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Abstract

As an alternative to cohort analysis we present an econometric approach for (long term) forecasting of population size and age distribution in cities.

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1 INTRODUCTION

The usual method of forecasting the population of cities and countries is the well known cohort analysis [see for example Wunsch and Termote (1978)]. However, applying this method on growing cities and new towns, where a relatively large part of the population growth is due to immigration, one faces the problem of how to forecast the future number and age distribution of the migrants, as the forecast of the future population size and age distribution heavily depends upon the present and future migration, especially if the forecast period is long. In applying cohort analysis on city level one quite often assumes that the age distribution of the migrants is the same as the age distribution of the present population[1)], so that migration only affects the population size but not the age distribution. If the forecast period is short, say a few years, and if the city growth is modest, this assumption will probably not affect the forecast performance too much, but this will not be the case with long term population forecasting for growing cities.

In this paper we propose an alternative and practical approach for long term population forecasting on city level based on an econometric model. This approach is appropriate if the city growth is planned in the sense that how many and what kind of houses will be build is decided by the municipal authorities. Our approach is based on a six-equation regression model relating the number of inhabitants in each of six age groups, relative to the total stock of dwellings, to the age distribution of the stock of dwellings, the share of resident owned dwellings, the share of apartments, the mean size of the dwellings and the mean income per income earner. The parameters of this model have been estimated using cross-section data of 131 Dutch cities and villages.

We shall also report on a long term forecasting experiment. On basis of the size and age distribution of the population and various characteristics of the stock of dwellings in 1971, and detailed information on house-building in the period 1971--1980, in the city of

Huizen, we made annual population forecasts up to 1980. These forecasts turn out to be reasonable close to their corresponding realizations.

In section 2 we describe the model and the estimation results. In section 3 we show how the model can be used for forecasting future population size and age distribution and moreover we present there the above mentioned forecasting experiment for Huizen. Finally in the appendix we pay attention to the estimation procedure and the results of two specification tests.

2 THE MODEL

For constructing the model we used 1977 (primo) data of 131 Dutch cities and villages with population size ranging from 2,200 to 739,000. The majority of these cities and villages, i.e. 80%, had a population size of less than 70,000. The variables considered are listed in the tables 1 and 2 below

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Insert the tables 1 and 2 here.

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The choice of the explanatory variables in table 2 is based on the following considerations:

Young families are often not yet in the position to buy a house. Only after some years when family income has sufficiently increased one can afford to buy one. Thus families living in a resident owned dwelling will probably be more established than families living in a rented dwelling, and therefore we may expect that their children will be somewhat older.

Families with children usually prefer to live in a one family house. Therefore we may expect to find less children in apartments than in one family houses, ceteris paribus.

It is obvious that one will find more children in large dwellings than in smaller ones and that wealthy people usually live in a larger dwelling than less wealthier people. Thus given the size of the dwelling one will find less children the wealthier the residents are, and given the wealth of the residents on will find more children the larger the dwellings are.

The age distribution of the total stock of dwellings represented by the variables x_5 through x_9 corresponds with the age distribution of the residents in that the families grow older together with the dwellings they live in. This is, of course, not a causal relationship but merely a firm parallelism.

Apart from the direct impact of the variables x_1 through x_9 on each of the dependent variables y_i there may be some mutual impact of the dependent variables among each other. For instance the variables y_1 through y_4 will be related to y_5 since children generally belong to families with two parents in the age group 20-64 year. Moreover, the number of young children is negatively correlated with the number of elderly.

In view of the above considerations we specify our model as a linear simultaneous equation system, i.e.

(1) $y = By + \Gamma x + \delta + u,$

where y is a six component vector of dependent variables, x is a nine component vector of the variables x_1 through x_9, B is a 6 x 6 matrix of coefficients representing the mutual impact of the dependent variables, Γ is a 6 x 9 matrix of coefficients representing the direct impact of the explanatory variables on the dependent variables, δ is a six component vector of constant terms and u is a six component vector of disturbances with zero mean vector and finite covariance matrix. Assuming that the matrix I - B (where I is the identity matrix) is nonsingular we can write the reduced form of (1) as

(2) $y = c_0 + Cx + v,$

where

$$c_0 = (I-B)^{-1}\delta,$$

$$C = (I-B)^{-1}\Gamma$$

and

$$v = (I-B)^{-1}u.$$

Thus each equation of the system (2) has the form

(3) $y_i = c_{0,i} + \sum_{\ell=1}^{9} c_{\ell,i}x_\ell + v_i \quad , i=1,2,....,6.$

The reduced form coefficients $c_{\ell,i}$ (ℓ=0,1,....,9 ; i=1,....,6) have been estimated by using the robust M-estimation method of Bierens (1981, Section 3.2). [See also the appendix]. The results are reported in table 3 below

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Insert table 3 here.

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The numbers between brackets are the standard errors of the coefficients involved. R^2 is the well known multiple correlation coefficient. The statistic $\hat{\sigma}^2$ is the estimate of variance of the error v_i and $F(9, 121)$ is the F-statistic with 9 and 121 degrees of freedom corresponding with the test of the hypothesis that the regression coefficients c_1 through c_9 are all zero. In all cases this hypothesis is rejected at a significance level of less than 1%. We also applied two specification tests, i.e. the Chow-test [Chow (1960) and Fisher (1970)] of the stability of the relation and Test 1 of Bierens (1982) of the hypothesis $E(v_i|x) = 0$ with probability 1[2]. For all six equations the null hypotheses involved could not be rejected, hence the data do not provide evidence that the model is incorrect.

Since the results in table 3 involve reduced form coefficients, it is difficult if not impossible to interprete the relative magnitude and the signs of these coefficients, as they not only represent the direct impact of the variables x_1 through x_9 but also the intercorrelation among the dependent variables. The plausibility of the model should therefore not be judged by interpreting the estimation results but by its forecasting power.

3 FORECASTING

In order to use this model for forecasting the population size and age distribution of a particular city over a certain period we need the values of the explanatory variables x_1 to x_9 for each year of this period. Most of the Dutch cities keep statistics of their stock of dwellings, and since city growth in the Netherlands is planned by the municipal authorities sufficient data on the future stock of dwellings are also available. The only problem is to predict the future values of the variable x_4, the mean income (in constant prices) per income earner. In practical applications of our model [See Bierens et.al. (1982a, 1982b)] we have always assumed that the impact of this variable will not vary much in time, hence that we may treat this variable as a constant term.

Now suppose we want to forecast the population size and age distribution over the period 1982-1990, say, in a particular city and that 1981 is the last year for which the actual values of the dependent variables are available. We may then shift the constant term (including the terms $c_{i,4}x_4$) of the model such that the model fits perfectly for 1981. Fixing these constant terms on their new level we then plug in the values of the explanatory variables x_1 through x_3 and x_5 through x_9 for each year of the forecasting period involved to obtain "raw" forecasts. Obviously these raw forecasts do not take into account the possible change of the birth rate. Since the birth rate is changing over time we have to make some adjustment. The correction procedure we have applied is the following. Let b(t) be the (forecast of the) birth rate in year t. The raw forecast of the variable y_1 for, say, 1985 is now corrected by multiplying it by

$$\sum_{t=1980}^{1984} b(t) \Big/ \sum_{t=1976}^{1980} b(t),$$

for, at the beginning of 1985, the children in the age group 0-4 years have been born in the period 1980-1984. The birth rates in these years are thus compared with the birth rates of the period 1976-1980 in which the children in the age group 0-4 years, at the beginning of 1981, were born. More generally, let t_0 be year in which the constant terms have been fixed and let $\tilde{y}_1(t),\ldots.\tilde{y}_4(t)$ be the raw forecasts of the variables y_1 through y_4 for $t > t_0$. Then the corrected forecasts are:

$$\hat{y}_1(t) = \frac{\sum_{j=t-5}^{t-1} b(j)}{\sum_{j=t_0-5}^{t_0-1} b(j)} \cdot \tilde{y}_1(t),$$

$$\hat{y}_2(t) = \frac{\sum_{j=t-10}^{t-6} b(j)}{\sum_{j=t_0-10}^{t_0-6} b(j)} \cdot \tilde{y}_2(t),$$

$$\hat{y}_3(t) = \frac{\sum_{j=t-15}^{t-11} b(j)}{\sum_{t_0-15}^{t_0-15} b(j)} \cdot \tilde{y}_3(t),$$

$$\hat{y}_4(t) = \frac{\sum_{j=t-20}^{t-16} b(j)}{\sum_{j=t_0-20}^{t_0-11} b(j)} \cdot \tilde{y}_4(t).$$

As a test of the prediction ability of our model we applied the above procedure to a medium size Dutch city, namely Huizen. On basis of the actual situation in 1971 we predicted the population size in the six age groups over the period 1972-1980 and we compared these predictions with the corresponding realizations. The results are given in table 4.

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Insert table 4 here.

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The birth rate sequence we used to correct the raw forecasts concerned the number of live birth per 1000 women in the age group 15-49 years in the Netherlands, as reported by the Central Bureau of Statistics [CBS 1976]. It would be better, of course, to use the birth rate in Huizen itself, but this series was not available. Nevertheless the forecasting results provide evidence that our econometric approach is a useful alternative to cohort analysis.

Furthermore we apply another correction to the raw forecasts to take into account a social change in the Netherlands. In the beginning of the 1970's it was still not uncommon for maids, servants and "children" in the age group 20-64 years to live or stay living with their master or parents. Since then the situation has changed very much. Drawing on experience we assume that the percentage of people in the age group 20-64 years living with master or parents will linearly decrease from approximately 13% in 1971 [CBS 1971] to zero in 1977.

APPENDIX

1 Specification testing

The reduced form model (3) consists of linear regression equations. In order that these linear equations are legitimate representations of the structures involved, they should satisfy the usual regression assumption that the expectation of the error term v_i conditional on the vector x of regressors equals zero with probability 1. Thus the model is true if

$$H_0 : E(v_i|x) = 0 \quad \text{a.s.} \quad \text{for } i = 1,2,\ldots,6$$

In Bierens (1982) two consistent tests have been proposed for testing the above hypothesis. In the research under review we have applied one of these tests, i.e. test 1, to each of the six equations. In particular we have applied the same procedure as in the numerical example in Section 9 of Bierens (1982). The statistic of model specification test 1 is a random variable $\hat{m}(\varepsilon)$, say, depending on a test parameter $0<\varepsilon<\infty$, satisfying

$\limsup_{n\to\infty} P(\hat{m}(\varepsilon) \leq \alpha) \leq \alpha$ for any $\varepsilon \geq 0$ and any $\alpha > 0$ if the null hypothesis is true

and

$\lim_{n\to\infty} P(\hat{m}(\varepsilon) \leq \alpha) = 1$ for any $\varepsilon > 0$ and any $\alpha > 0$ if the null hypothesis is false,

where n is the number of observations. Thus conducting the test at, say, the 5% significance level we do not reject the null if for given $\varepsilon > 0$, $\hat{m}(\varepsilon) > 0.05$, and we reject the null if not. In table A 1 we present the values of this test-statistic $\hat{m}(\varepsilon)$ - for $\varepsilon = 1,2,3,4$ and 5 - for each of the six equations. Clearly these results show that there is no evidence that the linear specification is false.

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Insert table A 1 here.

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A second specification test we have applied is the test of the hypothesis that the model applies to all observations, using the Chow-test [Chow (1960) and Fisher (1970)]. In the case under review the test-statistic is distributed as F (10, 111)[3)] under the null hypothesis. The results in table A 2 indicate that if we perform this test at the 5% significance level we cannot reject the null hypothesis.

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Insert tabel A 2 here.

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2 Estimation

The usual method for estimating linear relationships is the well known Ordinary Least Squares (OLS) estimation method. Writing a particular equation of our model in matrix form as

$$y = X\beta + v$$

where $y' = (y_1,\dots,y_n)$, $v = (v_1,v_2,\dots,v_n)$, $\beta'=(\beta_1,\dots,\beta_k)$

$$X = \begin{pmatrix} x_{1,1},\dots,x_{1,k} \\ x_{n,1},\dots,x_{n,k} \end{pmatrix} ,$$

with n the number of observations and k the number of regressors (including the constant term), the OLS estimator of the parameter vector β is defined as

$$\hat{\beta} = (X'X)^{-1}X'y \ .$$

Under suitable regularity conditions we have

$$\sqrt{n}(\hat{\beta} - \beta) \to N_k[0, \sigma^2(\text{plim } \tfrac{1}{n}X'X)^{-1}] \text{ in distribution,}$$

where σ^2 is the variance of the $u_j^{\prime}$s, i.e.

$$\sigma^2 = Ev_j^2 \ .$$

Moreover, σ^2 can be estimated consistently by the variance of the OLS residuals

$$\hat{\sigma}^2 = \tfrac{1}{n}\textstyle\sum_{j=1}^{n}\hat{u}_j^2 \ .$$

However, in Bierens (1981, Chapter 3) it has been shown that if the distribution of the errors v_j has a positive kurtosis then the robust M-estimation method is more efficient. The robust M-estimation method of Bierens is a two-stage estimation procedure where an estimator $\tilde{\beta}(\gamma)$ is obtained by maximizing an objective function of the form

$$\tilde{R}(\beta|\gamma) = \frac{1}{n}\sum_{j=1}^{n}\rho\left(\frac{y_j - \sum_{i=1}^{k}\beta_i x_{ij}}{\gamma}\right) ,$$

where ρ is the density of the standard normal distribution and γ is a positive scale parameter. Under some regularity conditions we have

$$\sqrt{n}(\tilde{\beta}(\gamma) - \beta) \to N_k[0, h(\gamma)\ (\text{plim}\ \frac{1}{n}X'X)^{-1}] \text{ in distr..}$$

Now if the kurtosis of the error distribution, defined by

$$K = \frac{E\ v_j^4}{(E\ v_j^2)^2} - 3 ,$$

is positive then there exists a $\gamma > 0$ such that

$$h(\gamma) < \sigma^2 ,$$

hence in this case $\tilde{\beta}(\gamma)$ is more efficient than the OLS estimator $\hat{\beta}$. Note that a positive kurtosis implies that the tails of the error distribution are heavier than those of the normal distribution.

The function $h(\gamma)$ can be estimated uniform consistently by

$$\tilde{h}(\gamma) = \frac{\gamma^2\ \frac{1}{n}\sum_{j=1}^{n}\rho'(\hat{u}_j/\gamma)^2}{\{\frac{1}{n}\sum_{j=1}^{n}\rho''(\hat{u}_j/\gamma)\}^2}$$

where

$$\hat{u}_j = y_j - \sum_{j=1}^{n}\beta_i^* x_{ij}$$

with the β_i^* consistent initial estimates of the β_i, for example the OLS estimate or a robust M estimate with some fixed $\gamma_1 > 0$. It can be shown that if $\tilde{h}(\gamma)$ takes a minimum at $\tilde{\gamma}$ on an interval $[\gamma_*,\infty)$ with $\gamma_* > 0$ and if $h(\gamma)$ takes a minimum at γ_0 on the same interval, then

$$\text{plim}\ \tilde{\gamma} = \gamma_0 ,$$

$$\text{plim}\ \tilde{h}(\tilde{\gamma}) = h(\gamma_0)$$

and

$$\sqrt{n}(\tilde{\beta}(\tilde{\gamma}) - \beta) \rightarrow N[0,\ h(\gamma_0)(\text{plim}\ \frac{1}{n}X'X)^{-1}] \quad \text{in distr..}$$

Thus $\tilde{\beta}(\tilde{\gamma})$ is the efficient robust M-estimator of β.

All the six equations of our model have been estimated by using OLS as well as the above robust M-estimation method. From table A 3 below we see that in all cases the robust M-estimation method performs better, for $\tilde{h}(\tilde{\gamma})$ is always smaller than $\hat{\sigma}^2$. Only for equation 6, the equation concerning the elderly, the difference between $\tilde{h}(\tilde{\gamma})$ and $\hat{\sigma}^2$ is very small, so that for this equation the OLS estimator is nearly as efficient as the robust M-estimator. These results correspond with those in table A 4, for a positive kurtosis implies that robust M-estimation is more efficient than OLS estimation.

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Insert the tables A 3 and A 4 here.

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FOOTNOTES

1)This is common practice in the Netherlands

2)See the Appendix

3)This is the well known Fisher distribution with 10 and 111 degrees of freedom.The dataset was randomly splitted in two subsets of size 100 and 31, respectively.

TABLES

Table 1 Dependent variables

notation	description
y_1	Number of inhabitants in the age group 0-4 years, relative to the total stock of dwellings.
y_2	Idem in the age group 5-9 years.
y_3	Idem in the age group 10-14 years.
y_4	Idem in the age group 15-19 years.
y_5	Idem in the age group 20-64 years.
y_6	Idem in the age group 65 years and older.

Table 2 Explanatory variables

notation	description
x_1	Number of resident owned dwellings, relative to the total stock of dwellings.
x_2	Number of apartments, relative to the total stock of dwellings.
x_3	Mean number of rooms, including the kitchen, per dwelling.
x_4	Mean income per income earner.
x_5	Number of dwellings in the age group 0-4 year relative to the total stock of dwellings.
x_6	_Idem_ in the age group 5-11 years.
x_7	_Idem_ in the age group 12-16 years.
x_8	_Idem_ in the age group 17-31 years.
x_9	_Idem_ in the age group 32-64 years.

Table 3 Estimation results

y \ c	0	1	2	3	4	5	6	7	8	9	R^2	$\hat{\sigma}^2$	F(9,121)
1	.2225 (.0648)	-.0336 (.0391)	-.1139 (.0340)	.0230 (.0144)	-.2425 (.1222)	.2263 (.0669)	.1214 (.0533)	.0044 (.1267)	-.2524 (.0611)	-.4635 (.1252)	.6191	.0017	21.9683
2	.0517 (.0592)	.0508 (.0357)	-.1033 (.0311)	.0497 (.0132)	-.1445 (.1117)	.0906 (.0612)	.2422 (.0487)	.2086 (.1159)	-.2315 (.0559)	-.4416 (.1145)	.7156	.0014	39.4713
3	-.1578 (.0541)	.0814 (.0326)	-.0383 (.0284)	.0819 (.0120)	-.0221 (.1021)	.0037 (.0559)	.1461 (.0445)	.2914 (.1059)	-.1106 (.0511)	-.4099 (.1045)	.7180	.0013	35.3952
4	-.2962 (.0552)	.0443 (.0333)	.0186 (.0290)	.1141 (.0123)	.0793 (.1042)	-.0992 (.0570)	.0514 (.0455)	.0072 (.1081)	.04275 (.0521)	-.2850 (.1067)	.5388	.0020	16.8259
5	-.1176 (.1771)	.2802 (.1068)	.2375 (.0929)	.3591 (.0394)	-1.0630 (.3340)	.2495 (.1828)	.3340 (.1457)	.3881 (.3465)	-.1430 (.1672)	-.5844 (.3422)	.6561	.0144	26.2004
6	.2982 (.0833)	.0993 (.0502)	-.0073 (.0437)	.0309 (.0185)	.0886 (.1571)	-.4153 (.0860)	-.3430 (.0685)	-.5224 (.1629)	-.1093 (.0786)	.4245 (.1609)	.5145	.0025	14.2468

Table 4 Long term population forecasts for Huizen on basis of the situation in 1971

row 1 = actual number of inhabitants
row 2 = forecast
row 3 = percentage forecast error.

year	number of inhabitants in the age group: 0-4	5-9	10-14	15-19	20-64	65+	total	stock of dwellings
1972	2400	2600	2340	2020	12910	1930	24200	6878
	2360	2670	2350	1960	12940	1900	24190	
	-1.6	2.8	0.7	3.1	0.2	-1.7	-0.1	
1973	2380	2630	2380	2080	13120	2020	24600	7005
	2270	2650	2390	2000	12880	1960	24150	
	-4.7	0.8	0.6	-3.8	-1.8	-2.8	-1.9	
1974	2170	2620	2500	2130	13100	2140	24640	7150
	2140	2650	2430	2050	12850	2000	24100	
	-1.5	1.4	-3.0	-3.8	-1.9	-6.6	-2.2	
1975	1970	2650	2520	2210	12980	2210	24540	7161
	1970	2620	2440	2090	12610	2000	23720	
	0.0	-1.2	-3.5	-5.2	-2.8	-9.5	-3.3	
1976	1740	2700	2690	2250	13960	2270	25600	7740
	2080	2860	2660	2320	13620	2030	25570	
	19.5	6.2	-1.3	3.4	-2.4	-10.5	-0.1	
1977	2050	2850	2810	2300	15210	2380	27600	8481
	2150	3080	2940	2670	14890	2360	28090	
	5.0	8.1	4.7	16.0	-2.1	-1.3	1.8	
1978	1930	2820	2780	2360	15360	2440	27690	8657
	2190	3140	3040	2760	15070	2190	28390	
	13.2	11.2	9.4	16.9	-1.9	-10.3	2.5	
1979	2080	2760	2910	2510	16170	2540	28980	9175
	2290	3090	3110	2800	15820	2240	29350	
	10.7	11.8	6.9	11.6	-2.2	-11.8	1.3	
1980	2040	2650	2960	2620	16500	2570	29340	9239
	2240	2880	3070	2770	15900	2260	29120	
	9.8	8.7	3.7	5.7	-3.6	-12.1	-0.7	

Table A 1 Values of the test-statistic of Bierens' specification test 1

Equation \ ε	1.	2.	3.	4.	5.
1	.74	.95	1.02	1.00	1.00
2	.50	.82	1.03	1.02	1.00
3	.73	1.00	1.07	1.03	1.01
4	.99	.98	.98	.99	.99
5	.94	1.06	1.02	1.01	1.00
6	.61	.81	.98	1.00	1.00

Table A 2 Chow-test statistics

Equation	F(10,111)
1	.6737
2	.8218
3	.9811
4	1.1335
5	1.0786
6	.1764

Table A 3 A comparison of the efficiency of OLS and robust M-estimates

Equation	$\hat{\sigma}^2$	$\tilde{h}(\tilde{\gamma})$
1	.001694	.001560
2	.001401	.001305
3	.001280	.001088
4	.001895	.001135
5	.013950	.011660
6	.002580	.002578

Table A 4 Estimated kurtosis*) of the error distribution

Equation	Estimated kurtosis of: OLS residuals	Robust M-residuals
1	1.556	1.777
2	1.348	1.430
3	2.259	3.933
4	6.711	9.709
5	4.315	7.474
6	0.093	0.102

*) Defined as $\frac{1}{n}\sum_{j=1}^{n}\hat{u}_j^4 / (\frac{1}{n}\sum_{j=1}^{n}\hat{u}_j^2)^2 - 3$, where the $\hat{u}_j$ are the - mean corrected - residuals.