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Substitution between energy and non-energy inputs in the Netherlands, 1950-1974

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ABSTRACT

This paper seeks to estimate factor demand relations in a three factor demand model that allows for considerable freedom in the variation of the substitution parameters.

Our starting point is a twice differentiable aggregate production function

$$Y = F[K,L,E.M,A] \ ,$$

which relates gross output Y to the services of capital (K), labor(L), energy (E), and all other materials (M). A is a technology index. Under certain conditions this function can be written

$$Y = F_1[H(K,L,E,A_1),M,A_2] \ .$$

For purposes of estimation we choose a specific functional form for H, viz. the Generalized Cobb-Douglas (GCD) function developed by Diewert. Further restrictions of constant returns and Hicks neutrality are needed to arrive at

$$Y = F_2[A_1.f(K,L,E), M,A_2] \ ,$$

where f also is a GCD function.

We are concerned with estimating the function f(K,L,E). Direct estimation of f is however impossible, since the value of f cannot be observed. This difficulty is solved by using the duality relationship between cost and production functions (Shephard, Arrow, Diewert).

Application of Shephard's lemma leads to the following system that is linear in the unknown parameters:

$$y_i = \sum_k \phi_{ik}\beta_{ik} \qquad (i=1,2,3),$$

where the y_i are cost shares, ϕ_{ik} is a function of the prices, and β_{ik} are the parameters to be estimated.

A major problem has been to collect adequate data on factor prices. Especially the price index of capital services, which is based on Christensen and Jorgenson, can easily be challanged.

We fit the model to data for the Dutch enterprise sector, 1950-1974, and find that energy and labor are substitutes, and energy and capital complements (after 1960). This justifies the inclusion of energy as a seperate input in the production function. The results can be used to assess the effect of energy price changes on energy use and total output.

SUBSTITUTION BETWEEN ENERGY AND NON-ENERGY INPUTS IN THE NETHERLANDS 1950 - 1974*)

By Jan R. Magnus

1. Introduction

The oil crisis of 1973 brought home the immediate dependence of economic activity on energy supplies and reinforced the anxieties about the long-term prospects raised by the Meadows report [1972]. Today, the general view that scarcity of energy will affect output or at least impede its growth seems so obvious as to require no argument. The effects of the embargo as a physical restriction were of course readily demonstrated by imposing bottleneck constraints in an input-output model.

It is not so easy, however, to enlarge on a longer-term perspective. This contains the threat of a continued increase in the real costs of energy rather than of sudden disruptions of supply, and it is clear that the response to this will involve substitution. For if the popular view is that energy scarcity reduces economic growth just as cheap, abundant supplies favour it, the underlying belief is that these effects arise precisely because the economic process responds to these stimuli by adaptation.

One should wish for an economic model that corresponds to these simple beliefs and demonstrates inter alia why output should react to variations in factor prices. The present paper falls short of this objective, as it is limited to the substitution between factors in response to price changes without explicit consideration of the level of output. We do however include energy amongst the factors of production, and thus open the way to the introduction of non-factor inputs in the aggregate production function.

The first three sections that follow present the economic model which is based on the Generalized Cobb-Douglas Cost function developed in Diewert [1973]. In section 5 we discuss the construction of the relevant annual aggregate data for the Dutch economy 1950-1974. Section 6 is devoted to estimation and prediction problems. In the last two sections we assess the validity of the model by fitting it to the data.

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2. The problem of value added; weak separability.

Classical economists recognized three primary factors of production, viz. labor, capital and land or natural resources. The latter was quietly dropped when Douglas introduced the first explicit production function in 1928 and it passed into oblivion until it was recalled by Meadows. The vast majority of production function studies by economists deal with labor and capital only and refer to value added rather than to output. Material or "non-factor" inputs are omitted from the production function and substracted from its result. This elimination of particular inputs implies certain assumptions about their rôle in production, which we will now investigate.

Our starting point is a twice differentiable aggregate production function

$$Y = F[K,L,E,M,A] \quad , \tag{1}$$

which relates gross output Y to the services of four aggregate inputs: capital (K), labor (L), energy (E), and all other materials (M). A is a technology index. The formulation of such a model presupposes the existence of consistent aggregate indices for the inputs K,L,E and M, that is it requires that every input is weakly separable from all other inputs[1]. It is, however, by no means certain that such indices exist. Berndt and Christensen [1973a, 1974] have investigated the existence of consistent aggregate indices of labor and capital in US manufacturing. They found that equipment and structures could be consistently aggregated, but that this was not possible for production workers and non-production workers.

In the present case we use K and L without a further examination of this issue; while a case can be made for disaggregating either variable, we have decided not to follow this route since it would increase sensibly the number of parameters[2], which is undesirable from the point of view of estimation. The reason why we distinguish between M and E will be clarified in the sequel.

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1) See Green [1964] and Berndt/Christensen [1973b] for a further discussion of consistent aggregation and weak separability. See also Leontief [1947a,b] and Solow [1956].

2) In our Generalized Cobb-Douglas model we have three inputs and six parameters. Four inputs would imply ten parameters and five inputs fifteen.

In the context of a production function the concept of real value added has an economic interpretation only if one of the following three conditions holds[3]:

1. The prices P_E, P_M and P_Y move in fixed proportions;
2. The quantities E, M and Y move in fixed proportions;
3. K and L are weakly separable from E and M.

Condition 1) is discussed in Diewert [1972] in the context of Hicks' aggregation theorem. It does not apply in the instance we have in mind, since the price of energy fell quite considerably relative to the price of produced goods as is shown in fig. 1. Likewise the second condition - Leontief's aggregation theorem[4] - is not applicable as energy consumption rose much faster than output, as shown in fig. 2. Condition 3) finally means that we can write:

$$F[K,L,E,M,A] = F_1[g(K,L),E,M,A] \qquad (2)$$

where g(K,L) is identified as real value added. Berndt and Christensen [1973b] have shown that this assumption of separability leads to severe restrictions on Allen partial elasticities of substitution[5] between pairs of inputs. If condition 3) is satisfied, it turns out that the elasticity of substitution between energy and any capital or labor input must be the same. This seems highly implausible: we would expect energy and certain types of capital services to be complements (i.e. have negative partial elasticities of substitution), energy and certain types of maintenance workers to be complements, and energy and unskilled labor to be substitutes (i.e. have positive partial elasticities of substitution).

We conclude that there is no theoretical justification for the use of real value added as a measure of production, and that we should therefore include E and M in the production function.

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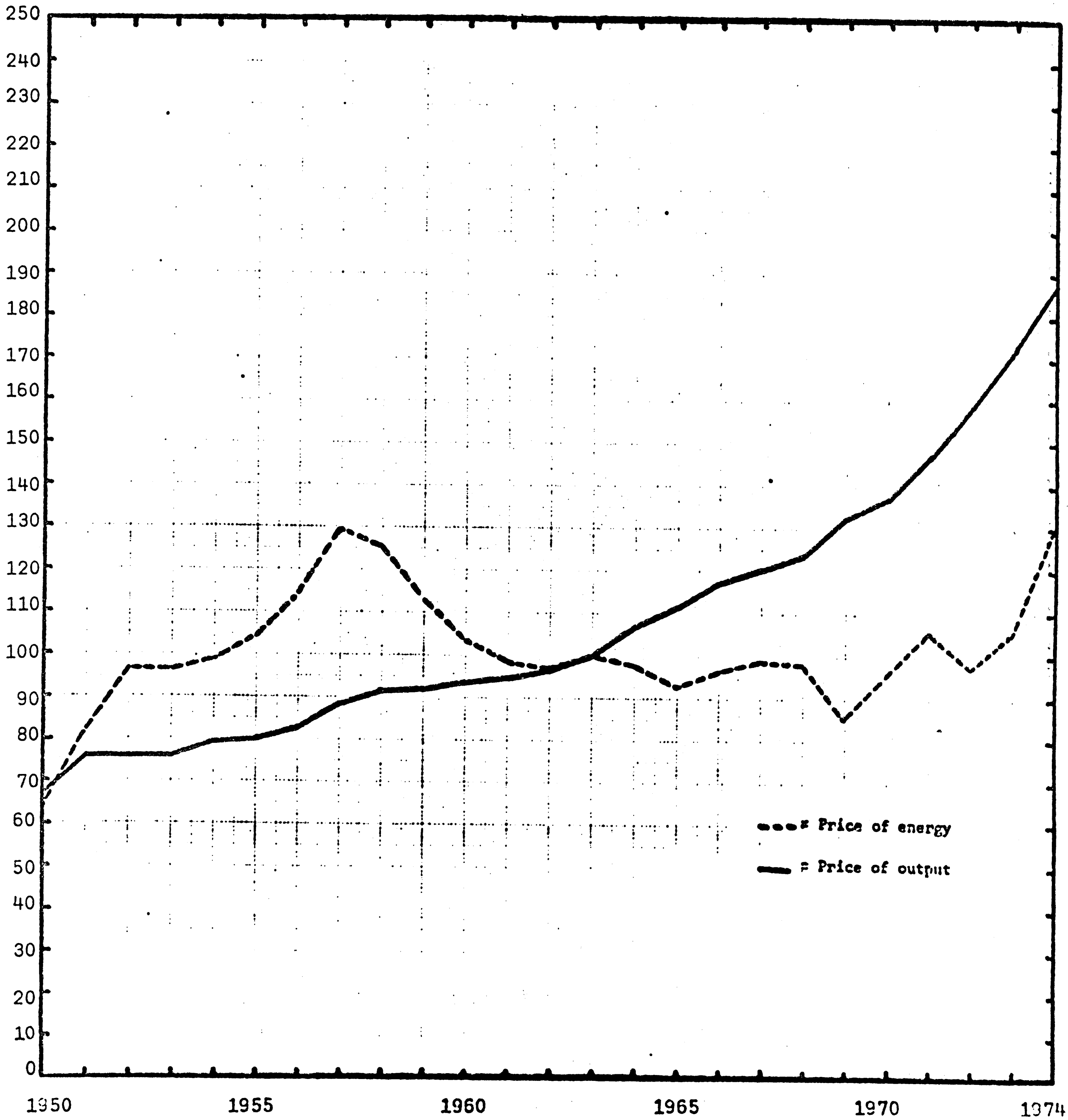
3) See Berndt and Wood [1975 p. 265]

4) Leontief [1936 p. 55].

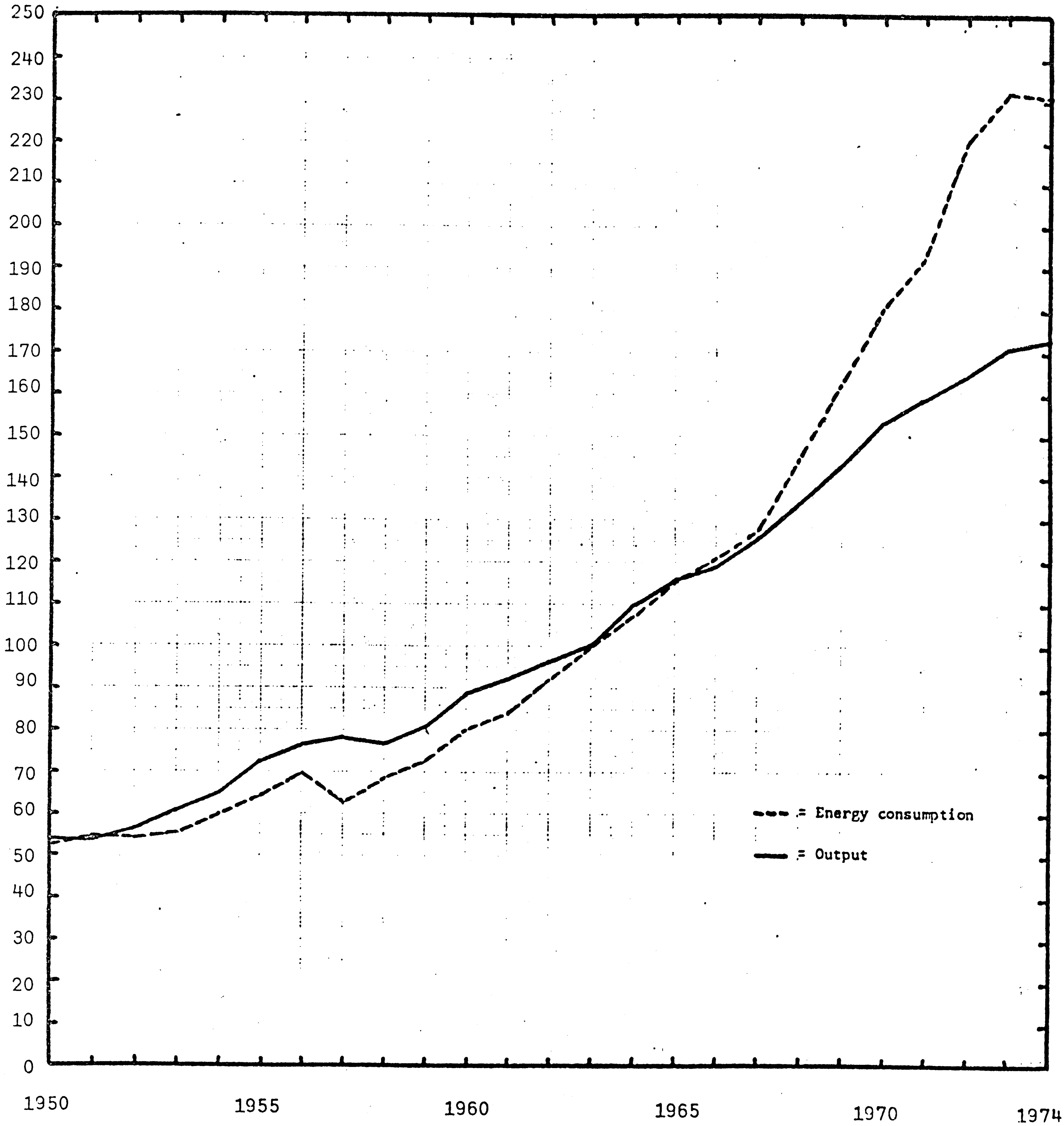
5) See Allen [1938 pp. 503-509] for a definition.

Figure 1. Prices of output and energy in the Netherlands 1950-74
(index 1963 = 100)



Source: see section 5

Figure 2. Output and energy consumption in the Netherland 1950-74
(index 1963 = 100)



Source: see section 5

Yet limitations in data collection prevent us from taking this course. We are able to construct the relevant data for E, but to include M would entail a series of estimates of the annual use in production of raw materials, both imported and home produced; in the latter case the net contribution of land and other natural resources must be assessed. This task is beyond us. Reluctantly we act as though one of the above three conditions is valid with respect to M, though not with respect to E. This enables us to write:

(3) $$Y = F[K,L,E,M,A] = F_2[H(K,L,E,A_1),M,A_2]$$

3. The Generalized Cobb-Douglas Production Function.

For purposes of estimation we must employ a specific functional form for H. We have searched for a highly general functional form, one that places no a priori restrictions on the Allen partial elasticities of substitution (AES), and one that can be interpreted as a second order approximation to an arbitrary twice-differentiable production function.
Several functional forms satisfy these requirements[6]: the translog function[7], the Generalized Leontief[8] and the Generalized Cobb-Douglas. The choice between these functions is a matter of taste. We opted for the Generalized Cobb-Douglas (GCD) function developed by Diewert [1973], because it is the most natural generalization of the celebrated Cobb-Douglas function. No study using the GCD function is known to us, but the translog function and the Generalized Leontief have been empirically applied several times[9].

The n-factor GCD production function is defined as:

$$V(X_1,X_2,\ldots,X_n) = \Theta \prod_{i=1}^{n} \prod_{j=1}^{n} (\tfrac{1}{2}X_i+\tfrac{1}{2}X_j)^{\alpha_{ij}}, \quad \Theta > 0, \quad \alpha_{ij} = \alpha_{ji}$$

where X_i is the quantity of the ith input and V is output.
Treating one variable as the technology index[10], we may write:

$$V(X_1,\ldots,X_n,A) = \Theta \prod_{i=1}^{n} \prod_{j=1}^{n} (\tfrac{1}{2}X_i+\tfrac{1}{2}X_j)^{\alpha_{ij}} \prod_{h=1}^{n} (\tfrac{1}{2}X_h+\tfrac{1}{2}A)^{\alpha_{0h}} A^{\gamma}$$

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6) See Diewert [1974].

7) See Christensen - Jorgenson - Lau [1971, 1973].

8) Diewert [1971].

9) See for instance Parks [1971] and Lau - Tamura [1972] for applications of the Generalized Leontief and Christensen-Jorgenson-Lau [1973] and Berndt-Christensen [1973a, 1974] for applications of the translog function.

10) The technology index is treated in a similar way in Berndt-Christensen [1973a p. 83-84].

We assume that production is characterized by constant returns to scale and that any technical change affecting the inputs is Hicks-neutral. This leads to the following conditions:

1) $V(\lambda X_1,\lambda X_2,\ldots,\lambda X_n,A) = \lambda V(X_1,\ldots,X_n,A)$

2) $V(X_1,\ldots,X_n,A) = AV(X_1,\ldots,X_n)$

which imply that $\sum_{i,j=1}^{n} \alpha_{ij} = 1.$

From 2) we see that individual inputs are transformed into an aggregate input $V(X_1,\ldots X_n)$, which in turn is transformed into output by the scalar technology index A. We may therefore consider the input function $V(X_1,\ldots,X_n)$ alone, disregarding the technology index.
Consider again the aggregate production function

$$Y = F(K,L,E,M,A).$$

Suppose that there exists an input function $H(K,L,E,A_1)$ with constant returns to scale in input quantities, which is weakly separable from the other inputs in the production function F. Suppose also that any technical change affecting K,L and E is Hicks-neutral, and that H is a GCD function. Then it follows from (3) and (4) that we can write the production function F as:

$$(5) \qquad Y = F[K,L,E,M,A] = F_2[H(K,L,E,A_1),M,A_2] = \\ = F_2[A_1.f(K,L,E),M,A_2] \;,$$

where f also is a GCD function.
In the remainder of this paper we shall be concerned with estimating the function f(K,L,E). As the value of the function f (denoted as z) cannot be observed, direct estimation of f is impossible. In the next section we will indicate how we can get round this difficulty.

4. The Generalized Cobb-Douglas Cost Function; cost shares; the AES.

In recent years many attempts have been made to estimate production functions indirectly by means of cost-minimizing or profit-maximizing conditions that hold at given input prices. The optimal values of input quantities or of costs can then be expressed as a function of these prices. The estimation of Cobb-Douglas production function elasticities from income shares is the oldest example of such an approach. It has, however, become particularly widespread since Arrow et al. [1961] regressed value added per worker on the wage rate in order to estimate the elasticity of substitution. The use of cost functions, which is equally based upon an assumption about cost minimization for each given output, has been much stimulated by the clarification in Shephard [1953] and later works on the duality relationship between cost and production functions.

The method that we will follow is due to Arrow [1972][11]. The dual to the production function $f(K,L,E)$ is a cost function $C(z,p)$, which denotes the total minimum cost for the production of z at price vector p. In our case $C(z,p)$ factors into[12]: $C(z,p) = c(p)z$ where $c(p)$ is the unit cost function. Moreover $c(p)$ satisfies the same regularity conditions as f (see footnote 12)) and takes the following form:

$$(6) \qquad c(p) = \Theta^* \prod_{i=1}^{3} \prod_{j=1}^{3} \psi_{ij}^{\beta_{ij}}$$

where $\psi_{ij} = \frac{1}{2}p_i + \frac{1}{2}p_j$, $\beta_{ij} = \beta_{ji}$, $\Theta^* > 0$, $\sum_{i,j}\beta_{ij} = 1$.

As $c(p)$ is differentiable with respect to input prices p and $C(z,p)$ satisfies the appropriate regularity conditions[13], we may apply Shephard's lemma:

$$x_i(z,p) = \frac{\partial C(z,p)}{\partial p_i} \qquad (i=1,2,3),$$

where $x_i(z,p)$ is the cost minimizing demand for input i needed to produce z. Since $C(z,p) = z\, c(p)$ it follows that

$$x_i(z,p) = z\,\frac{\partial c(p)}{\partial p_i}\ .$$

We further note that $c(p)$ is linearly homogeneous in p, so application of Euler's theorem gives:

$$c(p) = \sum_i p_i \frac{\partial c(p)}{\partial p_i}\ .$$

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11) See also Shephard [1970, p 145-6] and Diewert [1973 , p 5-6].

12) This is so because $f(K,L,E)$ satiefies the following regularity conditions: f is a positive, nondecreasing, (positively) linear homogeneous, concave function over the positive orthant in three dimensional space (see Diewert [1973 , p 4]).

13) $C(z,p)$ is positive and real valued; defined and finite for all finite $z > 0$, $p > 0$; nondecreasing left continuous in z; nondecreasing in p; (positive) linear homogeneous in p for every $z > 0$; concave in p for every $z > 0$; $\lim_{z\to\infty} C(z,p) = \infty$ for every $p > 0$. See Diewert [1971] for further discussion.

We find:

$$p_i x_i = p_i z \frac{\partial c(p)}{\partial p_i}$$

$$\sum_i p_i x_i = z\, c(p).$$

Eliminating the (unknown) output z we find expressions for the cost shares y_i:

$$y_i \equiv \frac{p_i x_i}{\sum_i p_i x_i} = \frac{p_i}{c(p)} \frac{\partial c(p)}{\partial p_i} .$$

In our case $\frac{\partial c(p)}{\partial p_i} = c(p) \sum_k \frac{\beta_{ik}}{\psi_{ik}}$ from which we derive the following system that is linear in the unknown parameters:

(7) $$y_i = \sum_k \phi_{ik} \beta_{ik} \qquad (i=1,2,3),$$

where $$\phi_{ik} = \frac{p_i}{\psi_{ik}} = \frac{p_i}{\frac{1}{2}p_i + \frac{1}{2}p_k} .$$

We shall estimate the parameters of (7) by applying these equations to time series of aggregate annual data on cost shares and factor prices for the Netherlands. Many assumptions are involved in the passage from the theoretical optimum conditions that are reflected in (7) to their direct empirical verification by means of annual aggregates, and several of these are easily challenged. Thus the estimation procedure treats factor prices as predetermined variables, even though we are dealing with aggregates, and implies instantaneous adjustment in respect to all factors of production. Equally strong assumptions are involved in the construction of the capital data, as we shall note below. As matters stand we have not yet been able to remedy these shortcomings in our analysis. We shall again draw attention to their presence when we discuss our results.

Before we turn to the construction of data on cost shares and price indices in the next section, we will relate the parameters β_{ij} to the AES σ_{ij} between inputs i and j.
Uzawa[14)] [1962] showed that $\sigma_{ij} = \frac{c\, c_{ij}}{c_i c_j}$, where c is a linearly homogeneous unit cost function and

$$c_i = \frac{\partial c}{\partial p_i}, \quad c_{ij} = \frac{\partial^2 c}{\partial p_i \partial p_j} .$$

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14) See also Berndt-Christensen [1973b] for a proof under weaker conditions.

In our case $c(p) = \Theta^* \prod_i \prod_j \psi_{ij}^{\beta_{ij}}$ and it may be verified that

$$c_i = c \sum_k \frac{\beta_{ik}}{\psi_{ik}} \quad \text{and}$$

$$c_{ij} = c\left[\left(\sum_k \frac{\beta_{ik}}{\psi_{ik}}\right)\left(\sum_k \frac{\beta_{jk}}{\psi_{jk}}\right) - \tfrac{1}{2}\frac{\beta_{ij}}{\psi_{ij}^2} - \tfrac{1}{2}\,\delta_{ij}\sum_k \frac{\beta_{ik}}{\psi_{ik}^2}\right],$$

where δ_{ij} is the Kronecker δ: $\delta_{ij} = 1$ when $i=j$
0 when $i \neq j$

Thus σ_{ij} may be expressed in the parameters β_{ij} as follows:

$$(8) \qquad \sigma_{ij} = 1 - \tfrac{1}{2}\,\frac{\dfrac{\beta_{ij}}{\psi_{ij}^2} + \delta_{ij}\sum_k \dfrac{\beta_{ik}}{\psi_{ik}^2}}{\left(\sum_k \dfrac{\beta_{ik}}{\psi_{ik}}\right)\left(\sum_k \dfrac{\beta_{jk}}{\psi_{jk}}\right)}\,.$$

5. The data.

The data consist of annual time-series for the Netherlands, 1950-1974. As to labor and capital, they refer to the enterprises sector as defined in the National Accounts, exclusive of the production of crude oil, natural gas and coal. This ensures that no energy is generated within the aggregate thus defined, although it is of course transformed from one form into another, as in electricity generation. All primary energy can therefore properly be treated as an input.

We require data on cost shares and prices of labor, energy and capital. The cost shares are easily obtained once we have a volume or quantity series and a price series that refers to the price per unit of measurement of volume or quantity. We shall note these as we go along.

We have freely drawn on official statistics and on studies of the Central Planning Bureau, the Ministry of Economic Affairs and the Ministry of Finance. The labor data were readily available, the energy figures required some adjustment and the capital data are based on extensive theoretical considerations. We shall briefly indicate the main points of each series below; a full documentation is available from the author on request.

For labor the quantity is defined as the total labor force employed in the enterprises sector, mining excluded, corrected for the length of the working year and the incidence of sick leave. Labor input thus corresponds to the number of man-hours worked. The price is derived from average wage costs per employee (including social security contributions), reduced to the price in guilders of a man-hour for the enterprises sector as a whole, and has been deflated by means of the cost-of-living index.

For energy the input of the enterprises sector is obtained by reducing the known total consumption of primary energy by the estimated gross consumption of private households. This includes the consumption of coal, oil and gas for domestic use (mainly heating), the gross energy counterpart of domestic electricity demand and private petrol consumption. All energy quantities are expressed in 10^{12} Kcal. For the price of energy we used an existing index of the average purchase price of primary energy by industry, which was converted to a price in guilders per Kcal. Again prices have been deflated by means of the cost-of-living index.

The method of measuring real capital input and capital service prices is derived from Christensen and Jorgenson [1969][15] with some adaptations. This calls for the construction of rather delicate indices, and we shall explain the procedure at some length.

To begin with construct a series for the volume of capital stock, again excluding the mining sector. We distinguish three types of capital goods, viz. equipment, transport equipment (vehicles and ships) and buildings. For each type the capital stock is constructed from past investment in 1963 guilders as

(9) $$K_i(t) = I_i(t) + (1-\mu_i)\, K_i(t-1) \qquad (i=1,2,3)$$

where $K_i(t)$ capital stock volume at end of year t;

$I_i(t)$ volume of investment during year t;

μ_i depreciation rate[16].

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15) See also Coen [1968], Hall and Jorgenson [1967], Christensen and Jorgenson [1970] and Berndt and Christensen [1973a].

16) For μ_i we took μ_1 (equipment) = 0,06; μ_2 (transport) = 0,10 and μ_3 (buildings) = 0,03.

In constructing the price of capital service input for each type, we assume that the investment price of an asset equals the present value of its future services, evaluated at the price we wish to ascertain. This presupposes perfect foresight on the part of the firm. Also it is assumed that the service flow from a given asset declines geometrically over time. Disregarding taxes on the capital service yield, we can write the equality at issue as

$$(10) \qquad q(t) = \sum_{j=t}^{\infty} \left\{ (1-\mu)^{j-t} \, p(j+1) \prod_{s=t+1}^{j+1} \frac{1}{1+r(s)} \right\},$$

where q(t) price index of investment, 1963 = 1;

p(t) capital service price;

r(t) discount rate

all at year t.

From (10) follows the well-known expression

$$(11) \qquad p(t) = q(t-1)\, r(t) + q(t)\, \mu - \left(q(t)-q(t-1)\right).$$

Allowing now for taxes that are levied on the capital services value as it is obtained by a firm, (10) must be replaced by

$$(12) \qquad q(t) = \sum_{j=t}^{\infty} \left\{ \left((1-\mu)^{j-t} \, p(j+1) - u(j+1)\{(1-\mu)^{j-t} \, p(j+1) - D_{j+1}(t) q(t)\} \right) \prod_{s=t+1}^{j+1} \frac{1}{1+r(s)} \right\} + a(t)\, q(t)$$

where the meaning of the new variables is as follows:

u(t) effective corporate profits tax rate;

a(t) investment tax credit;

$D_j(t)$ proportion of the original cost of an investment in year t that may be deducted from income for tax purposes in year j.

Defining $\phi(t) = \sum_{j=t}^{\infty} u(j+1) D_{j+1}(t) \prod_{s=t+1}^{j+1} \frac{1}{1+r(s)}$ [17], we may derive from (12) an explicit expression for p(t):

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17) Thus ϕ is the discounted value of the tax savings generated by the depreciation allowance.

$$(13) \quad p(t) = \frac{1-a(t)-\phi(t)}{1-u(t)} \Big[r(t)\, q(t-1) + \mu q(t) - \big(q(t) - q(t-1)\big)\Big] + $$
$$+ \frac{1+r(t)}{1-u(t)} \big(a(t) - a(t-1) + \phi(t) - \phi(t-1)\big) q(t-1).$$

We now approximate $\phi(t)$ as follows:

$$\phi(t) = \sum_{j=t}^{\infty} u(j+1) D_{j+1}(t) \prod_{s=t+1}^{j+1} \frac{1}{1+r(s)} \simeq u(t+1) \sum_{j=t}^{\infty} D_{j+1}(t) \prod_{s=t+1}^{j+1} \frac{1}{1+r(s)}$$
$$\equiv u(t+1)\, B(t),$$

where $B(t) = \sum_{j=t}^{\infty} D_{j+1}(t) \prod_{s=t+1}^{j+1} \frac{1}{1+r(s)}$ is the discounted value of depreciation charges stemming from a current guilder of capital expenditures.

The expression for $p(t)$ then becomes:

$$(14) \quad p(t) = \frac{1-a(t)-u(t+1)B(t)}{1-u(t)} \Big[r(t)q(t-1) + \mu q(t) - \big(q(t) - q(t-1)\big)\Big] + $$
$$+ \frac{1+r(t)}{1-u(t)} \big(a(t) - a(t-1) + u(t+1)B(t) - u(t)B(t-1)\big) q(t-1).$$

Deleting the second term and writing $u(t)$ instead of $u(t+1)$ we arrive at the formula employed by Christensen and Jorgenson [1969 p 304], adapted to the Dutch tax system:

$$(15) \quad \pi(t) = \frac{1-a(t)-u(t)B(t)}{1-u(t)} \Big[r(t)q(t-1) + \mu q(t) - \big(q(t) - q(t-1)\big)\Big]$$

The data employed for the calculation of $K_i(t)$, $p_i(t)$ and $\pi_i(t)$ are presented in table 1. In table 2 we give the resulting values of K, p and π and we also agrregate equipment and transport (index 12) and equipment, transport and buildings (index 123). We found that π developes much more smoothly than p and that some values of π (and more values of p) are negative. Since, however, the model presupposes positive prices, π is a more appropriate index than p. We further want to include all three capital sectors and therefore select π_{123} as an estimate of the price of capital services. The development of the price and quantity indices of K_{123}, L and E is graphed in figures 3 and 4 respectively. It is inconceivable that the instant adjustment to price changes implied by our model would in effect take place with such fluctuations in capital price. In table 3 finally, we present the cost shares and price indices we have searched for.

Table 1. Time-series for r^*, u, I_i, q_i, B_i^{**} and a_i.

t	r	u	I_1	I_2	I_3	q_1	q_2	q_3
49						0.6526	0.7695	0.4915
50	0.0819	0.4000	1901.0000	653.0000	1407.0000	0.6782	0.7929	0.5176
51	0.0851	0.5000	1889.0000	543.0000	1303.0000	0.7599	0.8767	0.5955
52	0.0849	0.5000	1675.0000	541.0000	1333.0000	0.8549	0.8890	0.6442
53	0.0825	0.4600	1870.0000	681.0000	1678.0000	0.8400	0.8844	0.6199
54	0.0827	0.4600	2179.0000	995.0000	1677.0000	0.8277	0.9003	0.6427
55	0.0829	0.4300	2486.0000	1375.0000	1593.0000	0.8414	0.9270	0.6962
56	0.0855	0.4300	2946.0000	1578.0000	1946.0000	0.8859	0.9468	0.7710
57	0.0936	0.4700	2880.0000	1903.0000	2142.0000	0.9348	0.9235	0.8397
58	0.0933	0.4700	2326.0000	1483.0000	2018.0000	0.9471	0.9393	0.8626
59	0.0942	0.4700	2615.0000	1651.0000	2142.0000	0.9345	0.9704	0.8412
60	0.0954	0.4700	3059.0000	2113.0000	1693.0000	0.9445	0.9488	0.8763
61	0.0935	0.4700	3610.0000	2001.0000	1801.0000	0.9485	0.9664	0.9068
62	0.0955	0.4700	3779.0000	2153.0000	1848.0000	0.9534	0.9686	0.9375
63	0.0956	0.4500	3873.0000	2010.0000	1785.0000	1.0000	1.0000	1.0000
64	0.1003	0.4500	4275.0000	1893.0000	2310.0000	1.0627	1.0436	1.0904
65	0.1040	0.4500	4421.0000	2114.0000	2308.0000	1.1019	1.0454	1.1583
66	0.1111	0.4700	5011.0000	2157.0000	2645.0000	1.1458	1.0529	1.2359
67	0.1091	0.4700	5157.0000	2191.0000	3000.0000	1.1322	1.1032	1.2812
68	0.1147	0.4600	5718.0000	2569.0000	3293.0000	1.1151	1.0987	1.3205
69	0.1236	0.4600	5857.0000	2536.0000	3250.0000	1.1549	1.0913	1.4052
70	0.1242	0.4600	7205.0000	2855.0000	3524.0000	1.2334	1.1543	1.5266
71	0.1209	0.4700	6843.0000	3167.0000	3435.0000	1.3393	1.2240	1.6702
72	0.1221	0.4800	6813.0000	2800.0000	3043.0000	1.3809	1.2887	1.8351
73	0.1295	0.4800	7309.0000	3623.0000	3225.0000	1.3704	1.3438	1.9396
74	0.1467	0.4800	7764.0000	3290.0000	3808.0000	1.5304	1.4343	2.2051

* As an estimate of r we used the interest rate on Dutch government consols, including a risk factor and adjusting for changes in the ratio of internal investment funds to total investment.

** For the calculation of B we used the straight-line depreciation formula, taking into account accelerated depreciation allowances.

Table 1. (continued)

t	B_1	B_2	B_3	a_1	a_2	a_3
49	0.6579	0.5983	0.3385	0.0000	0.0000	0.0000
50	0.7089	0.6576	0.4820	0.0000	0.0000	0.0000
51	0.7078	0.6560	0.4796	0.0000	0.0000	0.0000
52	0.7065	0.6540	0.4772	0.0000	0.0000	0.0000
53	0.7029	0.6498	0.4732	0.1181	0.1181	0.1181
54	0.6983	0.6499	0.4686	0.1566	0.1566	0.1566
55	0.6903	0.6466	0.4606	0.1553	0.1553	0.1553
56	0.6738	0.6312	0.4429	0.1309	0.1386	0.1309
57	0.6719	0.6290	0.4399	0.0000	0.0385	0.0000
58	0.6689	0.6258	0.4364	0.0934	0.1076	0.0779
59	0.6655	0.6343	0.4466	0.1398	0.1398	0.1398
60	0.6584	0.6276	0.4202	0.1050	0.1137	0.1050
61	0.6502	0.6174	0.4031	0.0873	0.1013	0.0873
62	0.6431	0.6100	0.3972	0.0871	0.1011	0.0871
63	0.6342	0.6009	0.3489	0.0866	0.0901	0.0866
64	0.5849	0.5701	0.2615	0.0861	0.0895	0.0072
65	0.5704	0.5648	0.2560	0.0856	0.0736	0.0000
66	0.5637	0.5588	0.2517	0.0855	0.0735	0.0000
67	0.5541	0.5499	0.2630	0.0848	0.0729	0.0305
68	0.5458	0.5420	0.3214	0.0841	0.0723	0.0841
69	0.5408	0.5483	0.3190	0.0158	0.0286	0.0158
70	0.5349	0.5469	0.3166	0.0000	0.0186	0.0000
71	0.5260	0.5380	0.3124	0.0000	0.0185	0.0000
72	0.5160	0.5394	0.3070	0.0000	0.0235	0.0000
73	0.5083	0.5440	0.3024	0.0000	0.0267	0.0297
74	0.5083	0.5491	0.3041	0.0508	0.0593	0.1275

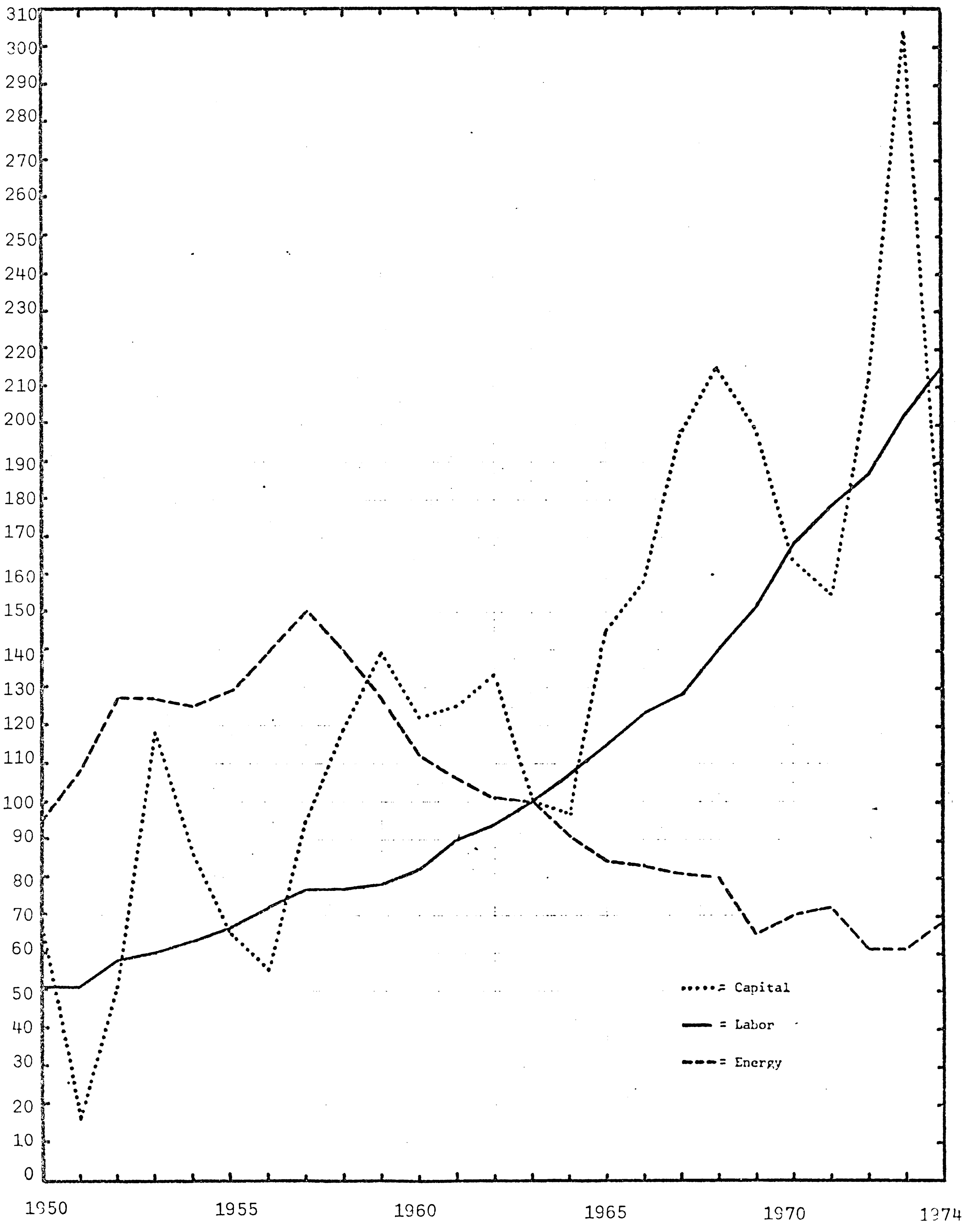
Table 2. The capital stock and the price of capital services.

	equipment			transport			buildings		
t	K_1	p_1	π_1	K_2	p_2	π_2	K_3	p_3	π_3
50	16659.0000	0.1812	0.0819	7853.0000	0.2572	0.1461	16539.0000	0.1312	0.0400
51	17548.4600	0.0271	0.0279	7610.7000	0.0944	0.0958	17345.8300	-0.0258	-0.0244
52	18170.5524	-0.0197	0.0268	7390.6300	0.1595	0.2033	18158.4551	0.0069	0.0323
53	18950.3193	0.3402	0.1405	7332.5670	0.3867	0.1796	19291.7014	0.2683	0.1181
54	19992.3001	0.1580	0.1270	7594.3103	0.1874	0.1485	20389.9504	0.0844	0.0555
55	21278.7621	0.0938	0.1013	8289.8793	0.1351	0.1398	21371.2519	0.0177	0.0235
56	22948.0364	0.0709	0.0819	8966.8913	0.1563	0.1595	22676.1143	-0.0097	0.0093
57	24451.1542	-0.1247	0.1163	9973.2022	0.0589	0.2566	24137.8309	-0.1678	0.0428
58	25310.0849	0.3247	0.1472	10458.8820	0.3143	0.1855	25431.6960	0.2422	0.1100
59	26406.4798	0.2507	0.1631	11063.9938	0.2339	0.1638	26810.7451	0.2756	0.1569
60	27881.0910	0.0762	0.1501	12070.5944	0.1745	0.2331	27699.4227	0.0118	0.0940
61	29818.2256	0.1198	0.1618	12864.5350	0.1589	0.1926	28669.4401	0.0608	0.1073
62	31808.1321	0.1360	0.1646	13731.0815	0.1886	0.2161	29657.3569	0.0961	0.1152
63	33772.6441	0.1107	0.1193	14367.9733	0.1583	0.1874	30552.6362	0.0369	0.0785
64	36021.2855	0.0745	0.1200	14824.1760	0.1627	0.1915	31946.0571	-0.1696	0.0678
65	38281.0084	0.1709	0.1644	15455.7584	0.2393	0.2582	33295.6754	0.1194	0.1291
66	40995.1479	0.1730	0.1804	16067.1826	0.2616	0.2679	34941.8051	0.1416	0.1465
67	43692.4390	0.2315	0.2552	16651.4643	0.1998	0.2207	36893.5509	0.2908	0.2044
68	46788.8926	0.2528	0.2634	17555.3179	0.2931	0.3026	39079.7444	0.4222	0.2895
69	49838.5591	0.0641	0.2279	18335.7861	0.2427	0.3361	41157.3521	-0.0035	0.1871
70	54053.2455	0.1609	0.1939	19357.2075	0.2483	0.2540	43446.6315	0.1155	0.1563
71	57653.0508	0.1769	0.1754	20588.4867	0.2650	0.2642	45578.2326	0.1498	0.1466
72	61006.8678	0.2824	0.2963	21329.6381	0.3098	0.2946	47253.8856	0.1447	0.1542
73	64655.4557	0.3836	0.3947	22819.6742	0.3531	0.3371	49061.2696	0.4134	0.3037
74	68540.1283	0.3337	0.1802	23827.7068	0.4284	0.3254	50597.4310	0.5408	0.1189

Table 2. (continued)

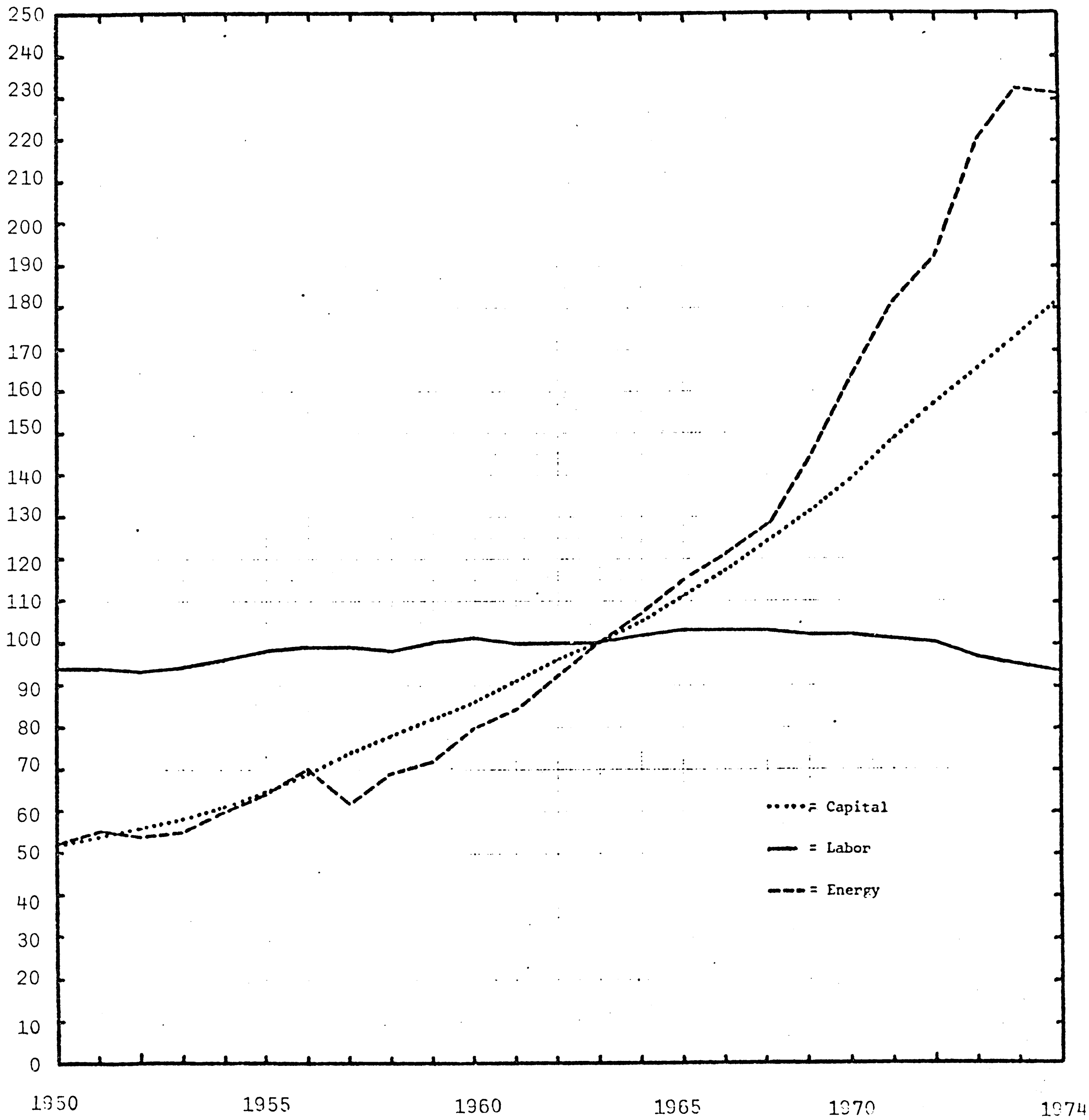
t	equipment + transport			equipment + transport + buildings		
	K_{12}	p_{12}	π_{12}	K_{123}	p_{123}	π_{123}
50	24512.0000	0.2056	0.1025	41051.0000	0.1756	0.0773
51	25159.1600	0.0475	0.0484	42504.9900	0.0176	0.0187
52	25561.1824	0.0322	0.0778	43719.6375	0.0216	0.0589
53	26282.8863	0.3531	0.1514	45574.5877	0.3172	0.1373
54	27586.6104	0.1661	0.1329	47976.5608	0.1314	0.1000
55	29488.6414	0.1053	0.1120	50859.8933	0.0685	0.0748
56	31914.9277	0.0949	0.1037	54591.0420	0.0515	0.0645
57	34424.3564	-0.0715	0.1569	58562.1873	-0.1112	0.1099
58	35768.9669	0.3217	0.1584	61200.6629	0.2886	0.1383
59	37470.4736	0.2458	0.1633	64281.2187	0.2582	0.1606
60	39951.6855	0.1059	0.1752	67651.1082	0.0674	0.1419
61	42682.7606	0.1316	0.1711	71352.2006	0.1032	0.1455
62	45539.2135	0.1519	0.1801	75196.5704	0.1299	0.1545
63	48140.6175	0.1249	0.1397	78693.2536	0.0908	0.1159
64	50845.4615	0.1082	0.1408	82791.5185	-0.0039	0.1127
65	53736.7667	0.1906	0.1914	87032.4421	0.1630	0.1676
66	57062.3304	0.1979	0.2051	92004.1355	0.1765	0.1828
67	60343.9033	0.2227	0.2457	97237.4542	0.2486	0.2300
68	64344.2105	0.2638	0.2741	103423.9549	0.3237	0.2497
69	68174.3452	0.1122	0.2570	109331.6972	0.0686	0.2307
70	73410.4530	0.1818	0.2098	116857.0845	0.1572	0.1899
71	78241.5375	0.2001	0.1988	123819.7701	0.1816	0.1796
72	82336.5058	0.2895	0.2958	129590.3914	0.2367	0.2442
73	87475.1299	0.3757	0.3797	136536.3990	0.3892	0.3524
74	92367.8352	0.3581	0.2177	142965.2661	0.4228	0.1827

Figure 3. Price indices (deflated) of capital, labor and energy
Dutch enterprises 1950-74



Source: see section 5

Figure 4. Quantity indices of capital, labor and energy Dutch enterprises 1950-74



Source: see section 5

Table 3. Price indices and Cost shares of capital, labor and energy.
Dutch enterprises 1950-74

t	Price indices			Cost shares			total cost *
	$p_K(=\pi_{123})$	p_L	p_E	capital share	labor share	energy share	
1950	0.6667	0.5576	0.9533	.1692	.7577	.0731	18.74
51	0.1615	0.5558	1.0813	.0481	.8538	.0981	16.62
52	0.5084	0.5775	1.2723	.1352	.7657	.0991	19.08
53	1.1847	0.6016	1.2659	.2653	.6530	.0818	23.60
54	0.8629	0.6261	1.2515	.2064	.7051	.0886	23.26
55	0.6453	0.6697	1.2853	.1591	.7463	.0946	23.89
56	0.5565	0.7206	1.3946	.1374	.7584	.1042	25.62
57	0.9481	0.7683	1.5004	.2169	.6965	.0866	29.69
58	1.1933	0.7701	1.4045	.2669	.6497	.0835	31.74
59	1.3856	0.7819	1.2686	.3027	.6233	.0739	34.09
60	1.2244	0.8228	1.1220	.2761	.6529	.0710	34.77
61	1.2548	0.8999	1.0579	.2784	.6561	.0655	37.28
62	1.3331	0.9450	1.0081	.2920	.6440	.0641	39.80
63	1.0000	1.0000	1.0000	.2334	.6963	.0704	39.08
64	0.9719	1.0719	0.9120	.2232	.7127	.0641	41.81
65	1.4455	1.1534	0.8355	.2949	.6515	.0536	49.44
66	1.5772	1.2270	0.8273	.3113	.6378	.0509	54.03
67	1.9842	1.2801	0.8131	.3665	.5866	.0469	61.03
68	2.1540	1.3989	0.7960	.3812	.5721	.0467	67.73
69	1.9900	1.5059	0.6470	.3616	.5970	.0414	69.75
70	1.6382	1.6779	0.6987	.3097	.6418	.0484	71.64
71	1.5492	1.7772	0.7155	.2998	.6493	.0510	74.16
72	2.1064	1.8718	0.6094	.3732	.5834	.0434	84.78
73	3.0402	2.0199	0.6051	.4626	.5003	.0371	104.02
74	1.5764	2.1486	0.6809	.3086	.6404	.0510	84.64

* in billions of 1963 guilders

6. Estimation and prediction.

Having collected data on cost shares and input prices during T years, we may now ask how to estimate the parameters β_{ij} from the system of equations (7), if we suppose the prices to be exogeneous.
Imposing the symmetry constraints $\beta_{ij} = \beta_{ji}$ and adding a stochastic disturbance vector, we obtain

$$(16) \qquad \begin{pmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & \phi_{12} & \phi_{13} & 0 \\ 0 & 1 & 0 & \phi_{21} & 0 & \phi_{23} \\ 0 & 0 & 1 & 0 & \phi_{31} & \phi_{32} \end{pmatrix}_t \begin{pmatrix} \beta_{11} \\ \beta_{22} \\ \beta_{33} \\ \beta_{12} \\ \beta_{13} \\ \beta_{23} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{pmatrix}$$

(t = 1.......T)

For short: $y_t^* = X_t^* \beta^* + \varepsilon_t^*$ (t=1...T)

The following two characteristics of the model should be noted:

$$(17) \qquad s'y_t^* = 1 \qquad \text{with } s' = (1\ 1\ 1)$$

$$(18) \qquad q'\beta^* = 1 \qquad \text{with } q' = (1\ 1\ 1\ 2\ 2\ 2),$$

where (17) stems from the fact that the y_i are shares and (18) is the linear homogeneity condition.

Now suppose $E\varepsilon^* = 0$ and $E\varepsilon^*\varepsilon^{*'} = \sigma^2 V^*$,

where $\varepsilon^{*'} = (\varepsilon_1^{*'}, \varepsilon_2^{*'}, \ldots, \varepsilon_T^{*'})$.

Then $1 \quad = s'y_t^* = s'X_t^*\beta^* + s'\varepsilon_t^*$.

Also $0 \quad = Es'\varepsilon_t^* = 1 - s'X_t^*\beta^*$.

Therefore $s'X_t^*\beta^* = 1$ and $s'\varepsilon_t^* = 0$, so that V^* must be singular.

Thus there are two problems involved in the direct estimation of β^* from (16), viz. a singular disturbance covariance matrix and one linear restriction on the parameter vector.

Theil [1971. pp 274-289] has outlined a way of dealing directly with these problems. We will follow a simpler route that yields the same results.

First notice that $s'X_t^* = q'$, so that conditions (17) and (18) imply that one of the equations in (16), say the third, becomes supenfluous. We therefore delete the third equation and write:

$$(19) \qquad \begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} 1 & 0 & \phi_{12} & \phi_{13} & 0 \\ 0 & 1 & \phi_{21} & 0 & \phi_{23} \end{pmatrix}_t \begin{pmatrix} \beta_{11} \\ \beta_{22} \\ \beta_{12} \\ \beta_{13} \\ \beta_{23} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$

$(t=1....T)$

In the above system y_{3t} and β_{33} do not appear, so that both constraints have been resolved.

We abbreviate (19) to:

$$(20) \qquad y_t = X_t\beta + \varepsilon_t \qquad (t=1....T)$$

Combining year observations we may write

$$(21) \qquad y = X\beta + \varepsilon,$$

where $y' = (y_1', y_2', \ldots, y_T')$; $X' = (X_1', X_2', \ldots, X_T')$; $\varepsilon' = (\varepsilon_1', \varepsilon_2', \ldots, \varepsilon_T')$.

Suppose $E\varepsilon = 0$ and $E\varepsilon\varepsilon' = \sigma^2 V$, then application of Aitkens theorem[18)] to (21) leads to the following estimators:

$$(22) \qquad \hat{\beta} = (X'V^{-1}X)^{-1}X'V^{-1}y$$

$$(23) \qquad V(\hat{\beta}) = \sigma^2(X'V^{-1}X)^{-1}$$

$$(24) \qquad \hat{\sigma}^2 = \frac{1}{2T-5}(y-X\hat{\beta})'V^{-1}(y-X\hat{\beta})$$

On closer inspection of (19) we see that the two equations are related, since β_{12} appears in both. More important, the two equations may be disturbance-related, that is the neglected variables in the two equations may be the same (or at least correlated). Certain a priori restrictions as to the structure of V are however needed: we assume that the disturbances are distributed independently over time and that the covariance matrix for each period is constant over time.

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18) See Theil [1971 p. 238].

This implies that V is a block-diagonal matrix, that is $V = I \otimes V_D$, where $\otimes$ denotes the Kronecker product operator and V_D is a symmetric, positive semidefinite 2×2 matrix that, without loss of generality, may be written

$$(25) \qquad V_D = \begin{pmatrix} 1+\xi & \nu \\ \nu & 1-\xi \end{pmatrix} ,$$

the normalization then being $trV_D = 2$.

The statistics (22) to (24) now take the following forms:

$$(22') \qquad \hat{\beta} = \left(\sum_{t=1}^{T} X_t' V_D^{-1} X_t \right)^{-1} \left(\sum_{t=1}^{T} X_t' V_D^{-1} y_t \right)$$

$$(23') \qquad V(\hat{\beta}) = \sigma^2 \left(\sum_{t=1}^{T} X_t' V_D^{-1} X_t \right)^{-1}$$

$$(24') \qquad \hat{\sigma}^2 = \frac{1}{2T-5} \sum_{t=1}^{T} e_t' V_D^{-1} e_t \text{ , where } e_t = y_t - X_t \hat{\beta}$$

For any initial values (ν,ξ) we may calculate $\hat{\beta}$ and the residuals $e = y - X\hat{\beta}$. The statistic

$$(26) \qquad E = \frac{2}{e_{(1)}' e_{(1)}' + e_{(2)}' e_{(2)}'} \begin{pmatrix} e_{(1)}' e_{(1)} & e_{(1)}' e_{(2)} \\ e_{(1)}' e_{(2)} & e_{(2)}' e_{(2)} \end{pmatrix} ,$$

where $e_{(i)}$ is the T × 1 vector containing the residuals of the i-th equation (i=1,2), is used as an estimator of V_D [19].
We replace V_D by E and again calculate $\hat{\beta}$, e and E. This procedure is repeated until E stabilizes.

---

19) The asymptotic properties of E are discussed in Theil [1971, p. 399-402]. The procedure originates with Zellner [1962].

Another troublesome question is how to take account of the loss of degrees of freedom caused by the estimation of V. As the number of parameters to be estimated has increased by two (ν and ξ) we replace (24') by

$$(24'') \qquad \hat{\sigma}^2 = \frac{1}{2T-7} \sum_{t=1}^{T} e_t' V_D^{-1} e_t \ .$$

We finally wish to compose the covariance matrix of the total β^* vector. This may be done in the following way:
Define:

$$(27) \qquad A = \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ -1 & -1 & -2 & -2 & -2 \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}$$

where all undesignated elements are zero,

then $\text{Cov}\ \beta^* = \text{Cov}\ A\beta = A(\text{Cov}\ \beta)A'$.

7. Empirical results.

Our first object was to search for stable values of ν and ξ. Deleting the capital equation [20] in the passage from (16) to (19), we found a unique stability point at $\nu = -.1046$ and $\xi = -.9852$, independent of the initial values of ν and ξ. We also investigated the convergence points emerging from the time-series where the first two years, the last two years and both were deleted, since we observed that these years were very non-typical.
The resulting convergence points and beta vectors were very similar to the ones we found with the complete set of data. We therefore decided on the following V_D matrix

$$V_D = \begin{bmatrix} .0148 & -.1046 \\ -.1046 & 1.9852 \end{bmatrix}$$

Below we present the estimated β^* vector, i.e. the parameter estimates of the GCD unit cost function c(p) of (6), with their asymptotic standard errors

20) The results proved to be completely independent from the equation deleted, as of course they should.

parameter[21)]	estimate	asymptotic standard error
β_{EE}	.0046	.0032
β_{LL}	.4270	.0748
β_{KK}	-.0302	.0755
β_{EL}	.0217	.0081
β_{EK}	.0453	.0073
β_{LK}	.2323	.0739

For each equation (including the capital equation) we computed R^2 as one minus the ratio of the residual sum of squares to the total sum of squares. The R^2's are:0.9944 for the E equation, 0.9917 for the L equation and 0.9576 for the K equation.

In table 4 we present the estimated Allen partial elasticities of substitution (σ_{ij}) as formulated in (8). These estimates are the essence of this paper.
We will make a few comments:

(i) Energy and labor appear to be slightly substitutable (σ_{EL} is about .76) No doubt energy and certain types of maintenance workers are complements, but energy is a substitute for the mass of (unskilled) labor.

(ii) Labour and capital are only slightly substitutable (σ_{LK} is about .34). This is not in congruence with the traditional two-input (capital-labor) studies, where one usually finds strong substitutability. However Parks [1971], applying a five-input model to Swedish manufacturing, finds σ_{LK} = .12.

(iii) Quite unexpectedly, energy and capital appear to be very slightly substitutable before 1960 (with the exception of 1951; σ_{EK} is about .06). In the last fifteen years energy and capital are complementary (σ_{EK} is about -.38).

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21) We shall use E,L and K insted of 1,2 and 3 for easy reference.

(iv) The "own" AES σ_{ii} (i=E,L,K) also have an interesting economic interpretation[22]. When we define τ_i as the "own" price elasticity of factor demand, then $\tau_i = y_i \sigma_{ii}$. The estimates of τ_i are presented in table 5. We see that energy is quite responsive to a change in its own price (τ_E is about -.45),that capital also is responsive to its own price (τ_k is about -.22), and that labor is the least responsive (τ_L is about -.15).

(v) We notice with regret that σ_{KK} (and therefore τ_K) is positive in 1951, which means that the substitution matrix (σ_{ij}) is not negative semidefinite in that year. Our cost model and the resulting factor demand equations are therefore not consistent with cost minimization[22] in 1951. In all other years this inconsistency does not arise. This is a drawback[23], resulting from the non-typical value of p_K in 1951 and, more fundamentally, from the grotesque way in which capital prices have been computed.

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22) See Parks [1971, p. 135].

23) Deaton [1974, p. 343] and Parks [1971, p. 136] report on similar inconsistencies.

Table 4. Estimated Allen elasticities of substitution (σ_{ij})
Dutch enterprises 1950-74

	σ_{EE}	σ_{LL}	σ_{KK}	σ_{EL}	σ_{EK}	σ_{LK}
1950	-6.32	-.24	-.82	.82	.01	.32
51	-6.33	-.11	1.69	.89	-.06	-.17
52	-6.26	-.19	-.74	.86	.13	.21
53	-6.46	-.36	-.89	.80	.13	.46
54	-6.32	-.27	-.85	.83	.09	.36
55	-6.27	-.20	-.77	.85	.05	.25
56	-6.27	-.17	-.67	.86	.05	.18
57	-6.30	-.24	-.83	.83	.07	.32
58	-6.38	-.29	-.86	.80	.06	.40
59	-6.50	-.32	-.87	.78	.04	.44
60	-6.47	-.28	-.85	.77	-.05	.40
61	-6.50	-.26	-.84	.76	-.12	.39
62	-6.56	-.26	-.84	.74	-.15	.40
63	-6.43	-.19	-.77	.78	-.28	.31
64	-6.48	-.18	-.76	.77	-.39	.30
65	-6.78	-.23	-.84	.71	-.31	.39
66	-6.88	-.23	-.85	.70	-.32	.40
67	-7.22	-.27	-.87	.66	-.24	.45
68	-7.42	-.27	-.87	.65	-.26	.45
69	-7.78	-.24	-.88	.65	-.39	.43
70	-7.32	-.18	-.85	.71	-.63	.36
71	-7.24	-.17	-.83	.72	-.73	.34
72	-8.28	-.21	-.89	.67	-.55	.40
73	-9.68	-.26	-.90	.60	-.30	.47
74	-7.54	-.14	-.84	.74	-.96	.32

Table 5. Estimated "own" price elasticities of factor demand (τ_i)
Dutch enterprises 1950-74

	τ_E	τ_L	τ_K
1950	-.46	-.18	-.14
51	-.62	-.09	.08
52	-.62	-.15	-.10
53	-.53	-.23	-.24
54	-.56	-.19	-.18
55	-.59	-.15	-.12
56	-.65	-.13	-.09
57	-.55	-.17	-.18
58	-.53	-.19	-.23
59	-.48	-.20	-.26
60	-.46	-.18	-.23
61	-.43	-.17	-.23
62	-.42	-.17	-.25
63	-.45	-.14	-.18
64	-.42	-.13	-.17
65	-.36	-.15	-.25
66	-.35	-.15	-.26
67	-.34	-.16	-.32
68	-.35	-.16	-.33
69	-.32	-.14	-.32
70	-.35	-.12	-.26
71	-.37	-.11	-.25
72	-.36	-.12	-.33
73	-.36	-.13	-.42
74	-.39	-.09	-.26

The assumption $E\varepsilon\varepsilon' = \sigma^2(I \otimes V_D)$ in the two-equation-model (19) is equivalent to the assumption $E\varepsilon^*\varepsilon^{*\prime} = \sigma^2(I \otimes V_D^*)$ in the original three-equation-model (16), where V_D^* is a symmetric, positive semidefinite (in fact singular) matrix of order three that is easily derived from V_D as defined in (25):

$$V_D^* = \begin{pmatrix} 1+\xi & \nu & -(1+\nu+\xi) \\ & 1-\xi & -(1+\nu-\xi) \\ & & 2(1+\nu) \end{pmatrix}$$

The estimates of each block $\sigma^2 V_D^*$ of the disturbance covariance matrix $\sigma^2(I \otimes V_D^*)$ are

$$10^{-6} \times \begin{array}{c} \\ \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{ccc} E & L & K \end{array} \\ \begin{pmatrix} 32 & -229 & 197 \\ & 4347 & -4118 \\ & & 3921 \end{pmatrix} \end{array} \begin{array}{c} \\ E \\ L \\ K \end{array}$$

From this we derive the corresponding correlation matrix. We find a very high negative correlation between the residuals of the capital equation and those of the labor equation: $\rho(e_K,e_L) = -.997$.
Further $\rho(e_E,e_K) = .551$ and $\rho(e_E,e_L) = -.610$.

The variance of the residuals of the first equation (the one that "explains" y_1) is $32 * 10^{-6}$. As the mean of y_1 equals .068, we find an unexplained margin of 2 * .0057 = .011 (i.e. 16% of .068). For the second and third equation we find unexplained margins of .132 (20%) and .125 (47%) respectively.

Finally we estimated the unit cost function $c(p)$ and since we know total cost $C(z,p)$ we can compute an index of z $\left(\text{since } C(z,p)=zc(p)\right)$. In figure 5 we confront the estimated z with the known data of output in the Netherlands. The result is disappointing.

8. Concluding remarks

This paper has sought to estimate factor demand relations in a three factor demand model that allows for considerable freedom in the variation of the substitution parameters. One of the major problems has been to collect adequate data on factor prices. Especially the price index of capital services is by no means ideal (see figure 3) and requires improvement[24]. Other shortcomings in our analysis are that the GCD model requires predetermined factor prices and instantaneous adjustment in respect to all factors of production. Besides it is found that in one year the substitution matrix is not negative semidefinite, thus violating the underlying theory of the model.

Nevertheless the study finds evidence that energy interacts with labor (substitutable) and capital (complementary after 1960) in different ways. This justifies the inclusion of energy as a separate input in the production function.
The result can be used to assess the effect of energy price changes on energy use and total output.
To carry out these projections in the context of our model we must predict future factor prices and apply the appropriate formulas[25]
We then find unbiased predictions of the cost shares. From these we can derive projections of energy consumption, labor quantity and the volume of capital stock, if we have projections of output z[26].

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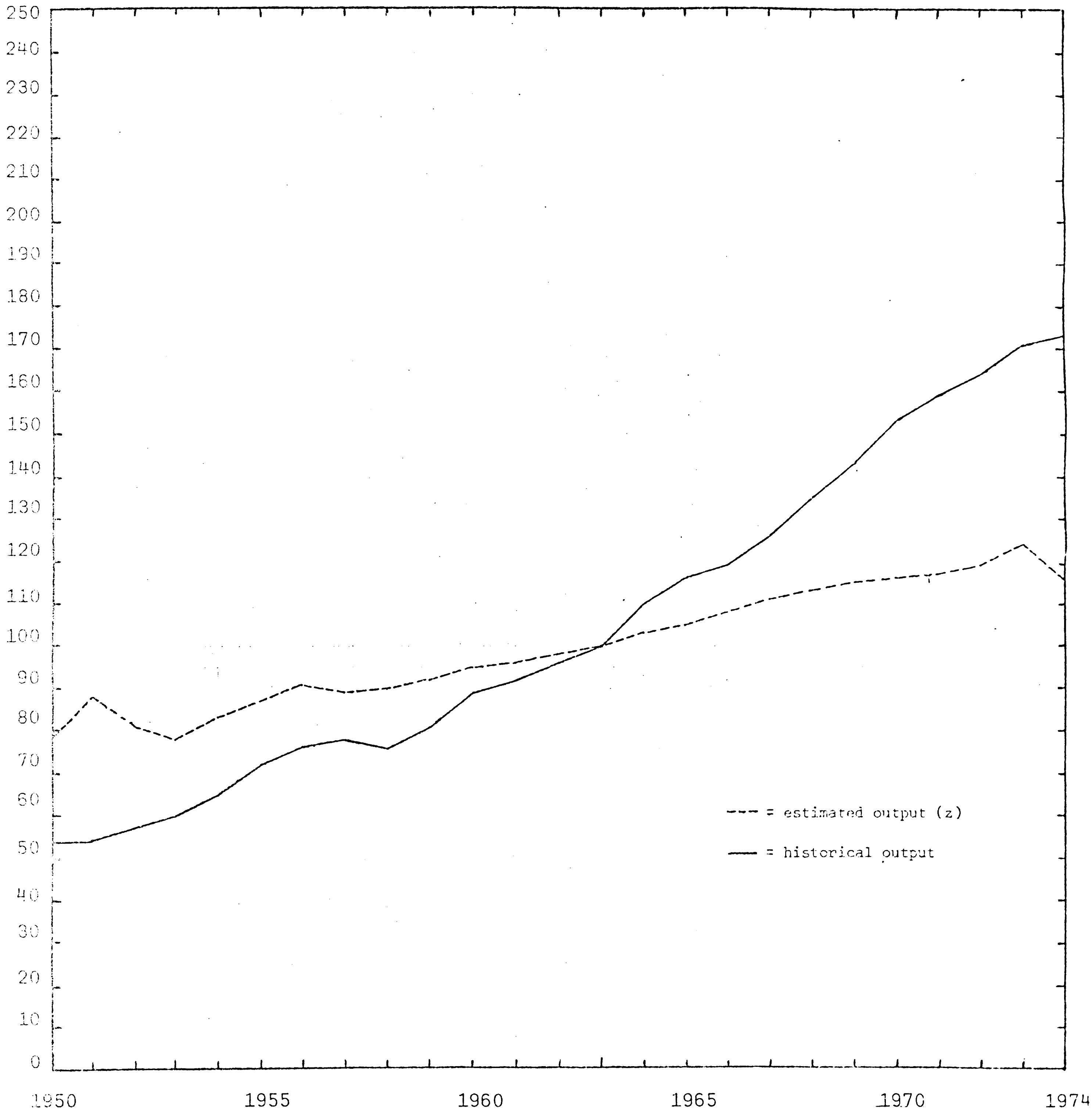
24) One way out is to study only marginal effects. We can then use the price of investment goods. Such an approach would, however, require a completely new model.

25) See Theil [1971, pp. 280-288] for the best linear unbiased predictor and the covariance matrix of its prediction error.

26) Also sufficient are projections of the costs or quantities of any of the factors.

Figure 5. Output (historical and estimated) in the Netherlands 1950-74
(index 1963 = 100)



The complementarity (after 1960) between energy and capital has two interesting implications for investment policy.
First, higher priced energy will - ceteris paribus - dampen the demand for new plant and equipment[27] and, to the extent that productivity gains are embodied in new plant and equipment, this may slow down the rate of productivity growth. Secondly, investment incentives like accelerated depreciation allowances and investment tax credits result in increased demand for energy.
This calls for a cautious use of these instruments.

One final remark: we have reported on all relevant findings, positive and negative, of our research. This enables the reader to form a clear notion of the strength of our conclusions.

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27) but it will stimulate employment, as $\sigma_{EL} > 0$.

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