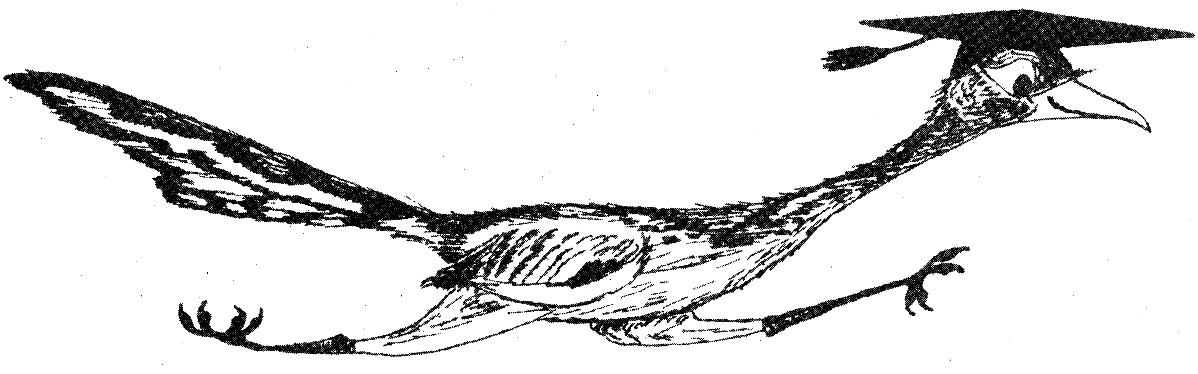
Consumption

STAFF REPORT

CONSUMER'S SURPLUS IN

QUALITY SPACE

Staff Report Number 31



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[1981?]

CONSUMER'S SURPLUS IN QUALITY SPACE

by

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Abstract

Monetary measures of the individual's welfare arising from commodity price changes as approximated by areas beneath estimatible demand equation systems have been rigorously developed in the literature. However, although similar welfare measures associated with changes in (environmental) quality have been rigorously developed (Maler), their applicability to estimatible demand systems has been limited. The present paper attempts to develop a rigorously justifiable measure of individual benefits by which policy-induced environmental quality changes for an interrelated set of commodities can be practically evaluated from potentially estimatible demand functions. This benefit measure is then applied to the special case of a system of commodity demand equations linear in both prices and qualities.

Consumer's Surplus in Quality Space

I. Introduction

A major objective of environmental resource management is to maximize the efficiency (benefits) associated with the use of environmental resources. Given the typically non-market or public good character associated with services of such resources, estimates of the demand for and benefits from related investments can be a valuable basis for improved environmental decisionmaking.

In recent years, there has been considerable applied research interest in estimating the benefits from investments in improved environmental quality. Categorizing by resource type, there are studies in air quality by Brookshire, et al. [3], Lave [10], Nelson [12], and Randall, et al. [14]; in water quality by Portney [13], Dornbusch [6], Binkley and Hanemann [1], Reiling, et al. [16]; in waterfowl by Hammack and Brown [8]; and in forest quality by Walsh and Olienyk [17], just to name a few. In general, the approach used in these studies has been to evaluate environmental benefits by estimating demand function of the affected individuals for related environmental goods.

Given the importance of developing environmental benefit measures which can be rigorously derived from the individual's preference set, some associated theoretical research has emerged. In this regard, the work of Maler [11] which develops environmental quality benefits measures within a rigorous expenditure function context has received much attention.

However, in spite of the recognized importance of basing empirical environmental demand/benefits assessments on measures which can be rigorously tied back to individual preferences, little such integrative research has been done in this area.

For example, in the context of evaluating environmental quality improvements for a single commodity (such as an individual's recreational trips to a give site), a recommended procedure for evaluating the consumer's associated payoff is to measure benefits as the increased area between the commodity's shifted out demand curve due to the quality improvement (Freeman, [7] Maler [11]). However, for this case of environmental quality improvement this method has not been rigorously defended in an expenditure function framework (whereas for the case of commodity price changes the work of Willig [18, 20] has justified use of areas beneath the demand function as measuring the change in the value of the expenditure function). There has been even less integrative research developing theoretically justifiable measures of benefits from environmental quality changes for multiple commodities. Freeman [7, pp. 199-201] develops a procedure by which areas between two interrelated site demand functions measures the individual's benefits from quality changes at both sites, but the method is not rigorously defended.

It would appear then that there is a need for work which develops rigorously justifiable methods by which policy-induced environmental quality improvement benefits for several interrelated commodities can be evaluated through the use of observable demand functions.

The present paper attempts to address this perceived need, and is organized into three remaining sections. Section II develops a measure of benefits to the individual consumer associated with a policy-induced change in an environmental quality parameter associated with the consumption of each of $\underline{n}$ private goods (e.g. angling success rates at $\underline{n}$ sites. This measure is based on the individual's expenditure function, and is theoretically consistent with the underlying preference function; yet it is potentially empirically measurable from the related n commodity demand functions. Section III develops a formula which allows the calculation of the associated benefits of the improved environmental quality parameter for one or more of the n goods, when the individual's demand system exhibits linearity in both "own" and "cross" prices and qualities, a formula which can be evaluated through use of ordinary computer programs. Section IV presents the concluding remarks.

The methods used in this paper will build on the foundations developed by Willig [18,20], who identified a theoretically rigorous measure of welfare change from price changes tied to the individual's private goods demand functions; Randall-Stoll [15] in which an extension was made of Willig's price measure to incorporate quantity changes; Maler [11], who identified the benefits of quality changes as dependent on the individual's expenditure function, and Willig [19], who identified the assumptions under which the benefits to the individual of a marginal quality change for a single good could be measured by marginal changes in areas under the associated ordinary demand function.

II. The Model

Following Maler [11] and Freeman [7], assume that the individual's utility function depends on the consumption flows of n private goods, and at the most general level, an exogenous "environmental quality" index associated with the consumption of each private good. (Endogenous environmental quality is addressed in Bockstael and McConnell [2].) An example would be that of utility associated with trips to n recreation sites, for which a single exogenous environmental quality attribute at each site (such as harvest per hour of angler effort) is a separate argument in the individual's utility function.

Using the framework of the expenditure function i.e. the dual to the traditional utility maximizing problem, the individual seeks to minimize

$$(1) \qquad m = \sum_i P_i X_i$$

where m is the expenditure of money income, P_i and X_i, respectively prices and quantities consumed of the ith private good. (1) is to be minimized subject to

$$(2) \qquad U(X_1, \ldots X_n; Q_1, \ldots Q_n) - U^\circ = 0$$

where the level of utility, U(.) which is reached for any given vector of P's and qualities (Q's) is constrained to equal that utility, U° reached under an initial price, quality, money income set.

The solution to this constrained minimization problem yields the well-known (minimum) expenditure function, e.g. Freeman, [7] which shows the minimum expenditure, m*, needed to sustain U° under any arbitrary price-quality set, i.e.

$$(3) \quad m^* = m^* (P_1, \ldots P_n; Q_1, \ldots Q_n/U^\circ) = m^*(P_1, \ldots P_n; Q_1, \ldots Q_n/ P_1^o, \ldots P_n^o; Q_1^o, \ldots Q_n^o ; m^o)$$

where the zero superscripts refer to initial values.

Benefits from Price Changes for a Given Quality. It is well known e.g. Willig [18] that the derivative of (3) with respect to P_i (the marginal expenditure change needed to maintain utility at U° per unit change in P_i) equals the Hicksian income compensated demand for X_i at that level of utility. If (2) does not exhibit strong separability as defined by Maler [7] between the X's and the Q's, then these Hicksian demand functions depend on the Q's, i.e.,

$$(4) \quad \frac{\partial m^*}{\partial P_i} = X_i^*(P_1, \ldots P_n; Q_1, \ldots Q_n; m^*) = X_i^*(\tilde{P}, \tilde{Q}, U^\circ)$$

where X_i^* is the ith compensated demand function, m* is defined in (3), and $\tilde{P}, \tilde{Q}$ are the vectors of prices and qualities respectively.

Willig [20] shows that the multiproduct compensating variation (CV) attributable to a vector of non-marginal changes in $\tilde{P}$ from $\tilde{P}^o$ to $\tilde{P}^1$ is equal to Δm^* and in the context of the present quality-dependent demand system, Willig's measure can be expressed as

$$(5) \qquad CV = \Delta m^* = \int_{\tilde{P}^o}^{\tilde{P}^1} \sum_i \frac{\partial m^*}{\partial P_i} dP_i = \int_{\tilde{P}^o}^{\tilde{P}^1} \sum_i X_i^* (.) dP_i$$

where $X_i^*(.)$ is as defined in (4). Hence the CV equals the area under the compensated demand functions between the relevent price horizontals. The CV in (5), a line integral, is unique (independent of the price adjustment path between $\tilde{P}^o$ and $\tilde{P}^1$) because of Young's theorem [5], i.e. $\partial^2 E/\partial P_i \partial P_j = \partial^2 E/\partial P_j \partial P_i$, which implies $\partial X_j^*/\partial P_i = \partial X_i^*/\partial P_j$, for all i≠j, and a corresponding symmetry of the pure substitution cross price effects matrix.

Benefits from a Change in Quality. Often in the arena of environmental benefits evaluation, e.g. Cicchetti, et. al. [4], one would use (5) to assess the benefit to the individual of a policy which introduced a vector of new commodities (sites), at a zero or nominal price vector, $\tilde{P}^o$ rather than the status quo situation where the individual does not have access to the commodities at all, i.e. must pay the price vector $\tilde{P}^1$ associated with the zero consumption vector. It is the goal of this section to extend (5) to account for commodity quality changes in such a way that the rigor of the expenditure approach function is maintained, but at the same time the benefits are measurable from systems of demand equations.

Following the current state of the art as developed by Maler [11], if one wished to rigorously evaluate the compensating variation due to quality changes, one would directly evaluate the increment in the expenditure function (3) due to the increment in one or more of the commodity (site) qualities, Q_j. Using this direct expenditure function approach, depending on circumstances, one or more of the following three measures of benefits of an increment in (3) would thus be sought

$$(6a) \quad dm^* = \frac{\partial m^*}{\partial Q_j} dQ_j = m_{qj}(\tilde{P},\tilde{Q},U^o)\,dQ_j \quad \text{or}$$

$$(6b) \quad dm^* = \sum_j \frac{\partial m^*}{\partial Q_j} dQ_j = \sum_j m_{qj}(\tilde{P},\tilde{Q},U^o)\,dQ_j \quad \text{or}$$

$$(6c) \quad \Delta m^* = \int_{\tilde{Q}^o}^{\tilde{Q}^1} \sum_j m_{qj}(\tilde{P},\tilde{Q},U^o)\,dQ_j$$

where m_{qj} (.)indicates the functional dependence on $\tilde{P}$, $\tilde{Q}$, and U^o of $\frac{\partial m^*}{\partial Q_j}$.

Equation (6a) measures the differential approximation to the gain in benefits resulting from an increment dQ_j in the jth commodity's quality.

This approximation, dm^* equals dQ_j times the marginal demand price of (marginal benefits from) an increment to the jth commodity's quality, m_{qj}. The second equation, (6b) is the same differential approximation concept, and also measures the gain in benefits, dm^* but now is measured relative to changes in several commodity qualities, $d\tilde{Q}$, times corresponding marginal demand prices. Equation (6c) measures the exact increment to benefits, Δm^* from a change in the multiple commodity quality vector $\Delta\tilde{Q}$, from $\tilde{Q}^o$ to $\tilde{Q}^1$, and is a line integral.

Equations (6) in their present form, although offering a theoretically rigorous measure of benefits, would for most empirical work, not be capable of evaluation, unless the expenditure function (3) could be found. Whereas Maler [11, pp 123-25, 187-91] indicates methods for finding this multicommodity expenditure function from a set of ordinary (non income-compensated) demand functions, it requires the solution of a complicated system of partial differential equations, which would be impossible to apply in most empirical situations.

The present paper, on the other hand attempts to develop a general formula for the theoretically desirable measures in (6), which can, under certain circumstances be empirically derived directly from the commodity (site) demand functions, rather than having to be derived from the expenditure function, itself usually not capable of evaluation. Thus, the approach described in this paper makes benefit measurement easier by bypassing derivation of the expenditure function.

The benefit evaluation procedure proposed in this paper will make direct use of (5). Although (5) has traditionally been used in the literature as a demand-function-based measure of benefits induced by a price vector change brought about by a $\Delta\tilde{P}$ from $\tilde{P}^o$ to $\tilde{P}^1$, (e.g., Willig

[18,20],) in this paper (5) will be modified in order to be used to assess the benefits of a change in quality from $\tilde{Q}^o$ to $\tilde{Q}^1$.

As in (6) an evaluation will be made of the following three increments to the CV in (5): (1) the approximate change in CV due to a marginal quality change of a single commodity, (2) the approximate change in CV from marginal changes in several commodity qualities, and (3) the exact CV change due to several nonmarginal commodity qualit changes.

To find the first measure, we differentiate the CV in (5) with respect to the environmental quality index associated with the jth commodity, Q_j, to obtain the marginal demand price of Q_j, and then multiply by that policy-induced change in quality, dQ_j. This yields a differential approximation to the exact change in CV and is

$$\text{(7a)}\quad dCV = \frac{\partial(CV)dQj}{\partial Q_j} = \left[\int_{\tilde{P}^o}^{\tilde{P}^1} \sum_i \frac{\partial X_i^*}{\partial Q_j} dP_i + \sum_i X_i^*(\tilde{P}^1,\tilde{Q}^o,U^o)\frac{dP_i^1}{dQ_j} - \sum_i X_i^*(\tilde{P}^o,\tilde{Q}^o,U^o)\frac{dP_i^o}{dQ_j}\right] dQ_j$$

$$= \left[\int_{\tilde{P}^o}^{\tilde{P}^1} \sum_i X^*_{Q_{ij}}(\tilde{P},\tilde{Q},U^o)dP_i + R_j\right] dQ_j$$

where differentiation under the integral sign is permitted by Leibnetz' Rule, (Danese [5]), and yields an integral plus two remainder terms, the latter of which we denote as R_j. The term inside the integral $X^*_{Q_{ij}}$ is the marginal effect on (compensated) demand X_i^* of a change in quality of commodity j. Since this term is to be summed over the relevent commodities and then integrated, the resultant line integral has the economic interpretation of a marginal gain in areas beneath relevent demand curves (those forwhich $X^*_{Q_{ij}} \neq 0$) per unit change in the jth commodity's quality, Q_j. The remainder term, R_j is zero because the vector of maximum prices, $\tilde{P}^1$ is by definition that vector which forces all the demands X^*_i to equal zero (hence the vanishing of the first half of R_j) and also the vector of minimum administered prices, $\tilde{P}^o$ would not normally depend on commodity quality (hence $dP^o_i/dQ_j = 0$ for all i, j and the second half of R_j vanishes).

Observe that (7a) the approximate change in CV from the th commodity's quality change, dCV, depends on the vector of existing commodity <u>price</u> spreads, $\Delta\tilde{P}$, spreads which equal the difference between administered prices $\tilde{P}^o$ and the prices at which the commodity consumption vector equals zero, $\tilde{P}^1$. This should make sense since greater administered prices, <u>ceteris paribus</u> (lower $\Delta\tilde{P}$) would tend to restrict consumption of all commodities and hence would reduce the benefits of a <u>given</u> increased quality of the single commodity in question. Note also that (7a) shows that even in cirumstances when only a <u>single</u> commodity's quality changes, the resultant marginal effect on areas under <u>all</u> related demand schedules must be accounted for (those for which $X^*_{Q_{ij}} \neq 0$) in order to capture the entire effect on benefits. This is in marked contrast to Freeman's [7, pp. 199-201] proposed method in which it is suggested that for a given commodity (site) quality change, only changes in areas beneath that commodity's own demand function should be included in benefits, and changes in areas beneath related demand functions are to be ignored.

As to the second measure, one can obtain the differential approximation to the change in CV associated with several simultaneous commodity quality changes by finding the total differential of (5) over each relevant commodity (site) for which quality undergo a marginal change, dQ_j, and yields

$$(7b)\quad d(CV) = \sum_j \frac{\partial(CV)}{\partial Q_j} dQ_j = \sum_j \left[\int_{\tilde{P}^o}^{\tilde{P}^1} \sum_i X^*_{Q_{ij}}(\tilde{P},\tilde{Q},U^o)\,dP_i \right] dQ_j$$

Equation (7b) is merely the aggregation of (7a) over the relevent commodities for which quality changes.

Both (7a) and (7b) provide approximations to the change in CV associated with changing quality, approximations which are good for small changes. However for larger quality changes these equations are progressively more inexact. From the Fundamental Theorem of Integral Calculus, [5] one can evaluate the exact change in CV, i.e. $CV = CV(\tilde{Q}^1) - CV(\tilde{Q}^o)$, by finding the definite integral of dCV in (7b), which should be evaluated at the upper and lower quality levels, respectively $\tilde{Q}^1$ and $\tilde{Q}^o$. This equals

$$(7c)\quad \Delta(CV) = \int_{\tilde{Q}^o}^{\tilde{Q}^1} d(CV) = CV(\tilde{Q}^1) - CV(\tilde{Q}^o) = \int_{\tilde{Q}^o}^{\tilde{Q}^1} \sum_j \frac{\partial(CV)}{\partial Q_j} dQ_j.$$

where the term inside the integral is defined by (7b)[1].

Observe that with the integral (7b) inserted into the integral (7c), a double integral results. As often happens with double integrals, one would expect the limits of integration on the inside integral (the prices) to be a function of the values taken on by the variable associated with the limits of the outside integral, i.e. the qualities. This holds true in the present context; one would expect that as the exogenous quality vector increases (decreases), the vector of demands would tend to shift out (in) and the associated maximum price vector, $\tilde{P}^1$ would generally increase (decrease) as a result.

The benefits of a nonmarginal change in $\tilde{Q}$ shown in (7c) could be evaluated by use of the following three step procedure. First, evaluate multiple commodity (site) benefits as the summed areas beneath the existing vector of commodity demand functions for existing quality, $\tilde{Q}^o$, which should be unique if the cross price effects matrix is symmetrical (footnote #1). Second evalute the benefits given the subsequent quality vector, $\tilde{Q}^1$ as equivalent areas beneath the vector of demand functions. Third, subtract the step one results from those of step two, the difference of which is equal to the ΔCV in (7c). The change in CV due to multiple commodity quality changes is measured as the net difference in areas beneath the relevent system of demand functions with as compared to without the quality changes. This finding again, is in marked contrast to the results of Freeman, [7, p. 200], and will be further addressed in Section III of this paper.

Equations (7) respectively measure the theoretically desired terms in equations (6). But equations (7) have the desirable attribute of being much more empirically tractable than methods which would attempt the burdensome task of directly estimating (6) by finding the expenditure function. Equation (7) estimates the benefits of quality change directly from the commodity demand functions themselves, and thus allow the researcher to bypass the expenditure function.[2]

III. Special Empirical Considerations for Linear Systems

Equations (7) empirically measure the payoff associated with policies which change the environmental quality attribute in question for a vector of commodities. In the present section a practical formula is developed for the general measure (7c) in the special case of a demand system which is linear in both commodity prices and qualities. In addition, this linear system formula is applied for the special cases of (1) two interdependent commodities (2) only a single independent commodity. For both of these sub cases, the benefits measure is compared to the more ad hoc measures currently used by practitioners.

General Model for a Linear Demand System. Suppose that a system of commodity (site) demand equations has been estimated to depend linearly on both prices (travel costs) and "qualities", specifically assumed to take the following form:

$$(8) \qquad \underset{n\times 1}{\tilde{X}} = \underset{n\times 1}{\tilde{A}_Q} + \underset{n\times n}{\tilde{B}_p} \underset{n\times 1}{\tilde{P}} + \underset{n\times n}{\tilde{B}_Q} \underset{n\times 1}{\tilde{Q}}$$

where $\tilde{X}$ is the vector of consumption levels, $\tilde{A}_Q$ a vector of intercepts associated with zeros in both $\tilde{P}$ and $\tilde{Q}$, $\tilde{B}_p$ a symmetric matrix of own- and cross price coefficients[3], $\tilde{B}_Q$ a (not necessarily symmetric) matrix of own- and cross-quality coefficients, where the matrix dimensions are as indicated. For the ith commodity, (8) becomes

$X_i = A_{Qi} + \sum_j B_{p_{ij}} P_j + \sum_j B_{Q_{ij}} Q_j$; where j is the index of commodities whose quality affects X_i, and it is assumed that the demand system is closed, i.e. that the commodities in the i index equals those in j.

The system (8) can be viewed as a generalization into quality space of the linear demand system in price space. The latter, such as that used by Cicchetti, et al. [4], is

$$(9) \qquad \underset{n\times 1}{\tilde{X}} = \underset{n\times 1}{\tilde{A}} + \underset{n\times n}{\tilde{B}_p} \underset{n\times 1}{\tilde{P}}$$

where A is a vector of intercepts, i.e. consumptions at a zero price vector. Relative to (9) Cicchetti, et al. [4] showed that the benefits of introducing the vector of commodities (sites) at administered prices $\tilde{P}^o$ (possibly the zero vector), rather than having none of them available at the maximum price vector $\tilde{P}^1$ is, suppressing the matrix dimensions,

$$(10)\ \Delta W = W(\tilde{P}^1) - W(\tilde{P}^o) = \tfrac{1}{2}(\tilde{P}^{1\prime}\ \tilde{B}_p\ \tilde{P}^1 - \tilde{P}^{o\prime}\ \tilde{B}_p\ \tilde{P}^o) + \tilde{A}'(\tilde{P}^1 - \tilde{P}^o)$$

$$= \tfrac{1}{2}\ \Delta\tilde{P}'\tilde{B}_p\ \Delta\tilde{P} + \tilde{A}'\Delta\tilde{P}.$$

where W indicates the scalar welfare (Cicchetti, et. al. did not use the CV as their ΔW), and the primes indicates a matrix transpose. The validity of (10) as a unique measure of ΔW rests on assuming that the demand system (9) meets the path independence condition, i.e. symmetry of the $\tilde{B}_p$ matrix.

A (heuristic) generalization of (10) to account for non-marginal variations in commodity quality, equal to the present paper's ΔCV measure of ΔW (7c), is now developed. The procedure used follows three steps. First a quality-dependent generalization of the intercept term, $\tilde{A}$, in (9)-(10) is developed. Second, a quality-dependent generalization of the maximum price term, $\tilde{P}^1$, in (9)-(10) is derived. (It is assumed that $\tilde{P}^o$ is independent of quality.) Third, a quality-dependent generalization of the benefits formula (10) is derived, based on the above two generalizations of $\tilde{A}$ and $\tilde{P}^1$.

First from (8), the vector of consumption intercepts, $\tilde{I}_Q$, at zero $\tilde{P}$ but positive $\tilde{Q}$, equals

$$(11)\qquad \tilde{I}_Q = \tilde{A}_Q + \tilde{B}_p\ \tilde{O} + \tilde{B}_Q\tilde{Q} = \tilde{A}_Q + \tilde{B}_Q\tilde{Q}$$

Second, observe that the price vector which simultaneously drives all commodity consumptions to zero ($\tilde{P}^1$ in (10)) would in the present case, generally depend on the commodity quality vector, $\tilde{Q}>0$. Thus, for example, greater qualities at all sites would be expected to increase the $\tilde{P}^1$ vector as demands experience an outward shift. The $\tilde{P}^1$ in (10), becomes for the present more general case of non-zero commodity quality, from (8)

$$(12) \qquad \tilde{P}^{1Q} = (-\tilde{B}_p)^{-1}(\tilde{A}_Q + \tilde{B}_Q\tilde{Q}) = (-\tilde{B}_p)^{-1}(\tilde{I}_Q)$$

where $\tilde{P}^{1Q}$, the vector of quality-dependent maximum commodity prices, is found by solving the matrix expression (8) for that $\tilde{P}$ which forces $\tilde{X}$ to be a vector of zeros, and a -1 superscript indicates a matrix inverse.

Third, given the expressions (11) and (12) as quality-dependent generalizations for the consumption intercept vector and maximum price vector respectively, these terms can be substituted into (10), and the benefits due to the given $\tilde{Q}$-induced price spread, $P^{1Q} - P^o$, become

$$(13) \qquad W(\tilde{Q}) = \tfrac{1}{2}(\tilde{P}^{1Q\prime}\tilde{B}_p\tilde{P}^{1Q} - \tilde{P}^{o\prime}\,\tilde{B}_p\,\tilde{P}^o) + \tilde{I}_Q{}'(\tilde{P}^{1Q} - \tilde{P}^o)$$

where both $\tilde{P}^{1Q}$ and $\tilde{I}_Q$ are dependent on $\tilde{Q}$, as developed in (12) and (11) respectively, and $\tilde{P}^o$, the vector of administered prices (entry fees), is assumed to be independent of the vector of commodity qualities. (However, a quality dependent $\tilde{P}^o$ could easily be incorporated into (13).

It is, of course the increment to (13) brought about by changes in $\tilde{Q}$ that is of interest to researchers and environmental policymakers. Two cases will be addressed here.

First in the case of marginal changes in $\tilde{Q}$, or for an approximation of larger changes, the (scalar) total differential of W in (13) is

$$\text{(13a)} \quad dW = \sum_j \frac{\partial(W)}{\partial Q_j} dQ_j = \underset{1\text{xn}}{\frac{\partial W}{\partial \tilde{Q}}} \underset{\text{nx1}}{d\tilde{Q}} = \left(\underset{\text{nxn}}{\tilde{B}_p} \underset{\text{nx1}}{\tilde{P}^{1Q}} + \underset{\text{nx1}}{2\tilde{I}_Q} - \underset{\text{nx1}}{2\tilde{P}^o}\right)' \left(\underset{\text{nxn}}{(-\tilde{B}_p)^{-1}} \underset{\text{nxn}}{\tilde{B}_Q} \underset{\text{nx1}}{d\tilde{Q}}\right)$$

where the matrices after the third equality are obtained by matrix differentiation of (13) with respect to $\tilde{Q}$ and multiplying by the vector of quality changes, $d\tilde{Q}$. As would be expected, simplification of that last term will show that it is equal to the differential benefit approximation formula in (7b), for the special case of the linear demand system (8) (Derivation is available from the author).

Second, the exact change in W from a non-marginal $\Delta\tilde{Q}$ can be found by evaluating (13) at initial and terminal levels of $\tilde{Q}$, and is

$$\text{(13b)} \qquad \Delta W = CV(\tilde{Q}^1) - CV(\tilde{Q}^0)$$

which equals (7c) for the present case of the linear demand system (8), and is readily calculable by programming a computer. (Programs are available from the author.)

Two Interdependent Commodities. Suppose, following Freeman's example [7, p. 200], there is a demand equation system of two recreation sites which exhibit interdependent quality effects, and for which both sites undergo a policy-induced quality improvement. This section uses the formulas developed in (13a) - (13b) to assess the benefits of that quality improvement, compares the results with a procedure suggested by Freeman, and evaluates why and under what circumstances there may be a difference.

For the sake of the concreteness suppose that for the two-site case, (8) has been estimated to take the following form

$$\text{(14)} \quad \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -.5 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} + \begin{bmatrix} 2 & -.2 \\ -.1 & 1 \end{bmatrix} \begin{bmatrix} Q_o \\ Q_1 \end{bmatrix}$$

where the subscripts refer to the site index, and it is assumed, following Freeman and for simplicity in exposition, that the symmetrical $\tilde{B}_p$ matrix displays zero cross price effects. It is desired to find the change in benefits due to an assumed change in the quality vector from $[(Q_o^o \quad Q_1^o)]' = [(5 \quad 5)]'$ to $[(Q_o^1 \quad Q_1^1)]' = [(6 \quad 6)]'$ i.e. quality at each site increases from 5 to 6. Evaluating (13) for the above with-versus-without-quality-improvement levels in order to find the exact ΔW change (13b), ΔW becomes for this two-site system, a positive \$62.73.[4]

An intuitive understanding of how the \$62.73 benefit measure is derived can be had by referring to Figure 1, where the demand system (14) is illustrated by heavy solid lines associated with initial site qualities and light solid lines for terminal (improved) qualities. From equation (13b) \$62.73 is the (two-site summation of) W at terminal quality (\$453.48) less W at initial quality (\$390.75), where direct unambiguous summation across sites is permitted in the present case of (14) because of the assumed zero cross-price effects. Thus, from (13b) ΔW is evaluated by merely adding across sites the difference between terminal quality benefits (light solid lines) and initial quality benefits (heavy solid lines), without regard to how these functions shifted about during the process of quality change.

Freeman's proposed method [7, p. 200] for evaluating this quality-induced gain in benefits is fundamentally different that the method proposed in this paper, in that the intermediate adjustment path from the quality variations are explicitly considered in his method.

For example, beginning with site 1, his method would propose the following: The gain in benefits due to its quality increasing from 5 to 6, holding site 2 quality constant at its initial value of 5, is shown by site

1's demand shifting from the initial dark solid line out to the dotted line, and equals \$40. There is no need, according to Freeman, to subtract off the lost benefits at site 2 from its demand having shifted into the dotted line as a result of site 1's improvement (\$2.89). Next, his method suggests the following: Evaluate the change in site 2's benefits as its quality increases from 5 to 6, <u>holding site 1's quality constant at its new level of 6</u>, realizing that site 2's demand will as a result shift from the dotted line to the outside line. These benefits (\$29.80) at site 2, again do not, according to Freeman need to be adjusted by the lost benefits at site 1 (\$4.18) resulting from site 2's quality improvement.

Thus, Freeman's proposed method yields a quality-induced benefit gain across sites of \$69.80. However, if for each site's quality increase, lost benefits at substitute sites were netted out (\$2.89 + \$4.18 = \$7.07) his proposed method would yield the identical benefits of \$62.73 found by applying (13b) in this paper. Thus the key to the difference in the two methods lies in the treatment of (lost) benefits at substitute sites (for quality increases).

Freeman, in deriving his multiple site quality benefits evaluation procedure, applied a well-known result by Knetsch [9], which shows that there is no need to take into account shifting demands (changing recreation use) at existing sites, when evaluating the benefits of a new site (lowering its price from the maximum to the administered level). However, it would appear that while Knetsch's procedure is correct within his stated context, i.e., for evaluating the benefits of <u>price</u> change(s), it is not suitable for evaluating the benefits of <u>quality</u> changes.

The cause of this assymetry between price and quality evaluations can be seen by referring to equations (4) - (5) and (7a) - (7c). From equation

(4) it can be seen that the benefit from (change in the expenditure function due to) a marginal change in price of the ith commodity, or $(\partial m^* / \partial P_i)$, equals the quantity demanded of that commodity times that change in price, $X_i^* \, dP_i$. That is, to evaluate the benefits of a single marginal price change, dP_i, one only needs to apply that price change to the (compensated) demand function for that single good, with no need to consider resulting shifting demands for substitute goods. Similarly, for multiple non-marginal price changes, (5) shows that the associated (path independent) CV measure of benefits can be found by sequentially adding the area beneath each demand function in isolation, as long as for each good one takes into account the effect of "previous" goods' price-change-induced demand shifts. Thus, as seen from (4) - (5) the benefit evaluation procedure of Freeman [7] and Knetsch [9] is correct for the case of price changes, and a given site's benefits arising from its price change can be evaluated without regard to resultant changing demand at substitute sites.

However, for evaluating quality changes at a given site, resultant demand shifts at substitute sites must be accounted for. This can be seen most clearly in the case of only a single jth commodity's quality change, dQ_j by differentiating (5) with respect to Q_j, then multiplying by that dQ_j to find the differential approximation to the resulting change in CV. As seen from (7a) $\partial(CV)/\partial Q_j$, relative to (5), equals the integral of $\partial X_i^* / \partial Q_j$ summed over all i commodities for which the individual's demand shifts due to the change in Q_j. Only in circumstances when $\partial X_i^* / \partial Q_j = 0$ for all $i \neq j$ will one be able to ignore substitute sites in quality benefits evaluation.

This holds to an even greater extent when evaluating the benefits of quality changes for each of many commodities, as is shown for the marginal approximation and exact cases in (7b) - (7c) respectively. Thus, in a recreation benefits context, for each site the benefits of improved quality at that site equals the increased area beneath that site's demand function net of reduced area (if any) lost beneath substitute sites' demands.

<u>Only A Single Commodity Under Consideration</u>. When the environmental quality parameter for a single "independent" commodity (isolated site) is undergoing a potential policy change, we may assume that $\partial X_i / \partial Q_j = 0$ for all $i \neq j$; however, when $i=j$ $\partial X_i / \partial Q_i = B_Q$, (a scalar) if one assumes a linear system such as (8). Given this independence, $\partial X_i / \partial P_j = 0$ could similarly be assumed for all $i \neq j$ but when $i=j$ $\partial X_i / \partial P_i$ equals the scalar B_p. In such a simplified case, by substituting (11) and (12) into (13), one obtains a special case of W as dependent only on the given commodity's quality, Q i.e.

$$(15) \quad W(Q) = \tfrac{1}{2}[(1/B_p)(A_Q + B_Q Q)^2 - P_o^2 B_p] - [(1/B_p)(A_Q + B_Q)^2 - (A_Q + B_Q Q)P_o]$$

Referring to Figure 2, this becomes, assuming a zero administered commodity (site) price P_o, $W = -\tfrac{1}{2}(1/B_p)(A_Q + B_Q Q)^2$, which is the triangular area associated with a given quality, Q.

Comparing this benefit at a higher $Q=Q^1$ with that smaller area for a lower quality, $Q=Q^o$ yields the gain in CV for the single commodity (site) due to a quality improvement from Q^o to Q^1. In this special case, the ΔCV is shown in Figure 2, and equals the area between the two demand functions as identified by Maler [11, p. 185] and Freeman [7, p. 198]. Thus, in the case of the single independent commodity, linear in price and quality, the quality benefits evaluation methods developed in the present paper which are based on the theoretically correct expenditure function approach are consistent with that of Maler and Freeman, and with what has been traditionally applied by practitioners.

IV. Conclusions

Economists have commonly used areas beneath shifted private good demand functions to evaluate the benefits (costs) of the public good environmental improvement (degradation), when that improvement can be identified as bringing about a shifting of relevant demands. Some work has been done which evaluates the benefits of commodity quality changes in a rigorous expenditure function context, e.g. Maler [11]. However, little if any has been done which rigorously integrates the widely used demand function area approach to the theoretically correct expenditure function approach in the case of quality changes, particularly where several interdependent commodities are involved. This paper has attempted to develop a rigorously justifiable method by which environmental quality improvement benefits for several commodities can be evaluated through use of observable interdependent demand functions, and hopefully provides a point of departure for further related research.

ENDNOTES

[1]Equations (7) are all line integrals, for which the issue of their uniqueness must be addressed. Since the X_i^* are each income compensated demands, then the integrals (7a,b) are independent of the order in which the price changes are evaluated, because $\frac{\partial\left(\frac{\partial X_i^*}{\partial Q_j}\right)}{\partial P_k} = \frac{\partial\left(\frac{\partial X_k^*}{\partial Q_j}\right)}{\partial P_i}$ for all i,j,k, since $\frac{\partial X_i^*}{\partial P_k} = \frac{\partial X_k^*}{\partial P_i}$. However for (7c) to be independent of the path over which quality changes are evaluated, the necessary condition for uniqueness is

$\frac{\partial\left(\frac{\partial (CV)}{\partial Q_j}\right)}{\partial Q_i} = \frac{\partial\left(\frac{\partial (CV)}{\partial Q_i}\right)}{\partial Q_j}$, a condition which in general does not hold, even if the pure substitution (price) effects matrix is symmetrical. However, from (7a) it can be seen that uniqueness of the benefits measure of quality change (7c) can be assured if the following desired condition

$$\frac{\partial\left(\frac{\partial X_i^*}{\partial Q_j}\right)}{\partial Q_k} = \frac{\partial\left(\frac{\partial X_i^*}{\partial Q_k}\right)}{\partial Q_j}$$

holds for all i, j, k. (This is not the same as symmetry of the cross quality effects matrix, $\frac{\partial X_i^*}{\partial Q_j} = \frac{\partial X_j^*}{\partial Q_i}$.)

However, if each X_i^* is linear in the Q's, (i.e. $\tilde{X}=f(\tilde{P}) + \tilde{B}_Q \tilde{Q}$) then this desired condition holds and (7c) is unique. Again $\tilde{B}_Q$ does not need to be symmetric to ensure uniqueness of (7c).

[2]Technically the use of equations (7) to measure the ΔCV due to changing quality require knowledge of the system of income-compensated demands whereas given available data one can typically econometrically estimate only ordinary demand systems, although of course, a price symmetry constraint necessary for uniqueness (footnote #1) can be imposed on the estimated system. Relevant research could be directed toward deriving error bounds associated with the use of ordinary demands to estimate quality benefits, benefits which in this paper are technically only applicable to compensated demands. Such research might follow the approach of Willig [18, 20] for price changes and Randall - Stoll [15] for quantity changes.

[3]The demand system (8) should be income-compensated in order to be a representative case of (4). However, even if (8) were not income compensated, if a symmetry constraint in the $\tilde{B}_p$ price effects matrix were imposed as a side constraint when estimating (8), the validity of the results which follow in this section would be maintained. However, in general the resulting benefits measure would then be best interpreted as a consumer surplus rather than a CV.

[4]The differential approximation to the benefit gain dW, (13a) is $60.30, which is identical to the value obtained if (7b) is applied directly to (14).

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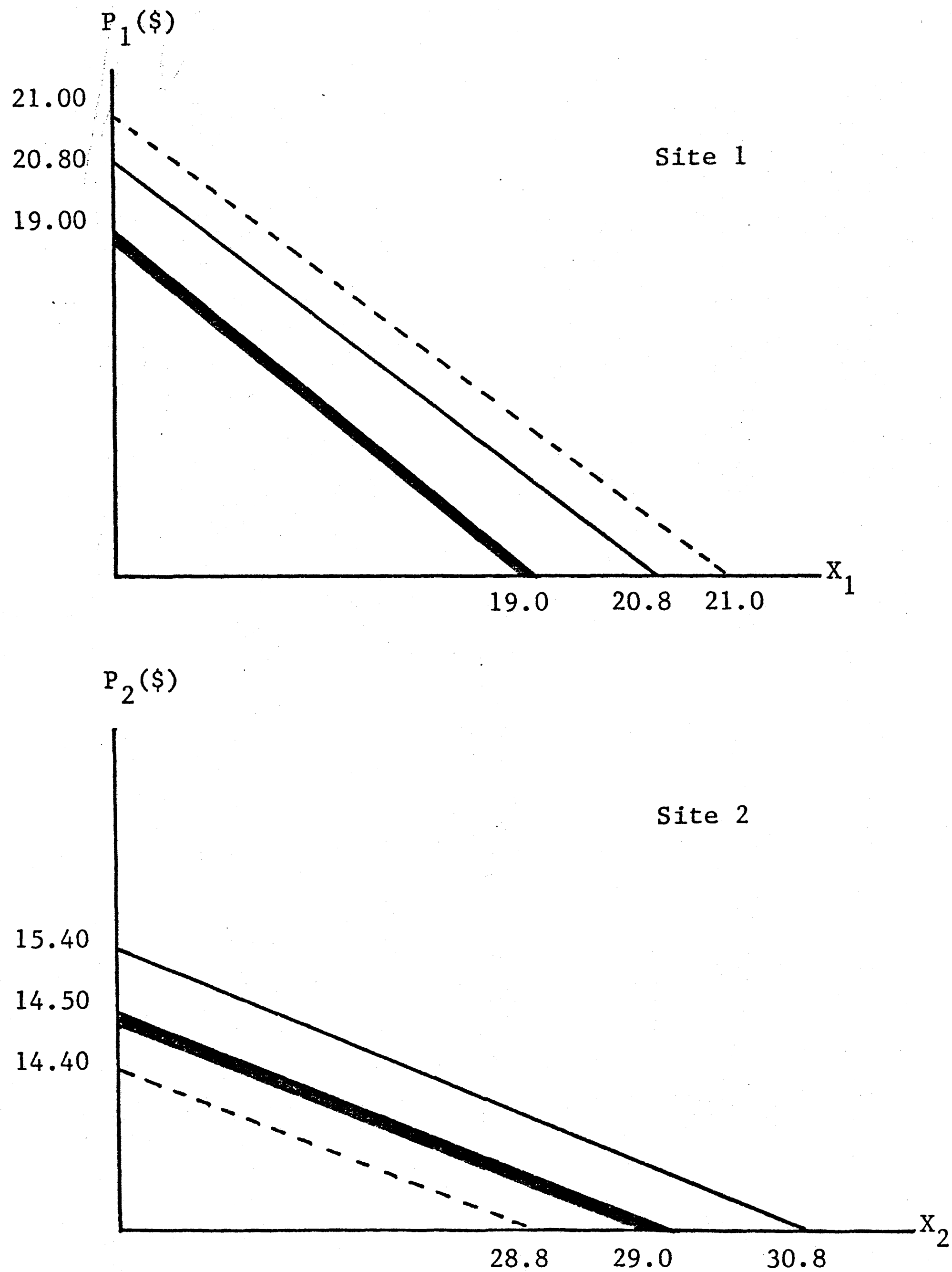

Figure 1. Benfits from improving environmental quality at two interdependent sites

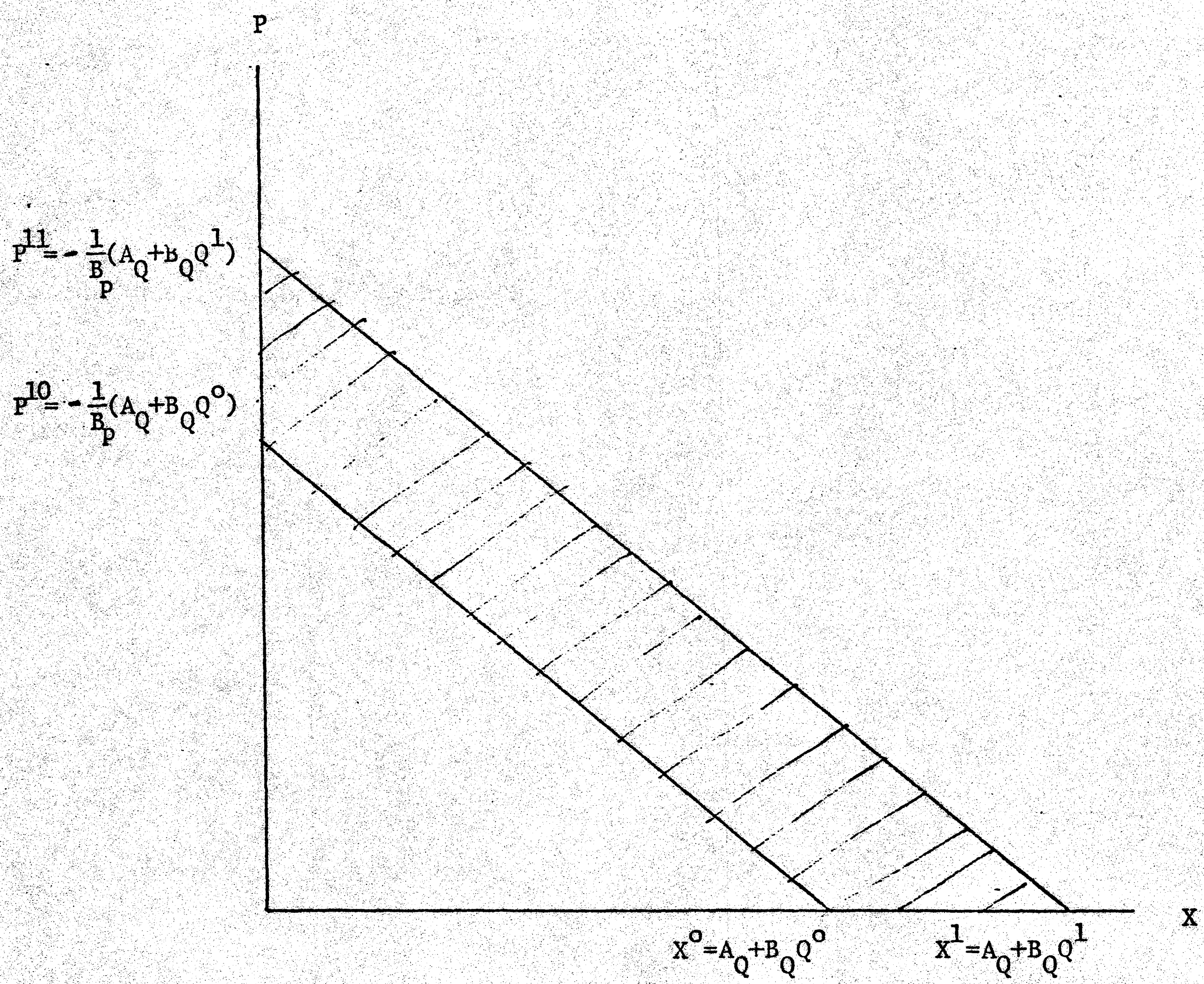

Figure 2. Benefits of A Single Independent Commodity Resulting from a Change in Environmental Quality