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Taxes, Unemployment and Welfare in a Harris-Todaro Economy

Margaret E. Grosh
Department of Economics
Cornell University

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Cornell University
486 Uris Hall
Ithaca, New York
14853-7601

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Despite the elegance of the Bhagwati-Srinivasan optimal wage subsidy theorem in a Harris-Todaro economy, it has not provided a useful policy tool. The financial and informational problems are too great to achieve the optimal subsidy. Hence marginal subsidies must be studied more thoroughly.

This paper presents the sufficient conditions for marginal, across-the-board employment-increasing wage taxes in an open Harris-Todaro economy. It is shown that the constellation of parameters required for the sufficient condition to hold is likely to be observed. Parallel results are derived for taxes on manufacturing wages.

The effects of taxes/subsidies on welfare are considered. Previous work has considered welfare to be only a function of GNP. Here unemployment is introduced explicitly into the welfare function. Preferences considered are Lexicographic, constant elasticity of substitution, and Cobb-Douglas with bargaining. In general, even when nondistortionary financing is assumed, wage subsidies are not sufficient to increase welfare. The sufficient conditions for each case are examined.

Margaret E. Grosh

Taxes, Unemployment and Welfare in a Harris-Todaro Economy[1]

The Harris-Todaro wage subsidy literature is extensive, but is deficient in two respects. It has failed to suggest adequate financing mechanisms for the proposed optimal wage subsidies, and it has dealt too simplistically and too briefly with the interplay between unemployment and welfare. Because no adequate financing mechanism has been proposed for the large across-the-board optimal subsidies, study of smaller suboptimal subsidies is important. Because across-the-board subsidies seem particularly unlikely, sectoral subsidies merit considerable attention. Most of the literature has been aimed exclusively at curing unemployment, and where welfare has been considered, it has been postulated as a function of national income alone. This denies unemployment's psychic, social and political costs. A broader definition of welfare is called for.

In this paper I examine marginal changes in single-sector and across-the-board wage subsidies. Their impact on unemployment and a variety of welfare functions is traced. A brief review of the literature is provided in Section 1. The model is set out in Section 2. In Section 3 the effects of subsidies on employment are detailed, and in Section 4 their effects on several welfare formulations are outlined. Section 5 concludes the discussion.

SECTION 1: The Literature

The Harris-Todaro (1970) model of labor migration has given development economists an elegant and simple explanation of two somewhat puzzling observances: the migration of large numbers of laborers away from rural areas of full employment into urban pockets of high unemployment, and the seeming impossibility of creating enough jobs in areas of high unemployment to reduce the numbers of jobless workers. The Harris-Todaro model holds the two phenomena to be results of the same root causes--an exogenously determined urban wage, and laborers' basing of their migration decisions on their wage expectations, rather than on actual market wages.

The original Harris-Todaro model, as set out in their 1970 AER paper, makes a number of extreme assumptions about the labor market. Among these are complete job turnover in the manufacturing sector in each period; risk neutrality on the part of migrants; lack of discrimination in hiring in terms both of education and training, and of non-performance related characteristics such as tribal, racial or religious affiliation; and the complete inability of a rural worker to find an urban job while still employed in the rural sector. These and other objections have been considered in subsequent work [Fields (1975), Corden and Findlay (1975), and McCool (1982)]. The modifications necessary to eliminate the assumptions have complicated the algebra somewhat but have not altered the essence of the simpler formulation of the model nor its conclusions.

Some have found objectionable the model's assumption of sector-specific capital. Corden (1974), Corden and Findlay (1975), and McCool (1982) are among those who have extended the basic model by allowing capital mobility. Once again its implications were unchanged. Corden and

Findlay (1975) and Khan (1980) have examined the results of increased endowments of capital and labor.

Empirical work has supported the relevance of the model. With the relaxation of some of the extreme labor market assumptions, Fields (1975) has shown that the unemployment rates predicted by the model are close to those actually observed. Todaro (1976b) has shown that the constellation of parameters necessary to produce the paradox of job creation inducing unemployment is not unlikely to be observed, especially in African LDCs. A good review of the empirical literature is found in Todaro (1976a).

Because the Harris-Todaro model is robust to details of formulation and has empirical support, its policy implications merit serious attention. Each of the papers cited above considers some of them. The most important policy implications can be grouped into two classes.

The first group addresses itself to the root cause of migration and unemployment--the rural-urban wage differential. These policies recommend a diminution of urban attraction either by eliminating or decreasing the fixed wage in the urban sector, or by increasing the wage or non-wage amenities in the countryside. The ramifications of these policies are well understood and because they seek to reduce the manufacturing wage to an appropriate level, they are the preferred way of lessening unemployment in a Harris-Todaro economy. The excessive fixed wage exists, however, because of the political strength of those receiving it or because it is efficient for the firm to offer it[2] Thus an economist's recommendation that the fixed wage be eliminated may not have much impact.

The second group of policies presumes that the manufacturing wage has been lowered to the extent possible but is still above the market-clearing level. Harris and Todaro (1970) show that an urban wage subsidy coupled with migration restrictions will achieve first best. Migration restrictions are, however, politically impalatable. Bhagwati and Srinivasan (1974) show that there exists an across-the-board subsidy which will also achieves first best and does not require migration restrictions. One of the difficulties with the Bhagwati-Srinivasan subsidy is that its calculation assumes knowledge of the optimal value marginal product in the urban sector. It being unlikely that this is known, the planner is aware that the optimum exists but does not know how to attain it. Basu (1980) solves the information problem by showing that any subsidy greater than the Bhagwati-Srinivasan optimum will guarantee first best. Furthermore, the existing manufacturing wage is a member of this set. The proof is elegant but a subsidy of this magnitude requires paying the whole cost of labor in the economy, an amount which is higher than the total initial labor cost and which may even be higher than the economy's total income. That unemployment can be so eliminated seems a proposition of limited value.

Basu's monotonicity theorem is far more helpful. Assuming non-distortionary financing in a perfectly competitive, open economy, gross national product will increase for any increase in subsidy level from zero to the optimum. This is the first truly useful prescription of the wage subsidy literature. The welfare function used, however, is somewhat restrictive and exploration of subsidies with other welfare functions is merited. Nor has Basu given full consideration to single-sector policies. The next section puts forth a model with which to address these issues.

SECTION 2: The Model

The most succinct exposition of an optimal across-the-board wage subsidy is found in Bhagwati and Srinivasan (1974), so I will follow closely their formulation of the model. Consider a small, open economy in which prices are taken from the international market and thus unresponsive to changes in the country's economy. We denote these fixed prices by P_m and P_a where m stands for manufacturing and a for agriculture. The economy is perfectly competitive and divided into two sectors--an urban manufacturing sector and a rural agricultural sector.[3] Each produces according to a constant returns to scale production function which is concave in each input. Thus

$$M = F(L_m, K_m) \quad (1)$$

and

$$A = G(L_a, K_a) \quad (2)$$

where L represents labor and K capital. The economy's endowment of the two factors of production is fixed, and the allocation of capital between sectors is fixed. Labor can work in either sector or be unemployed. By choice of units

$$1 = L_m + L_a + U . \quad (3)$$

The manufacturing wage is exogenously determined and exceeds the full employment equilibrium wage. The wage in the agricultural sector is competitively determined. Employers in each sector will hire until the value of marginal product of the last worker hired equals the wage. Thus

$$w_m = P_m F_L \quad (4)$$

and

$$w_a = P_a G_L \quad (5)$$

where the subscripts L denote the partial derivatives of output with respect to labor.

Laborers in the agricultural sector may not seek or obtain work in the urban sector. If they migrate and cannot find employment they remain unemployed. Because the agricultural wage is flexible there is full employment in that sector. Because the manufacturing wage is fixed, there can exist unemployment in the urban sector. Workers will migrate to the sector in which the expected wage is the highest. The system is in equilibrium when the expected wages are equal. The expected wage in each sector is the wage received for employment multiplied by the probability of finding employment. In agriculture the expected wage is the actual wage. In manufacturing the expected wage is the fixed wage times the percent of workers in the urban sector who have jobs. Accordingly, the equilibrium condition can be written as

$$w_a = w_m L_m/(L_m + U) \qquad (6)$$

Subsidy rates are represented by s, but the reader should bear in mind that a tax, denoted by t, is merely a negative subsidy, so $t = -s$. Subsidies are applied only to wages and the subscript denotes the sector's wage which is being subsidized. In hiring labor the firm will respond to the gross of subsidy wage while the laborer will react to the net of subsidy wage. Thus 4) and 5) become

$$w_m(1 - s_m) = P_m F_L \qquad (4')$$

$$w_a(1 - s_a) = P_a G_L \,. \qquad (5')$$

A uniform wage subsidy implies that $s_m = s_a = s$ and that $ds_m = ds_a = ds$. Most of the tax incidence literature assumes that initial taxes/subsidies are zero but this system is simple enough to allow for non-zero initial taxes with little complication in the algebra. If initial taxes exist, because a subsidy is a negative tax, an increase in a subsidy is identical in effect to a decrease in a tax. The paper uses the terminology of the literature, i.e. it talks about subsidization, but it should be borne firmly in mind that it could as well refer to a lowering of initial wage taxes. This provides an important way of addressing the financing problem.[4]

The four key equations of the model are 3, 4′, 5′ and 6. Given this system, determining the effects of subsidies is a simple exercise in comparative statics.

SECTION 3: Employment Effects

Across-the-Board Wage Subsidies

Employing the tax equivalence and solving for dU/ds reveals

$$\frac{dU}{ds} = -\frac{1}{J}\left\{P_a G_{LL} w_m + P_m F_{LL} w_a + \frac{w_m^2}{L_m + U}\left(1 - \frac{L_m}{L_m + U}\right)(1 - s_a)\right\} \tag{7}$$

where J denotes the determinant of the Jacobian matrix and is

$$J = P_m F_{LL}\left\{\frac{w_m L_m (1 - s_a)}{(L_m + U)^2} - P_a G_{LL}\right\} \tag{8}$$

Because the marginal product of labor diminishes as labor increases, F_{LL} and G_{LL} are non-positive. The remaining term is non-negative so the sign of dU/ds is indeterminate.

The implication of this indeterminacy is important. If an economy suffers from a fixed wage in its urban sector, it will not know what the optimal labor force allocation is, and therefore not be able to construct the optimal Bhagwati-Srinivasan subsidy. Unless financing greater that the economy's initial wage bill is available, so that Basu's result becomes applicable, the best that the planner could do is to hope to achieve full employment by sequential guesses. Since dU/ds is indeterminate, such a procedure is not only not guaranteed to end in the elimination of unemployment: it may not even approach it. The planner finds himself confronted with the standard difficulty of a second best world--given a distorted economy, actions which would normally increase employment may instead reduce it. It is also interesting to note that the indeterminacy of sign indicates that it may be possible to increase taxes and employment simultaneously. This, surely, would make fiscal policy decisions easier! It is important, then, to examine the conditions under which the sign of dU/ds is determinate.

The necessary condition for determinacy of dU/ds is that the numerator of the expression be determinate. The numerator is not easy to interpret. Analysis of the sufficient conditions is somewhat more fruitful. This leads to the following proposition:

<u>Proposition 1</u>[5]

If 1.i) $G_{LL} = 0$ and $(1 - w_a/w_m) < \eta_m$
or 1.ii) $F_{LL} = 0$ and $(1 - w_a/w_m)L_a/L_m < \eta_a$
then $dU/ds < 0$.

η is the elasticity of the wage with respect to the sector's employment.

When is one of these conditions likely to hold? The structure of migration (equation 6) implies that for a given manufacturing wage, and therefore a given level of employment in manufacturing, the wage ratio will vary from zero to unity as unemployment varies from unity to zero. The smaller is unemployment, the larger is the rural-urban wage ratio, and the more likely[6] it is that a subsidy will increase employment, all else equal. For a given level of unemployment, the wage ratio will be higher with a higher level of manufacturing employment, and thus the tendency for dU/ds to be negative will be greater.

On the other side of the inequality in condition 1.i is η_m. Its value is determined by the technology in the manufacturing sector. The assumption of a constant returns to scale production function has not constrained η very much. Examination of its value for three common production functions in the constant elasticity of substitution family will give an idea of its range. A linear production function has infinite substitutability of capital and labor, denoted here as σ, and an elasticity of labor's value of marginal product with respect to labor of zero[7] ($\sigma = \infty$, $\eta \rightarrow 0$). A Cobb-Douglas production function has unitary elasticity of substitution, and an elasticity of wage with respect to employment equal to the factor share of capital (σ=1, $\eta = \alpha_K$). A Leontief production function with zero substitutability has an elasticity which approaches infinity (σ = 0, $\eta \rightarrow \infty$). Thus we can say that all else equal, the less substitutability we have in the manufacturing technology, the more likely is a general wage subsidy to increase employment.

What sort of values of the various relevant parameters have been observed in developing countries? The values of the left hand side of condition 1.i may commonly range from .22 to .80 (see Table 1). Many computable general equilibrium models (for a survey see Shoven and Whalley (1984)) assume unitary elasticities of substitution of capital for labor. This implies that the right hand side of 1.i ranges from zero to unity.

Thus we cannot expect condition 1.i, sufficient for employment-increasing wage subsidies, to be fulfilled in all developing countries.

Non-fulfillment of condition 1.i does not, of course, mean that a wage subsidy will necessarily decrease employment. Condition 1.ii may hold. This seems unlikely. The model is constructed to apply to countries with relatively small manufacturing sectors so it is reasonable to expect the ratio of agricultural to manufacturing employment to be greater than unity. The value of the left hand side of 1.ii will be greater than in 1.i. The elasticity of factor substitution is frequently postulated to be greater in agriculture than that in manufacturing [Meier (1984), Eckaus (1955)] so that the right hand side of 1.ii would be less than that in 1.i.

The sufficient conditions of proposition 1 are necessary only if G_{LL} = 0, or alternately, F_{LL} = 0. Let us examine the case of G_{LL} = 0 first. This is the largest value it could attain. If, instead, the marginal product of labor in agriculture is still decreasing at the observed level of employment in agriculture, then the necessary condition for an unemployment-increasing subsidy may still hold, though the sufficient condition does not. Estimates of the value of G_{LL} are less common than those for other parameters, but it would not be surprising in densely populated areas where diminishing returns to labor have set in, to find small absolute values for G_{LL}. Indeed, some formulations of surplus labor models assume constant marginal product and thus G_{LL} = 0. It should also be noted that great substitutability of capital and labor is frequently presumed in agriculture. The extreme is a linear production function with G_{LL} = 0. More moderate functions give a small absolute G_{LL}. The smaller it is, of course, the more likely that violation of the sufficient condition implies violation of the necessary condition for employment-increasing wage subsidies.

The sufficient condition 1.ii is necessary only if $F_{LL} = 0$. Because manufacturing employment is small, it is almost certain that increased manufacturing employment will expand output. Thus $F_{LL} < 0$. It is impossible to say, however, whether its value will be such that the necessary condition for an employment-increasing across-the-board subsidy will be violated as well as the sufficient.

The possibility of violation of the necessary condition introduces an important ramification. If, indeed, the necessary condition for employment-increasing subsidies is violated, then employment can be increased by using a general wage tax. This follows because violation of the necessary condition implies that $dU/dt < 0$. Policy makers in such an economy would surely be pleasantly surprised to know that they could increase employment and collect revenue simultaneously. Indeed, the problem of financing across-the-board subsidies normally faced in a Harris-Todaro economy would be transformed into the rather more cheerful dilemma of where to spend the revenues raised from across-the-board wage taxes. Fortunately, the data required to determine if this will occur is fairly available. All the information needed to check the sufficient conditions is readily available. Verification of the necessary conditions requires estimates of G_{LL} and F_{LL}, which are not as frequently measured, but which could be approximated.

Thus the indeterminacy of the effect of the across the board subsidy on employment is not exceptionally troublesome. It allows a very useful perverse case, and perhaps equally important, it is possible for policy makers to distinguish whether the across the board subsidy will be useful in a particular economy.

Manufacturing Wage Subsidies

The standard remedy for an excessively high fixed wage in manufacturing is to provide a subsidy to the employer in order to reduce the difference between the fixed wage and the first best shadow wage. In the Harris-Todaro economy this is the same as a subsidy to the manufacturing wage. Algebraically we set $ds_m > 0$ and $ds_a = 0$. The normal result of increased employment is not, however, assured here. The expression for dU/ds_m is obtained by employing the implicit function theorem again and is given in equation 9.

$$\frac{dU}{ds_m} = - \frac{1}{J} \{P_a G_{LL} + \frac{w_m}{L_m + U}(1 - \frac{w_m}{L_m + U})\} \qquad (9)$$

J is negative, so the sign of dU/ds_m depends on the sign of the numerator. Its first term is negative and its second positive. This leads to

Proposition 2*

The necessary and sufficient condition for a subsidy to the manufacturing wage to increase employment is

If $(w_m/w_a - 1)(L_a/(1-L_a)) < \eta_a$, then $dU/ds_m < 0$.

If $(w_m/w_a - 1)(L_a/(1-L_a)) > \eta_a$, then $dU/dt_m < 0$.

Both the left and the right hand sides of this expression are positive. Since we cannot generally state whether the condition will obtain, consideration of its parts is in order. The smaller is the manufacturing wage relative to the agricultural wage, the greater the tendency for a subsidy in manufacturing to increase employment. The manufacturing wage is relatively small when unemployment is low, all else equal (eq. 6). The lower is employment in agriculture, ceteris paribus, the smaller will be the second term, and thus the more likely it will be that the manufacturing wage subsidy will increase employment. The lower the substitutability of labor for capital in agriculture, the higher will be η_a and the higher the likelihood that $dU/ds_m < 0$. In sum, initially low unemployment, a large manufacturing sector, and limited substitution opportunities in manufacturing will make it most likely that subsidization of the manufacturing wage will increase employment. The converse cases, of course, will make the corollary more likely to obtain, i.e. that wage taxation will increase employment.

It is possible to construct plausible values of the left hand side of the expression in proposition 2 (see Table 2). The elasticity of the wage in agriculture is likely to vary from zero to unity as was the case in manufacturing.

It is apparent that violation of the necessary and sufficient condition in proposition 2 is possible. In such an event the corollary will obtain. Thus LDC's with a small manufacturing sector may find that wage taxes in that sector will raise total employment. This is interesting in that it runs counter to traditional advice. Furthermore, income taxes in LDC's play a greater role in manufacturing than in agriculture. This occurs partly because the taxes are usually progressive and wages are higher in manufacturing, and partly because compliance with income taxes is low in the largely subsistence agricultural sector. Thus LDC's with small manufacturing sectors may have stumbled almost accidentally upon a means of raising employment. It is also valuable to know when the perverse case of employment-raising taxes will hold because it is much more readily financed than the normal case.

Agricultural Wage Subsidies

The subsidy for the agricultural wage is the simplest. The expression for its effect on unemployment is always negative.

$$\frac{dU}{ds_a} = \frac{1}{J} P_m F_{LL} w_a \tag{10}$$

Therefore,

Proposition 3

A wage subsidy in agriculture will increase employment.

That subsidizing labor in agriculture should raise employment is not surprising. After all, it is the wage dichotomy which causes unemployment. It becomes an important empirical issue to determine whether agriculture is favored over manufacturing in fiscal policy. Certainly the proponents of urban bias [Lipton, (1977)] would say not. Public finance economists, however, throw up their hands at taxing agriculture with its small-scale subsistence aspects and turn to manufacturing for their revenues [Goode (1984), Tait, Gratz and Eichengreen (1979), and Bird (1967)].

We have examined here only marginal sub-optimal wage subsidies or increments thereon. This is in keeping with the belief that only limited financing will be available and that the optimum is therefore unattainable.

The study has raised some interesting possibilities. It is entirely feasible that across-the-board and manufacturing wage subsidies may decrease employment rather than raise it. This suggests a solution for the financing problem ubiquitous in the Harris-Todaro literature. Tax rather than subsidize! Of course, only some economies will be so blessed as to fall into the perverse case where taxation raises both revenue and employment. The rest are left to balance wage distortions and their resulting unemployment with tax distortions.

SECTION 4: Welfare Effects

Consideration of welfare in a Harris-Todaro economy has used welfare and gross national product (GNP) synonymously.[9] When welfare is a function of GNP, Basu's theorem is very reassuring. National income increases monotonically with the wage subsidy up to the optimal level. The proof is quite simple. Welfare is set equal to GNP and partial derivatives with respect to the subsidy level are taken. Expressions for dL_m/ds and dL_a/ds are derived from a model such as that outlined in Section 2 above.

It should be noted that the monotonicity of welfare in subsidies depends critically upon the goods' prices being their weights in the welfare function. This allows the cancellation of terms in the derivation and gives a determinate sign. If the weights are for any reason different than the products' world prices, welfare may not increase with across-the-board wage subsidies. Prices may not be the most appropriate welfare weights if one good is a "merit" good or if there are externalities in production not captured in the pricing scheme. Introduction of market power could add similar qualifications to the welfare weighting scheme.[10]

The primary objection to defining welfare as GNP is not that prices are the wrong weights for sectoral output, but rather that societal welfare is not a function of a single argument. In addition to its effects on national income, unemployment is important in its own right. The psychic cost of unemployment is high even when the unemployed individual's income is maintained through transfers. Unemployment is frequently cited as a contributor to problems such as crime, drug abuse, and political unrest. Through undermining their confidence in future security, the presence of unemployment in an economy may lower the satisfaction and welfare of the employed. Lastly, poverty and unemployment are indivisible in the Harris-Todaro model. If there is any concern for poverty or income distribution, then unemployment must be explicitly added to the welfare function.

It is in this spirit that consideration of subsidies' effects on welfare are considered using several specifications of societal welfare as a function of national income and employment. Lexicographic preferences are treated first, followed by members of the constant elasticity of substitution (CES) family of welfare functions, and then by welfare bargaining models.

Lexicographic Preferences

The usual definition of lexicographic preference is $x \succ y$ if either 1) $x_1 > y_1$ or 2) $x_1 = y_1$ and $x_2 > y_2$ [Varian (1978, p. 83)]. Let x represent the welfare of the economy with wage subsidies and y the welfare without subsidies. Consider first the case where x_1 is GNP and x_2 is employment.[11] Employment is clearly of low priority. An increase in its level will not be preferred if it comes at the expense of even a minute loss in national income. This results in policy prescriptions very similar to those from welfare defined over only GNP. Across-the-board subsidies will always improve welfare, as will subsidies in either sector alone.

As the alternate case, let x_1 be employment and x_2 be GNP. Employment is of the first priority. Across-the-board subsidies may or may not increase it. The conditions under which this will occur are given in Section 3's treatment of unemployment. Manufacturing wage subsidies give similarly ambiguous results. Agricultural wage subsides will increase employment.

Use of lexicographic preferences serves to contrast the polar cases. Ironically, the only policy guaranteed to increase both employment and national income is the agricultural wage subsidy. The off-the-cuff answer that a manufacturing subsidy should be used to reduce the distortion caused by the inappropriately high manufacturing wage may be mistaken, as may be the only policy which will give first best, the across-the-board wage subsidy.

Constant Elasticity of Substitution Welfare Functions

Let welfare be a function of GNP and employment with constant elasticity of substitution. Then where W is welfare, and E employment,

$$W = [aGNP^{\rho} + bE^{\rho}]^{1/\rho} . \tag{11}$$

Determining the effect of a subsidy on welfare requires taking the derivative with respect to the subsidy

$$dW/ds = A\, dGNP/ds + B\, dE/ds \tag{12}$$

where A and B are the partial derivatives of welfare with respect to GNP and employment. National income is defined as GNP = $P_m M + P_a A$, and E = 1

- U. Equation 12 is then expressed in terms of the changes in the distribution of labor as

$$dW/ds = \underset{+}{A}\underset{+}{P_m}\underset{+}{F_L}\underset{+}{dL_m}/ds + \underset{+}{A}\underset{+}{P_a}\underset{+}{G_L}\underset{?}{dL_a}/ds - \underset{+}{B}\underset{?}{dU}/ds \qquad (13)$$

<u>Across-the-Board Wage Subsidies</u>

Applying the implicit function theorem to equations 3, 4′, 5′, and 6 yields expressions for dL_m/ds, dL_a/ds, and dU/ds for an across-the-board wage subsidy. As the signs underneath equation 13 suggest, dW/ds is of indeterminate sign. Manipulation of the expression yields four individually sufficient conditions for an across-the-board subsidy to increase welfare.

<u>Proposition 4</u>[12]

If 4.i) $(1 - w_a/w_m) < \eta_m$ then $dW/ds > 0$.

or 4.ii) $(1 - w_a/w_m)L_a/L_m < \eta_a$

or 4.iii) $$1 - \frac{B(w_m - w_a)}{A\, w_m w_a \eta_m} > s$$

or 4.iv) $$1 - \frac{B\, L_a(w_m - w_a)}{A(1-L_a)w_m w_a \eta_a} > s$$

Conditions 4.i and 4.ii are the same as 1.i and 1.ii found in Proposition 1. If either holds then employment increases with wage subsidization. GNP will always increase with wage subsidization, so both arguments in the welfare function are positive. Before proceeding to dissect conditions 4.iii and 4.iv, let us consider the A and B terms.

In the CES family of welfare functions with A representing the marginal welfare value of a change in GNP, and B the marginal welfare value of a change in employment,

$$\frac{B}{A} = \frac{b}{a}\left|\frac{GNP}{E}\right|^{1-\rho} . \qquad (14)$$

The expression is evaluated at the initial point, i.e. GNP and E are measured before the policy is introduced. The weights in the welfare function of GNP and employment are given by a and b, respectively. Let the elasticity of substitution between the two goods be σ. Then $\sigma = 1/(1-\rho)$. Choosing an appropriate value of ρ will yield as special cases linear, Cobb-Douglas and Leontief welfare functions. The Stone-Geary function also nests in the CES family. It seems particularly appropriate for a government's planning function. The minimums are the levels of GNP and employment which must be obtained to maintain the government. If they are not attained, the government will fall. The value of B/A for each case is given in Table 3.

The ratio B/A will in all cases be greater as b/a is greater, i.e. the more heavily weighted is unemployment in the welfare function. The ratio GNP/E is greater than unity,[13] so B/A increases as ρ increases. In the Leontief case $B/A \to \infty$.

Now let us examine condition 4.iii. The percent rate of subsidy is given by s. A complete wage subsidy is s = 1. Laissez-faire is s = 0. The relevant range of analysis is thus $0 < s < 1$. The greater is the left hand side of 4.iii, the more likely is the sufficient condition for a welfare-increasing subsidy to obtain.

The left hand side of 4.iii will be larger as B/A is smaller. B/A will be smaller (larger) the less (more) important is employment in the welfare function. If, for example, employment is not considered at all, then b = 0, which implies B = 0, and condition 4.iii collapses to $1 > s$.

This, of course, is a restatement of Basu's monotonicity theorem that GNP increases with subsidies up to the optimal range.

The degree of substitutability between employment and income in the welfare function has an even stronger effect on the magnitude of B/A. The greater the substitutability, the smaller will be B/A, and the more likely is the condition to hold. This is intuitively satisfying. If extra income easily compensates society for an increase in unemployment, as in the linear welfare function, then the extra income accruing from the subsidy's institution may be sufficient to increase total welfare. If, on the other hand, preferences are Leontief, $B/A \to \infty$, and condition 4.iii can never be met for a positive subsidy. Non-fulfillment of conditions 4.i and 4.ii implies that unemployment has increased as a result of wage subsidization. With Leontief preferences the increase in income will not compensate for decreased employment.

The influence of the other terms in 4.iii is parallel to their effect in Proposition 1. The smaller is the initial wage gap, or unemployment rate, and the lower the factor substitution elasticity in manufacturing, the more likely that an across-the-board wage subsidy will increase welfare.

Interpretation of condition 4.iv is analogous to that given for condition 4.iii and will be omitted to avoid repetition.

With the explicit introduction of employment into the welfare function, the unconditional desirability of across-the-board wage subsidies has been lost. Considerably more thought is required before the policy may be recommended.

Perhaps the most interesting aspect of the indeterminacy of dU/ds's sign is the role played by the elasticity of substitution between income and employment in the welfare function. It is far greater than the role played by the goals' weights in the welfare function.

A simple illustration of this is provided by looking at data from Kenya.[14] If $\rho \rightarrow -\infty$ (the Leontief case) condition 4.iii can never be met, as explained above. If $\rho = -.25$ condition 4.iii reduces to $1 - 13.29b/a > s$. Employment need only have a weight in the welfare function of $b > .07$ in order for the sufficient condition to be violated for all subsidy levels.[15] If $\rho = 0$ (the Cobb-Douglas case) condition 4.iii reduces to $1 - 1.61b/a > s$ and a welfare weight of $b > .38$ is needed before the sufficient condition is violated. If $\rho = 1$ (the linear case) the condition becomes $1 - .29b/a > s$ and a welfare weight of $b > .78$ is required for violation of the sufficient condition, for all subsidy levels.[16] Clearly, ρ has a greater role than b in determining whether the sufficient condition will be met.[17]

Having seen that employment in the welfare function may result in uncertainty as to the desirability of an across-the-board wage subsidy, let us turn to the single-sector subsidies.

Manufacturing Wage Subsidies

The same procedure as used above is employed to determine the effects of a wage subsidy in manufacturing. In this case the implicit function theorem is applied to equations 3, 4', 5 and 6. After due manipulation Proposition 5 is derived.

<u>Proposition 5'</u>*

If 5.i) $$\frac{L_a (w_m - w_a)}{(1-L_a) \quad w_a} < \eta_a \quad \text{then } dW/ds_m > 0$$

or 5.ii) $$\frac{B L_a (w_m - w_a)}{A \quad w_m} < \eta_a$$

or 5.iii) $$1 - \frac{B (w_m - w_a)}{A \quad w_a w_m} > s_m$$

or 5.iv) $$1 - \frac{B L_a (w_m - w_a)}{A(1-L_a) w_m w_a \eta_a} > s_m$$

There are small differences in their arrangement, but the same terms enter Proposition 4 and 5 in an analogous manner. The reader is spared the redundancy of reparsing the expressions. It is interesting to note how our numerical example changes, though. For a manufacturing wage subsidy condition 5.iii (the parallel to 4.iii) becomes $1 - 1.47b/a > s_m$ for Cobb-Douglas welfare and $1 - .26b/a > s_m$ for linear welfare. Given the elasticity of substitution of employment for GNP, a higher welfare weight for employment is necessary for condition 4.iii to be violated. Manufacturing wage subsidies will more often increase welfare than across-the-board subsidies of the same rate and are, of course, a much smaller financial burden.

<u>Agricultural Wage Subsidies</u>

The wage subsidy in agriculture is, again, the simplest case. Both income and employment are monotonically increasing in the agricultural wage subsidy, so

Proposition 6

A wage subsidy in agriculture will increase welfare.

Study of the effects of marginal wage subsidies in the CES family with welfare defined in terms of income and employment has shown that ambiguities are introduced. The crucial role of the elasticity of substitution has also been revealed. This is the parameter most difficult to estimate, and probably the most crucial in the model. Thus the policy maker is in a double quandary. Not only is there a theoretical possibility of his policies backfiring, he cannot readily determine if it is likely to occur in his country. It is easy to see why the literature has avoided more frequent use of multidimensional welfare functions. They do not lend themselves to neat, categorical results. But we need, from time to time, to remind ourselves that ambiguities exist, and in the case of employment- or welfare-increasing taxes the perverse cases that they allow should be cherished.

So far in this discussion of welfare we have made the convenient, if heroic, assumption of a single social welfare function. This can be reconciled with Arrow's impossibility theorem [Arrow, (1951)] by attributing the function to an all-powerful planner rather than considering it the consensus of the members of the society. Since such omnipotence is rare, let us take one more step toward the real world and consider the case where different factions have different goals.

Welfare with Bargaining

When society is composed of non-homogeneous individuals a realistic welfare function takes into account these differences. A convenient way of doing so is to use a variable-bargaining-power specification of the welfare function. The form employed here generalizes the Nash (1950) cooperative bargaining game by explicit introduction of bargaining power parameters as a determinant of the solution. It replaces Nash's axiom of symmetry with one of proportionality which states that the desired property is that the subject of the bargain be distributed among agents in proportion to their bargaining powers. The axioms of Pareto optimality, independence of irrelevant alternatives and independence of equivalent utility representations are maintained. Using these axioms it can be shown that the bargainers act as if they were maximizing the bargaining-power-weighted product of their individual utility functions. The proof of the theorem is provided in Svejnar (forthcoming). Illustrative applications are found in Svejnar and Smith (1984) and Svejnar (1982).

Using a variable-bargaining-power specification, societal welfare can be defined as a function of the welfare of its component groups, i.e.,

$$W = \prod_{i=1}^{n} W_i^{v_i} \tag{15}$$

where W is societal welfare, W_i is the welfare (utility) function of the i^{th} group, and v_i is its bargaining power. It is convenient to normalize the bargaining power variable so that $0 \leq v_i$ and $v_1 + v_2 + \ldots + v_n = 1$.

The members of a Harris-Todaro economy can be sensibly broken into factions is several ways, with variants also possible in defining the welfare function within each group and their bargaining powers. This section will consider two formulations.

Case 1

This is conceptually the simplest case. There are two factions within the society--capital and labor.[19] Capitalists desire to maximize the returns to capital, and laborers want to maximize their expected wage.[20] Bargaining powers are assumed to be the share of the two groups in national income,[21] so

$$W = W_1^{v_1} W_2^{v_2} \qquad (16)$$

$$W_1 = P_m M + P_a A - w_m L_m - w_a L_a \qquad v_1 = W_1$$

$$W_2 = w_a \text{ [22]} \qquad v_2 = W_2$$

Using these definitions for the welfare function the following propositions can be derived:

Proposition 7[23]

$dW/ds \gtrless 0$ as $s \lessgtr 1$.

Proposition 8[24]

$$dW/ds_m \gtrless 0 \text{ as } s_m \lessgtr \frac{1}{\frac{L_a}{L_m+U}\frac{1}{\eta_a} + 1}$$

Proposition 9[25]

$dW/ds_a \gtrless 0$ as $s_a \lessgtr 1$

When welfare is the product of the individual welfares of capital and labor weighted by their respective shares in GNP, a partial across-the-board wage subsidy will improve welfare. A partial agricultural wage subsidy will also increase welfare.

Manufacturing wage subsidies will either increase or decrease welfare depending principally on the size of the subsidy and the degree of factor substitution possible in the agricultural technology. The subsidy will draw labor to manufacturing and out of agriculture. If there is little flexibility in agriculture (σ low, η high), then with the withdrawal of labor, the wage will rise faster than if there is more substitution (σ high, η low), and labor will gain more. The greater the gain for labor, the more it will work to offset capital's loss and the more likely that $dW/ds > 0$. Low substitutability gives a larger range of welfare-improving subsidies.

It can be noted from the proofs of the above propositions that in each case capital loses and labor gains. This is not surprising given that it is labor that is being subsidized. It does, however, compel consideration of the bargaining powers assigned. The general simplicity of the results is partly driven by each party having identical bargaining weights and welfare functions. If labor has power greater than its share in income then aggregate welfare will increase even more than in the case where the bargaining power is the income share.

If it is capital that has a bargaining power greater than its income share, then its loss will be weighted more heavily and the effect of the subsidy will be indeterminate in all cases. Capital may well have a bargaining power greater than its share in GNP. It is generally more concentrated than labor, so its owners may organize more easily. Though, outside the confines of this formalization of the model, capital may flow abroad more easily than labor and can use this threat to increase its bargaining power. Furthermore, capitalists seem to be members of government more often than laborers and so increase their bargaining power.

The effect of financing the subsidy need also be mentioned here. It has been assumed that the subsidy is financed through a non-distortionary tax. The only non-distortionary tax in the Harris-Todaro economy is on capital.[26] This will, of course, add to the loss that capital suffers and create ambiguity in the results of the subsidies.

A final qualification to the desirability of wage subsidies is introduced if we step back from the static framework used here. If the well-being of capital is reduced, then a slower growth rate would be expected. A trade-off would then arise between current increases in welfare and lower future GNP.

<u>Case 2</u>

In the previous case qualified support for welfare-increasing subsidies was found. The reader will note, however, that each party was concerned only with its income. Unemployment was not considered in its own right. Let us reformulate the problem. Capitalists are still concerned only with their income and have bargaining power in proportion to it. Rural labor is concerned with its wage and bargains with its share of income. Urban labor is principally concerned with the unemployment rate. It is they who risk unemployment personally and they who live daily with the social problems that unemployment breeds. Their bargaining power is also their share of income. Thus we now have

$$W = W_1^{v_1} W_2^{v_2} W_3^{v_3} \qquad (17)$$

where

$$W_1 = P_m M + P_a A - w_a \qquad v_1 = W_1$$

$$W_2 = w_a \qquad v_2 = L_a w_a$$

$$W_3 = 1 - U \qquad\qquad v_3 = (1-L_a)w_a$$

Differentiating with respect to changes in the subsidies and simplifying leads to

<u>Proposition 10</u>[27]

If $s > \frac{L_m + U}{L_m + L_a}$ then $dW/ds < 0$

If $s < \frac{L_m + U}{L_m + L_a}$ then $dW/ds \gtrless 0$

<u>Proposition 11</u>[28]

If $s > \frac{L_m}{L_m + L_a}$ then $dW/ds < 0$

If $s < \frac{L_m}{L_m + L_a}$ then $dW/ds \gtrless 0$

<u>Proposition 12</u>[29]

$$dW/ds_a \gtrless 0 \quad \text{as} \quad s_a \lessgtr \frac{L_m + U}{L_m + L_a}$$

If subsidy levels are higher than the ratio of the percent of the work force in urban areas to the percent of the work force employed, then any of the three policies will lower aggregate welfare. At lower levels the effect of across-the-board and manufacturing wage subsidies is indeterminate, but the agricultural subsidy will raise welfare. Neat sufficient conditions for welfare improving across-the-board and manufacturing subsidies are not forthcoming.

In all cases capital is made worse off by the subsidy and agricultural labor gains. In the first two cases the effect on urban welfare is indeterminate. In the last case urban welfare increases.

The qualifications given for case 1 apply here as well. If capital's bargaining power is greater than its share of income, then the conditions given in Propositions 10, 11, and 12 become considerably more complicated and welfare will be less likely to increase with a subsidy. Consideration of financing sources and growth reinforce this qualification.

Having defined urban welfare to be a function of only the employment level may exaggerate its importance. Since the expected wage rises with the subsidies, if it is given some weight in urban labor's welfare function, it will increase the tendency of labor's gains to offset capital's losses. It should be borne in mind, however, that the unemployment rate directly affects the urban wage. This, together with the unpleasantness of living with the problems that unemployment causes, may justify the definition of urban labor's welfare to be the employment rate.

In concluding this section we note that the introduction of employment into the welfare function creates an even stronger tendency toward ambiguity in the variable-bargaining-power model than in the CES formulation. This results from the former capturing the conflicting interests of a heterogeneous society.

SECTION 5: Conclusions

The Harris-Todaro literature is replete with elegant, simple theorems dealing with the effects of wage subsidies. The theorems, alas, are rarely of practical value. They are mainly oriented toward large, optimal subsidies for which adequate financing is missing, or deal with welfare in only the narrowest of definitions. The moral of the Harris-Todaro story has always been that subsidies are good.

Because the financing problem must be solved before any subsidy recommendations can be useful to the planner, I have dealt only with marginal subsidy changes from a non-optimal initial position. The subsidy that I speak of may then be financed from existing revenue or from imaginable increments thereon. Indeed, "subsidy" is properly understood in this model to be a decrease in an existing tax.

Considering more practical levels of wage subsidization has shown that an across-the-board or manufacturing subsidy may not increase employment. Indeed, the necessary conditions for an employment-improving subsidy are violated for parameter values that are not unreasonable. This possibility calls for more careful consideration on the planner's part before a subsidy is recommended and may eliminate any financing problem by yielding a tax as the appropriate way to raise employment.

The introduction of the employment rate into the welfare function exposes the existing literature's naivete. With two arguments in the welfare function, the effects of wage subsidization become much more complex than previously shown. When a CES welfare function is used, the effects of across-the-board and manufacturing subsidies are ambiguous. The key parameter in determining the qualitative nature of the change is the elasticity of substitution of income for employment in the welfare function. If it is low then welfare will only increase if employment increases independently. Of a second order of importance is the actual weight of employment in the welfare function.

With consideration of society's heterogeneity, the CES function itself seems naive and a bargaining model captures better the way in which welfare is articulated. When this approach is taken, even when each faction is concerned only with its income, some ambiguity is introduced. Consideration of the simplifications made in assigning bargaining powers equal to shares in income, the use of a zero threat point and the static nature of the model further qualify the desirability of wage subsidies. Introducing employment into the welfare function strengthens the possibility of welfare-decreasing subsidies. At high levels this is guaranteed.

This paper has attempted to increase the applicability of the Harris-Todaro literature to policy making situations. In doing so much of the elegance and determinacy of the model's results have been sacrificed. I hope that the reader agrees that the added realism of the assumptions and the ability to derive verifiable sufficient conditions for welfare- and employment-improving policies justifies the sacrifice.

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1. I am indebted to Paul Weller, Nick Baigent and Jan Svejnar for their many helpful suggestions.

2. A union wage, legislative minimum wage, or highly paid government bureaucracy are among the explanations in the first class. An efficiency wage due to turnover [Stiglitz, (1974)], nutrition [Leibenstein, (1957)], or employee incentive [Garner, (forthcoming)] are of the second class.

3. The model seeks to explain that dichotomy in technology and productivity which is apparent to even the casual visitor to LDCs. The difference is not strictly urban/rural or manufacturing/agriculture but might perhaps best be described as modern/traditional. Here, however, I use the more common labels found in the literature.

4. The problem is not truly solved by considering a subsidy to be a reduction in wage taxes if the need to replace the revenue is considered.

5. For derivation see Appendix A.

6. In this paper "likelihood" and "probability" are not used in the sense of stochasticity. They are used in the sense that, all else equal, if the prior is met, the tendency for the stated outcome is greater than if the prior is not met.

7. For derivation see Appendix B.

8. For derivation see Appendix C.

9. The exception is two paragraphs in Anand and Joshi, (1979).

10. The most obvious case of world prices being inappropriate, trade distortions, cannot be rigorously treated here. That requires treatment of income effects in the demand functions which, if done properly, is quite laborious and yields results which are too complex to interpret.

11. Employment is used rather than unemployment in keeping with the standard practice of using desirable goals in the welfare function. Employment is defined in this paper as (1-U), or the rate of employment.

12. For proof see Appendix D.

13. Recall that the labor variables have been standardized through division by the size of the labor force. Because the production functions are homogenous of degree one, the GNP figures are thus also divided by the size of the labor force and should be interpreted as per capita income. Measured in dollars, GNP per capita is always observed to be greater than unity. It is divided by the percentage of the labor force which is employed. A number larger than unity divided by a fraction will, of course, yield a number greater than unity.

14. Kenya was chosen essentially arbitrarily. Among the countries with wage data available it was selected because Todaro worked there while formulating the Harris-Todaro model. It is assumed that production is

Cobb-Douglas. The parameter values used are α_k = .91, U = .07, L_m = .17, L = 5.4 million, income per member of the economically active population is 3852 Kenyan Schillings, w_m = 1162 and w_a = 835 Schillings. Data is from Kurian (1982) and ILO (1985).

15. These calculations were made assuming that a+b=1, or in other words, that welfare exhibits constant returns to scale. The general sense of the example is not affected by this assumption, though the precise numbers are.

16. Of course, slightly smaller values of b will violate the condition 4.iii for high subsidy levels.

17. It should be noted that condition 4.i obtains for this data.

18. See Appendix E for proof.

19. The identical results occur when capitalists and laborers are divided by sector and a four argument welfare function is used.

20. This implies a zero threat point, which is consistent with the model's assumption of the international immobility of factors. If the factors had alternate uses then the threat point should be defined as their welfare in the best alternate use, and the welfare function as the difference in welfare in the current use and that obtainable in the alternate use.

21. Explicit definition of bargaining powers is not required for a determinate solution to the maximization of welfare in a variable-bargaining-power model, but in this particular application it greatly enhances the clarity of results. The choice of v=W was made for the sake of logic and convenience in this application. It is not a general requirement of the model.

22. Note that $w_a = w_m L_m + w_a L_a$.

Proof: From (6) $w_a = w_m L_m / (L_m + U)$. Rearranging and substituting in (6) gives $w_m L_m + w_a L_a = w_a L_m (L_m + U)/L_m + w_a L_a = w_a (L_m + L_a + U)$. But from (3) $L_m + L_a + U = 1$.

23. For proof see Appendix F.

24. For proof see Appendix G.

25. For proof see Appendix H.

26. See the second essay in this dissertation.

27. For proof see Appendix I.

28. For proof see Appendix J.

29. For proof see Appendix K.

Table 1

Common Values for $1-w_a/w_m$

L_m	.05	.20	.35
U			
.1	.66	.33	.22
.15	.75	.43	.30
.20	.80	.40	.36

Calculated from data in Turnham(1971) and the World Bank (1984).

Table 2

Common Values for $(w_m/w_a - 1)(L_a/(1-L_a))$

L_a / U	.05	.20	.35
.10	11.32	1.16	.35
.15	12.00	1.39	.43
.20	12.00	1.50	.46

Calculated from data in Turnham (1971) and the World Bank (1984).

Table 3

Marginal Welfare Ratios in the CES Family

	Linear	Cobb-Douglas	Stone-Geary	Leontief
p	1	0	0	$\to -\infty$
$\frac{B}{A}$	$\frac{b}{a}$	$\frac{b}{a}\frac{GNP}{E}$	$\frac{b\,(GNP-GNP_{min})}{a\,(E-E_{min})}$	$\frac{b}{a}\left\lvert\frac{GNP}{E}\right\rvert^{\infty}$

Appendix A

Proof of Proposition 1

Application of the implicit function theorem to equations 1.3, 1.4′, 1.5′ and 1.6 of the model shows that

$$dU/ds = -\frac{1}{J}\left[P_a G_{LL} w_m + P_m F_{LL} w_a + \frac{w_m^2}{L_m+U}\left(1 - \frac{L_m}{L_m+U}\right)(1 - s_a)\right]. \quad (A.1)$$

J is negative. The first and second terms are negative and the third is positive. This leads to the statement of two alternately sufficient but not necessary conditions for an employment-increasing across-the-board subsidy.

Condition 1.i

Let us assume that the first term achieves its maximum value, i.e. $G_{LL} = 0$. Then dU/ds will be negative if

$$P_m F_{LL} w_a + \frac{w_m^2}{L_m+U}\left(1 - \frac{L_m}{L_m+U}\right)(1 - s_a) < 0. \quad (A.2)$$

Equation A.2 is not easily interpreted in its present form but algebraic manipulations resolve the problem. Rewriting A.2 and substituting in equation 6 yields

$$P_m F_{LL} w_m \frac{L_m}{L_m+U} + \frac{w_m}{L_m+U}(w_m - w_a)(1 - s_a) < 0. \quad (A.3)$$

Rearranging,

$$\frac{w_m - w_a}{P_m F_L}(1 - s_a) < -\frac{F_{LL} L_m}{F_L}. \quad (A.4)$$

Denoting the elasticity of the manufacturing wage with respect to manufacturing labor as η_m

$$\frac{(w_m - w_a)}{P_m F_L} (1-s_a) < \eta_m. \qquad (A.5)$$

Finally, recalling that $s_a = s_m$, substituting in equation 4', and rearranging yields

$$1 - \frac{w_a}{w_m} < \eta_m. \qquad (A.6)$$

Accordingly, if $G_{LL} = 0$, the sufficient condition may be stated: dU/ds will be negative if $1 - \frac{w_a}{w_m} < \eta_m$.

<u>Condition 1.ii</u>

In this case we assume that $F_{LL} = 0$ and combine the first and third terms. With F_{LL} reaching its maximum, the sufficient condition is

$$P_a G_{LL} w_m + \frac{w_m^2}{L_m+U} \left(1 - \frac{L_m}{L_m+U}\right) (1-s_a) < 0. \qquad (A.7)$$

Dividing out the like term, dividing by $P_a G_L$ and multiplying by L_a gives an expression for η_a in the first term. The second term is simplified by distributing w_m across the parentheses and substituting using equations 1.6 and 1.5'. Thus

$$- \eta_a + \frac{L_a}{L_m+U} \left(\frac{w_m - w_a}{w_a}\right) < 0. \qquad (A.8)$$

Using 1.6 again to replace the denominator of the second term and passing η_a to the right gives

$$\frac{L_a}{L_m} \left(\frac{w_m - w_a}{w_m}\right) < \eta_a . \tag{A.9}$$

Thus $dU/ds < 0$ if $F_{LL} = 0$ and

$$(1 - w_a/w_m)\ L_a/L_m < \eta_a .$$

Appendix B

Values of η for the CES Production Function

The general statement of the constant elasticity of production formula is

$$Q = [\alpha_K K^\rho + \alpha_L L^\rho]^{1/\rho}. \tag{B.1}$$

By definition

$$\eta = - \frac{Q_{LL}}{Q_L} L. \tag{B.2}$$

Thus for a CES function

$$\eta = (\rho-1)\alpha_L L^\rho [\alpha_K K^\rho + \alpha_L L^\rho]^{-1} - (\rho-1). \tag{B.3}$$

Case 1: The Linear Production Function

In the linear case ρ=1 and $\sigma\to\infty$ ($\sigma = \frac{1}{1-\rho}$ for all CES functions). Substituting ρ=1 into B.3 yields η=0.

Case 2: The Cobb-Douglas Production Function

In this case ρ=0, σ=1. Substituting into B.3 gives

$$\eta = - \alpha_L[\alpha_K + \alpha_L]^{-1} + 1 = \alpha_K.$$

assuming constant returns to scale, i.e., $\alpha_K + \alpha_L = 1$.

Case 3: The Leontief Production Function

For the Leontief function $\rho\to-\infty$, σ=0. Then substituting into B.3 rearranging and simplifying yields

$$\eta = \frac{1-\rho}{1 + \frac{\alpha_L}{\alpha_K}\left(\frac{L}{K}\right)^{\rho}} .$$

By choice of units of capital, let K=L

$$\eta = \frac{1-\rho}{1 + \frac{\alpha_L}{\alpha_K}} .$$

Thus as $\rho \to -\infty$, $\eta \to \infty$.

Appendix C

Proof of Proposition 2

Application of the implicit function theorem to equations 1.3, 1.4′, 1.5 and 1.6 of the model shows that

$$dU/ds_m = -\frac{1}{J}[w_m P_a G_{LL} + \frac{w_m^2}{L_m+U}(1 - \frac{L_m}{L_m+U})]. \tag{C.1}$$

J is negative. Thus the total derivative will be negative if the numerator is negative. This leads to the statement of the necessary and sufficient condition:

dU/ds_m will be negative if

$$w_m P_a G_{LL} + \frac{w_m^2}{L_m+U}(1 - \frac{L_m}{L_m+U}) < 0. \tag{C.2}$$

Equation C.2 is not easily interpreted in its present form, but algebraic manipulations simplify the matter. Rearrangement of terms and substituting in equation 1.6 yields

$$\frac{(w_m - w_a)}{P_a G_L} < -\frac{G_{LL}}{G_L} L_a (\frac{1-L_a}{L_a}). \tag{C.3}$$

Denoting the elasticity of the agricultural wage with respect to agricultural labor as η_a, and substituting in equation 1.5,

$$\frac{w_m - w_a}{w_a} < \eta_a \frac{1 - L_a}{L_a} . \tag{C.4}$$

Rearrangement gives

$$\left(\frac{w_m}{w_a} - 1\right)\left(\frac{L_a}{1 - L_a}\right) < \eta_a . \tag{C.5}$$

Thus the condition in C.2 may be more succinctly presented:

dU/ds_m will be negative if $\left(\frac{w_m}{w_a} - 1\right)\left(\frac{L_a}{1 - L_a}\right) < \eta_a$. (C.6)

Appendix D

Proof of Proposition 4

The point of departure is equation 1.12. Let us first derive the simplest expression for dGNP/ds. GNP = $P_m M + P_a A$, so

$$dGNP/ds = P_m F_L dL_m/ds + P_a G_L dL_a/ds. \quad (D.1)$$

Expressions for dL_m/ds and dL_a/ds are obtained by applying the implicit function theorem to equations 1.3, 1.4′, 1.5′, and 1.6. Substituting them into D.1 shows

$$\frac{dGNP}{ds} = \frac{1}{J}\left[\frac{w_m^2}{L_m+U}(1-s_a)\left(P_a G_L - P_m F_L \frac{L_m}{L_m+U}\right)^* + P_m F_L W_m P_a G_{LL} + P_a G_L P_m F_{LL} W_a\right]. \quad (D.2)$$

Substituting 1.4′ and 1.5′ into the starred expression and recognizing therein equation 1.6, the terms cancel, revealing that the expression equals zero. Thus

$$dGNP/ds = \frac{1}{J}\left[P_m F_L w_m P_a G_{LL} + P_a G_L P_m F_{LL} w_a\right]. \quad (D.3)$$

Substituting D.3 and dE/ds = - dU/ds into 1.12 gives

$$\frac{dW}{ds} = \frac{1}{J} [A(\overset{1}{P_m F_L w_m P_a G_{LL}} + \overset{2}{P_a G_L P_m F_{LL} w_a}) + \tag{D.4}$$

$$+ B (\underset{3}{P_a G_{LL} w_m} + \underset{4}{P_m F_{LL} w_a} + \frac{w_m^2}{L_m+U} \underset{5}{(1-\frac{L_m}{L_m+U})}(1-s_a)].$$

If the sum of 1 through 5 is negative, then divided by the negative J term, dW/ds >0. Terms 1, 2, 3, 4 are negative, while term 5 is positive. The alternate sufficient conditions combine term 5 with one of the other terms and derive the parameter values necessary for the combination to be negative.

Conditions 4.i and 4.ii

Condition 4.i combines terms 4 and 5 of D.4. Condition 4.ii combines terms 3 and 5. These are restatements of proposition 1's conditions 1.i and 1.ii, and are derived in Appendix A.

Condition 4.iii

Condition 4.iii is derived from terms 2 and 5 of D.4. Sufficiency is achieved by the combination being negative, i.e.,

$$AP_a G_L P_m F_{LL} w_a + B \frac{w_m^2}{L_m+U} (1- \frac{L_m}{L_m+U})(1-s_a) < 0. \tag{D.5}$$

Substitution of 1.5' and 1.6 into the first term, and distributing w_m in the second term gives

$$Aw_a(1-s_a) P_m F_{LL} \frac{w_m L_m}{L_m+U} + B \frac{w_m}{L_m+U} (w_m - \frac{w_m L_m}{L_m+U})(1-s_a) < 0. \tag{D.6}$$

Cancelling like terms, substituting 1.6 into the second term, dividing by $P_m F_L$, and recognizing the definition of η_m in the first term simplifies D.6 to

$$-Aw_a\eta_m = \frac{B\ (w_m - w_a)}{P_m F_L} < 0. \tag{D.7}$$

Moving the first term to the right hand side, substituting in 1.4' and dividing by Aw_a gives

$$\frac{B}{A}\frac{(w_m - w_a)}{w_a w_m (1-s)} < \eta_m. \tag{D.8}$$

Now we divide by η_m and multiply by 1-s. Then subtracting unity from each side of the expression and multiplying by negative one yields

$$1 - \frac{B\ (w_m - w_a)}{Aw_a w_m \eta_m} > s. \tag{D.9}$$

or, in other words, condition 4.iii.

Condition 4.iv

Condition 4.iv is derived by assuming that terms 1 and 5 of D.4 sum to be negative, i.e.

$$A\ P_m F_L P_a G_{LL} + \ B\ \frac{w_m}{L_m + U}\ (1 - \frac{L_m}{L_m + U})(1 - s_a) < 0. \tag{D.10}$$

Dividing each term by P_aG_L and multiplying each term by L_a will leave an expression for η_a in the first term. We further substitute equation 1.4′ into the first term and 1.5′ in the second. Then, distributing w_m over the brackets in the second term and substituting in equation 1.6, we write

$$- A\, w_m(1-s)\eta_a + B \frac{L_a}{L_m+U} \frac{(w_m - w_a)}{w_a} < 0. \qquad \text{(D.11)}$$

As above, we move the first term to the right hand side and divide to leave the subsidy standing alone.

$$\frac{B}{A} \frac{L_a}{L_m+U} \frac{(w_m - w_a)}{w_m w_a \eta_a} < 1-s. \qquad \text{(D.12)}$$

Subtracting unity from each side and multiplying by negative one leaves condition 4.iv,

$$1 - \frac{B}{A} \frac{L_a}{L_m+U} \frac{(w_m - w_a)}{w_m w_a \eta_a} > s. \qquad \text{(D.13)}$$

Appendix E

Proof of Proposition 5

To derive conditions sufficient for manufacturing wage subsidies to increase welfare, we start with equation 1.13, but use $s = s_m > 0$, $s_a = 0$. The implicit function theorem is applied to equations 1.3, 1.4′, 1.5, and 1.6 to generate the expressions for dL_m/ds_m, dL_a/ds_m, and dU/ds_m. Thus 1.13 can be written

$$\frac{dW}{ds_m} = \frac{1}{J}\Big[A\Big(\overset{1}{-P_m F_L \frac{w_m^2 L_m}{(L_m+U)^2}} + \overset{2}{P_m F_L w_m P_a G_{LL}} + \overset{3}{P_a G_{LL} \frac{w_m}{L_m+U}}\Big) + B\Big(\underset{4}{P_a G_{LL} w_m} + \underset{5}{\frac{w_m^2}{L_m+U}\Big(1-\frac{L_m}{L_m+U}\Big)}\Big)\Big]. \qquad \text{(E.1)}$$

If the sum of terms 1 through 5 is negative, then divided by J, which is negative, dW/ds_m will be positive. Terms 1 through 4 are negative, but the fifth term is positive. The alternate sufficient conditions are generated by combining term 5 with each of the other terms separately and deriving the parameter configuration necessary to assume that the sum is negative.

Condition 5.i

Condition 5.i combines terms 4 and 5 and is derived in appendix C.

Condition 5.ii

Condition 5.ii combines terms 3 and 5 of E.1. Sufficiency is obtained if their sum is negative, i.e.

$$AP_a G_{LL} \frac{w_m}{L_m+U} + B \frac{w_m^2}{L_m+U} (1- \frac{L_m}{L_m+U}) < 0. \qquad (E.2)$$

To transform this into something easier to interpret we first divide out the like terms, distribute w_m over the parenthesis in the second term, and replace with equation 1.6. Dividing by 1.5 and multiplying by L_a will give an expression for η_a in the first term. We now have

$$- A\eta_a + BL_a(\frac{w_m - w_a}{w_a}) < 0. \qquad (E.3)$$

Moving the first term to the right and dividing by A yields condition 5.ii,

$$\frac{B}{A} L_a (\frac{w_m - w_a}{w_a}) < \eta_a.$$

Condition 5.iii

Terms 1 and 5 of E.1 are combined in 5.iii. We assume that their sum is negative, so

$$- A\, P_m\, F_L \frac{w_m^2 L_m}{(L_m+U)^2} + B \frac{w_m^2}{L_m+U} \quad (1 - \frac{L_m}{L_m+U}) < 0. \qquad (E.4)$$

First distribute w_m over the parentheses in the second term. Divide out similar terms and substitute equation 1.6 into both terms. Then substitute in 1.4′ in the first term. E.4 is now expressed as

$$- Aw_m(1-s_m)\; w_a' + B\; (w_m-w_a) \;\; < 0. \qquad (E.5)$$

The first term moves to the right hand side. Dividing to isolate the subsidy term yields

$$\frac{B}{A} \frac{(w_m-w_a)}{w_m w_a} < 1-s_m. \qquad (E.6)$$

Then subtracting unity and multiplying by negative one yields 5.iii,

$$1 - \frac{B}{A} \frac{(w_m-w_a)}{w_m w_a} > s_m.$$

Condition 5.iv

This condition is the last combination, of terms 2 and 5 of E.1. We begin with

$$A\; P_m F_L w_m P_a G_{LL} + B \frac{w_m^2}{L_m+U} \; (1- \frac{L_m}{L_m+U}) < 0. \qquad (E.7)$$

As before w_m is distributed over the parentheses and equation 1.6 is used in the second term. The equation is divided by the remaining like term. Dividing by equation 1.5 and multiplying by L_a gives the definition of η_a in the first term. Lastly equation 1.4′ is substituted into the first term. E.7 is transformed:

$$- A\, w_m(1-s_m)\ \eta_a + B \frac{L_a}{L_m+U} \frac{(w_m-w_a)}{w_a} < 0. \tag{E.8}$$

Now the first term is moved to the right and the equation is divided to isolate the subsidy term.

$$\frac{B}{A} \frac{L_a}{(L_m+U)} \frac{(w_m-w_a)}{w_m w_a \eta_a} < 1-s. \tag{E.9}$$

Finally unity is subtracted from both sides and the equation multiplied by negative one, to yield condition 5.iv,

$$1 - \frac{B}{A} \frac{L_a}{(1-L_a)} \frac{(w_m-w_a)}{w_m w_a \eta_a} > s_m.$$

Appendix F

Proof of Proposition 7

Differentiating 1.15 gives

$$dW/ds = A_1 dW_1/ds + A_2 dW_2/ds. \tag{F.1}$$

where $A_1 = v_1 W_1^{v_1 - 1} W_2^{v_2}$ and $A_2 = v_2 W_1^{v_1} W_2^{v_2 - 1}$. (F.2)

These marginal welfare terms can be more compactly expressed as

$$A_1 = \frac{v_1}{W_1} W \quad \text{and} \quad A_2 = \frac{v_2}{W_2} W. \tag{F.3}$$

Let now see how each group's welfare is affected by the subsidy

Capital

Capital's welfare is defined in equation 1.16.

Differentiating with respect to an across-the-board subsidy gives

$$dW_1/ds = (P_m F_L - w_m) dL_m/ds + (P_a G_L - w_a) dL_a/ds - L_a dw_a/ds. \tag{F.4}$$

Substituting in for the derivatives and simplifying shows

$$\frac{dW_1}{ds} = \frac{1}{J}[-sP_a G_{LL} w_m^2 - sP_m F_{LL} w_a^2 - \frac{L_a}{L_m + U} P_m F_{LL} w_a^2 - \frac{L_a}{L_m + U} P_a G_{LL} w_m^2] < 0. \tag{F.5}$$

Each term inside the brackets is positive. Dividing by the negative Jacobian gives $dW_1/ds < 0$. Capital is made worse off by the introduction of the subsidy.

Labor

Labor's welfare is defined as its expected wage, w_a. Thus the change in its welfare caused by an across-the-board subsidy is

$$\frac{dW_2}{ds} = \frac{dw_a}{ds} = \frac{1}{J}\left[\frac{1}{L_m+U} P_m F_{LL} w_a^2 + \frac{1}{L_m+U} P_a G_{LL} w_m^2\right] > 0. \qquad (F.6)$$

Labor's welfare increases with the subsidy.

Society

To determine the overall effect of a subsidy, expressions F.5 and F.6 are substituted into F.1.

$$\frac{dW}{ds} = \frac{1}{J}\left[A_1\left(-sP_a G_{LL} w_m^2 - sP_m F_{LL} w_a^2 - \frac{L_a}{L_m+U} P_m F_{LL} w_a^2 - \frac{L_a}{L_m+U} P_a G_{LL} w_m^2\right) + A_2\left(\frac{1}{L_m+U} P_m F_{LL} w_a^2 + \frac{1}{L_m+U} P_a G_{LL} w_m^2\right)\right]. \qquad (F.7)$$

Factoring,

$$\frac{dW}{ds} = \frac{1}{J}\left[(P_a G_{LL} w_m^2 + P_m F_{LL} w_a^2)\left(-A_1 S - A_1 \frac{L_a}{L_m+U} + A_2 \frac{1}{L_m+U}\right)\right]. \qquad (F.8)$$

The sign of dW/ds depends on the sign of the term in the second set of parentheses. Let us investigate.

$$-A_1 s - A_1 \frac{L_a}{L_m+U} + A_2 \frac{1}{L_m+U} \gtrless 0. \qquad (F.9)$$

Substituting in from F.3,

$$-\frac{v_1}{W_1} Ws - \frac{v_1}{W_1} W \frac{L_a}{L_m+U} + \frac{v_2}{W_2} W \frac{1}{L_m+U} \gtrless 0. \qquad (F.10)$$

Dividing through by W, and substituting in equation 1.16 gives

$$- s + \frac{1-L_a}{L_m+U} \gtrless 0. \qquad (F.11)$$

Passing s to the right and using 1.3 to simplify gives

$$1 \gtrless s. \qquad (F.12)$$

In F.8 the second set of parenthesis encloses an expression which is positive (negative) when the subsidy level is partial (complete). The rest of the expression is positive so

$$\frac{dW}{ds} \gtrless 0 \text{ as } 1 \gtrless s.$$

Appendix G

Proof of Proposition 8

In the case of a manufacturing wage subsidy $s=s_m$ and $s_a=0$. Let us examine first how each group fares with the subsidy.

Capital

Capital's welfare is defined by 1.16 to be its share in GNP. Differentiating with respect to the subsidy,

$$dW_1/ds_m = (P_mF_L-w_m)\ dLm/ds_m + (P_aG_L-w_a)dL_a/ds_m-L_a/ds. \qquad (G.1)$$

Equation 1.5 shows that the dL_a/ds_m term is multiplied by zero and hence drops out. Substituting 1.4′ into the first term and using the expressions for the derivatives,

$$\frac{dW_1}{ds_m} = \frac{1}{J}\left[s_m \frac{w_m^2 w_a}{L_m+U} - s_m P_a G_{LL} w_m^2 - \frac{L_a}{L_m+U} P_a G_{LL} w_m^2\right] < 0. \qquad (G.2)$$

Labor

Labor's welfare is its expected wage, w_a.

$$\frac{dW_2}{ds_m} = \frac{dw_a}{ds_m} = \frac{1}{J}\frac{1}{L_m+U} P_a G_{LL} w_m^2 \quad > 0. \qquad (G.3)$$

Society

Social welfare is the change in the individual groups' welfare weighted by their marginal welfare weights (F.1). Accordingly,

$$\frac{dW}{ds_m} = \frac{1}{J}\left[A_1\left(s_m \frac{w_m^2 w_a}{L_m+U} - s_m P_a G_{LL} w_m^2 - \frac{L_a}{L_m+U} P_a G_{LL} w_m^2\right) + A_2 \frac{1}{L_m+U} P_a G_{LL} w_m^2\right]. \tag{G.4}$$

Rearranging,

$$\frac{dW}{ds_m} = \frac{w_m^2}{J}\left[A_1 s_m \frac{w_a}{L_m+U} - A_1 s_m P_a G_{LL} + (A_2 - A_1 L_a) \frac{1}{L_m+U} P_a G_{LL}\right]. \tag{G.5}$$

Substituting in for A_1 and A_2 from F.3 and 1.16,

$$\frac{dW}{ds_m} = \frac{w_m^2 W}{J}\left[s_m \frac{w_a}{L_m+U} - s_m P_a G_{LL} + \frac{1-L_a}{L_m+U} P_a G_{LL}\right]. \tag{G.6}$$

The sign of dW/ds_m thus depends on the sign of the expression in brackets. Rewriting it gives

$$s_m \frac{w_a}{L_m+U} \gtrless - P_a G_{LL}(1-s_m). \tag{G.7}$$

Dividing by equation 1.5 and multiplying by L_a shows an expression for η_a on the right.

$$s_m \frac{L_a}{L_m+U} \gtrless \eta_a - \eta_a s_m. \tag{G.8}$$

Isolating the subsidy term gives

$$s_m \gtrless \frac{1}{\frac{L_a}{L_m+U}\frac{1}{\eta_a} + 1}. \qquad (G.9)$$

In G.6 the bracketed term is divided by a negative, so

$$\frac{dW}{ds_m} \gtrless 0 \text{ as } s \lessgtr \frac{1}{\frac{L_a}{L_m+U}\frac{1}{\eta_a} + 1}.$$

Appendix H

Proof of Proposition 9

In the case of an agricultural wage subsidy $s=s_a$ and $s_m=0$. Let us examine how each group fares with the subsidy.

Capital

Capital's welfare is defined by 1.16 to be its share in GNP. Differentiating with respect to the subsidy,

$$dW_1/ds_a = (P_mF_L-w_m)dL_m/ds_a + (P_aG_L-w_a)dL_a/ds_a - L_a dw_a/ds_a. \quad (H.1)$$

Recognizing equation 1.4 in the first term, it cancels. The remaining terms, when their values are substituted in give

$$dW_1/ds_a = \frac{1}{J}\left[-s\, P_mF_{LL}w_a^2 - \frac{L_a}{L_m+U} P_mF_{LL}w_a^2\right] < 0. \quad (H.2)$$

Labor

Labor's welfare function is defined by its wage.

$$\frac{dW_2}{ds_a} = \frac{1}{J}\frac{1}{L_m+U} P_mF_{LL}w_a^2 > 0. \quad (H.3).$$

Society

Combining expressions H.2 and H.3 gives

$$\frac{dW}{ds_a} = \frac{1}{J} [P_m F_{LL} w_a^2 (-A_1 s_a + (A_2 - A_1 L_a) \frac{1}{L_m + U})]. \qquad (H.4)$$

The sign of H.4 depends upon the sign of the term is parentheses. Thus we examine

$$-A_1 s_a + (A_2 - A_1 L_a) \frac{1}{L_m + U} \gtrless 0. \qquad (H.5)$$

Substituting in for the A terms (see Appendix F, eq. F.3)

$$1 \gtrless s. \qquad (H.6)$$

The sign of H.4 is the same as the sign a H.6, i.e.

$$\frac{dW}{ds_a} \gtrless 0 \text{ as } 1 \gtrless s.$$

Appendix I

Proof of Proposition 10

The most enlightening way to determine the effect of a subsidy on welfare as defined in 1.17 is to examine the effects on each of the factions.

Capital

Capital's welfare is defined by its share of GNP. Differentiating the definition of GNP with respect to an across-the-board subsidy,

$$dW_1/ds = (P_m F_L - w_m)\, dL_m/ds + (P_a G_L - w_a)\, dL_a/ds - L_a dw/ds. \qquad (I.1)$$

Which is fully defined as

$$\frac{dW_1}{ds} = \frac{1}{J}\left[-s\, P_a G_{LL} w_m^2 - s\, P_m F_{LL} w_a^2 - \frac{L_a}{L_m+U} P_a G_{LL} w_m^2 - \frac{L_a}{L_m+U} P_m F_{LL} w_a^2\right] < 0. \qquad (I.2)$$

Agricultural Labor

The welfare of agricultural labor is defined by its expected wage.

$$\frac{dW_2}{d_s} = \frac{dw_a}{d_s} = \frac{1}{J}\left[P_m F_{LL} w_a^2 \frac{1}{L_m+U} + P_a G_{LL} w_m^2 \frac{1}{L_m+U}\right] > 0. \qquad (I.3)$$

Urban Labor

Urban labor's welfare is defined as the employment rate

$$\frac{dW_3}{d_s} = - \frac{dU}{d_s} = \frac{1}{J} [P_a G_{LL} w_m + P_m F_{LL} w_a + \frac{w_m^2}{L_m+U} (1 - \frac{L_m}{L_m+U})(1-s_a)] \gtrless 0. \qquad (I.4)$$

Society

The change in the society's welfare is the weighted sum of the changes of the factions' welfare, where the weights are their marginal welfare values.

$$\frac{dW}{ds} = \frac{1}{J} [P_a G_{LL} w_m^2 (-A_1 s + (A_2 - A_1 L_a) \frac{1}{L_m+U} + A_3 \frac{1}{w_m})$$

$$+ P_m F_{LL} w_a^2 (-A_1 s + (A_2 - A_1 L_a) \frac{1}{L_m+U} + A_3 \frac{1}{w_a}) \qquad (I.5)$$

$$+ \frac{w_m^2}{L_m+U} (1 - \frac{L_m}{L_m+U})(1-s_a)].$$

The last term in brackets is positive. If the other terms are positive, then divided by the negative Jacobian, dW/ds <0. Let us examine then the sign of the term in parentheses:

$$-A_1 s + (A_2 - A_1 L_a) \frac{1}{L_m+U} + A_3 \frac{1}{w_m} \gtrless 0; \quad -A_1 s + (A_2 - A_1 L_a) \frac{1}{L_m+U} + A_3 \frac{1}{w_a} \gtrless 0. \qquad (I.6)$$

Substituting in the definitions for the A terms,

$$-s + (L_a - L_a)\frac{1}{L_m+U} + \frac{(1-L_a)}{1-U}\frac{w_a}{w_m} \gtrless 0; \quad -s + (L_a - L_a)\frac{1}{L_m+U} + \frac{(1-L_a)}{1-U}\frac{w_a}{w_a} < 0.$$

Passing the subsidies to the right and substituting in equations 1.3 and 1.6 gives

$$\frac{L_m}{L_m+U} \gtrless s; \qquad \frac{L_m+U}{L_m+U} \gtrless s. \tag{I.8}$$

The condition on the right is stricter than that on the left. When it holds with a less than sign, the three groups of terms in I.5 are all positive, which divided by the negative Jacobian, gives $dW/ds < 0$.

Appendix J

Proof of Proposition 11

If the reader has persevered this far he knows the procedure.

Capital

Capital's welfare is defined by its share of GNP. Differentiating the definition of GNP with respect to a manufacturing subsidy,

$$dW_1/ds_m = (P_mF_L-w_m)dL_m/ds_m + (P_aG_L-w_a)dL_a/ds_m - L_a\ dw_a/ds_m. \quad (J.1)$$

There is no subsidy in agricultural so the second term goes to zero.

$$\frac{dW_1}{ds_m} = \frac{1}{J}\left[s_m \frac{w_m^2 w_a}{L_m+U} - s_m P_a G_{LL} w_m^2 - \frac{L_a}{L_m+U} P_m F_{LL} w_a^2 - \frac{L_a}{L_m+U} P_a G_{LL} w_m^2\right] < 0. \quad (J.2)$$

Agricultural Labor

The agricultural wage defines agricultural welfare

$$dW_2/ds_m = dw_a/ds_m = \frac{1}{J}\frac{1}{L_m+U}\ P_a G_{LL} w_m^2 > 0. \quad (J.3)$$

Urban Labor

The employment rate defines urban labors' welfare.

$$\frac{dw_3}{ds_m} = - \frac{dU}{ds_m} = \frac{1}{J} [P_a G_{LL} w_m + \frac{w_m^2}{L_m+U} (1- \frac{L_m}{L_m+U})] \gtrless 0. \tag{J.4}$$

Society

Weighting the three factions' welfare according to their marginal welfare weights gives

$$\frac{dW}{ds_m} = \frac{1}{J} [P_a G_{LL} w_m^2 (-A_1 s_m + (A_2 - A_1 L_a) \frac{1}{L_m+U} + A_3 \frac{1}{w_m}) \tag{J.5}$$

$$+ A_1 s_m \frac{w_m^2 w_a}{L_m+U} + A_3 \frac{w_m^2}{L_m+U} (1- \frac{L_m}{L_m+U}) - A_1 P_m F_{LL} w_a^2 \frac{L_a}{L_m+U}].$$

The last three terms are all positive. If the chain of terms in the outer parentheses is negative, the whole numerator is positive, and the denominator negative, hence dW/ds_m is negative. Let us, then, examine the terms in parentheses.

$$-A_1 s_m + (A_2 - A_1 L_a) \frac{1}{L_m+U} + A_3 \frac{1}{w_m} \gtrless 0. \tag{J.6}$$

Substituting in the definitions for the A terms, and passing the subsidy to the right,

$$(L'_a - L_a) \frac{1}{L_m+U} + (\frac{1-L_a}{1-U}) \frac{w_a}{w_m} \gtrless s_m. \tag{J.7}$$

Substituting in equation 1.3 and 1.6 gives

$$\frac{L_m}{L_m+L_a} \gtrless s_m. \tag{J.8}$$

If s_m is greater than the manufacturing portion of the employed labor force then J.6 is negative and $dW/ds_m < 0$.

Appendix K

Proof of Proposition 12

The effect of an agricultural subsidy can be broken into the effects on each of the three factions.

Capital

Differentiating the share of capital in GNP, capital's welfare function, with respect to an agricultural subsidy gives

$$dW_1/ds_a = (P_m F_L - w_m)\, dL_m/ds_a + (P_a G_L - w_a)\, dL_a/ds_a - L_a\, dw_a/ds_a. \tag{K.1}$$

The first term is zero because from 1.4, the marginal value product is its wage. Substituting in expressions for dL_a/ds_a and dw_a/ds_a from application of the implicit function theorem to equations 1.3, 1.4, 1.5′, and 1.6 yields

$$\frac{dW_1}{ds_a} = \frac{1}{J}\left[-P_m F_{LL} w_a^2 - \frac{L_a}{L_m+U} P_m F_{LL} w_a^2\right] < 0. \tag{K.2}$$

Agricultural Labor

Agricultural labor's welfare is defined by its wage.

$$\frac{dW_2}{ds_a} = \frac{dw_a}{ds_a} = \frac{1}{J} P_m F_{LL} w_a^2 \frac{1}{L_m+U} > 0. \tag{K.3}$$

Urban Labor

Urban labor's welfare is defined by the employment rate,

$$\frac{dW_3}{ds_a} = - \frac{dU}{ds_a} = \frac{1}{J} P_m F_{LL} w_a > 0. \tag{K.4}$$

Society

Weighting the factions by their marginal welfare terms,

$$\frac{dW}{ds_a} = \frac{1}{J} P_m F_{LL} w_a^2 \left(-A_1 s_a + (A_2 - A_1 L_a) \frac{1}{L_m+U} + A_3 \frac{1}{w_a}\right). \tag{K.5}$$

Looking at the term in parentheses

$$-A_1 s_a + (A_2 - A_1 L_a) \frac{1}{L_m+U} + A_3 \frac{1}{w_a} \gtrless 0. \tag{K.6}$$

Substituting in for the A terms from their definitions (see 1.17 and F.3) and passing the subsidy to the right

$$\frac{1-L_a}{1-U} \frac{w_a}{w_a} \gtrless s_a. \tag{K.7}$$

Simplifying with the aid of 1.3 reduces K.7 to

$$\frac{L_m+U}{L_m+L_a} \gtrless s . \tag{K.8}$$

dW/ds_a bears the same relation to zero as the ratio of urban labor to employed labor bears the agricultural subsidy level.