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by

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Abstract. This paper explores the role of wage structure as implicit insurance on human capital. It is shown that smaller wage differentials in the developed world can be welfare-enhancing by providing implicit insurance while larger wage differentials in underdeveloped countries make investments in human capital riskier. In other words, the students in a developed country are insured against poor educational outcomes through the existence of well-paid alternative employments which are not present in the economy of a less developed country. These results arise in a general equilibrium model when there are no insurance markets for human capital.

Keywords: Wage differentials, human capital, implicit insurance, economic development.

JEL classification code: J31, O15.

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1. Introduction

A salient feature distinguishing underdeveloped countries from developed countries is differences in wage structure. For example, Psacharopoulos [1973] found that in less developed countries university graduates on the average earned 6.4 times as much as those who only completed primary school while the corresponding figure for the developed countries was 2.4. Another reflection of the disparate wage structures is the differences in rates of return on education. Psacharopoulos [1985] summarizes extensive empirical work finding significantly higher rates of return on education in less developed countries when compared to the developed world. A natural question becomes then, how can market forces give rise to so different outcomes? Is it possible that the answer lies in market imperfections that plague the accumulation of human capital? Both Schultz [1961] and Friedman [1962] discuss how the market forces fail to provide neither credit nor insurance to investors in human capital. The observed differences in wage structures across countries can also be rationalized with a missing credit market for human capital as demonstrated by Ljungqvist [1992]. Since poor families in less developed countries have a high marginal utility of consumption, it is shown how they choose optimally to remain uneducated despite high rates of return on education. The present paper explores the other market imperfection mentioned by Schultz and Friedman. A missing insurance market for human capital and its consequences are examined in a general equilibrium model by introducing uncertainty about educational outcomes. As pointed out by Schultz [1961], one important reason for underinvestment in human capital is that "individuals face serious uncertainty in assessing their innate talents when it comes to investing in themselves." However, my analysis brings out the fact that the amount of uncertainty or risk depends on the economy's wage structure. Specifically, smaller wage differentials in the developed world are shown to provide implicit insurance on human capital while larger wage differentials in the less developed countries make investments in human capital riskier.

Education serves both the purpose of creating human capital and resolving uncertainties about individuals' abilities. It is indisputable that many agents investing in human capital face serious uncertainty about the future outcome of their endeavor. One obvious observation of this is that there are always agents dropping out from ongoing educational programs. This phenomena unfortunately cannot be avoided despite numerous testings the students have gone through. Through education, students learn about their abilities and some may graduate with flying marks while others marginally fulfill the requirements. The question becomes then how various economies allocate these individuals with different abilities between tasks. In developed countries, it is common to find individuals with lengthy educations even in relatively 'low' positions. For example, the incompetent law graduate may choose an administrative position which does not really require a law education while leaving the practice of law to more brilliant classmates. In contrast, you can find even the most incompetent law graduates practicing law in the underdeveloped world. The explanation seems to be simply that there is a shortage of law graduates in these countries. But my analysis brings out still another aspect of this observation. The incompetent law graduate in a developed country who chose the administrative position instead of practicing law, did so because of a more favorable earning in that alternative employment. My model demonstrates how the existence of these well-paid alternative employments in a developed country offers implicit insurance on human capital or, equivalently, how smaller wage differentials in such an economy reduce the riskiness associated with an education.

On the one hand, some wage differentials are needed to motivate educational investments but, on the other hand, larger wage dispersion increases also the riskiness of such investments. My model formally demonstrates that these two aspects of wage dispersion can generate multiple equilibria. In particular, an equilibrium in an underdeveloped country marked by a vicious cycle with large wage differentials and a high expected rate of return on education, and an equilibrium in a developed country with smaller wage differentials and a lower expected rate of return on education. The intuition is as follows. Consider first a developed country with a large percentage of educated individuals. The abundance of educated labor drives down the wage premium for occupations requiring an education. Due to smaller wage differentials, an investment in education is then associated with less risk. It follows that individuals are willing to obtain an education at an expected rate of return not too much in excess of the risk-free interest rate. A low expected rate of return on education, in turn, implies that wage differentials between occupations are relatively small, i.e., a less dispersed wage distribution. This circular argument captures the notion of a market equilibrium in a developed country. Contrarily, a small percentage of educated individuals in an underdeveloped country gives rise to large wage differentials. In that case, a failed education constitutes a big drop in future income. As a result, individuals will only obtain an education if there is a considerable risk premium associated with such investments. This high expected rate of return on education implies that wage differentials between occupations are relatively large which completes the circular argument for an equilibrium in an underdeveloped country.[1] To understand the welfare implications of different equilibria, let us consider an ideal (first-best) world where

[1] The existence of multiple wage structures is due to pecuniary externalities as discussed by Scitovsky [1954] which operate through the market mechanism as opposed to technological externalities. For an interesting analysis of pecuniary externalities and their welfare implications, see Greenwald and Stiglitz [1986]. The possibility of multiple equilibria is examined by Cooper and John [1988].

there were an insurance market for educational outcomes. In such an economy, all individuals would be fully insured against different educational outcomes, and since there is no aggregate risk, the expected rate of return on education would be equal to the risk-free interest rate. This outcome is unattainable in a market economy *without* the insurance market. Individuals are then forced to bear the risk of their own education. But my analysis shows that the lower expected rates of return on education in developed countries are an indication of these economies being closer to attain the first-best outcome when compared to underdeveloped countries.

The structure of my model is set out in the following section, and Section 3 describes a competitive equilibrium. Section 4 proves the existence of stationary equilibria and shows how multiple equilibria can be ex ante Pareto-ranked with welfare increasing in the share of educated workers in the labor force. A numerical example of multiple equilibria is provided in Section 5, and Section 6 concludes with a brief discussion. Proofs of propositions are deferred to the appendix.

2. The Model

Consider an economy populated by overlapping generations. Each generation consists of a continuum of agents distributed over the interval $[0,1]$. Agents live for three periods and they are all identical with respect to preferences and innate abilities. Agents are maximizing the expected utility of consuming two different goods; one capital-intensive good, denoted c, and one labor-intensive good, denoted s. For simplicity, agents are assumed to consume nothing in the first period of life, and the preferences of agent i born at time t are given by

$$E_t U\left(c^i_{t2} + c^i_{t3},\, s^i_{t2} + s^i_{t3}\right), \tag{1}$$

where E_t is the mathematical expectation operator conditioned on information at time t and x^i_{tj} is the agent's consumption of good $x \in \{c, s\}$ in his j:th period of life. The simple intertemporal structure of the preferences is chosen in order to focus on agents' risk aversion.[2] The utility function is strictly concave, strictly increasing, homothetic, and it satisfies two Inada conditions:

$$\lim_{c \to 0} U_1(c,s) = \lim_{s \to 0} U_2(c,s) = \infty, \tag{2}$$

where $U_j(\cdot)$ is the partial derivative of $U(\cdot)$ with respect to its j:th argument.

[2] The preference specification in (1) is associated with a high elasticity of savings at a gross risk-free interest rate of one. An alternative interpretation of my model would therefore be a partial equilibrium analysis of a small country facing a world market interest rate of one. The assumption of no discounting is only made for ease of exposition.

All agents have the same ability to acquire an education in the first period of life. They then work in the second period and retire in the third period of life. The production technologies of the two nonstorable goods exhibit constant returns to scale with labor as the only input. Both educated and uneducated agents can produce the labor-intensive good with the same productivity, let us say one unit of good s per worker. However, only educated agents can be employed in the production of the capital-intensive good. An educated agent can produce one unit of good c with probability $\pi \in (0,1)$, and with probability $1-\pi$ the productivity is only $\gamma \in (0,1)$ units of good c. An agent learns about his productivity after completing an education. Finally, the education technology is also constant returns to scale. The education of a young agent requires an input μ of the capital-intensive good (or, equivalently, educated workers must be employed teaching the young). The parameters of the model are assumed to satisfy[3]

$$\mu < \pi \in (0,1) \qquad \text{and} \qquad \mu < \gamma \in (0,1). \tag{3}$$

3. Description of an Equilibrium

Since all production technologies exhibit constant returns to scale, agents can be thought of as being self-employed. There are then three markets in each period; markets for capital-intensive goods and labor-intensive goods, and a credit market. A competitive equilibrium will be a sequence of prices, consumption and labor allocations such that:

a) given prices, agents maximize expected utility subject to their budget constraints,

b) all markets clear.

To formulate an agent's optimization problem, let p_t denote the price of the labor-intensive good in terms of the capital-intensive good at time t while R_t is the gross risk-free interest rate in terms of the capital-intensive good between periods t and $t+1$. An agent i born at time t must then choose whether or not to obtain an education, and contingency plans for next period's employment and future consumption in order to

$$\text{maximize} \qquad E_t\, U\left(c^i_{t2} + c^i_{t3},\, s^i_{t2} + s^i_{t3}\right) \tag{4}$$

$$\text{subject to} \qquad c^i_{t2} + \frac{c^i_{t3}}{R_{t+1}} + p_{t+1}\, s^i_{t2} + \frac{p_{t+2}\, s^i_{t3}}{R_{t+1}} \le I_{t+1}\,,$$

$$c^i_{t2},\, c^i_{t3},\, s^i_{t2},\, s^i_{t3} \ge 0\,,$$

[3] The parameter restrictions in (3) are imposed at the outset of the paper to streamline the analysis. $\mu < \pi$ is a necessary condition for the existence of multiple stationary equilibria as can be seen in Proposition 1. The additional assumption of $\mu < \gamma$ ensures that educated agents can always repay non-contingent educational loans in a stationary equilibrium.

$$
\text{where} \quad I_{t+1} = \begin{cases} p_{t+1} & \text{uneducated agent producing the labor-intensive good,} \\ p_{t+1} - R_t\,\mu & \text{educated agent producing the labor-intensive good,} \\ 1 - R_t\,\mu & \text{educated agent with high productivity producing the capital-intensive good,} \\ \gamma - R_t\,\mu & \text{educated agent with low productivity producing the capital-intensive good.} \end{cases}
$$

I_{t+1} is the agent's disposable income net of educational expenses which depends on his choice of employment and his productivity.

Let N_t be the share of educated workers in the labor force at time t, who received their education at time $t-1$. The fraction of educated workers with low productivity employed in the production of the capital-intensive good at time t is denoted ϕ_t. The two market-clearing conditions in the goods markets at time t become

$$
\pi\,N_t + \phi_t\,(1-\pi)\,\gamma\,N_t = \int_0^1 c^i_{t-1,2}\,di + \int_0^1 c^i_{t-2,3}\,di + \mu\,N_{t+1}\,, \tag{5.a}
$$

$$
1 - N_t + (1-\phi_t)(1-\pi)\,N_t = \int_0^1 s^i_{t-1,2}\,di + \int_0^1 s^i_{t-2,3}\,di\,. \tag{5.b}
$$

The supply of the capital-intensive good on the left-hand side of (5.a) reflects that a fraction π of all educated workers N_t are experiencing high productivity. Equilibrium prices must be such that these agents strictly prefer to produce the capital-intensive good since it otherwise would not be worthwhile to obtain an education. The remaining educated workers with low productivity, $(1-\pi)N_t$, are split between the production of the capital-intensive good and the labor-intensive good according to the proportions ϕ_t and $1-\phi_t$, respectively. All uneducated workers, $1-N_t$, are only capable of producing the labor-intensive good. The demand sides of (5) sum up the consumption of all agents in their second and third periods of life. An additional demand for the capital-intensive good comes from young agents investing in an education, μN_{t+1}. This amount is financed with loans from agents in their second period of life at the market interest rate R_t.

4. Stationary Equilibria

Let us now study stationary equilibria in which consumption and labor allocations are the same across generations. In such an equilibrium, the interest rate R is equal to one since $R<1$ would fail to generate any savings and $R>1$ would result in savings in excess of a time-invariant demand for student loans. Given $R=1$, an agent's indirect utility function can then be written as $V(I,p)$

where I stands for the agent's disposable income and p is the relative price of the labor-intensive good. The indirect utility function satisfies

$$V_1(I,p) > 0\,, \qquad \text{and} \qquad V_2(I,p) < 0\,. \tag{6}$$

Nonsatiation explains why the indirect utility function is increasing in disposable income, and the derivative with respect to p is strictly negative due to the second Inada condition in (2). Moreover, all agents will spend the same fraction of their incomes on any particular good because of the assumption of homothetic and identical preferences. The relative price of the labor-intensive good in terms of the capital-intensive good is therefore positively related to the ratio of the aggregate consumption of the capital-intensive good to that one of the labor-intensive good. Since the consumption of the capital-intensive good increases in N and ϕ while the opposite is true for the labor-intensive good,[4] the market-clearing price can be written as a function $p(N,\phi)$ where

$$p_1(N,\phi) > 0\,, \qquad \text{and} \qquad p_2(N,\phi) > 0\,. \tag{7}$$

In an equilibrium with both educated and uneducated workers, it must be true that agents are indifferent between acquiring an education or remaining uneducated since they are all identical in the first period of life, i.e.,

$$\pi V\Big(1-\mu,\, p(N,\phi)\Big) + (1-\pi)V\Big(\max\{\gamma,\, p(N,\phi)\} - \mu,\, p(N,\phi)\Big)$$
$$= V\Big(p(N,\phi),\, p(N,\phi)\Big)\,. \tag{8}$$

The left-hand side is the expected utility of an education where the first and second term captures the outcomes of high and low productivity, respectively. An educated agent chooses to work in the sector where he can earn the highest income while an uneducated worker can only produce the labor-intensive good as shown in the expression for his utility on the right-hand side of (8). The following proposition says that equilibrium condition (8) may have multiple solutions, i.e., the model is consistent with multiple stationary equilibria.[5]

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[4] A higher N will both increase the production of the capital-intensive good and its usage in education for maintaining that higher fraction of educated agents. However, the assumption in (3) that $\mu < \pi$ guarantees an increase in the consumption of the capital-intensive good even if all educated workers with low productivity are employed in the labor-intensive sector.

[5] We restrict our attention to the nondegenerate set $N \in (0,1]$. A trivial equilibrium can otherwise be found at $N = 0$ since the nonexistence of educated agents would make it physically impossible for the economy to attain any other production allocation.

Proposition 1. There will always exist a stationary equilibrium with $N \neq 0$,

i) if $\pi(1-\gamma) \leq \mu$, the stationary equilibrium is unique,

ii) if $\pi(1-\gamma) > \mu$, there may be multiple stationary equilibria.

Proof: See appendix.

The possibility of multiple equilibria arises from the fact that the utility of an uneducated agent is increasing in the price of the labor-intensive good while the expected utility of an educated agent may be both decreasing and increasing in that price depending on its level. Specifically, at very low values of N, the supply of the capital-intensive good is small which drives down the relative price of the labor-intensive good. All educated workers, both those with high and low productivity, prefer then to produce the capital-intensive good and the expected utility of an educated agent is therefore decreasing in the relative price of the labor-intensive good. Contrarily, at very high values of N, the labor-intensive good is more scarce and its high price induces educated workers with low productivity to seek employment in the labor-intensive sector. The relationship between the expected utility of an education and the price of the labor-intensive good is then ambiguous. This can be seen by taking the derivative of the left-hand side of (8) with respect to N, given that the relative price of the labor-intensive good exceeds γ (implying $\phi = 0$),

$$p_1(N,0)\left[\pi V_2(1-\mu,p) + (1-\pi)\frac{dV(p-\mu,p)}{dp}\right]. \tag{9}$$

The first term in square brackets is negative while the second term is positive. On the one hand, a higher price of the labor-intensive good reduces the utility of an educated worker with high productivity producing the capital-intensive good. On the other hand, the higher relative wage in the labor-intensive sector serves as an insurance on human capital by increasing the utility of an educated worker with low productivity. The riskiness of an education is reduced through the existence of well-paid jobs in the labor-intensive sector.

The necessary condition for multiple equilibria in Proposition 1, i.e., $\pi(1-\gamma) > \mu$, can be interpreted intuitively. The restriction is a lower bound on the difference between high and low productivity in the capital-intensive sector, $1-\gamma$. Educated agents with low productivity must be sufficiently less productive so that it may preferable to move them into the labor-intensive sector. The cost of an education enters into the condition since a lower μ means that it is less costly to educate new agents to replace the workers with low productivity who are moving into the labor-intensive sector. This cost argument is also strengthened the more likely it is that the newly educated agents turn out to be of high productivity, i.e., the higher the probability π is. After this interpretation of the necessary condition for multiple equilibria, it may not be surprising that a social planner facing this parameter region would prefer to allocate the educated agents with low productivity to the labor-intensive sector as stated in the following proposition.

Proposition 2. The labor allocation in a first-best stationary allocation is such that

i) if $\pi(1-\gamma) \leq \mu$, all educated workers, both those with high and low productivity, are producing the capital-intensive,

ii) if $\pi(1-\gamma) > \mu$, only educated workers with high productivity are producing the capital-intensive good (unless that good is so highly demanded that all agents are educated).

Proof: See appendix.

Finally, in the case of multiple equilibria, Proposition 3 below makes two statements about welfare ranking and return on education. Due to the missing insurance market, there will be no equilibrium attaining a first-best allocation but the expected utility of agents is increasing in the equilibrium share of educated workers in the labor force. Another relationship is that welfare is decreasing in the expected rate of return on education. A small risk premium associated with education indicates that the wage structure is good at providing implicit insurance on human capital while a high expected return on education signals that the economy is less successful in dealing with the market imperfection.

Proposition 3. Multiple stationary equilibria can be ex ante Pareto-ranked with welfare weakly increasing in the educated labor force N (strictly increasing in N if the relative price p differs between equilibria). An equilibrium with higher welfare is also associated with a lower expected rate of return on education.

Proof: See appendix.

5. A Numerical Example of Multiple Equilibria

The numerical example is based on the following model specification;

$$U\left(c_{t2}^{i}+c_{t3}^{i},\, s_{t2}^{i}+s_{t3}^{i}\right) \;=\; \log\left(c_{t2}^{i}+c_{t3}^{i}\right) \,+\, \log\left(s_{t2}^{i}+s_{t3}^{i}\right),$$

$$\pi = .5\,, \qquad \gamma = .2\,, \qquad \text{and} \qquad \mu = .17\,. \tag{10}$$

The parameters satisfy the necessary condition for multiple stationary equilibria in Proposition 1 and Figure 1 demonstrates the existence of three such equilibria. Let us start in the leftmost part of that figure to understand the shapes of the utility curves. A small share of educated workers in the labor force (N) is associated with a low relative price of the labor-intensive good and all educated workers, both those with high and low productivity, would therefore choose to produce the capital-intensive good. It follows that the expected utility of educated agents is decreasing in N while the utility of uneducated agents is increasing since each class of workers is better off the fewer they are relative to the other class. The two utility curves reach a flat segment when the relative price of the labor-intensive good has increased all the way to γ, i.e., to the low productivity level in the capital-intensive sector. Educated workers with low productivity are then indifferent to their choice of employment, a higher N will therefore not increase the relative price of the labor-intensive good but just cause educated workers with low productivity to move from the capital-intensive sector to the labor-intensive sector. When all these workers are employed in the labor-intensive sector, a higher N will once again be associated with a higher relative price of the labor-intensive good and a higher utility of uneducated agents. However, this time the expected utility of educated agents is also increasing in N. The reason is that the higher utility of an educated worker with low productivity, who produces the labor-intensive good, outweighs the loss of utility of an educated worker with high productivity. As a consequence, there are three stationary equilibria; E(1) with $N = .27$, E(2) with $N = .53$ and E(3) with $N = .94$.[6]

As pointed out in Proposition 3, agents are better off in an equilibrium with a higher share of educated workers in the labor force. This can be seen from the uneducated agents' utility being weakly increasing in N in Figure 1 or that these agents attain successively higher indifference curves in Figure 2. The latter figure depicts a stationary net production possibility frontier (PPF) where the production point for equilibrium j is denoted E(j) which in turn is connected to the corresponding indifference curve for an uneducated agent (also representing the expected utility of an educated agent).[7] The attained indifference curve lies strictly below the production point, i.e., the economy's average consumption bundle, because of no risk sharing. Another welfare cost is the distance between the production point and the first-best allocation that would arise in the presence of an insurance market for human capital. It is interesting to note that equilibrium E(1) is quite close to the first-best allocation while equilibrium E(3) is far away. The production in the

[6] The middle equilibrium E(2) is "unstable" if we make the ad hoc assumption that the share of educated workers is adjusted so that agents are moved into the educational status with the higher level of expected utility.

[7] The kink in the PPF in Figure 2 follows from Proposition 2. Since $\pi(1-\gamma) > \mu$, it is efficient to first allocate *only* educated workers with high productivity in the capital-intensive sector. But when all agents are educated, production of the capital-intensive good can only be expanded by also employing educated workers with low productivity. The marginal rate of transformation is then less favorable for the capital-intensive good, reflected in the flater slope of the PPF.

latter equilibrium takes even place in the interior of the PPF since all educated workers are then producing the capital-intensive good but, according to Proposition 2, an efficient labor allocation would only employ educated workers with high productivity in the capital-intensive sector.

6. Discussion

This paper has demonstrated how smaller wage differentials in the developed world provide implicit insurance on human capital while larger wage differentials in underdeveloped countries make investments in human capital riskier. The endogenous positive relationship between the dispersion of the wage distribution and the riskiness of human capital is obtained in a simple general equilibrium model where the only market imperfection is that individuals cannot insure their human capital. This relationship should also hold with additional economic imperfections in a more complicated framework. And my terminology of "uneducated" and "educated" workers should not be taken literally. Another interpretation can be general education versus professional education. The purpose of the model is to study how various economies allocate individuals with *different* educational levels and abilities between tasks. It is then shown that students in a developed country are insured against poor educational outcomes through the existence of well-paid alternative employments which are not present in the economy of a less developed country.

The case of a missing insurance market for human capital has already received much attention in the literature and been used to rationalize government redistribution policies, see for example Eaton and Rosen [1980], Varian [1980], and Loury [1981]. The idea has been that risk-averse individuals who are uncertain about their future income might favor a redistributing transfer system which reduces the variance of disposable income. Surprisingly, my model suggests that the market forces in a developed country is actually accomplishing something similar to these transfer policies. The compressed wage structure in a developed country is better at providing implicit insurance on human capital than larger wage differentials in an underdeveloped country. Borjas [1987] offers some supportive empirical evidence on the existence of such implicit insurance in a developed country. His conclusion could have been taken out of my paper: "The United States, in a sense, 'insures' low-income workers against poor labor market outcomes while 'taxing' high-income workers" compared to an underdeveloped country with a high level of income inequality.

Appendix

Proof of Proposition 1

The assumption in (3) that $\gamma > \mu$ ensures that even an educated agent with low productivity can more than repay the educational loan at the stationary interest rate $R = 1$. The existence of a stationary equilibrium with educated workers is then guaranteed by the first Inada condition in (2) which implies that the relative price of the capital-intensive good goes to infinity when the supply goes to zero. A closer characterization of a stationary equilibrium will now be obtained by finding a functional relationship between N and ϕ so that equilibrium condition (8) can be expressed in only one endogenous variable. This is done by deriving a function $\hat{p}(N)$ defined as the relative price of the labor-intensive good consistent with educated workers with low productivity choosing rationally what good to produce. First of all, the first Inada condition in (2) implies that the limit of $p(N,\phi)$ is zero when N goes to zero for any $\phi \in [0,1]$, it follows that educated workers with low productivity will be producing the capital-intensive good at low values of N, i.e., $\phi = 1$. The relative price $p(N,1)$ will then be increasing in N until the price takes on the value γ, let say at $N = N_0$. The relative price $p(N_0,1) = \gamma$ makes the educated agents with low productivity indifferent to what good they produce. Any additional small increase in N will therefore not affect the price since already educated agents with low productivity will just move into the labor-intensive sector. This continues until all educated workers with low productivity are producing the labor-intensive good, let say at $N = N_1$ such that $p(N_1,0) = \gamma$. After this point, further increases in N will once again mean a higher relative price throughout the remaining range of N. One last qualification must be added, the variable N may reach its upper bound of one at any point in the argument. The function $\hat{p}(N)$ can be summarized as

$$\hat{p}(N) = \begin{cases} p(N,1) \leq \gamma & \text{for } N \in (0,\, N_0]\,, \\ \gamma & \text{for } N \in [N_0,\, N_1]\,, \\ p(N,0) \geq \gamma & \text{for } N \in [N_1,\, 1]\,, \end{cases} \tag{11}$$

where $N_0 \leq N_1 \leq 1$.

The relative price $p(N,\phi)$ in (8) is now replaced by $\hat{p}(N)$. It can then be seen that the right-hand side, the utility of an uneducated agent, is strictly increasing in N except for $N \in [N_0, N_1]$ where it is constant. The expected utility of an educated agent, the left-hand side of (8), is strictly decreasing in N for $N \in (0, N_0]$ while it is also constant in the interval $[N_0, N_1]$. An ambiguity arises for $N > N_1$, i.e., when an educated agent with low productivity strictly prefers to produce the labor-intensive good. As discussed and shown in expression (9), the expected utility of an educated agent may then start to increase in N. This raises the possibility of multiple solutions to equation (8) and a numerical example of multiple equilibria is provided in Section 5. Since the left-hand (right-hand) side of (8) is strictly decreasing (increasing) for $N < N_0$, a sufficient condition for ruling out multiple equilibria is to show that a necessary condition for an equilibrium is violated for $N \geq N_0$. One such necessary condition is that the expected income of an educated agent net of educational expenses is strictly greater than the deterministic income of an uneducated worker,

$$\pi(1-\mu) + (1-\pi)\left(\max\{\gamma, p\} - \mu\right) > p. \tag{12}$$

After noticing from (11) that $p \geq \gamma$ for $N \geq N_0$, condition (12) can be reduced to read $\pi(1-p) > \mu$. After invoking $p \geq \gamma$ one more time, a necessary condition for multiple equilibria is therefore that $\pi(1-\gamma) > \mu$, or a sufficient condition for ruling them out is $\pi(1-\gamma) \leq \mu$. ◇

Proof of Proposition 2

In the case of no discounting, a generation faces the following resource constraints:

$$c_2 + c_3 \equiv c(N,\phi) = \pi N + \phi(1-\pi)\gamma N - \mu N, \tag{13.a}$$

$$s_2 + s_3 \equiv s(N,\phi) = 1 - N + (1-\phi)(1-\pi)N. \tag{13.b}$$

To trace out the stationary net production possibility frontier (PPF), let us first find the trade-off between the labor-intensive good and the capital-intensive good when varying N for a constant ϕ;

$$-\frac{c_1(N,\phi)}{s_1(N,\phi)} = \frac{\pi + \phi(1-\pi)\gamma - \mu}{1 - (1-\phi)(1-\pi)} \equiv \Psi(\phi), \tag{14}$$

$$\text{and} \quad \frac{d\Psi(\phi)}{d\phi} = (1-\pi)\frac{\mu - \pi(1-\gamma)}{\left[1 - (1-\phi)(1-\pi)\right]^2} \begin{cases} > 0 & \text{if } \pi(1-\gamma) < \mu, \\ = 0 & \text{if } \pi(1-\gamma) = \mu, \\ < 0 & \text{if } \pi(1-\gamma) > \mu. \end{cases}$$

Now, start out at the end-point of the PPF with no production of the capital-intensive good, i.e., $N = 0$. The first production of the capital-intensive good requires then an increase in N which will take place at the best possible trade-off in terms of lost production of the labor-intensive good. The maximal trade-off as a function of ϕ, $\Psi(\phi)$, depends on the model's parameters. If $\pi(1-\gamma) < \mu$, the trade-off is most favorable at $\phi = 1$ while $\phi = 0$ is the optimal choice when $\pi(1-\gamma) > \mu$. On the other hand, if this condition on the parameters holds with equality, it does not matter in which order N and ϕ are increased when producing more of the capital-intensive good. ◇

Proof of Proposition 3

This proposition follows immediately from the proof of Proposition 1. In an equilibrium, all agents will obtain the same expected utility which is equal to the utility of an uneducated agent (and maybe larger if the equilibrium occurs at $N = 1$). The Pareto-ranking is then implied by the fact that the utility of an uneducated agent is weakly increasing in N (strictly increasing in N when the relative price p also changes).

The expected rate of return on education in a stationary equilibrium is given by

$$\pi \frac{1-p}{\mu} + (1-\pi) \frac{\max\{\gamma - p, 0\}}{\mu}, \tag{15}$$

where the return on education is equal to wage income in excess of the wage in the labor-intensive sector. After invoking the relative price $\hat{p}(N)$ in (11) and the argument in the previous paragraph, it can be seen that the expected rate of return on education is inversely related to the welfare level in a stationary equilibrium. ◊

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Figure 1: Expected utility of an educated agent and an uneducated agent as a function of the share of educated workers in the labor force. The model specification is given by (10).

Solid line	expected utility of an educated agent.
Dashed line	utility of an uneducated agent.
$E(j)$	$j \in \{1,2,3\}$, three stationary equilibria.

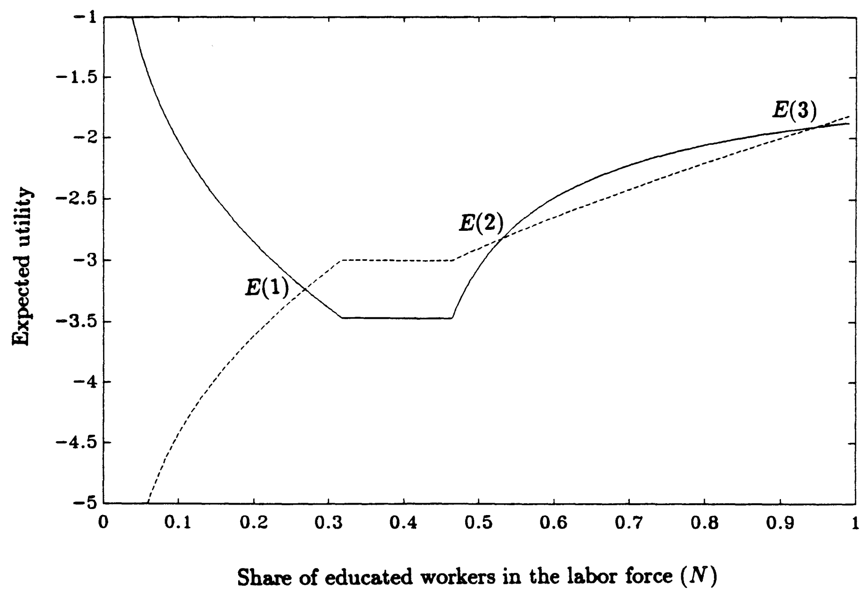
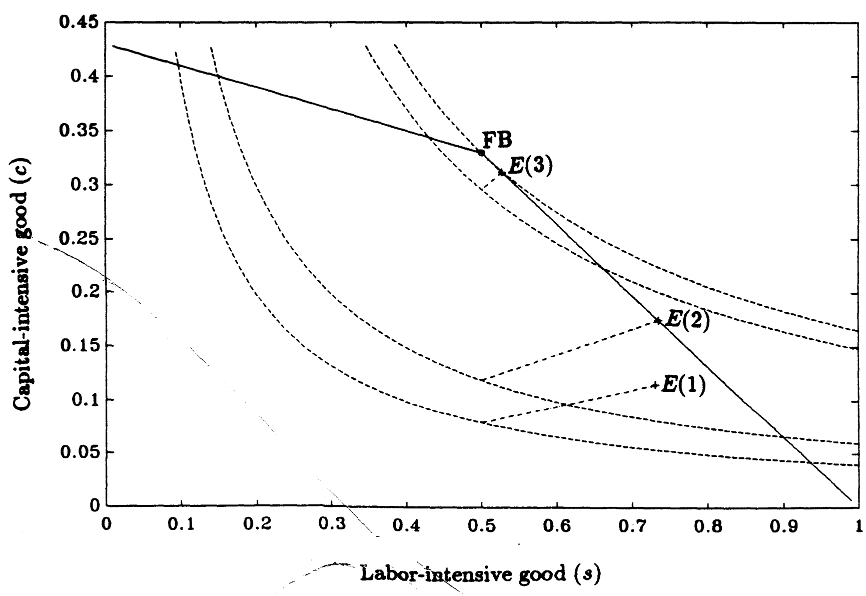


Figure 2: Production and consumption in different equilibria. The model specification is given by (10).

Solid line	stationary net production possibility frontier.
Dashed lines	indifference curves.
FB	net production in the first-best stationary allocation intersected by the attained indifference curve.
$E(j)$	net production in stationary equilibrium j connected via a dashed-dotted line to the corresponding indifference curve of an uneducated agent (also representing the expected utility of an educated agent).



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