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A SIMPLE MODEL OF PRIVATE FIRMS IN A CENTRALLY PLANNED ECONOMY

Daniel Berkowitz*
Assistant Professor
Department of Economics
University of Wisconsin-Madison
August 1991

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A SIMPLE MODEL OF PRIVATE FIRMS IN A CENTRALLY PLANNED ECONOMY

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Abstract

This paper models the behavior of private firms in a parallel consumer goods market. The state sector is characterized by a fixed quantity, quality and price of output while the private sector strategically chooses its quality and quantity. The private sector response to a marginal cut-back in state provision of consumer goods is analyzed for the possible regimes of an excess-demand equilbrium. It is shown that this cut-back, unaccompanied by transfers of assets or materials to the private sector, may increase public welfare.

1. INTRODUCTION

The reforming economies of the Soviet Union and Eastern Europe have both state and private provision of consumer goods. Because prices, qualities and quantities of state-provided goods are determined administratively rather than by market forces, there are persistent imbalances in local markets. The Soviet press contains much anecdotal evidence of consumers using connections (blat) and side-payments (vzyatky) in order to obtain scarce state goods. Yet, because of the low quality of these goods, there are reports of excessive accumulation of worthless state retail inventories (see Hewett, 1988, p.78-80 and Skurski, 1983, chpt 5).

Because of the state sector's persistent disequilibrium, local governments have always depended upon the private provision of consumer goods. In the Soviet Union, private farmers - as well as state and collective farm members - have always supplied a large share of total food goods (see, for example,Grossman 1977, p.3; Shlapentokh, 1989, p.191). Furthermore, private producers have been significant suppliers of clothing, housing construction, handicrafts, small electronics and second-hand consumer durables (see Grossman, 1977 and Simes 1975 for the USSR; for Hungary see Kornai, 1990; for Poland see Aslund 1983 and Rostenkowski 1989). In contrast to the state sector, private sector goods' prices are market clearing. While the quality of state goods is fixed, that of the private sector is reactive to local demand. The operation of local state retail shops simultaneously with flexible private merchants is an example of a parallel market (Simes 1975).

This paper analyzes the behavior of an imperfectly competitive private sector and local government incentives to reduce state provision of consumer goods in a highly stylized model of a parallel market. Since many

Soviet and East European private merchants face bureaucratic obstacles, poorly developed sources for material and capital supplies and exorbitant transport costs, the barriers to market entry by non-state firms are formidable. Thus, an imperfectly competitive private sector is assumed. To capture the private sector's flexibility relative to the state sector, it is assumed that private firms can change the quality of its product at any point in time while state quality is fixed. It is demonstrated that, because of private sector responses to market forces, there are numerous situations in which a local government can increase the welfare of its constituency by cutting back public provision at the margin.

Local governments in the USSR and Eastern Europe traditionally have been responsible both for receiving consumer goods from central supply organs and selling these goods at centrally determined prices in state stores. These stores sell goods to both residents and non-residents. Recently, local governments have come under increasing pressure to first satisfy the demands of their local population. Thus, it is assumed that local governments have an incentive to reduce state provision when this increases overall surplus. In computing welfare no specific weights are assigned to consumer and producer surplus. Following Arnott and Grieson (1981), it is assumed that although a local government knows the percentage of residential and non-residential participation in its market, it cannot discriminate between them.[1] Thus, a higher rate of residential consumer participation leads a local government to place a greater weight on consumer surplus. A higher rate of residential private producer participation makes the local government to value producer surplus more highly.

This paper focuses on the strategic behavior of private firms in a parallel market and relates to three literatures. Following Ericson (1984)

and Stahl and Alexeev (1985), the literature on parallel markets has assumed that private entrepreneurs are price takers (see Davis, 1988, pp.78-85 for a survey). Wellisz and Findlay (1986) model the efficiency gains of illegal private production. Panagariya analyzes the impact of state price-quantity controls and private firms' higher efficiency relative to large state firms on illegal private activity in a static (1989) and dynamic setting (1990). Polterovich (1986) provides the most general treatment of the simultaneous operation of state and private activity in which private activity at flexible prices is legal once state quotas are satisfied at fixed prices. In all of these treatments, the private sector is perfectly competitive. In this paper, private firms know the distribution of consumer demand and the quality, quantity and price of state goods. Taking state behavior as fixed, private firms qualitatively differentiate themselves from each other and the state in order to maximize profits.

This paper generalizes the work of Gabszewicz and Thisse (1980), Shaked and Sutton (1983) and, in particular, Gal-Or (1985). These papers develop models of oligopoly markets in which firms engage in quality competition and either price or quantity competition. They characterize Nash equilbria in which prices are market clearing. By incorporating a fixed state outlet, this paper is able to study the characteristics of Nash equilbria with excess demand or excess supply for the state good.

Finally, Ireland (1991) looks at the optimal mix of public and private provision in a world in which private consumer goods and services are qualitatively different than those publicly provided. He takes public and private prices as given and explores the impact of different financing schemes on consumer demand. This paper uses a framework, similiar to Ireland's, of vertical product differentiation between the state and

private sectors and adds production into the private sector. The benefits of less state provision given the reaction of the private sector are examined.

The paper is structured as follows: Section 2 introduces the model. Section 3 characterizes possible Nash equilibria. Sections 4-7 analyze equilibria in which the state is the high, middle and low quality producer. Section 8 concludes the paper.

2. THE MODEL[2]

There is a continuum of consumers indexed by a taste parameter x. For simplicity, x is assumed to be distributed uniformly over the compact interval [0,X]. Quality is indexed by m with higher values of m denoting higher quality. Quality is restricted to one dimension - say, fashion or durability - and is perfectly observed by producers and consumers.

Each consumer purchases either one or no good. The pay-off to a consumer type x purchasing product type m is represented by the utility function U(m,x):

$$U(m,x) = m(a + bx), \quad a, b > 0 \tag{1}$$

(1) implies that consumers are better off purchasing higher quality products and that consumers with a higher taste parameter, x, value high quality products more those with a low x. Because income effects are held fixed, this function measures a consumer's willingness to pay for a product.

The cost to consumer x of buying product m is denoted $\zeta(p,e)$:

$$\zeta(p,e) = p + t, \tag{2}$$

where p denotes the goods price, t is a side-payment which, with the posted goods price, is necessary to obtain the good. In this cost specification, disutility of side-payments is independent of consumer type[3]:

$$t(x) = t \tag{3}$$

Consumer surplus derived from buying a product is the pay-off net of

cost. If a consumer buys a product, he/she must receive a non-negative surplus, $U(m,x) - \zeta(p,e)$:

$$m(a + bx) - p - t \geq 0 \qquad (4)$$

There is a state firm and two private firms. For simplicity, *each firm produces only one good.* State behavior is administratively determined and is described by the triplet $S = \{m_s, q_s, p_s\}$ which is a fixed quality, quantity and price. The state price is set so that every consumer is willing to buy a state good *in the absence of side-payments:*

$$am_s > p_s \qquad (6)$$

Private firms produce goods of quality m_i. The *unit production cost* in the private sector, $c(m)$, is constant in quantity, increasing in quality and symmetric. The following functional form is tractable and captures all of these features:

$$c(m) = m_i^2 \qquad (7)$$

Each private firm chooses product quality and quantity at the same point in time so as to maximize its profits.

3. NASH EQUILIBRIA

In equilibrium, each consumer takes a price vector, $p = \{p_s, p_h, p_l\}$, and a side-payments vector, $t = \{t_s, t_h, t_l\}$, which describe the cost of obtaining high and low quality private goods and state goods. A consumer buys either a unit of a good or withdraws from the market in order to maximize consumer surplus. Consumer surplus is denoted $v(x,m,p,t)$:

$$v(x,m) = \max_i \{ U(m_i,x) - \zeta(p_i,t_i) \}: \qquad (8)$$

$$= \max_i \{ m_i(a + bx) - p_i - t_i \}: \quad i = h, l \text{ or } s$$

In equilibrium, each private firm chooses its own profit maximizing quality-quantity pair given state behavior, the optimal response of the

other private firm and the best choice of each consumer. Prices in the private sector clear the market. However, there may be a necessary side-payment, t_s, in addition to the posted price, p_s, in the state sector.

In order to formally define the Nash equilibrium, let σ_i denote the quality-quantity strategy, let π_i denote profits of private firm i (i = h, l) and let S describe state behavior: $S = \{m_s, q_s, p_s\}$.

DEFINITION 1. The two tuple $\{\sigma_h^*, \sigma_l^*\}$ and triplet S consitute a Nash equilibrium if:

I. $\sigma_h^* = \text{argmax}_{\sigma_h} \pi_h(\sigma_h, \sigma_l^*, S) \qquad \sigma_h^*, \sigma_h \in R_+^2$

$\sigma_l^* = \text{argmax}_{\sigma_l} \pi_l(\sigma_l, \sigma_h^*, S) \qquad \sigma_l^*, \sigma_l \in R_+^2$

subject to:

if $v(x) \geq 0$ a consumer buys some good i_x,

if $v(x) < 0$ a consumer withdraws from the market, in which

$i_x = \text{argmax} \{ m_i(a + bx) - p_i - t_i \}$

$v(x) = \max_i \{ m_i(a + bx) - p_i - t_i \}, \; i = h, l, s.$

II. $p \in R_+^3, \quad t = \{t_s, 0, 0\}$ so that

$q_h^* = \{x | i_x = h\},$

$q_l^* = \{x | i_x = l\},$

$q_s \geq \{x | i_x = s\},$

Definition 1 extends Gal-Or's (1985, p.314) definition to incorporate a fixed state retail outlet. The first part of Definition 1 is the standard best response requirement. The second partindicates that prices are market clearing in the private sector and that there may be excess supply in the state sector. Furthermore, consumers in the private sector pay only a money price: but, those in the state sector pay a money price and perhaps an

additional side-payment. This second part follows from the uniformity assumption made about the taste parameter x.

Two equilibria of interest are now defined. An excess-demand equilibrium holds when an endogenous side-payment plus the state price clear the state sector. An excess-supply equilibrium holds when there are surplus goods and no side-payments in the state sector. This is formally defined below:

DEFINITION 2. The two tuple $\{\sigma_h^*, \sigma_l^*\}$ and triplet S constitute an excess-demand equilibrium if conditions I and II from Definition 1 hold and:

III. $q_s = \{x | i_x = s\}$ and $t_s > 0$.

DEFINITION 3. The two tuple $\{\sigma_h^*, \sigma_l^*\}$ and triplet S constitute an excess-supply equilibrium if conditions I and II from Definition 1 hold and:

III. $q_s > \{x | i_x = s\}$ and $t_s = 0$.

The next three lemmas characterize Nash equilibria. All proofs are in the appendix. Without any loss of generality, brands one, two and three denote the high, middle and low quality products.

LEMMA 1. *Suppose the brands are ordered:* $m_1 \geq m_2 \geq m_3$. *In equilibrium, there is a maximum of four regions as follows: consumers in taste groups* $x \in [x_1, X]$, $[x_2, x_1]$ *and* $[x_3, x_2]$ *purchase brands one, two and three respectively and consumers in taste group* $x \in [0, x_3]$ *refrain from consumption, such that* $X > x_1 > x_2 > x_3 > 0$.

By lemma 1, since the price plus side-payments exceeds their willingness to pay, consumers with the lowest taste parameters may withdraw

from the market. Consumers that buy a product divide into a maximum of three groups in which those with a higher taste parameter buy a higher quality product.

By lemma 1, the state may produce the high, intermediate or low quality good. Thus, there are three possible cases:

$$\text{Case I:} \quad m_s \geq m_h \geq m_l \tag{9}$$

$$\text{Case II:} \quad m_h \geq m_s \geq m_l$$

$$\text{Case III:} \quad m_h \geq m_l \geq m_s$$

The next lemmas characterize excess-demand and excess-supply equilibria:

LEMMA 2. *If an excess demand equilibrium exists, then the market is not saturated, i.e.,* $X - q_s - q_h - q_l > 0$.

Therefore, in an excess-demand equilibrium, a group of consumers with the lowest index taste parameters withdraw from the market.

LEMMA 3. *If an excess supply equilibrium exists, then the market is saturated.*

In an excess-supply equilibrium, there are no side-payments in the state sector. Since the state price is set so that all consumers are willing to buy state goods in the absence of side-payments, lemma 3 follows.

The remainder of the paper will focus on the properties of excess-demand equilibria. All proofs will be placed in the appendix.

4. EXCESS DEMAND EQUILIBRIA

Consider an excess demand equilibrium in which the brands are ordered:

$m_1 \geq m_2 \geq m_3$. Since all markets clear and by lemmas 1 and 2, there exist cut-off points x_1, x_2 and x_3:

$$x_1 = X - q_1 \tag{10}$$

$$x_2 = X - q_1 - q_2$$

$$x_3 = X - q_1 - q_2 - q_3 > 0$$

Equilibrium prices and the side-payments are determined so that consumer type x_1 is indifferent to buying good one and two; consumer type x_2 between buying good two and three; and finally, consumer type x_3 between buying good three and withdrawing from the market:

$$p_1 + t_1 = p_2 + t_2 + (a + bx_1)(m_1 - m_2) \tag{11}$$

$$p_2 + t_2 = p_3 + t_3 + (a + bx_2)(m_2 - m_3)$$

$$p_3 + t_3 = m_3(a + bx_3)$$

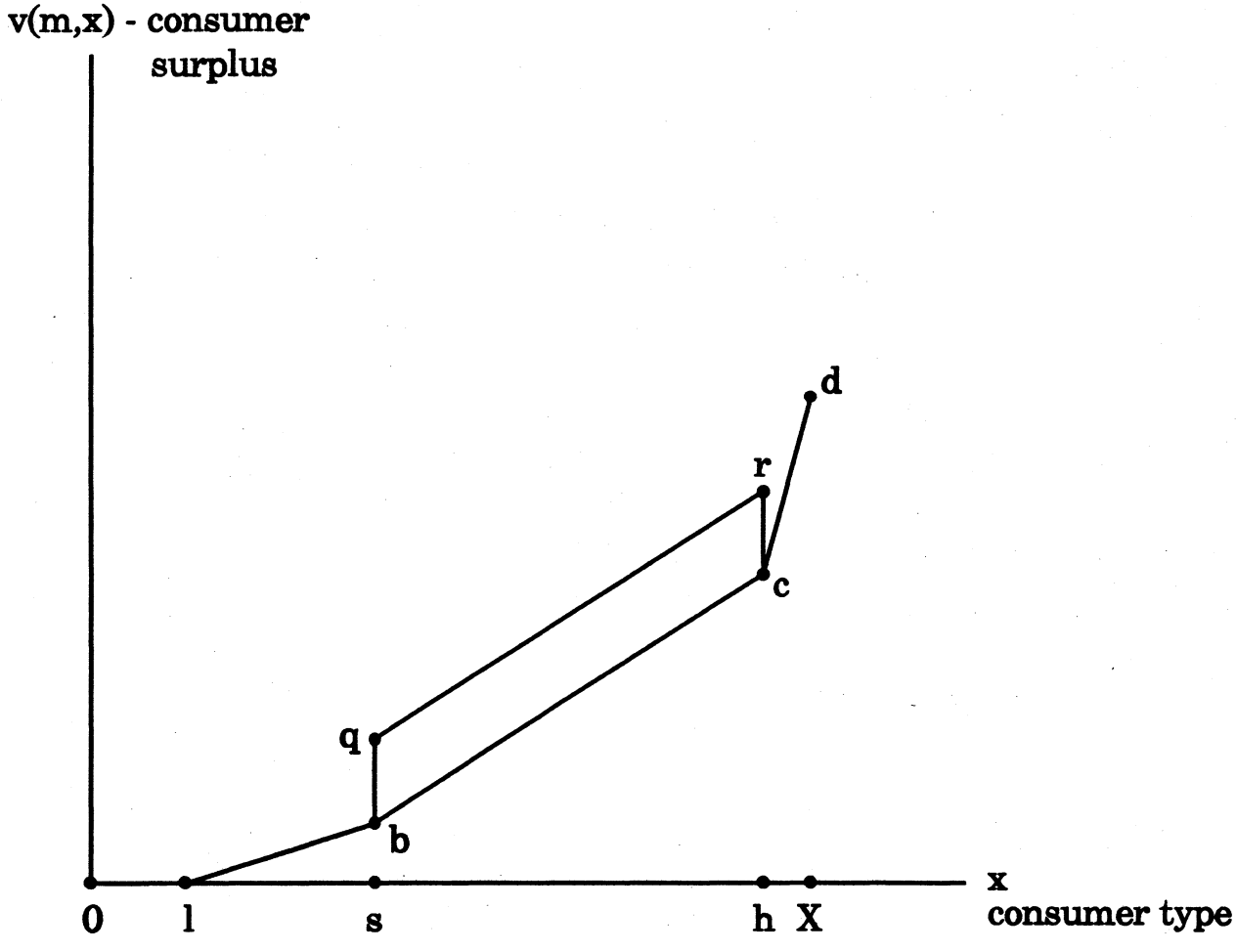
where

$t_l = t_h = 0$, $t_s > 0$, and $S = \{m_s, q_s, p_s\}$ is given.

An excess-demand equilibrium with the state as intermediate producer ($m_h > m_s > m_l$) is portrayed in figure 1. Segments ls, sh and hX represent goods sold by the low quality, state and high quality firms while segments lb, bc and cd measure the distribution of consumer surplus derived from buying low quality, state and high quality goods. The slopes of segments lb, bc and cd are the quality levels m_l, m_s and m_h respectively. Thus, the distribution lbcd is piece-wise linear and convex in the taste parameter x. Segment qb = segment rc is the additional cost above p_s which consumers must pay to buy state goods and the parallelogram qbrc is the welfare loss of these side-payments.

The next lemma puts an open lower bound on side-payments.

FIGURE 1
STATE AS INTERMEDIATE PRODUCER



LEMMA 4. *If an equilibrium exists in which the market is not saturated, then state sector transactions costs,* t_s, *are greater than* $am_s - p_s > 0$.

COROLLARY TO LEMMAS 2 AND 4. *An excess demand equilibrium exists if and only if the market is not saturated.*

By lemma 2, if an excess demand equilbrium exists, the market is not saturated. According to lemma 4, if the market is not saturated in equilibrium, then side-payments must be paid in the state sector. An equilbrium with side-payments is, by definition, an excess demand equilbrium. Thus, the corollary follows.

5. THE STATE AS HIGH QUALITY PRODUCER

This section characterizes an equilbrium in which the state is the high quality producer: $m_s \geq m_h \geq m_l$. It analyzes the impact of a cut-back in state provision of consumer goods. A small cut-back results in an increase in both private supply quantity and quality and an increase in intra-private product differentiation. Furthermore, there is an increase in producer surplus and a decline in consumer surplus.

By (11) the private sector profit functions are:

$$\pi_h = \{ m_h[a + b(X - q_s - q_h)] - bm_l q_l - m_h^2 \}q_h \qquad (12)$$

$$\pi_l = \{ [a + b(X - q_s - q_h - q_l)] - m_l \}m_l q_l$$

The first order conditions (FOC) necessary for optimality are:

$$d\pi_h/dq_h = \{ [a + b(X - q_s)]m_h - bm_l q_l - m_h^2 - 2bm_h q_h \} = 0 \qquad (13)$$

$$d\pi_h/dm_h = q_h\{ a + b[X - q_s - q_h] - 2m_h \} = 0$$

$$d\pi_l/dq_l = m_l\{ [a + b(X - q_s - q_h)] - m_l - 2bq_l \} = 0$$

$$d\pi_l/dm_l = q_l\{ a + b[X - q_s - q_h - q_l] - 2m_l \} = 0$$

(13) is a system of four equations in four unknowns. This reduces to the unique system:

$$m_h = 9[a + b(X - q_s)]/23 \qquad (14)$$

$$m_l = 2m_h/3$$

$$bq_h = a + b(X - q_s) - 2m_h$$

$$bq_l = 2(m_h - m_l)$$

In appendix 3, it is proven that (14) satisfies conditions necessary and sufficient for optimality. In appendix 6 conditions on parameters are derived such that the equilibrium is globally stable. That is, neither the low or high quality private firm has an incentive to make a large deviation and change its rank.

Using (12)-(14) the following characteristics of the equilbrium can be shown to hold.

PROPOSITION 1. *The following properties hold in an excess demand equilbrium when* $m_s \geq m_l \geq m_h$*:*

(a) $m_h > m_l$

(b) $q_h < q_l$

(c) $\pi_h > \pi_l$

By Proposition 1 there is product differentiation within the private sector. The high quality private firm makes more profit and has a smaller market share than the low quality private firm.

The next propositions analyze the impact of a cut-back in state provision on the response of the private sector.

PROPOSITION 2. *As a result of a cut-back in state provision, both high and low quality private firms increase the quality of goods supplied. The range of intra-private product differentiation increases.*

PROPOSITION 3. *As a result of a cut-back in state provision, both high and low quality private firms increase quantity of goods supplied. However, there is a decline in the participation of consumers and an increase in side-payments,* t_s.

Suppose the local government can accurately predict the response of the private sector to a marginal cut-back in the provision of state consumer goods. Under what circumstances would the local government have an incentive to cut back state provision? Since the cost of producing state goods is unknown, the impact on state profits is ignored. No specific weights to consumer and producer surplus are assigned since these would depend upon the local situation. For example, in a region where many private producers commute from another area, one would expect the weight on private producer surplus to be low. In a region in where there are many locally based private producers and where many consumers commute from other regions, one would expect the weight on private producer surplus to be high relative to consumer surplus. The next propostion analyzes private producer surplus.

PROPOSITION 4. *A cut-back in state provision increases the profits of each private firm and, therefore, overall producer surplus.*

PROPOSITION 5. *A cut-back in state provision increases consumer surplus in the private sector, and decreases consumer surplus in the state sector. Overall consumer surplus falls.*

Since a cut-back in state provision results in an increase in the quality and quantity of privately provided goods and an increase in state sector side-payments, private sector consumer welfare improves and state sector consumer welfare deteriorates. The quantity response from the private sector is not sufficient to compensate for the withdrawal of high quality state goods and overall consumer surplus falls. A local government that valued producer surplus more than consumer surplus would have an incentive to cut back the provision of state goods at the margin.

6. THE STATE AS THE INTERMEDIATE PRODUCER

This section analyzes the intermediate case: $m_h \geq m_s \geq m_l$. It is demonstrated that a small cut-back in state provision increases the aggregate private supply of goods and shrinks intra-private quality differentiation. Producer surplus increases while the impact on consumer surplus is ambiguous. Conditions under which consumer surplus might increase are derived.

By (11) the private sector profit functions are:

$$\pi_h = \{ m_h[a + b(X - q_h)] - bm_s q_s - bm_l q_l - m_h^2 \} q_h \qquad (15)$$

$$\pi_l = \{ [a + b(X - q_s - q_h - q_l)] - m_l \} m_l q_l$$

The FOC necessary for optimality are:

$$d\pi_h/dq_h = \{ [a + b(X - 2q_h)]m_h - bm_s q_s - bm_l q_l - m_h^2 \} = 0 \qquad (16)$$

$$d\pi_h/dm_h = q_h\{ a + b[X - q_h] - 2m_h \} = 0$$

$$d\pi_l/dq_l = m_l\{ [a + b(X - q_s - q_h)] - m_l - 2bq_l \} = 0$$

$$d\pi_l/dm_l = q_l\{ a + b[X - q_s - q_h - q_l] - 2m_l \} = 0$$

This system can be reduced:

$$m_h(a + bX) + bm_s q_s = 3m_h^2 - m_l^2 \tag{17}$$

$$3m_l = (2m_h - bq_s)$$

$$bq_h = a + bX - 2m_h$$

$$bq_l = m_l$$

In appendix 4, it is demonstrated that (17) reduces to a unique system satisfying necessary and sufficient conditions:

$$m_h(m_l) = \frac{a + bX + \sqrt{(a + bX)^2 + 12(bm_s q_s + m_l^2)}}{6} \tag{18}$$

$$3m_l = (2m_h - bq_s)$$

Furthermore, in appendix 6, it is shown that the equilbrium is globally stable when the parameters of the model are appropriately restricted.

The next proposition uses (18) to characterize the equilibria.

PROPOSITION 6. *In an excess demand equilibrium with $m_h \geq m_s \geq m_l$, there is strict intra-private product differentiation.*

Propositions 7-10 analyze the impact of a cut-back in state output.

PROPOSITION 7. *As a result of a small cut-back in state provision, the quality range in the private sector shrinks.*

PROPOSITION 8. *As a result of a cut-back in state provision, both high and low quality private firms supply more goods. However, the overall consumer participation level falls and t_s increases.*

PROPOSITION 9. *A cut-back in state provision results in an increase in the profits of both private firms and in producer surplus.*

By Proposition 9, a local government would have an incentive to curtail state provision when a high value is placed on private producer surplus.

Intuitively, one could argue in favor of or against the consumer welfare benefits of a decline in state output. An argument against this policy is that the private sector quantity response is not sufficient to compensate for the withdrawal of the state from the market. Therefore, an exacerbation of the shortage of state goods occurs, side-payments increase and welfare falls. An argument in favor of this policy is that the private sector *quality response* more than compensates for the increased shortage of state goods. Even though the high quality private firm decreases its quality, it serves a larger consumer group with a product that is still qualitatively better than the state product. Furthermore, the low quality private firm now serves a larger consumer group with a better product.

PROPOSITION 10. *As a result of a cut-back in state provision, consumer welfare in the low quality private goods sector increases. The impact on consumers buying from both the state and the high quality firm is ambiguous. The change in consumer welfare is higher when:*

(a) the cut-back elicits a strong increase in m_l *and a strong decrease in* m_h*;*

(b) the cut-back elicits a large increase in both q_l *and* q_h.

7. THE STATE AS LOW QUALITY PRODUCER

This section analyzes the case in which the state is the low quality producer. It is demonstrated that a small cut-back in state provision results in an increase in private output and a decline in the average private quality level. While the impact of this policy on consumer and producer surplus is ambiguous, producer surplus is likely to increase when the initial quantity and quality of state provision is high.

By (11), when $m_s \leq m_l \leq m_h$ the private sector profit functions are:

$$\pi_h = \{ m_h[a + b(X - q_h)] - bm_l q_l - bm_s q_s - m_h^2 \} q_h \tag{19}$$

$$\pi_l = \{ m_l[a + b(X - q_h - q_l)] - bm_s q_s - m_l^2 \} q_l$$

The FOC necessary for optimality are:

$$d\pi_h/dq_h = \{ [a + bX]m_h - bm_l q_l - bm_s q_s - m_h^2 - 2bm_h q_h \} = 0 \tag{20}$$

$$d\pi_h/dm_h = q_h\{ a + b[X - q_h] - 2m_h \} = 0$$

$$d\pi_l/dq_l = \{ [a + b(X - q_h)]m_l - bm_s q_s - m_l^2 - 2bm_l q_l \} = 0$$

$$d\pi_l/dm_l = q_l\{ a + b[X - q_h - q_l] - 2m_l \} = 0$$

The system in (20) is one with four equations in four unknowns. In the appendix, it is shown that this system can be reduced:

$$m_h(a + bX) = 3m_h^2 - m_l^2 \tag{21}$$

$$2m_l m_h = 3m_l^2 - bm_s q_s$$

$$bq_h = a + bX - 2m_h$$

$$bq_l = 2(m_h - m_l)$$

Note that if we can solve for m_h and m_l, then q_h and q_l are determined. In the appendix, it is proven that there is a unique solution satisfying the necessary and sufficient conditions:

$$m_h(m_l) = \frac{a + bX + \sqrt{(a + bX)^2 + 12m_l^2}}{6} \qquad (22)$$

$$m_l(m_h) = \frac{2m_h + \sqrt{4m_h^2 + 12bm_s q_s}}{6}$$

Furthermore, in appendix 6, restrictions on parameters are derived such that the equilibrium is globally stable.

Using (20)-(22), we demonstrate the following characteristics of an excess demand equilbrium in which the state is the low quality producer.

PROPOSITION 11. *The following properties hold in an excess demand equilbrium when* $m_s \leq m_l \leq m_h$*:*

(a) $m_h > m_l$

(b) $q_h < q_l$

(c) $\pi_h > \pi_l$

Using (14) and (15), it is simple to analyze the impact of a cut-back in state provision.

PROPOSITION 12. *As a result of a small cut-back in state provision, the high and low quality private firms cut-back their quality levels and intra-private product differentiation increases.*

PROPOSITION 13. *As a result of a small cut-back in state provision, both private firms supply more goods. However, there is a decline in overall consumer participation in the market and* t_s *increases.*

PROPOSITION 14. *As a result of a small cut-back in state provision, the profits of the high quality firm increase while the impact on the low quality firm's profits is ambiguous. The impact on producer surplus is amboguous. Producer surplus is more likely to increase when:*

(a) $m_h - m_l$ *is low;*

(b) q_l *is low;*

(3) q_s *is high;*

By Proposition 14 producer surplus is likely to increase when state provision is cut back under the following initial circumstances: intra-private sector differentiation is low; the low quality private producer has a small market share; the state has a large market share.

Intuitively, a cut-back in public provision has an ambiguous effect on consumer welfare. Consumers buying state goods are worse off because there are less goods available and side-payments are higher. Consumers buying in the private sector may be either worse or better off because there are more private goods available, but the average quality level falls.

8. CONCLUSIONS

The paper's analysis suggests several important lessons for studying the behavior of private firms in a parallel market with excess demand. When the state is either the high or low quality producer, a cut-back in state provision increases intra-private product differentiation. However, when the state is the intermediate quality producer, a cut-back in state provision decreases intra-private quality differentation. In all three cases, a cut-back in public provision induces an increase in aggregate private supply, a decline in consumer participation and an increase in state

sector side-payments.

Assuming that a local government has no control over state prices or quality level, it may have an incentive to reduce the provision of state goods. When the state is the high or intermediate quality producer, a reduction in state provision increases private producer surplus. When the state is the low quality producer, a reduction of state provision under plausible conditions may also increase private producer surplus.

When the state is the high quality producer, consumer surplus always decreases when there is cut-back in state provision. However, when the state is the intermediate or low quality producer, the impact is ambiguous. Thus, depending upon the value that a local government assigns to consumer and producer surplus, a small withdrawal of the state from the market unaccompanied by a transfer of assets to the private sector can be welfare improving.

While this analysis is at the level of an example, it is conducted at about the same degree of generality as papers by Gabszewicz and Thisse (1980), Shaked and Sutton (1983) and Gal-Or (1985). In this simple framework, this paper teaches that, in an excess demand equilibrium, a marginal cut-back of state provision unaccompanied by transfers of assets or materials to the private sector may be beneficial. Future research should examine the extent to which this lesson is robust to a world with free entry of private firms and a more general distribution of consumer tastes.

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[1]In the Soviet Union, many local governments are trying to discriminate against non-residential buyers in state shops by issuing ration coupons to its residents. Because these coupons can be bought very cheaply in the private sector, it is believed that this discrimination is not very effective.

[2]The modelling in this section generalizes Gal-Or (1985) to incorporate a passive state supplier.

[3]In a more realistic set up, consumers might be characterized not only by a willingness to make side-payments, but also by a willingness to pay the official posted price.

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