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Abstract

This paper presents a theoretical analysis of the effects of imperfect price discrimination in a differentiated products Bertrand oligopoly. Three types of discrimination are considered: that based on differences in industry demand elasticity (i.e. sensitivity to changes in a common industry price), that based on differences in brand elasticity (i.e. sensitivity to price differentials between firms), and spatial discrimination. Limit results are obtained which show that, when markets are approximately competitive, industry demand elasticity discrimination always increases total output and surplus, while brand elasticity discrimination has the opposite effects. An example is presented in which these results hold for more general degrees of competitiveness. In this example it is shown that spatial discrimination always results in lower prices for all consumers, which increases total surplus but decreases industry profits.

1. Introduction

This paper presents a theoretical analysis of the effects of imperfect price discrimination in an oligopoly in which prices are determined non-cooperatively. The forerunner of this research is the analysis of the monopoly case begun by Pigou (1929) and Robinson (1934) and recently investigated by Schmalensee (1981) and Varian (1985). The main result of this literature is that while perfect price discrimination increases total output and total surplus relative to the inefficient uniform price monopolist, imperfect price discrimination has ambiguous effects on total output and surplus. Imperfect discrimination results in lower prices in some markets and higher prices in other markets than would occur otherwise. The gains in output and surplus from lower prices in the former markets may or may not outweigh the losses in the latter markets. The direction of the effect depends on a condition on the relative curvature of demand in the various markets.

In this paper it is shown that the results for the polar monopoly case are inapplicable to the more general case in which the market power of firms is limited. For example, it is shown that when discrimination is based on differences in the underlying reservation price for the product of the industry and when markets are approximately perfectly competitive, then discrimination has an unambiguously positive impact on total output and surplus, in contrast to the Robinson-Schmalensee result. In fact, there are two additional types of discrimination that can occur with competition which do not occur with collusion. It is found that in the non-cooperative case welfare conclusions generally depend on the type of discrimination practiced rather than on any condition on the relative curvature of industry demand in the various markets.

Analysis of discrimination in markets which are somewhat competitive is important because the practice is prevalent. Witness, for example, the common use of coupons, periodic sales, and senior citizen discounts in retailing and the use of super-saver airfares. (Phlips (1983) discusses other common business practices.) Recently Beilock (1985) has empirically documented price discrimination in the unregulated trucking industry in Florida. The early writers Clemens (1950-51) and Wright (1965) stressed the importance of analyzing price discrimination in a competitive context. The profession has belatedly begun to recognize this fact as there are a variety of recent theoretical analyses of the subject including Katz (1984), Borenstein (1985), Spulber (1979, 1984), Oren, Smith and Wilson (1983), and Panzar and Postlewaite (1984).

The present analysis is conducted in a model in which the products of competing firms are exogeneously differentiated and prices are determined by Bertrand competition. The buying public is segmented into n markets (for example, into the senior citizen market and the under 65 market). Firms are assumed to be able to base price on market membership giving rise to imperfect discrimination in the non-cooperative equilibrium. Three types of discrimination are classified. First, buyers in different markets may tend to differ in the strength of their underlying reservation price for the product of the industry. Price discrimination occurs on this basis as it does in the collusive case. This type is referred to as industry demand elasticity discrimination. Second, buyers in different markets may tend to differ in the degree to which they find competing products to be substitutes giving rise to what is referred to as brand elasticity discrimination. Third, buyers in one market may have a strong preference for a particular firm while buyers in another market have a strong preference for a second firm. The resulting discrimination is denoted spatial discrimination. With collusion only the first type is found.

In the analysis, total output, total surplus, and profit in the discriminatory regime are compared with their corresponding values in a regime in which discrimination is infeasible. General results are obtained for the case in which all markets are approximately perfectly competitive, i.e. the case in which competing products are close substitutes. The analysis complements the works of Robinson and Schmalensee which analyze the opposite polar case of pure monopoly. It is shown that in approximately competitive markets discrimination based solely on differences in brand elasticity has unambiguously negative consequences on total output and total surplus while that based solely on differences in industry demand elasticity has positive effects. To extend these results an example with linear industry demand is constructed in which explicit solutions for equilibrium variables are obtained. For this demand structure Robinson has shown that monopoly price discrimination leaves output unchanged and decreases total surplus. It is shown here, however, that this surplus decrease occurs only at a relatively high degree of monopoly power. For a wide range of parameters, industry demand discrimination increases total output and surplus. The last part of the analysis uses the linear demand model to analyze spatial discrimination. For this case the surprising result is obtained that discrimination decreases the equilibrium price paid by all consumers, increasing total surplus while decreasing industry profits.

Of the recent analyses of price discrimination, Katz (1984) and Borenstein (1985) are the most relevant to the present one. Katz uses the Salop and Stiglitz (1981) "Bargains and Ripoffs" model to investigate the welfare consequences of the ability of firms to discriminate against buyers with poor price information. In his analysis demand is inelastic and quantity purchased is independent of regime. Total surplus is dependent only on the efficiency of the number of firms. In contrast, in the present analysis, surplus is dependent on the size and distribution of output while the number of firms is exogenous. Borenstein presents a generalization of the Salop (1979) circle model to determine the effects of discrimination on free entry zero profit equilibria. He also makes the distinction between discrimination based on industry demand elasticity and that based on brand elasticity. He advances conclusions on their relative merits based on numerous simulations which corroborate the analytical results presented here.

The remainder of the paper is organized as follows. Section 2 presents the model and defines imperfect discrimination and the concepts of industry demand and brand elasticity. Section 3 presents a construction in which the state of the industry varies by a parameter representing the degree of market power. Conditions determining the effects of industry demand and brand elasticity discrimination when markets are approximately competitive are presented. Section 4 presents the example of linear industry demand and generalizes the results of Section 3 for that case. Section 5 analyzes spatial discrimination. Section 6 concludes.

2. The Model

The industry considered is comprised of m firms. The variant of the product sold by each firm is in general differentiated from the variants sold by the other firms and is assumed to be exogenously determined.[1] The cost of producing x units for each firm is cx, i.e. all the firms in the industry share the constant marginal cost of c.

Each buyer purchases at most one unit of the product from at most one firm. Buyers are heterogenous. The type of each buyer is denoted by the m-tuple $(r^1, r^2, \ldots, r^m) = \underline{r} \in R^m$, where r^j is the dollar valuation or reservation price of the buyer for product j. Suppose that a buyer of type $\underline{r}$ faces the price vector $(p^1, p^2, \ldots, p^m)$ where p^j is the price of product j. The decision rule of this utility maximizing consumer is to purchase from firm j with probability one if $r^j - p^j \geqslant 0$ and if $r^j - p^j > r^k - p^k$ for $k \neq j$. In the event that the buyer's maximum utility is obtained by purchase from one of several firms, it is assumed that the buyer randomizes somehow between these firms.[2]

Firms are said to be able to practice _perfect_ or _first-degree_ price discrimination if each firm can observe the type of each buyer and set price on that basis. Denote $p^j(\underline{r})$ as the price set by firm j to type $\underline{r}$ in the regime in which perfect price discrimination is feasible. By the familiar Bertrand argument, the following is the unique equilibrium outcome of price competition

$$p^j(\underline{r}) = c \quad \text{for} \quad j \neq j'$$

$$p^{j'}(\underline{r}) = c + r^{j'} - \max[0, r^{j''}]$$

$$\text{where } j' = \underset{j}{\text{argmax}}\ r^j \text{ and } j'' = \underset{j \neq j'}{\text{argmax}}\ r^j.$$

In this regime the price charged by each firm to each buyer is equal to cost except for the price set for the product most valued by the buyer. The latter price is set so that the buyer is indifferent to purchasing this product and his next highest valued product at cost. As in the monopoly case, perfect price discrimination exhausts all gains from trade. The difference here is the allocation of the surplus.

The practice of perfect price discrimination entails enormous informational requirements, presents many logistical difficulties, and is likely to be illegal. However, it may be feasible for firms to practice imperfect price discrimination. To model this, it is assumed here that the population of all buyers is partitioned into n subsets which are termed "groups of buyers" or "markets". The density of type $\underline{r}$ in market i is denoted by the continuously differentiable function $f_i(\underline{r}) \geq 0$, $\underline{r} \in R^m$. Firms are said to be able to practice imperfect or third-degree price discrimination if each firm can observe the market each buyer belongs to and set price on that basis.[3]

For example, the population of buyers may be partitioned into the two subsets of those buyers over and under 65 years of age. Firms may be able to discriminate by offering senior citizen discounts. Because each group of buyers is generally made up of a wide variety of buyer types, membership in a market is not a perfect indicator of buyer type. However, because the distribution of types generally differs across markets, membership in a market is informative. In this analysis, three ways in which markets may differ are distinguished.

First, buyers in one market may tend to have a lower reservation price for the product of any firm than buyers in another market. One might argue that senior citizens have low incomes and generally may not purchase from any firm if all firms set a uniformly high price. Such a market will be said to have high industry demand elasticity.

Second, buyers in one market may tend to view the competing firms' products as close substitutes while buyers in another market find them to be poor substitutes. One might argue that a retired senior citizen has lower time costs to travel across town than an employed middle-aged buyer. A market containing such buyers will be said to have high brand elasticity because of the readiness of these buyers to switch brands when there is only a small price differential.

Third, the buyers in one market may have a stronger preference for the product of one particular firm than buyers in another market. For example, in the spatial models buyers may be classified as those located near one firm and those located near a second firm. Because of transportation costs, the former buyers have a stronger preference for the first firm and a weaker preference for the second firm, while the latter buyers' preferences are reversed.

The above three ways in which markets may differ result in three categories of imperfect price discrimination, defined respectively as industry demand elasticity, brand elasticity, and spatial discrimination. For most of this paper, a symmetry condition on the position of each firm in each market is imposed. Since asymmetry of market position is the essence of spatial discrimination, this form does not occur under the assumption. In Section 5, this assumption is relaxed and spatial discrimination is explored.

The symmetry assumption is the following.

Assumption 1.

$$f_i(r^1,r^2,\dots,r^m) = f_i(r^m,r^1,r^2,\dots,r^{m-1}),\ i = 1,\dots,n.$$

Note that this assumption is more general than it appears since firms can be renumbered so that it holds.

Throughout this analysis, it will be convenient to use reduced form demand curves rather than cumbersome integrals over the densities f_i. Let p_i^j be the price set by firm j in market i. Let $x_i^j(p_i^1,\dots,p_i^m)$ be the demand of firm j in market i given these prices. Based on the behavior of buyers described above and the distribution of buyer types in market i, this function is determined as follows (it simplifies notation to calculate the demand of firm 1).

$$(1)\qquad x_i^1(p^1,\ldots,p^m) = \int_{p^1}^{\infty} \int_{-\infty}^{r^1-p^1+p^2} \cdots \int_{-\infty}^{r^1-p^1+p^m} f_i(r^1,r^2,\ldots,r^m)dr^m\ldots dr^2dr^1.$$

By differentiating (1) and evaluating assuming equal prices $p^1 = p^2 = \ldots p^m = p$ one obtains the following symmetry condition on the cross partials.

$$(2)\qquad \frac{\partial x_i^j\,(p,\ldots,p)}{\partial p^k} = \frac{\partial x_i^k\,(p,\ldots,p)}{\partial p^j}$$

This does not depend on the symmetry assumption. Assumption 1, however, is needed to establish the following symmetry property. (It simplifies notation to express it in terms of firms 1 and 2.)

$$(3)\qquad x_i^1\,(p,\tilde{p},\tilde{p},\ldots,\tilde{p}) = x_i^2\,(\tilde{p},p,\tilde{p},\ldots,\tilde{p})$$

This just states that the demand in market i of firm j when it sets some price p while all other firms set another price $\tilde{p}$ is the same as the demand of any other firm k when it sets price p and all other firms set price $\tilde{p}$. This symmetry property allows the convenience of conducting the entire analysis and defining all concepts in terms of the demand of a single firm, which is taken to be firm 1.

The industry demand $z_i(p)$ is defined as the total industry output in market i when all firms set the symmetric price p, i.e.
$z_i(p) = \sum_{j=1}^{m} x_i^j(p,\ldots p) = mx_i^1(p,\ldots,p)$. Using the fact that $f_i(\ ) \geqslant 0$ all $\underline{r} \in R^m$ and by differentiating equation (1), it can be shown that the following restrictions can be placed on $x_i^j(p^1,\ldots,p^m)$ and $z_i(p)$.

(4a) $\dfrac{\partial x_i^j(p^1,\ldots,p^m)}{\partial p^j} \leqslant 0$ Downward sloping individual firm demand curve.

(4b) $\dfrac{\partial x_i^j(p^1,\ldots,p^m)}{\partial p^k} \geqslant 0, \; j \neq k$ Firm's products are substitutes.

(4c) $z_i'(p) = m \sum_{j=1}^{m} \dfrac{\partial x_i^1 (p,\ldots,p)}{\partial p^j} \leqslant 0$ Downward sloping industry demand curve.

It is useful to write the slope of the individual demand curve as follows

(5) $$\frac{\partial x_i^j(p,\ldots,p)}{\partial p^j}$$

$$= \frac{\partial x_i^j(p,\ldots,p)}{\partial p^j} + \sum_{k\neq j} \frac{\partial x_i^j(p,\ldots,p)}{\partial p^k} - \sum_{k\neq j} \frac{\partial x_i^k(p,\ldots,p)}{\partial p^j}$$

$$= \frac{1}{m} z_i'(p) - \sum_{k\neq j} \frac{\partial x_i^k(p,\ldots,p)}{\partial p^j} .$$

This representation uses (2) the equality of the cross partials of the demand functions at symmetric prices. The equation states that the decline in the volume of sales incurred by a firm which raises its price can be interpreted as having two parts.

The first part, the slope of the industry demand curve weighted by the firm's share of industry demand, accounts for the buyers who no longer purchase from any firm in the industry because of the price increase, who otherwise would have purchased. The tendency of buyers in market i to act this way can be measured by the following elasticity.

(6) $$D_i(p) = \frac{\frac{-1}{m} z_i'(p)p}{\frac{1}{m} z_i(p)} = \frac{-z_i'(p)p}{z_i(p)}$$

This quantity will be referred to as the industry demand elasticity of market i. If the buyers in market i tend to have low reservation prices for the products of all the firms then market i tends to have a high industry demand elasticity.

The second part of the firm's decline of sales in (5) is the loss of buyers who substitute to products of competitors of the firm because their prices become relatively lower. This tendency can be measured by the following elasticity:

(7) $$B_i(p) = \frac{\sum_{j \neq k} \frac{\partial x^k(p,\ldots,p)}{\partial p^j}}{\frac{1}{m} z(p)} p$$

This will be referred to as the brand elasticity of market i. If the market has relatively high brand elasticity then buyers who view competing firms' products as close substitutes (i.e. have reservation prices for competing firms which are close in value) tend to be relatively more dense in population weight than buyers who view the firms products as poor substitutes.[4]

The equilibrium price in each market when firms can imperfectly discriminate and compete in each market separately is now determined. Suppose that in market i firm 1 sets price p^1 and all other firms set price p. The profit of firm 1 in market i is then

$$(8) \qquad \pi_i(p^1,p) = (p^1-c)x_i^1(p^1,p,\ldots,p).$$

Firm 1's first-order condition is

$$(9) \qquad \frac{\partial \pi_i(p^1,p)}{\partial p^1} = x_i^1(p^1,p,\ldots,p) + (p^1-c)\frac{\partial x_i^1(p^1,p,\ldots,p)}{\partial p^1} = 0.$$

In a symmetric equilibrium, $p^1 = p$. In this case (9) reduces to

$$(10) \qquad \frac{\partial \pi_i(p,p)}{\partial p^1} = \frac{1}{m} z_i(p) + (p-c)\frac{\partial x_i^1(p,\ldots,p)}{\partial p^1} = 0.$$

It is assumed that a price p^* exists which solves (10) and that it is the globally optimal response of firm 1 when all other firms offer p^*, i.e. $\pi(p^*,p^*) > \pi(p,p^*)$ for $p \neq p^*$. This implies that p^* is a symmetric Nash equilibrium of Bertrand competition. It is further assumed that this equilibrium is the unique equilibrium. It is well known that an equilibrium does not always exist in models such as this. However, we note that the above equilibrium assumptions (e.g. uniqueness) are not vacuous as examples can be provided in which they hold.

It is useful to rewrite (10) in elasticity terms. Substituting (5) into (10) yields

$$(11) \qquad \frac{1}{m}z_i(p) + (p-c)\left[\frac{1}{m}z_i'(p) - \sum_{j\neq 1} \frac{\partial x_i^j(p,\ldots,p)}{\partial p^1}\right] = 0.$$

Dividing through by $\frac{1}{pm}z_i(p)$ and using (6) and (7) yields the following equilibrium condition

$$(12) \qquad p - (p-c)\left[D_i(p) + B_i(p)\right] = 0,$$

which can be rewritten as

$$(12') \qquad \frac{p - c}{p} = \frac{1}{D_i(p) + B_i(p)}.$$

The markup over cost is determined by the familiar inverse demand elasticity rule noting that the individual firm demand elasticity is the sum of the industry demand and brand elasticities. As the brand elasticity in market i approaches zero, the formula reduces to exactly the standard monopoly formula. In this limiting case all population weight is on buyers who find the competing firms products not substitutable at all.[5] In fact, taking an approach similar to Triffen (1956), one could define pure monopoly as what occurs in this limiting case.[6]

The other limiting case is the one in which the brand elasticity is infinitely high. In this case the firms' products are perfect substitutes and by the familiar Bertrand argument, price equals marginal cost. If all markets are infinitely brand elastic, there is no price discrimination even if markets differ by industry demand elasticity since perfectly competitive prices are determined on the supply side, not the demand.

When $0 < B_i(p) < \infty$, market i is neither purely monopolistic nor perfectly competitive. Since, in general, demand and brand elasticity differ across markets, from (12') there is, in general, a different equilibrium price in each market, meaning that there is price discrimination in the imperfectly competitive equilibrium.

3. The Output and Welfare Effects of Discrimination

The equilibrium discriminatory price in each market determines the level of output, profit, and total surplus in each market. Summing across markets results in the level of aggregate output, profit and surplus. In this section these levels are compared with their corresponding values in a regime in which firms are constrained to offer a non-discriminatory price.

In the limit at which brand elasticity is zero in all markets, the analysis is analytically equivalent to the pure monopoly analysis of Robinson and Schmalensee. In this case any form of discrimination is necessarily based on differences in industry demand elasticity. Robinson and Schmalensee have shown that in this case discrimination has ambiguous effects on total output and surplus. The direction of the effect on output depends on the relative shapes of the industry demand curves in the different markets. A necessary but insufficient condition for total surplus to increase is that output increase. This follows from the allocational inefficiency caused by discrimination. The practice can only increase profits because prices are determined by the solution to a single optimization problem. Expanding the choice set can only make the maximized value of the objective function higher. Since these results hold in the limit of pure monopoly power, a continuity argument can be used to show that they hold when the brand elasticity in each market is arbitrarily small.

When the market power of competing firms is limited, the Robinson-Schmalensee analysis is not applicable. Differences in brand elasticity are an important basis for discrimination in the competitive case which must be accounted for in any analysis of discrimination. In addition, it is shown below that even when discrimination is based solely on differences in industry demand elasticity the effects are markedly different with competition.

Of course, for general preferences a cutoff point of some measure of market power cannot be obtained below which discrimination has certain competitive characteristics and above which it has certain monopoly characteristics. What is accomplished is a demonstration of the features of discrimination in markets close enough to the limit of perfect competition.

In the analysis a construction is presented in which the monopoly power of firms in the industry is interpreted as varying by a parameter α. In the limit as α approaches zero the competing firms' products are regarded as perfect substitutes by buyers in all markets. At this limit point, price is equal to marginal cost in all markets regardless of the regime and output and total surplus are identical whether or not discrimination is feasible. Comparative statics are conducted to determine the effect on prices of an increase in α (market power) at $\alpha = 0$. The effect of the change in prices on total output and total surplus is then determined and conditions are obtained under which, for small α, these variables are higher with discrimination than without it.

Analysis

Let the parameter $\alpha \in [0,\bar{\alpha})$ $\bar{\alpha} > 0$ index a particular realization of the model described above, i.e., a particular "world." The unit cost c and the number of markets n are assumed to be independent of α but the distribution of types in each market is assumed to vary with α. Let $f_i(r^1,\ldots,r^m,\alpha)$ denote the density of type $(r^1,\ldots,r^m)$ in market i in world $\alpha, i=1,\ldots,n$. The individual firm demand curve of firm j in market i, $x_i^j\ (p^1,\ldots,p^m,\alpha)$, is derived from this density using equation (1). All derived functions, e.g. $z_i(p,\alpha)$ and $D_i(p,\alpha)$, are now written to denote functional dependence on α. To simply the presentation, all assumptions are stated in terms of these derivative concepts rather than the primitives. The following definitions are needed.

Definitions

$$(13) \qquad \phi_i(p,\alpha) = \frac{\frac{1}{m} z_i(p,\alpha)}{\sum_{k \neq j} \frac{\partial x_i^k(p,\ldots,p,\alpha)}{\partial p^j}} \qquad i = 1,\ldots,n$$

$$(14a) \qquad RD_{ij}(p,\alpha) = \frac{D_i(p,\alpha)}{D_j(p,\alpha)}$$

$$(14b) \qquad RB_{ij}(p,\alpha) = \frac{B_i(p,\alpha)}{B_j(p,\alpha)}$$

$\phi_i(p,\alpha)$ is the inverse of the brand elasticity divided by the price which is used rather than the brand elasticity for analytical convenience. RD_{ij} and RB_{ij} are respectively the ratios of the demand and brand elasticities between markets i and j.

The following are assumed to hold

Assumption 2. $z_i(p,0) < \infty$, $\phi_i(p,0) < \infty$ for $p \geqslant 0$.

Assumption 3. $z_i(p,\alpha)$ and $\phi_i(p,\alpha)$ are continuously differentiable for $p \geqslant 0$ and $p \geqslant 0$ and $\alpha \in [0,\bar{\alpha})$.

Assumption 4. $z_i(c,0) > 0$, $\frac{\partial z_i(c,0)}{\partial p} < 0$, and

$$\sum_{i=1}^{n} z_i(c,0) = z_{ND}(c,0) = 1 \qquad \text{(Normalization)}.$$

<u>Assumption 5.</u> $\phi_i(p,0) = 0, \quad \frac{\partial\phi_i(p,0)}{\partial p} > 0 \quad \text{for } p \in [c,\bar{p}) \text{ some } \bar{p} > c.$

<u>Assumption 6.</u> $0 < RB_{ij}(p,0) < \infty$

$$\text{where } RB_{ij}(p,0) = \lim_{\alpha\to 0} RB_{ij}(p,\alpha).$$

Assumptions 2 and 3 state that the functions $\phi_i(p,\alpha)$ and $z_i(p,\alpha)$ are real valued continuously differentiable functions over the entire range of α. Assumption 4 states that at price equal to marginal cost and $\alpha = 0$, industry demand is strictly positive and strictly downward sloping. Without loss of generality, the industry demand summed over all markets is assumed to be unity at price equal to marginal cost. Thus z_i can be interpreted as the proportion of total output at the competitive price made up of purchases in market i. Assumption 5 makes precise the assumption that increases in α are associated with decreases in brand elasticity. In the limit as α goes to zero, $\phi_i(p,0) = 0$, or, equivalently, in the limit brand elasticity is infinite. As α is increased, ϕ_i increases above zero. Assumption 6 states that even though the brand elasticity in each market approaches infinity, the ratio of the brand elasticities between markets approaches a finite limit.

To understand the above construction it is helpful to think in terms of the following sketch of a special case of it. Consider a generalized version of the standard Hotelling (1929) spatial model with m firms at equidistant locations around a circle. Suppose that a market consists of buyers with various transport cost functions and various reservation prices for a good <u>delivered</u> at their location and that the various kinds of buyers within a market are uniformly distributed around the circle. Holding the willingness to pay for a delivered good as fixed, suppose the transport cost function of each consumer is varied by a multiplicative parameter α. In the limit as α goes to zero, transport costs are zero for all buyers who then regard the competing firms products as perfect substitutes. A version of this example is presented in Section 4.

Equilibrium Prices

The symmetric first-order equilibrium condition (9) determining the equilibrium discriminatory price in market i, after manipulation and use of the definition of $\phi_i(p,\alpha)$, can be rewritten as

$$(15) \qquad EC_i(p,\alpha) = \phi_i(p,\alpha) - (p-c)[\frac{D_i(p,\alpha)}{p} \phi_i(p,\alpha) + 1] = 0.$$

At $p = c$ and $\alpha = 0$, $EC_i(c,0) = 0$, $i = 1,\ldots,n$, i.e. when competing firms' products are perfect substitutes price equals marginal cost. Let $p_i(\alpha)$ solve $EC_i(p_i(\alpha),\alpha) = 0$.

In the regime in which discrimination is infeasible, each firm is constrained to offer one price in all markets. In effect, the buyers in the n different markets are pooled into one aggregate market with aggregate or non-discriminatory demand defined by $x^j_{ND}(p^1,\ldots,p^n) = \sum_{i=1}^{n} x^j_i(p^1,\ldots,p^n)$ for firm j. The associated industry demand and brand elasticities $D_{ND}(p,\alpha)$ and $B_{ND}(p,\alpha)$, as well as $\phi_{ND}(p,\alpha)$ can be calculated from x_{ND}. The equilibrium non-discriminatory price is then determined from (15) with i = ND, and is denoted $p_{ND}(\alpha)$.

In order to compare output and surplus in the two regimes for small α, the effect of a change in α on prices at α equal to zero must be evaluated. Totally differentiating $EC_i(p,\alpha)$ and evaluating at $p = c$ and $\alpha = 0$ yields

$$(16) \qquad p_i'(0) = \frac{\partial \phi_i(c,0)}{\partial \alpha} \qquad i = 1,\ldots,n,ND$$

Thus the first-order effect of a change in α on price at α equal to zero is the change in ϕ, which is the change in the inverse of the brand elasticity (without the price term). This change is positive by Assumption 5.

The greater degree of market power captured by higher α results in higher prices in both regimes reducing total surplus relative to the efficient allocation. The relative efficiency of the two regimes depends in a sense on how the effect of α on the non-discriminatory price compares with the average of the effects on the discriminatory prices. Using L'Hopital's rule, it is shown in Appendix A that the following relationship between the non-discriminatory and the discriminatory prices holds.

$$(17) \qquad p_{ND}'(0) = \frac{\partial \phi_{ND}}{\partial \alpha} = \frac{\sum_{i=1}^{n} z_i}{\sum_{i=1}^{n} \frac{z_i}{\frac{\partial \phi_i}{\partial \alpha}}} = \frac{1}{\sum_{i=1}^{n} \frac{z_i}{p_i'}}$$

Not that functions of p and α are implicitly evaluated at $p = c$ and $\alpha = 0$.

Recalling the interpretation of z_i as the proportion of total sales at the competitive price made up of sales in market i, the relationship states that the change in the non-discriminatory price is the harmonic mean of the change in the discriminatory prices. By Jensen's inequality, this is less than the arithmetic mean of the change in the discriminatory prices. Thus brand elastic buyers have a disproportionate effect in causing the non-discriminatory equilibrium price to be at a low level. Intuitively, the presence of the those buyers causes the marginal benefit of slightly undercutting competitors to be relatively high because these buyers are so sensitive to slight price differentials.[7]

Having determined the relationship between equilibrium prices in the two regimes, total output and surplus in the two regimes can be compared. In the discriminatory regime, the measures can be written as the following functions of α.

Output $$Q_D(\alpha) = \sum_{i=1}^{n} z_i(p_i(\alpha),\alpha).$$

Profit $$\pi_D(\alpha) = \sum_{i=1}^{n} (p_i(\alpha)-c)z_i(p_i(\alpha),\alpha)$$

Consumers Surplus $$C_D(\alpha) = \sum_{i=1}^{n} \int_{p_i(\alpha)}^{\infty} z_i(q,\alpha)dq$$

Total Surplus $$TS_D(\alpha) = C_D(\alpha) + \pi_D(\alpha)$$

The analogous quantities in the non-discriminatory regimes, $Q_{ND}(\alpha)$, $\pi_{ND}(\alpha)$, $C_{ND}(\alpha)$, $TS_{ND}(\alpha)$ are calculated by substituting $p_{ND}(\alpha)$ for each $p_i(\alpha)$ in the corresponding definitions above.

At $\alpha = 0$ prices are marginal cost in all markets in both regimes which implies that all of the above measures are identical in both regimes. Therefore for small α, the comparison depends on how the derivitives of these measures differ in the two regimes when evaluated at $\alpha = 0$.

The first proposition compares the two regimes in the general case in which markets differ by brand elasticity in the limit (i.e. $RB_{ij}(c,0) \neq 1$ for some $i \neq j$).

Proposition 1. Suppose for some i and j, $RB_{ij}(c,0) \neq 1$. Then the following hold:

(ia) $\pi_D'(0) > \pi_{ND}'(0)$

(ib) $TS_D'(0) = TS_{ND}'(0)$

(ic) $C_D'(0) < C_{ND}'(0)$

(ii) If $RD_{ij}(c,0) = 1$, all i,j ($D_i(c,0) = D_j(c,0)$ all i,j), then

$Q_D'(0) < Q_{ND}'(0)$ and $TS_D''(0) < TS_{ND}''(0)$.

(iii) Let n = 2 and without loss of generality assume that $RB_{12} > 1$.

(a) $Q_D'(0) \gtreqless Q_{ND}'(0)$ as $RD_{12} \gtreqless RB_{12}$.

(b) $TS_D''(0) > TS_{ND}''(0)$ if $RD_{12} > \dfrac{RB_{12}+(RB_{12})^2}{2}$.

Proof. $RB_{ij}(p,\alpha) = \dfrac{\phi_j(p,\alpha)}{\phi_i(p,\alpha)}$. Since $\phi_i(p,\alpha) = 0$, all i, by L'Hopital's rule

$$RB_{ij}(c,0) = \frac{\frac{\partial\phi_j(c,0)}{\partial\alpha}}{\frac{\partial\phi_i(c,0)}{\partial\alpha}} = \frac{p_j'(0)}{p_i'(0)}$$

Since $RB_{ij} \neq 1$ some i,j, $p_i' \neq p_j'$ for such i,j.

Proof of (ia). $\pi_D'(0) - \pi_{ND}'(0) = \sum_{i=1}^{n} z_i(p_i' - p_{ND}')$

$$= \sum_{i=1}^{n} z_i p_i' - \frac{1}{\sum_{i=1}^{n} \frac{z_i}{p_i'}}$$

> 0 by Jensen's inequality.

Note that the second equality follows from (17). Proofs of (ib) and (ic) are left to the reader.

Proof of (ii). Let $D_i = \frac{-z_i'}{z_i} = D$ which is constant for all i by hypothesis.

$$Q_D'(0) - Q_{ND}'(0) = \sum_{i=1}^{n} z_i'(p_i' - p_{ND}')$$

$$= D[\sum_{i=1}^{n} -z_i p_i' + \frac{1}{\sum_{i=1}^{n} \frac{z_i}{p_i'}}]$$

$$< 0 \text{ by Jensen's inequality.}$$

The fact that $TS_D''(0) < TS_{ND}''(0)$ is proved similarly. The proof of (iii) is in Appendix B. Q.E.D.

The premise of this proposition is that at least two markets differ by brand elasticity in the limit. Because of this, the effect of a change in α on the price differs in these markets, i.e. $p_i'(0) \neq p_j'(0)$, some i,j. As discussed above, the average increase in price is higher in the discriminatory regime. This results in higher profits by a first-order difference, as stated by Proposition 1(i). Because total surplus is maximized at price equal to marginal cost, by the envelope theorem there is no first-order difference in surplus in the two regimes. The first-order increase in surplus attained by the firms is then at the expense of a first-order loss in surplus by consumers.

Proposition 1(ii) states that if markets have identical industry demand elasticity, then discrimination results in an unambiguous first-order decline in output and a second-order decline in total surplus. The intuition is straightforward. Discrimination results in an average price which is higher than the non-discriminatory price, implying the price increase from discrimination in the low brand elasticity markets is proportionally greater than the price decrease in the high brand elasticity markets. Since all markets have the same industry demand elasticity, the decline in output in the former markets is greater than the increase in output in the latter markets, resulting in a net decrease in output. Total surplus decreases because of the output decrease and the allocational inefficiencies resulting from discrimination.

Proposition 2(iiia) presents an intuitive condition under which output increases with discrimination for the case of two markets. If industry demand elasticity and brand elasticity are positively correlated, and if the ratio of the high to low industry demand elasticities is greater than the corresponding ratio of brand elasticities, output increases with discrimination by a first-order difference. Even though discrimination results in a proportionally greater price increase to the low brand elasticity buyers than a price decrease to the high brand elasticity buyers, the former have lower industry demand elasticity and the net effect on output is positive. The condition under which total surplus increases is that the ratio of industry demand elasticities be larger than the average of the ratio of the brand elasticities and the square of that term. This is a stronger condition than is needed for output to increase, which is in accord with the Schmalensee result that an output increase is a necessary condition for a total surplus increase.

In the monopoly analysis of Robinson and Schmalensee, brand elasticity is identically zero in all markets and discrimination is based entirely on differences in industry demand elasticity. Analogous to this in the competitive case is the situation in which brand elasticity is identical in all markets in the limit as α goes to zero, i.e. $RB_{ij}(c,0) = 1$ for all i,j. This is assumed to hold in Proposition 2 below. By L'Hospital's rule this is equivalent to the assumption that

$$\frac{\partial \phi_i(c,0)}{\partial \alpha} = \frac{\partial \phi_j(c,0)}{\partial \alpha} \text{ for all } i,j.$$

(since $\phi_i(c,0) = 0$ for all i). From (17) the first-order effect on prices is the same in all markets (including the non-discriminatory case) and is denoted p'. Because of this, price discrimination has no first-order effects on output and profits and no second-order effect on total surplus. Differences between regimes are then accounted for by differences in the second-order effect of α on prices. Differentiating $EC_i(p,\alpha)$ from (15) twice results in

$$(18) \qquad p_i''(0) = 2\,\frac{\frac{\partial z_i}{\partial p}}{z_i}\,p'^2 + k \qquad i = 1,\ldots,n,ND$$

$$= \frac{-2D_i p'^2}{c} + k$$

where k is a term which depends on second derivatives of ϕ_i which will be assumed to be identical for all i. The above equation shows that the second-order effect of a change in α on prices is higher the lower the industry demand elasticity, D_i, which is used in the following result.

<u>Proposition 2</u>. Suppose that $RB_{ij} = 1$ for all i,j (which is equivalent to the assumption that $\frac{\partial \phi_i}{\partial \alpha} = \frac{\partial \phi_i}{\partial \alpha}$, all i,j at p=c and α=0). Suppose that the second derivatives of $\phi_i(p,\alpha)$ are identical at this point for all i. Suppose, however, that $D_i(c,0) \neq D_j(c,0)$ some i,j. Then the following hold. (In addition to the fact that $Q_D'(0) = Q_{ND}'(0)$, $TS_D''(0) = TS_{ND}''(0)$, and $\pi_D'(0) = \pi_{ND}'(0)$).

(a) $\qquad Q_D''(0) > Q_{ND}''(0)$

(b) $\qquad TS_D'''(0) > TS_{ND}''(0)$

(c) $\pi_D''(0) = \pi_{ND}''(0)$.

Proof of (a). From (18)

$$p_i'' - p_{ND}'' = 2p'^2 \left[\frac{\frac{\partial z_i}{\partial p}}{z_i} - \frac{\frac{\partial z_{ND}}{\partial p}}{z_{ND}} \right]$$

Twice differentiating $Q_D(\alpha) - Q_{ND}(\alpha)$ and evaluating at $\alpha = 0$ yields

$$Q_D''(0) - Q_{ND}''(0) = \sum_{i=1}^{n} \frac{\partial z_i}{\partial p} (p_i'' - p_{ND}'')$$

$$= 2p'^2 \left[\sum_i \frac{\left(\frac{\partial z_i}{\partial p}\right)^2}{z_i} - \frac{\left(\sum_i \frac{\partial z_i}{\partial p}\right)^2}{(\sum_i z_i)} \right]$$

$$> 0$$

where the inequality follows from the fact that $D_i \neq D_j$ some i,j and because it can be shown that the function $\frac{a^2}{b}$, $b > 0$, is strictly convex (when $D_i \neq D_j$ some i,j which holds by assumption).

Proof of (b). Thrice differentiating $TS_D(\alpha) - TS_{ND}(\alpha)$ and evaluating at $\alpha = 0$ yields

$$TS_D'''(0) - TS_{ND}'''(0) = 3k[Q_D''(0) - Q_{ND}''(0)]$$

$$> 0.$$

Proof of (c). Twice differentiated $\pi_D(\alpha) - \pi_{ND}(\alpha)$ yields

$$\pi_D''(0) - \pi_{ND}''(0) = \sum_{i=1}^{n} z_i(p_i'' - p_{ND}'')$$

$$= 2p'^2 \left[\sum_{i=1}^{n} \frac{\partial z_i}{\partial p} - \frac{\partial z_{ND}}{\partial p} \right]$$

$$= 0.$$

Q.E.D.

Proposition 2 contains the surprising result that when markets are approximately competitive (i.e. α is small) and when markets differ in the limit only in industry demand elasticity, then discrimination results in an unambiguous increase in total output and surplus, albeit a small one. The Robinson and Schmalensee condition determining the direction for the monopoly case plays no role here. In contrast to that case, the output increasing effects of lower prices in some markets always outweigh the output decreasing effects from higher prices in the other markets. An intuitive explanation of this result will be presented in the next section.

Another surprising result of Proposition 2 is that there is no second-order change in profits even though there is a second-order difference in prices. The third-order difference in profits is a complicated expression including third derivatives of the price functions (which do not appear in $TS_D'''(0) - TS_{ND}'''(0)$). In numerical examples considered, $\pi_D(\alpha) > \pi_{ND}(\alpha)$ always held for small enough α. Yet in several examples, the inequality was reversed for relatively low values of α, bounded above zero.

4. Linear Industry Demand

In this section a variation of the Hotelling (1929) spatial model is introduced serving two purposes. First it is an example satisfying the assumptions of Section 3 and it illustrates the limit results obtained there. Second, because of its simple structure, explicit solutions for prices can be obtained, which makes it possible to extend the results for more general degrees of market power.

Consider a duopoly in which the two firms may be thought of as retailers located at two separate towns selling the same type of commodity. Marginal cost c is assumed to be zero. Buyers live in one of the two towns and must bear a transport cost to purchase from the out-of-town firm. The buying public in each town is divided into markets (perhaps on an age basis). The number of buyers in market i in each city whose reservation price for the product of their hometown firm is no less than p is given by the following linear function

$$\text{(19)} \qquad \frac{1}{2}z_i(p) = w_i(V_i - p).$$

The total industry demand, combining the two cities, is then $z_i(p)$. Without loss of generality it is assumed that $\sum_i w_i = 1$, implying that w_i can then be thought of as a population weight.

If a buyer in market i chooses to purchase out-of-town he bears a transport cost t which is uniformly distributed on the interval $[0,\alpha T_i]$ and is independent of the reservation price for the product. Therefore if firm 1 sets price p in market i greater than price $\tilde{p}$, the out-of-town price, then firm 1's market i demand is

$$\text{(20a)} \qquad x_i^1(p,\tilde{p},\alpha) = \frac{1}{2}z_i(p)\left(\frac{\alpha T_i - p + \tilde{p}}{\alpha T_i}\right), \text{ for } p \geq \tilde{p}.$$

Since $p \geq \tilde{p}$, firm 1 sells only to hometown buyers. Of these, only those with reservation price greater than p and transport cost greater than $p-\tilde{p}$ purchase from firm 1. When firm 1 undercuts the out of town firm the formula is slightly different

$$\text{(20b)} \qquad x_i^1(p,\tilde{p},\alpha) = \frac{1}{2}z_i(p) + \int_0^{\tilde{p}-p} \frac{1}{2\,\alpha T_i} z_i(p+t)dt, \text{ for } p \leq \tilde{p},$$

reflecting the fact that in this case firm 1 captures some out-of-town purchases in addition to all purchases made by buyers living in the firm's hometown.

In spite of the fact that the demand formula changes at $p=\tilde{p}$, it is continuously differentiable at this point. It can be shown that there exists an equilibrium discriminatory price in each market which is the unique equilibrium. If the support of transport costs is not too diverse across markets then a non-discriminatory equilibrium exists.

The results of Section 3 can be illustrated as follows. $\phi_i(p,\alpha)$, the brand elasticity inverse term, in this case is simply $\phi_i(p,\alpha) = \alpha T_i$, which can be shown by differentiating (20) and using (13). It can be easily verified that this satisfies Assumptions 2-6, in particular that $\phi_i(p,0) = 0$ and that it is increasing in α. The results of Propositions 1 and 2 can thus be applied. Note first that the ratio of the brand elasticities is simply $RB_{ij} = \phi_j/\phi_i = T_j/T_i$. Note second that at price equal to marginal cost (which is zero) the ratio of industry demand elasticities is $RD_{ij} = V_j/V_i$. Therefore, from Proposition 1, if α is small enough and if $T_i \neq T_j$ some i,j but $V_i = V_j$ all i,j, then discrimination results in a decrease in total output and total surplus. On the other hand if $n = 2$ and $1 < T_2/T_1 < V_2/V_1$ then discrimination increases output for small enough α. Proposition 2 holds when markets are identical in brand elasticity which in this case occurs when $T_i = T_j$ for all i,j. If $V_i \neq V_j$ some i,j then from this proposition discrimination results in a strict increase in output and total surplus for small enough α.

In fact, because the equilibrium condition in the above example is quadratic, equilibrium prices can be solved for explicitly and global comparisons can be made. For example, it can be shown that if $T_i \neq T_j$ some i,j while $V_i = V_j$ all i,j, then discrimination always decreases total output and surplus.

The most interesting result holds for the case in which brand elasticity is identical in all markets. Suppose T_i is constant for all i at $T_i = 1$ meaning that the average transport cost in each market to make an out-of-town purchase is $\frac{\alpha}{2}$. It can be shown that as α is made arbitrarily large, the equilibrium price converges to the monopoly level. For this limiting case, with linear demand, Robinson has shown that output is identical in both regimes and that total surplus is lower with discrimination because of the allocational inefficiency. By continuity, for large enough α, $TS_D(\alpha) < TS_{ND}(\alpha)$ while from Proposition 2 for small enough α, $TS_D(\alpha) > TS_{ND}(\alpha)$. It is interesting to determine the point at which the welfare evaluation changes sign. Proposition 3 obtains a lower bound for this point.

Proposition 3. Suppose that transport costs are uniformly distributed on the interval $[0,\alpha]$ in each market. Suppose that c = 0 and that the markets are ordered so that $V_1 < V_2 < \cdots < V_n$.

(i) $Q_D(\alpha) > Q_{ND}(\alpha)$ for $\alpha > 0$

(ii) $TS_D(0) = TS_{ND}(0)$

$TS_D(\alpha) - TS_{ND}(\alpha)$ is a strictly increasing function of α for α such that

$$\alpha \leq \frac{1}{2}V_1.$$

Sketch of Proof

The equilibrium price in a market with industry demand intercept V can be written as a function of V and α as follows.

$$p(V,\alpha) = \alpha + \frac{V}{2} - \left(\alpha^2 + [\tfrac{V}{2}]^2\right)^{1/2}$$

The discriminatory price in market i is $p_i = p(V_i,\alpha)$. Aggregate industry demand behaves as if it were linear with demand intercept $\bar{V} = \sum_i w_i V_i$ and the non-discriminatory price is $p_{ND} = p(\bar{V},\alpha)$ (assuming this price is low enough so that there is positive demand in all markets). The difference in output between the regimes is

$$Q_D(\alpha) - Q_{ND}(\alpha) = \sum_{i=1}^{n} w_i[V_i - p_i] - \sum_{i=1}^{n} w_i[V_i - p_{ND}]$$

$$= -\sum_{i=1}^{n} w_i p_i + p_{ND}.$$

Therefore to show (i) it is sufficient to show that $p(V,\alpha)$ is strictly concave in V, which is left to the reader. (In the pure monopoly case p is linear in V and output is the same in both regimes).

To show (ii), the total surplus in each regime is calculated explicitly by using the assumption of linear demand in the formula for total surplus. The difference between the surplus in the two regimes can be shown to be

$$TS_D - TS_{ND} = \frac{1}{2}[p_{ND}{}^2 - \sum_i w_i p_i^2]$$

Substituting the formulas for p_i and p_{ND} into the above and substituting $\bar{U}$ for $\bar{V}/2$ and U_i for $V_i/2$ yields

$$2[TS_D - TS_{ND}] = \bar{U}^2 - (\alpha+\bar{U})^2(\alpha^2 + \bar{U}^2)^{\frac{1}{2}} - \sum_i w_i[U_i^2 - (\alpha+U_i)^2(\alpha^2 + U_i^2)^{\frac{1}{2}}].$$

Let $g(U,\alpha) = \frac{1}{2}[U^2 - (\alpha+U)^2 (\alpha^2 + U^2)^{\frac{1}{2}}]$ The above is then equivalent to

$$TS_D - TS_{ND} = g(\bar{U},\alpha) - \sum_{i=1}^{n} w_i g(U_i,\alpha).$$

Note that at $\alpha = 0$, $g(\bar{U},0) = \sum_{i=1}^{n} w_i g(U_i,0)$ meaning that $TS_D = TS_{ND}$. To show that $TS_D = TS_{ND}$ is an increasing function of α for $\alpha \leq U_1 = \frac{V_1}{2}$ it is sufficient to show that $\frac{\partial g}{\partial \alpha}(U,\alpha)$ is a concave function of U for such α, which is left to the reader. Q.E.D.

Proposition 3(i) states that when industry demand is linear discrimination always increases output in the non-cooperative equilibrium. However, for α large enough, monopoly is approached and in the limit quantities are the same.

Proposition 3(ii) presents a lower bound such that if α is no greater than this level, then the total surplus increase from discrimination is an increasing function of α. The criterion is that the average transport cost be no greater than one half the average reservation price in the weakest demand market. To put this into perspective, at transport costs equal to this bound the discriminatory price and profit in the weakest market are respectively 59% and 82% of their pure monopoly levels. Above this bound the change in surplus from discrimination eventually decreases and ultimately becomes negative[8].

To understand this result, it is helpful to consider the pure monopoly case. In this case the price in a market with demand intercept V is simply $\frac{V}{2}$, the average reservation price. In the non-discriminatory case, price is set as though the demand intercept were $\sum_i w_i V_i$, (assuming it is optimal to have positive sales in all markets). Discrimination results in price increases in demand inelastic markets which are relatively as large as price decreases in demand elastic markets and the net effect on output is zero.

In contrast, the non-cooperative price function is a strictly concave rather than linear function of the demand intercept V. At low levels of V, transport costs are high relative to reservation value and the firm acts in a relative sense like a monopolist. If V is increased, the firm is able to extract this additional surplus by raising the price significantly. At high levels of V transport costs are low relative to reservation value and the firm acts relatively like a competitor. Further increases in V result in negligible increases in price. Paradoxically, the firm acts like a monopolist in markets with low prices and a competitor in markets with high prices. The impact of this is that in moving from a non-discriminatory to a discriminatory regime, price decreases in markets with high industry demand elasticity are relatively large compared with price increases in inelastic markets resulting in a postive net effect on output.

The above discussion suggests a means of distinguishing whether observed discrimination in this model is based on differences in industry demand elasticity or brand elasticity. Suppose $V_1 < V_2 < \cdots < V_n$ but $T_1 = T_2 = \cdots = T_n$. In this case firms act like competitors in the high V markets and like monopolists in the low V markets. Specifically, if there is a cost increase it is passed along almost dollar for dollar in the higher V, higher price markets, while it is partially absorbed by the firms in the lower V, lower price, markets. It can easily be shown that a given cost increase has a larger positive effect on price, the larger the original price in the market. On the other hand if $V_1 = V_2 = \cdots V_n$ but $T_1 < T_2 < \cdots < T_n$, then the firms act like monopolists in the high price markets and like competitors in the low price markets. A given cost increase has a smaller positive effect on price, the larger the original price in the market.

It is interesting at this point to contrast the Bertrand case above with a Cournot oligopoly. Consider the simplist case with homogeneous products, m firms, linear demand and zero marginal cost. In this case the Cournot price formula has the familiar form $\frac{V}{1+m}$ which is linear in the demand intercept V. Price as a percentage of average total surplus is independent of the size of the surplus. Output is the same in the two regimes regardless of the number of firms implying discrimination results in a strict decline in total surplus. The welfare analysis of discrimination in a Cournot oligopoly appears to be essentially the same as the monopoly analysis, in contrast to the above results for the Bertrand case.

5. Spatial Discrimination

This section briefly explores spatial discrimination in a competitive context. This form would occur, for example, if the two firms in the model of Section 4 could distinguish the hometown of a buyer. In the non-cooperative equilibrium each firm recognizes its inferior position in the out-of-town market and offers those buyers a discount.

There are many papers which analyze spatial models in which discriminatory prices are set by a monopolist or collusive oligopoly including Holahan (1975), Norman (1981), Spulber (1981) and a large literature cited by Phlips (1983). Yet the form of discrimination discussed in these papers is better classified as that based of differences in industry demand elasticity. Buyers who live further from a firm have higher per unit transport costs which cause their industry demand to be more elastic. A discriminatory monopolist sets a lower price. In contrast, if firms act collusively in the model discussed here, there is no discrimination in equilibrium.

One paper which does present a model of competitive spatial discrimination of the type discussed here is Macleod, Norman, and Thisse (1985). The contribution of that paper is a proof of the existence of a free entry equilibrium in a simple model in which demand is inelastic and transport costs are identical at each location.

For simplicity the example of section 4 will be discussed with the generalization that industry demand is concave, $z'(p) < 0$, $z''(p) \leq 0$, rather than linear. Transport costs are distributed on the interval [0,1]. Firms can distinguish two markets, buyers in town 1 and 2. Let the price offered by firm j to market i be p_i^j. Clearly in equilibrium $p_1^1 > p_1^2$ and $p_2^1 < p_2^2$. If $p_1^1 \leq p_1^2$, for example, no buyer in town 1 would purchase from firm 2. But firm 2 could always capture some sales in market 1 by slightly undercutting firm 1 in that market. Assuming that $p_1^1 > p_1^2$, the demand of firm 1 in market 1 is then

$$x_1^1(p_1^1, p_1^2) = \tfrac{1}{2}z(p)(1-p_1^1+p_1^2)$$

This formula is identical to (20a), which is the demand of firm 1 in the combined markets when discrimination is not feasible and when firm 1 sets a higher price than firm 2. This holds because in this case the only buyers who would purchase from firm 1 are those that live in town 1. This simple fact plays an important role in the result below. The demand of firm 2 in market 1 can be written as

$$x_1^2(p_1^1, p_1^2) = \int_0^{p_1^1-p_1^2} \tfrac{1}{2}z(p_1^2 + t)dt$$

It can be shown that in this example there exists a unique discriminatory equilibrium in which each firm offers a price p_h to hometown buyers and a price p_o to out-of-town buyers. If discrimination is not feasible, there exists a unique non-discriminatory equilbrium price denoted p_{ND}. The surprising relationship between p_n, p_o, and p_{ND} is stated in the following proposition.

Proposition 4. Both discriminatory prices are less than the non-discriminatory price, i.e. $p_0 < p_h < p_{ND}$. Therefore all consumers are better off with spatial discrimination, while profits are lower and total surplus and output are higher.

A proof of this is sketched here. Let p^i be the price offered by firm i and let $\pi_i(p^1,p^2)$ be the profit of firm 1 in market i in the discriminatory regime and $\pi_{ND}(p^1,p^2)$ be the profit of firm 1 from the combined markets in the non-discriminatory regime. As discussed above, when $p^1 \geq p^2$, $\pi_1^1(p^1,p^2) = \pi_{ND}^1(p^1,p^2)$. If can be shown that in the relevant range

$$\frac{\partial\pi_{ND}^1(p^1,p^2)}{\partial p^1 \partial p^1} < 0 \text{ and } \frac{\partial\pi_{ND}^1(p^1,p^2)}{\partial p^1 \partial p^2} > 0.$$

This implies that the reaction function of the firm is upward sloping in the other firm's price, i.e. in the non-discriminatory case, if firm 2 offers a lower price, firm 1's optimal response is to set a lower price. Since the reaction function of firm 1 in its hometown market in the discriminatory regime is the same as its reaction function in the nondiscriminatory regime and since p_{ND} is the optimal reaction to p_{ND}, if the out-of-town price p_0 is less than p_{ND}, the optimal response of the hometown firm, p_h, is then also less than p_{ND}. Thus, to obtain the result, it only has to be shown that in equilibrium the out-of-town price is less than p_{ND}. It is intuitive that this would hold and the proof is left to the reader.

The above result could be extended to the more general model with many firms. Spatial discrimination would be said to occur if firms, though not able to observe the type $\underline{r} = (r^1, r^2, \ldots, r^m)$ of a buyer, could observe the ranking of the firms by the individual. Again, the reaction function of the most preferred firm is the same as it is in the non-discriminatory case. If the reaction function in the non-discriminatory case is an increasing function of the other firms' prices, as it usually is in these models, then the non-discriminatory price is greater than the discriminatory price set by the most preferred firm and therefore by all the firms.

Though not quite a Pareto improvement (firms are worse off) competitive spatial discrimination has the appealing consequence of bringing all prices closer to marginal cost. Note that this holds for general degrees of monopoly power and is not a limiting result as are those in Section 3.

6. Conclusion

This paper analyzed the effects of price discrimination in a model of a differentiated products oligopoly. It was shown that brand elasticity discrimination has negative consequences on output and total surplus in markets in which competing firms products are close substitutes. Conversely, industry demand discrimination has positive consequences in such markets. These conclusions were extended for more general degrees of market power for the special case of linear industry demand which is surprising since monopoly discrimination decreases total surplus for that case. Finally it was shown that spatial discrimination results in lower prices in all markets and lower industry profits.

Appendix A

It has to be shown that at $p = c$ and $\alpha = 0$

$$\frac{\partial \phi_{ND}}{\partial \alpha} = \frac{\sum_{i=1}^{n} z_i}{\sum_{i=1}^{n} \frac{z_i}{\frac{\partial \phi_i}{\partial \alpha}}} .$$

$\phi_{ND}(p,\alpha)$ can be written as

$$\phi_{ND}(p,\alpha) = \frac{\sum_{i=1}^{n} z_i(p,\alpha)}{\sum_{i=1}^{n} \frac{z_i(p,\alpha)}{\phi_i(p,\alpha)}} .$$

Differentiating yields

$$\frac{\partial \phi_{ND}}{\partial \alpha} = \frac{\sum_{i=1}^{n} \frac{\partial z_i}{\partial \phi} \phi_1}{\sum_{i=1}^{n} z_i \frac{\phi_i}{\phi_i}} - \frac{(\sum_i z_i)(\sum_i \frac{\phi_i}{\phi_i} \frac{\partial z_i}{\partial \alpha}) \phi_1}{(\sum_i \frac{\phi_i}{\phi_i} z_i)^2}$$

$$+ \frac{(\sum_i z_i)(\sum_i z_i (\frac{\phi_i}{\phi_i})^2 \frac{\partial \phi_i}{\partial \alpha})}{(\sum_i \frac{\phi_i}{\phi_i} z_i)^2}$$

where the first term is multiplied by $\frac{\phi_i(p,\alpha)}{\phi_i(p,\alpha)}$ and the second and third terms are multiplied by $\frac{\phi_i(p,\alpha)^2}{\phi_i(p,\alpha)^2}$.

By Assumption 6,

$$0 < \lim_{\alpha \to 0} \frac{\phi_i}{\phi_i} < \infty.$$

Since $\lim_{\alpha \to 0} \phi_i(p,\alpha) = 0$, the limit of the first two terms is zero.
By L'Hospital's rule

$$\lim_{\alpha \to 0} \frac{\phi_i(c,\alpha)}{\phi_i(c,\alpha)} = \frac{\frac{\partial \phi_i}{\partial \alpha}(c,0)}{\frac{\partial \phi_i}{\partial \alpha}(c,0)} .$$

Therefore,

$$\frac{\partial \phi_{ND}(c,0)}{\partial \alpha} = \frac{(\sum_i z_i)\left(\sum_i \frac{z_i}{\frac{\partial \phi_i}{\partial \alpha}}\right)\left(\frac{\partial \phi_i}{\partial \alpha}\right)^2}{\left(\sum_i \frac{z_i}{\frac{\partial \phi_i}{\partial \alpha}}\right)^2\left(\frac{\partial \phi_i}{\partial \alpha}\right)^2} = \frac{\sum_i z_i}{\sum_i \frac{z_i}{\frac{\partial \phi_i}{\partial \alpha}}} .$$

Q.E.D.

Appendix B. Proof of Proposition 1(iii)

(iiia) Let $z_i' = \frac{\partial z_i}{\partial p}$.

$$Q_D'(0) - Q_{ND}'(0) = z_1'p_1' + z_2'p_2' - (z_1' + z_2')p_{ND}'$$

Dividing through by p_{ND}' and using the formula for p_{ND}' derived in Lemma 2 yields

$$\frac{1}{p_{ND}'}[Q_D'(0) - Q_{ND}'(0)] = z_1'z_1 + z_1'z_2\frac{p_1'}{p_2} + z_2'z_1\frac{p_1'}{p_2'} + z_2'z_2 - z_1' - z_2'$$

$$= -z_1'z_2\frac{(p_2'-p_1')}{p_2'} + z_2'z_1\frac{(p_2'-p_1')}{p_1'}$$

where the second inequality uses the fact that $z_1 + z_2 = 1$. By assumption, $RB_{12} > 1$. Since $RB_{12} = \frac{p_2'}{p_1'}$, $p_2' > p_1'$. Therefore

$$Q_D'(0) \gtreqless Q_{ND}'(0) \text{ as } \frac{-z_1'z_2}{p_2'} \gtreqless \frac{-z_2'z_1}{p_1'}$$

or as

$$RD_{12} = \frac{\frac{-z_1'}{z_1}}{\frac{-z_2'}{z_1'}} \gtreqless \frac{p_2'}{p_1'} = RB_{12}.$$

(iiib) $TS_D''(0) - TS_{ND}''(0) = z_1'p_1'^{\,2} + z_2'p_2'^{\,2} - (z_1' + z_2')p_{ND}'^{\,2}$

$$= z_1'p_1'^{\,2} + z_2'p_2'^{\,2} - \frac{(z_1'+z_2')}{(\frac{z_1}{p_1'} + \frac{z_2}{p_2'})^2}$$

By dividing the above by $-z_1'p_1'^{\,2}$, setting $\frac{z_1'}{z_2'} = z'$, $\frac{z_1}{z_2} = z$, and recalling that $\frac{p_2'}{p_1'} = RB_{12}$, it follows that $TS_D''(0) > TS_{ND}''(0)$ is equivalent to

$$-1 - \frac{RB_{12}^{\;2}}{z'} > \frac{-1 - \frac{1}{z'}}{(z_1 + \frac{z_2}{RB_{12}})^2} = \frac{(1 + \frac{1}{z_1})(1 + \frac{1}{z'})^2}{(1 + \frac{1}{zRB_{12}})^2} .$$

After manipulation of the above, it can be shown that the inequality holds if the following holds

$$\frac{z}{z'} (1-RB_{12}^{\;2}) > 2(\frac{1}{RB_{12}} - 1) + \frac{1}{z} (\frac{1}{RB_{12}} - 1) + \frac{2}{z'} (RB_{12} - 1)$$

A set of conditions sufficient for this to hold is

(*) $\frac{z}{z'} (1 - RB_{12})^2 > 2(\frac{1}{RB_{12}} - 1)$

(**) $\frac{1}{z}(\frac{1}{RB_{12}} - 1) + \frac{2}{z'} (RB_{12} - 1) < 0$

It can be shown that (*) is equivalent to

$$RD_{12} > \frac{RB_{12} + (RB_{12})^2}{2}$$

and that (*) implies (**) which completes the proof. **Q.E.D.**

Footnotes

1. See Holmes (1985, chapter 4) and Borenstein (1985) for an analysis which uses the Salop (1979) circle model with free entry.
2. The actual randomization does not matter in calculating the demand of the firm because the set in which such ties occur has measure zero.
3. See Holmes (1985, chapter 5) for an analysis of this model in which firms cannot directly observe the market a buyer belongs to, but can use quality to sort buyers by level of industry demand. The analysis of Mussa and Rosen (1978) for the monopoly case is extended to a Bertrand oligopoly.
4. Borenstein (1984) presents an analysis of elasticities which is suggestive of the one above.
5. If the price in market i is determined cooperatively rather than non-cooperatively then the price cost margin is the inverse of the industry demand elasticity and is independent of brand elasticity.
6. I am grateful to Don Hester for this reference.
7. As an illustrative example suppose that there are buyers with perfect brand elasticity with positive measure in the market. In this case there cannot be a symmetric equilibrium with price above marginal cost because of the familiar Bertrand reasoning, regardless of the number of brand inelastic buyers. In this case price equal to marginal cost solves the symmetric first-order condition. Of course, this is not an equilibrium because a firm can set a small positive profit margin and salvage positive profits from sales to the brand elastic buyers. In general, when brand elasticity is extremely diverse an equilibrium does not exist. However, I have found that in examples as long as it is not too diverse, an equilibrium does

exist. For instance, in a spatial model example, as long as unit transport cost parameters differed by no more than a factor of 10, an equilibrium existed.

8. If a monopolist does not serve a market in the non-discriminatory allocation then total surplus may increase with discrimination.

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