OPTIMAL USE OF QUALITATIVE MODELS: AN APPLICATION TO COUNTRY GRAIN ELEVATOR BANKRUPTCIES

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Abstract

Qualitative models can be used for decision making under uncertainty. This provides a useful framework for evaluating the models. If the costs for every action/state of nature combination are known, decisions made using a well-calibrated model would result in actual costs being close to expected costs. In addition, the actual cost can be compared to the cost of perfect foresight actions, giving a bound on the value of a better model. Application of these procedures is made using a logit model developed to predict Missouri country grain elevator bankruptcies.

Key words: bankruptcy, country grain elevators, decision rule, elevator regulation, qualitative models.

Qualitative models, characterized by a categorical or discrete endogenous variable, have become more prominent in the economics literature during the past decade. Amemiya provides many examples of economic topics requiring qualitative analysis. Categorical variables appearing in recent agricultural economics literature include dichotomous variables indicating whether an event has occurred (e.g., bankruptcies), turning points (Naik and Leuthold; Kaylen; Harris and Leuthold; Brandt and Bessler), and direction of movement (Feather and Kaylen). Probability estimates from a qualitative model can aid the decision maker who must choose one of a finite number of actions when the cost of an action depends on the value of the random qualitative variable (state of nature). Early warning models for country grain elevator bankruptcies (Siebert) and credit scoring functions for loans (Fischer and Moore) are examples of such decision making situations: a decision maker (regulator or loan officer) must choose among alternative actions (scheduling or not scheduling an extra audit, making or not making a loan) when the state of nature is unknown (the firm will either survive or go bankrupt, the loan will be collectible in full or result in a loss).

This paper differs from previous studies in that it explicitly recognizes that choice of action is a standard decision under uncertainty problem. It is assumed that the decision maker chooses actions which yield the highest expected value of the objective function. In contrast, other studies have considered only that state of nature which has the highest probability of occurrence. The chosen action is then the one which would result in the highest payoff if the modal state of nature is realized. Still other studies have neglected to consider the costs of all the action/state of nature combinations. They have implicitly assumed that the best actions for each state of nature have zero costs.

The principles advocated in this paper are illustrated with an application to Missouri country grain elevator bankruptcies. The issue is an important one; the stressful environment of the early 1980's has resulted in many firms leaving the industry due to insolvency. State regulators attempt to control bankruptcy costs by scheduling extra audits for particular elevators. The more effectively extra audits are scheduled, the lower the total costs to society.

The application builds on earlier work by Siebert. A logit model is developed to provide bankruptcy probabilities. These probabilities are then used to optimally decide whether to schedule extra audits. The decision rule differs from Siebert's because it is recognized

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that even correctly scheduled audits incur costs and that extra audits may not result in zero cost bankruptcies.

The paper proceeds as follows. First, the expected cost minimizing decision rule is presented. This discussion is followed by a brief description of a logit bankruptcy model developed for Missouri country grain elevators. Application of the decision rule is then demonstrated by using it to evaluate the usefulness of the logit model.

THE DECISION RULE

Following Anderson et al., the generalized (discrete) decision problem may be characterized by several components. These are the set of possible actions, the set of states of nature, the probabilities of the states, the consequences associated with each state/action combination, and the decision maker's objective function. In this paper, it is assumed that the decision maker minimizes expected loss. The components of the decision problem are displayed in the general loss matrix in Table 1, and the solution is that action which yields the lowest expected loss as given by the bottom row. The table illustrates the value of a qualitative model to the decision maker; it supplies the probabilities for the states of nature.

Curiously, results from qualitative models have not been explicitly used within this type of a framework. For example, Fischer and Moore used a logit model to classify loans as low risk (action a_1) or high risk (action a_2). The authors wanted these classifications to be similar to those made by experts using other information. Thus, the two states of nature corresponded to expert classifications of low risk (state Θ_1) or high risk (state Θ_2). Letting p denote the model-computed probability of low risk and C_{ij} denote the cost associated with state of nature i and action j, the expected loss for classifying a loan as low risk would be $pC_{11}+(1-p)C_{21}$, and the expected loss for classifying a loan as high risk would be $pC_{12}+(1-p)C_{22}$. Rather than explicitly considering the expected losses associated with the classifications (actions), Fischer and Moore followed the prediction procedure suggested by Pindyck and Rubinfeld. That is, a loan was classified as low risk if the probability of its being classified low risk by the experts was 0.5 or greater. This decision rule is consistent with the minimization of classification error; it would also be consistent with the minimization of expected loss if correct actions resulted in zero losses ($C_{11}=C_{22}=0$) and misclassifications incurred equal positive costs ($C_{12}=C_{21}$).

In his country grain elevator bankruptcy study, Siebert also considered two states of nature: survival (state Θ_1) and bankruptcy (state Θ_2). In contrast to Fischer and Moore, Siebert explicitly considered two actions: whether to forego (action a_1) or schedule (action a_2) an extra audit. His decision rule was based on a cut-off value for the probability of survival. Any firm with a probability of survival less than this cut-off value would be scheduled for an extra audit. The cut-off value was chosen so as to "minimize the combined costs of bankruptcy (type I errors) and bankruptcy detection (type II errors)" (p. 565). This procedure minimizes the expected costs of errors using the model. Let p represent the probability of a firm surviving, C_a represent the cost of an extra audit for this firm, and C_b represent the cost of this firm going bankrupt. If the chosen action is to schedule an extra audit, the expected cost of error is the probability of survival times the cost of the unnecessary audit, pC_a. If the chosen action is to forego an extra audit, the expected cost of error is the probability of bankruptcy times the cost of the bankruptcy, $(1-p)C_b$. Thus, an extra audit should be scheduled if $pC_a < (1-p)C_b$. Note that if $C_a = C_b$ then this yields the same decision rule as minimizing the probability of error.

The Siebert procedure can be improved upon by considering the costs associated with every action/state of nature combination rather than just the costs associated with errors. There is no cost associated with foregoing an extra audit of a survivor firm (i.e., $C_{11}=0$), but the auditing of a firm incurs costs even if it is correct to audit the firm (i.e., $C_{22} \neq 0$).

The Fischer and Moore classification rule and the Siebert action choice rule are consistent with special cases of the decision rule suggested in this paper: choose that action with the lowest expected cost. This more general rule forces the analyst to explicitly consider the costs associated with every possible action/state combination. This procedure also provides a convenient method for conducting an ex-post evaluation of the probability generator (e.g., qualitative model or expert opinion). For a set of data, the expectation and variance of cost following the decision rule may be calculated using the generated prob-

TABLE 1. GENERAL LOSS MATRIX

State of Nature	Probability	Action a_1	a_2	...	a_m
		---Losses---			
θ_1	P_1	C_{11}	C_{12}	...	C_{1m}
θ_2	P_2	C_{21}	C_{22}	...	C_{2m}
.	.	.			.
.	.	.			.
.	.	.			.
θ_n	P_n	C_{n1}	C_{n2}	...	C_{nm}
Expected Loss		$\sum_{i=1}^{n} P_i C_{i1}$	$\sum_{i=1}^{n} P_i C_{i2}$	...	$\sum_{i=1}^{n} P_i C_{im}$

abilities for the states of nature. Using the notation of Table 1, and letting A_j denote that set of observations for which action j is chosen by the decision rule (j=1,...,m), the expected cost is

$$(1)\ E(C) = \sum_{j=1}^{m} \sum_{k \epsilon A_j} \sum_{i=1}^{n} p_i^k C_{ij}^k,$$

where the superscript k identifies the k-th observation. Assuming independence between the observations, the variance is

$$(2)\ V(C) = \sum_{j=1}^{m} \sum_{k \epsilon A_j} \sum_{i=1}^{n} p_i^k (C_{ij}^k)^2 - [E(C)]^2.$$

Once the state of nature for each observation is known, the actual cost is

$$(3)\ A(C) = \sum_{j=1}^{m} \sum_{k \epsilon A_j} \sum_{i=1}^{n} \delta_i^k C_{ij}^k,$$

where δ_i^k is one if the i-th state of nature occurred for the k-th observation; otherwise, it is zero.

A probability generator would be "good," in the sense of being well-calibrated, if the difference between actual and expected costs is small relative to the standard error given by the square root of the variance in equation (2). The formulas developed in this section of the paper may also be used to place bounds on the value of better probability generators. The best possible generator would assign a probability of one to the correct state of nature and zero to all others. In an ex-post analysis, the perfect probability generator would have been worth at most the difference between the actual cost and the cost which would have been realized using perfect foresight. Analyses such as this can give an idea of the optimal amount of resources to devote to improving the probability generator.

A BANKRUPTCY MODEL FOR MISSOURI COUNTRY GRAIN ELEVATORS

The usefulness of qualitative models within the decision under uncertainty framework is demonstrated with an application to Missouri grain elevator bankruptcies. The depressed agricultural economy of recent years has had a severe impact on agribusiness firms which sell production inputs to farmers or market farmers' produce; among the most affected were country grain elevators. During the 1980's decreasing livestock numbers, droughts, and government programs which curtailed production caused sales levels to decline for many of these firms. High interest rates and bad debt losses also affected their profit positions (Devino). These factors resulted in a large number of firms leaving the industry. Many actually went bankrupt; for example, about 25 percent of the 140 Missouri grain elevators which exited the industry between 1980 and 1985 were insolvent (Coday).

Any time a country elevator becomes bankrupt there is a potential for farmers with grain stored in the elevator to sustain losses. Government entities, both state and federal, attempt to mitigate these losses through licensing and regulation of grain warehouses. Applicants typically supply financial statements to the licensing agency. Bonding requirements are established on the basis of firm size and its financial condition.

Regulatory agencies also attempt to control losses by identifying those elevators with the potential for bankruptcy so that these firms may be audited more frequently. Timely regulatory intervention may prevent bankruptcy or result in lower losses than would otherwise occur. It is assumed that an extra audit has the effect of reducing bankruptcy costs. The problem is to formulate an optimal rule for deciding whether to schedule an extra audit.

The decision under uncertainty approach to this problem requires knowledge of the bankruptcy probability. Qualitative response models can provide this information. Early efforts at bankruptcy modeling involved discriminant analysis or linear probability models (Altman; Beaver). More recently, there have been an abundance of studies comparing these approaches to logit models (Maddala; Collins and Green; Amemiya; Press and Wilson). The strong theoretical and empirical support these studies provided for the logit approach led to Siebert's adoption of this model for Midwest country grain elevator bankruptcies.

In this application, Siebert's logit model is adapted to Missouri. The available data consisted of financial records—balance sheets and profit and loss statements. These were available for essentially all of the elevators licensed by the state of Missouri during 1985. Since only three of these 224 elevators went bankrupt during 1985, the data set was augmented by including firms which went bankrupt during the five years 1980 through 1984. While 35 firms actually became insolvent during 1980-85 (Coday), data limitations prior to 1985 precluded including all of them in the analysis. Consequently, the data set consists of 239 observations—18 of the 35 bankrupt elevators and 221 survivor firms.

Following Siebert, explanatory variables from financial ratio groups reflecting liquidity, solvency, activity, and profitability were considered for inclusion in the model. Several models were estimated using the SAS Supplemental Procedure LOGIST as described by Harrell. These were evaluated on the basis of the signs and statistical significance of coefficients. The final model chosen for this study is

$$(4)\ P_i = \{1 + \exp[-(\beta_0+\beta_1 X_{i1}+\beta_2 X_{i2})]\}^{-1},$$

where P_i is the probability of elevator i surviving the year, X_{i1} is the debt-to-assets ratio for the i-th elevator at the start of the year, and X_{i2} is the net income-to-assets ratio for the i-th elevator at the start of the year. The estimation results are shown in Table 2.

As noted, the final model was chosen partly on the basis of the significance of the coefficients; all have magnitudes well over twice their approximate standard errors. In addition, the likelihood ratio test rejects the joint hypothesis that both of the coefficients on the explanatory variables are zero. The model suggests the probability of survival decreases as the debt-to-assets ratio increases and the probability of survival increases as the income-to-assets ratio increases. Hence, the signs of the coefficients agree with a priori expectations.

APPLICATION OF THE DECISION RULE TO MISSOURI GRAIN ELEVATOR BANKRUPTCIES

For each elevator, there are two possible "states of nature": firm survival or bankruptcy. The action choices are to either forego or

TABLE 2. ESTIMATION RESULTS FOR LOGIT MODEL OF COUNTRY GRAIN ELEVATOR BANKRUPTCIES, MISSOURI, 1980-1985

Variable	Estimated Coefficient	Approximate Standard Error	Significance Level[a]
Constant	6.027	0.953	0.00
Debt-to-Assets Ratio	−4.574	1.043	0.00
Net Income-to-Assets Ratio	4.887	2.144	0.02
Model Chi-square Statistic[b] = 43.43			
Number of Observations = 239			

[a]The significance level is that level at which the null hypothesis of a zero coefficient would just be rejected. For example, the hypothesis of a zero coefficient on the net income-to-assets ratio would be rejected at the five percent level, but not at the one percent level.

[b]This statistic is used to perform the likelihood ratio test of the null hypothesis that both of the explanatory variables jointly have zero coefficients. Under the null hypothesis, it is approximately distributed as a chi-squared random variable with two degrees of freedom. The reported value is significant at the one percent level, leading to rejection of the null hypothesis.

schedule an extra audit. Scheduling an extra audit would be the best action for firms with a low survival probability and not scheduling an audit would be the best action for those with a high probability of survival. The problem is to choose a cut-off probability value to use in determining the appropriate action.

Siebert's decision rule was based on minimizing the expected cost of errors. In contrast, the decision under uncertainty rule proposed in this paper recognizes that even correct actions may result in costs. Thus, if an extra audit is scheduled, the cost of the audit (C_a) is incurred regardless of the state of nature. Further, the firm which goes bankrupt is likely to impose costs on society even if it has been audited. Let C_b represent the cost of an unaudited bankruptcy and $\alpha(0 < \alpha < 1)$ denote the proportion by which this cost would be reduced by an extra audit. Table 3 then depicts the societal loss matrix for the problem. Expected social cost is minimized by scheduling the firm for an extra audit if

(5) $C_a + (1-p)(1-\alpha)C_b < (1-p)C_b$, or
$p < 1 - C_a/\alpha C_b$,

where p is the probability of survival.

Since foregoing an extra audit is a natural "default" action for this problem, the logit model will be assessed in terms of the expected savings using the audit decision rule as opposed to never scheduling extra audits. This necessitates developing analogues to equations (1) through (3). From the decision rule, if $p > 1 - C_a/\alpha C_b$, then the firm is not scheduled for an extra audit, and there are no savings. But, if $p < 1 - C_a/\alpha C_b$, the expense of an audit is incurred, and there is a $(1-p)$ chance of the firm going bankrupt. Thus, expected savings due to conducting the extra audit are $(1-p)\alpha C_b - C_a$, and the variance of savings is $p(1-p)(\alpha C_b)^2$.

The audit and bankruptcy costs, as well as the percentage savings associated with the extra audit, may depend upon firm specific characteristics (e.g., firm size). For a set of firms, the expected savings due to the audit decision rule would be

$$(6)\ E(S) = \sum_{k \epsilon A} \{(1-p^k)\alpha^k C_b^k - C_a^k\},$$

where the superscript k identifies the k-th firm, and A indexes the set of firms scheduled for an extra audit,

$$A = \{k \mid p^k \leq 1 - C_a^k/\alpha^k C_b^k\}.$$

Assuming independence, the variance of total savings would be

$$(7)\ V(S) = \sum_{k \epsilon A} p^k (1-p^k)(\alpha^k C_b^k)^2.$$

Once the outcome is observed, the actual savings due to the model would be

$$(8)\ A(S) = \sum_{k \epsilon B} \alpha^k C_b^k - \sum_{k \epsilon A} C_a^k,$$

where B is the set indexing all of those firms which went bankrupt after having had extra audits.

For illustrative purposes, this study assumes C_a, C_b, and α are the same for all firms. After dividing through by the bankruptcy cost savings due to an extra audit, αC_b, reference can be made to normalized audit costs and normalized savings. Figure 1 shows how the percentages of correct actions for in-sample elevators vary with normalized audit costs. As $C_a/\alpha C_b$ approaches zero, the decision rule would suggest auditing all firms. Thus, all of the bankrupt firms would be correctly audited while all of the surviving elevators would be incorrectly audited. As the normalized audit cost increases, fewer firms are audited. For example, at a normalized audit cost of 0.27, the model correctly audits 50 percent of the bankrupt firms and correctly

TABLE 3. SOCIETAL LOSS MATRIX FOR COUNTRY GRAIN ELEVATOR ACTION CHOICES[a]

State of Nature	Probability	No Audit	Schedule Audit
Survival	p	0	C_a
Bankruptcy	$1-p$	C_b	$C_a + (1-\alpha)C_b$
Expected Loss		$(1-p)C_b$	$C_a + (1-p)(1-\alpha)C_b$

[a] C_a = cost of an extra audit
C_b = cost of bankruptcy without extra audit
α = proportion by which cost of bankruptcy is reduced by extra audit

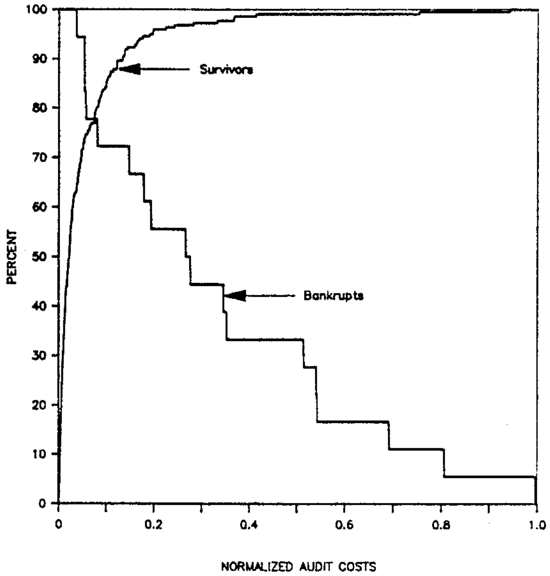

Figure 1. Percentage of In-Sample Elevators Correctly Classified Versus Normalized Audit Costs.

fails to audit about 97 percent of the survivor firms. At unity ($C_a = \alpha C_b$), an audit cannot save money, so no firms would be audited.

Figure 2 shows how actual and expected in-sample savings vary with normalized audit costs. With free audits, all of the firms are audited. Since 18 elevators actually went bankrupt, equation (8) shows actual savings would be $18\alpha C_b$. Since the dichotomous logit model always has the expected in-sample number of events equal to the actual number of events, equation (6) shows the expected savings with free audits is also $18\alpha C_b$. At the other extreme, normalized audit costs of unity imply no firms are audited. In this case, equations (6) and (8) show the expected and actual savings due to the decision rule would be zero. The audits are simply too expensive relative to their potential savings.

The adequacy of the probability generator can best be assessed by considering its performance at the most relevant normalized audit cost ranges. For example, Siebert implicitly assumed his early warning model would result in zero-cost bankruptcies (i.e., $\alpha = 1$). He also used audit costs of $2,100 and an average bankruptcy cost of $535,000. These values result in a normalized auditing cost of about 0.004. From equation (5), this suggests that all of those firms with a survival probability less than 0.996 should be audited. This cut-off value results in 184 of the 221 survivor firms being audited along with the 18 firms which actually went bankrupt. While the percentage of incorrect actions is high, the normalized audit cost is extremely low. All of the unnecessary audits combined would cost only 73.6 percent of the cost of one bankruptcy. This explains why the normalized savings is 17.2, about 95 percent of the maximum possible, even though so many incorrect audits are suggested.

The in-sample results suggest the adapted version of Siebert's bankruptcy model works well. The actual savings which could have been realized agree quite closely with the expected savings. For most normalized audit costs, actual normalized savings are well within the two standard deviation limits calculated with equation (7). The limits are exceeded only for values of $C_a/\alpha C_b$ in the ranges (.69, .75) and (.81, .94). Figure 1 shows actual savings are low for these ranges because one or two survivor elevators are incorrectly scheduled for extra audits.

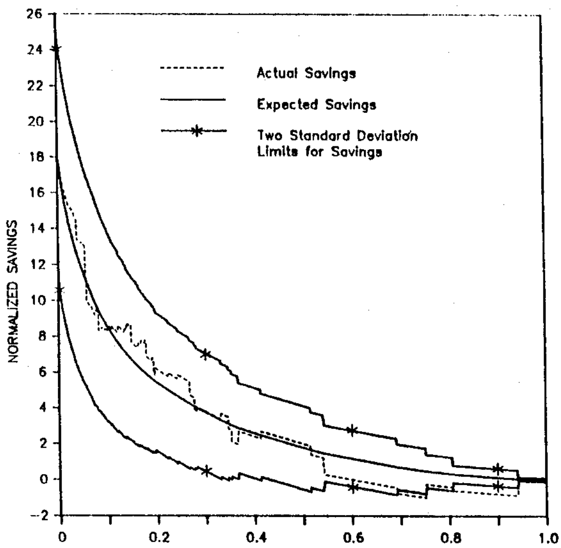

Figure 2. Expected and Actual Normalized In-Sample Savings Due to Model Versus Normalized Audit Costs.

SUMMARY AND CONCLUSIONS

This paper has proposed using qualitative model results within a decision making under uncertainty framework. It has been shown that this framework can be useful in assessing the qualitative model. The procedures were demonstrated using a logit model of Missouri country grain elevator bankruptcies to decide whether to schedule extra audits.

The empirical application improves upon previous work done by Siebert. Most importantly, his decision rule has been altered to more fully account for societal costs. This has been done by noting that extra audits don't result in zero cost bankruptcies, as well as noting that even correctly scheduled audits incur costs. In addition, this paper has shown how to analytically determine the optimal value so that elevators with a survival probability less than this cut-off would be audited. In contrast, Siebert used a search procedure.

The logit model presented in this paper performed well. For the most relevant ranges of normalized audit costs, it was shown that the normalized savings which could have been realized by using the model were within two standard errors of their expected values. In fact, using Siebert's suggested audit and bankruptcy costs, the actual savings which could have been realized using the early warning model were close to the maximum possible.

This paper has demonstrated the usefulness of evaluating qualitative models within the decision under uncertainty framework. While beyond the scope of this paper, future empirical studies should concentrate on estimation of the costs associated with action/state of nature combinations. This could be especially useful for assessing the possible returns to better model development.

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