

# **Perfect Substitutes: A Note on the Alchian-Allen Hypothesis**

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# **Perfect Substitutes: A Note on the Alchian-Allen Hypothesis**

## **Abstract**

Alchian and Allen hypothesized that a country will export high rather than low quality goods. Here we show that if the inherent attributes of the commodities are identical the country will only export the high-quality good.

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### **Introduction**

In 1964, Alchian and Allen postulated that a country or region would export relatively high valued products. They argued that when identical per unit fees such as transportation costs are added to the prices of both high and low quality goods, the relative price of the high quality product declines, and its relative consumption increases in markets in which those fees must be paid. Borcharding and Silberberg (1978) argued that when markets are competitive and both commodities are exported from region A to region B, relative exports of the high quality good increase as transportation costs increase. They also found that this effect becomes stronger when the two goods are closer substitutes. In re-examining the Alchian-Allen proposition, Umbeck (1980) focused on the quality attributes of the two competing commodities to obtain his results.

In this note, Umbeck's discussion of quality attributes is elaborated by introducing the concept of hedonic prices into the analysis. We show that, when the two commodities are effectively perfect substitutes in terms of their mix of quality attributes, trade will take place only in one commodity—the “high quality” good—if both commodities are consumed in both countries. Alternatively, if both commodities are shipped from the exporting country to the importing country then only one commodity—the “low quality” good—will be consumed in the exporting country. We believe that neither of these findings has been previously reported in the literature.

## The Model

Lancaster (1966) and Rosen (1974) developed the concept of implicit or hedonic prices to account for the fact that consumers do not derive utility from a commodity itself, but from its quality-determining characteristics or attributes. Here, we consider two goods that have two common quality attributes  $q$  and  $r$ , but the high quality good,  $h$ , has more units of each quality attribute than the low quality good,  $l$ . Both commodities are produced in each of two countries or regions, Country 1 and Country 2. In each country, buyers have the same perceptions of and preferences with respect to the quality attributes of the two commodities. In addition, markets are assumed to be competitive.

Suppose that each commodity has the two quality attributes,  $q$  and  $r$ , and that units of their attributes are identical. In general, given competitive markets, the relationships between the observed market prices and the hedonic prices for the quality attributes of each commodity in each country will then be as follows:

$$p_i^h = w_i q^h + v_i r^h, \quad (1)$$

and

$$p_i^l = w_i q^l + v_i r^l, \quad (2)$$

where  $p_i^h$  and  $p_i^l$  are the prices of the high and low quality goods,  $w_i$  and  $v_i$  are the hedonic prices of the quality attributes,  $q^j$  and  $r^j$  are the quantities of the quality attributes in each commodity, where  $i = 1, 2$  denotes the country of interest and  $j = h, l$  denotes the commodity. Thus, when both commodities are consumed in each country, in equilibrium the relative market prices of the two commodities will be:

$$\frac{p_i^h}{p_i^l} = \frac{w_i q^h + v_i r^h}{w_i q^l + v_i r^l}, \text{ where } i = 1, 2. \quad (3)$$

In the absence of trade, there is no link between prices in Country 1 and prices in Country 2. However, if trade is feasible, then that is no longer the case. Following Alchian and Allen, we assume that a fixed per-unit transport cost  $t$  is incurred when either commodity is shipped from Country 1 to Country 2. A competitive equilibrium that allows for trade then implies that:

$$p_2^j \leq p_1^j + t, \text{ for } j = h, l. \quad (4)$$

If *both* goods are traded internationally, then equations (4) hold as equalities, and it follows that:

$$\frac{p_2^h}{p_2^l} = \frac{p_1^h + t}{p_1^l + t}. \quad (5)$$

Since  $t > 0$ , it must be the case that:

$$\frac{p_2^h}{p_2^l} < \frac{p_1^h}{p_1^l}. \quad (6)$$

This is the key Alchian-Allen result which states that the relative price of the high quality commodity is lower in the importing country, Country 2, than in the exporting country, Country 1. It holds if both commodities are exported from Country 1 to Country 2.

We now show that this cannot be the case if the commodities are perfect substitutes. The proof is straightforward. Given that attributes  $q$  and  $r$  are each measured in the same units, if

$$\frac{q^h}{q^l} = \frac{r^h}{r^l} = \alpha, \text{ where } \alpha > 1, \quad (7)$$

then *one unit of the high quality good and  $\alpha$  units of the low quality good* are perfect substitutes.

This interpretation of perfect substitutes is slightly different from the usual textbook definition,

which implies that individual units of each commodity are perfect substitutes, that is,  $\alpha=1$ . If

$\frac{q^h}{q^l} \neq \frac{r^h}{r^l}$ , then the two commodities are not perfect substitutes because they contain different

proportions of the two attributes. From equations (7) and (3), we obtain:

$$\frac{p_i^h}{p_i^l} = \alpha, \quad i = 1, 2. \quad (8)$$

This equation implies that  $\frac{p_1^h}{p_1^l}$  must be equal to  $\frac{p_2^h}{p_2^l}$ . However, this contradicts the inequality in

equation (6), which must hold if both goods are being exported from Country 1 to Country 2. Thus,

when the two commodities are perfect substitutes and both are consumed in both countries, only one

of the two commodities can be traded.

The above result also implies that if both commodities are to be exported from Country 1 to

Country 2 then the two commodities must be imperfect substitutes for one another. When one unit

of the high quality good and  $\alpha$  units of the low quality good are close substitutes, but not perfect

substitutes,  $\frac{q^h}{q^l} \neq \frac{r^h}{r^l}$ . Thus, based on equation (3), the price ratios may be different across the

countries because the hedonic prices (which are determined by consumer tastes and producer costs

[Rosen, 1974]) are not identical in the two countries, so that corresponding TERMS in (3) do not

cancel. As condition (6) may hold in equilibrium, both goods may be traded internationally.

### Is the Alchian/Allen Proposition Correct in the Case of Perfect Substitutes?

If the two commodities are perfect substitutes in the sense defined above, and both commodities are consumed in each country, does the Alchian/Allen proposition hold? The answer is yes. When all local markets are in equilibrium from (8) it follows that:

$$\frac{p_2^h}{p_2^l} = \frac{p_1^h}{p_1^l} = \alpha. \quad (9)$$

According to inequality (6), however, if both goods are traded internationally, and in Country 1,  $\frac{p_1^h}{p_1^l}$  is equal to  $\alpha$ , then in Country 2,  $\frac{p_2^h}{p_2^l}$  must be less than  $\alpha$ . As we have seen, this condition cannot be satisfied if both commodities are consumed in Country 2. Therefore, it follows that both commodities cannot be exported from Country 1 to Country 2.

To determine which commodity will be exported from Country 1, we consider the two possible cases: (1) Country 1 exports only the low quality commodity, and (2) Country 1 exports only the high quality commodity. If only the low quality commodity is exported, then the following commodity price relationships must hold:

$$p_2^h < p_1^h + t, \text{ and } p_2^l = p_1^l + t. \quad (10)$$

This implies that

$$\frac{p_2^h}{p_2^l} < \frac{p_1^h + t}{p_1^l + t} \quad (11)$$

Given that both commodities are being consumed in both countries, equation (9) must be satisfied.

However, substituting equation (9) into equation (11) implies that  $(p_1^h - p_1^l) < 0$ .

This is clearly not feasible; the higher quality product cannot sell at a discount relative to the low quality commodity. When Country 1 exports only the high quality commodity, then it follows that:

$$p_2^h = p_1^h + t, \text{ and } p_2^l < p_1^l + t. \quad (12)$$

Thus,  $\frac{p_2^h}{p_2^l} > \frac{p_1^h + t}{p_1^l + t}$ , and it can also be shown that  $(p_1^h - p_1^l) > 0$ , which is perfectly feasible.

It is also straightforward to show that if both commodities are perfect substitutes and both are exported from Country 1 to Country 2 then only the low quality commodity will be consumed in Country 1. Given that both commodities are exported from Country 1 and, consequently, consumed in Country 2, it follows from equation (8) and equation (5) that:

$$\frac{p_2^h}{p_2^l} = \alpha, \text{ and } \frac{p_2^h}{p_2^l} = \frac{p_1^h + t}{p_1^l + t}. \quad (13)$$

This implies that  $\frac{p_2^h}{p_2^l} < \frac{p_1^h}{p_1^l}$  and, therefore, that  $\frac{p_2^h}{p_2^l} > \alpha$ . This result implies that in Country 1 the hedonic prices of the quality attributes in the high quality commodity are greater than in the low quality commodity. This is only feasible if the high quality commodity is not consumed in Country 1. Thus, Country 1 must export all its production of the high quality commodity to Country 2.



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## Conclusion

We have shown that when two commodities are perfect substitutes in the sense that one unit of the high quality good and more than one unit of the low quality good are perfect substitutes, and if both countries consume both commodities, then only the high quality commodity will be traded. Borcharding and Silberberg relied on an assumption that both goods are exported to obtain their comparative statics result that an increase in transport costs would lead to an increase in the relative exports of the high quality commodity. Their statement that as the two goods “become closer and closer substitutes, ... we should expect to see Alchian and Allen’s hypothesis confirmed” should be interpreted carefully. When the two goods become closer substitutes, a corner solution certainly becomes more likely. However, when they become perfect substitutes, a corner solution becomes a certainty; that is, either exports of the low quality good will cease or the exporting country will cease to consume the high quality commodity.

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