Multi-Stage Optimization Using Separable Programming

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ABSTRACT

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This paper presents a case study of an integrated poultry firm with a plant location problem where two distinct processing functions are necessary and each process is subject to size economies. The problem was approached using separable programming. Results showed that separable programming is effective for problems involving multi-stage processing.

Introduction

The economic problem of plant location, numbers and size has been addressed frequently in the literature. With an objective function minimizing combined transportation and processing costs, the problem has been addressed using a transportation or transshipment model in the vein of Stollsteimer or Chern and Polopolus. Later studies (King and Logan, Hurt and Tramel) considered economies of scale and the intermediate product problem.

Earlier efforts dealing with economies of scale used discrete activities for each plant size. Computational difficulties limited the number of size alternatives considered when numerous potential locations were available. Studies of dairy processing plants (Kloth and Blakely, Stennis) overcame some of the difficulties by using separable programming to approximate a nonlinear cost function. However, these studies considered only one type of plant and did not address the problem of a firm producing distinct but related products requiring unique facilities for each product.

Relatively little empirical work has been forthcoming related to firms processing two products, one of which is an input in the process for the other, but each has a separate cost function exhibiting size economies. A typical example of this characteristic is an integrated poultry firm assimilating feed ingredients which are processed into feed. The feed is distributed to broiler growers and the broilers are in turn assimilated by the processing plant and then shipped as dressed broilers.

The economic problem is the typical plant location problem for each product. The usual costs of acquiring inputs and distribution of product to final destinations are apparent. In addition, the firm faces two distinct processing cost curves, each of which may be non-linear and subject to economies and/or diseconomies of size. In some cases it should be useful to consider each of these cost curves simultaneously.

This paper presents an example of a plant location problem where two distinct processing functions are necessary and each process is subject to size economies. The problem is a case study of an integrated poultry firm and is approached by using separable programming. The separable programming routine developed by UNIVAC (Sperry Univac) and similar to the more widely used mathematical programming system of IBM was used for this purpose. Results will be presented for the feed producing, and distribution segment along with the meat processing segment.

The Model

Rodriguez used separable programming to study a feedmill location problem for an integrated poultry firm. His objective was to determine the size, number and location of feed mills that would minimize the cost of assembling feed ingredients, processing feed and distributing feed to growers. The feed processing cost function was segmented using separable programming.

This study extends Rodriguez's model to include the cost of assembling the grown out broilers, processing the birds and shipping dressed birds to demand points. The broiler processing function is also subject to economies of size and can be appropriately approximated by using the separable routine. The model can be specified as follows:

(1) $\text{Min TCC} = \sum_{j=1}^{o} \sum_{i=1}^{n} C_{ij}X_{ij} + \sum_{i=1}^{n} f_i(X_i) + \sum_{k=1}^{p} \sum_{i=1}^{n} \sum_{h=1}^{\theta} T_{ki}^{h} X_{ki}^{h} +$

$\sum_{j=1}^{o} \sum_{c=1}^{d} M_{jc}P_{jc} + \sum_{c=1}^{d} f_c(P_c) + \sum_{c=1}^{d} \sum_{y=1}^{r} P_{cy}Z_{cy}$

Subject to the following constraints:

(2) $\sum_i X_{ij} = R_j$

(3) $\sum_i X_{ki}^{h} \leq S_k^h$

(4) $\sum_h \sum_k S_k^h \geq \sum_j R_j$

(5) $\sum_k \sum_{h=1}^{\alpha} X_k^h = .6 \sum_k \sum_{h=1}^{\theta} X_k^h$

(6) $\sum_k \sum_{h=\alpha+1}^{\beta} X_k^h = .2 \sum_k \sum_{h=1}^{\theta} X_k^h$

(7) X_{ij}, X_{ki}^{h}, P_{jc}, and $P_{cy} \geq 0$

where:

TCC = total combined cost for assembly, processing and distribution of poultry feed; and for assembly, processing and transporting of broilers;

C_{ij} = per unit transportation cost of delivering feed from the feed mill i(i=1, ..., n) to the grower j(j=1, ..., o);

X_{ij} = quantity of processed feed delivered from feed mill i to grower j;

$f_i(X_i)$ = non-linear function expressing the total costs of processing quantity X_i in feed mill i;

T_{ki}^{h} = per unit assembly cost of shipping raw feed material h(h=1, ..., α for raw material one, h= +1, ...,β for raw material two, h= +1, ..., e for raw material three) from supply area K(k=1, ..., p) to feed mill i;

X_{ki}^{h} = quantity of raw material h shipped from supply area k to feed mill i;

R_j = quantity of poultry feed required by grower j;

S_k^h = quantity of raw material h available in supply area k;

M_{jc} = per unit transfer cost of shipping live birds from grower j to meat processing plant c;

P_{jc} = quantity of birds shipped from grower j to broiler processing plant c; where .46296296 is the conversion rate of a pound live bird per pound of feed fed (1÷ 2.16);

$f_c(P_c)$ = non-linear function expressing the total costs of processing quantity P_c in broiler processing plant c; where .72 is the amount of dressed meat obtained per pound of live bird;

P_{cy} = quantity of processed meat shipped from meat processing plant c to demand area y;

Z_{cy} = per unit transfer cost of shipping processed meat from broiler processing plant c to demand area y.

The first term in equation (1), $\sum_{j=1}^{o} \sum_{i=1}^{n} C_{ij} X_{ij}$, expresses the cost of shipping feed to the growers. Feed manufacturing costs are represented by $\sum_{i=1}^{n} f_i(X_i)$, a continuous function segmented by the separable routine. Raw material assembly costs were represented by $\sum_{k=1}^{p} \sum_{i=1}^{n} \sum_{h=1}^{e} T_{ki}^{h} X_{ki}^{h}$. Rodriguez's model was complete with this term.

The constraints required the total feed and allocated proportions among the three ingredients. The $\sum_{j=1}^{o} \sum_{c=1}^{d} M_{jc} P_{jc}$ term calculates the cost of shipping the birds to the broiler processing plants. The cost of processing broilers per unit of processed meat is represented by $\sum_{c=1}^{d} f_c (P_c)$. This cost curve was also segmented by the separable routine. The final term $\sum_{c=1}^{d} \sum_{y=1}^{r} P_{cy} Z_{cy}$, represents the cost of shipping processed broilers to final destinations. A sample matrix with two feed ingredients from four supplying regions, two feed mills, four growers, two broiler processing plants and two demand points is shown in Table 1. The other ingredient was not included in the sample matrix but was incorporated in the complete model to insure proper proportion of feed ingredients.

Data and Procedures

The initial problem addressed by Rodriguez considered optimal location, size and number of feed mills. Rodgriguez's analysis examined the location problem under different assumptions as to grower location and concentration. His study considered a ten year planning horizon. The situation would be representative of a firm replacing old milling facilities and at the same time expanding broiler production into new areas. Thus, grower feed demands were fixed and growth was proportionately increased in each period. The model was not allowed to select expansion region, only to select feed mill location.

The broiler processing and distribution sections were incorporated in the Rodriguez model in order to obtain a least-cost solution for the entire problem. Results are presented only for the base period and the final period of the ten year planning horizon of the original Rodriguez problem. This paper compares the results for the two periods where

Table 1. Sample matrix for two feed ingredients, four feed ingredient supplying regions, two feed mills, four growers, two broiler processing plants and two dressed broiler demand points.

		P1SOY1	P1SOY2	P1COR3	P1COR4	P2SOY1	P2SOY2	P2COR3	P2COR4	P1COST	P1D1	P1D2	P1D3	P1D4	P2COST	P2D1	P2D2	P2D3	P2D4	MP1COST	MP1D1	MP1D2	MP2COST	MP2D1	MP2D2	ENDCOST	P1C1	P1C2	P1C3	P1C4	P2C1	P2C2	P2C3	P2C4	CH1	CH2	CH3	CH4	C1MP1	C2MP1	C3MP1	C4MP1	C1MP2	C2MP2	C3MP2	C4MP2	MP1CHI	MP2CHI	MP1LOS	MP2LOS	RHS	
OBJ	N	C	C	C	C	C	C	C	C	.	C	C	C	C	.	C	C	C	C	.	C	C	.	C	C	.	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	.	
SOY1	L	1				1																																													A	
SOY2	L		1				1																																												A	RAW MATERIAL
COR3	L			1				1																																											A	SUPPLY
COR4	L				1				1																																										A	
P1ROW	E	-1	-1	-1	-1						p	p	p	p																																					0	FEED PROCESSING
P2ROW	E					-1	-1	-1	-1							p	p	p	p																																0	SEGMENTATION
P1	E	1	1	1	1																						-1	-1	-1	-1																					0	FEED TRANSFER
P2	E					1	1	1	1																						-1	-1	-1	-1																	0	
P1PRO1	E	-1	-1	Z	Z																																														0	
P1PRO2	E	k	k	-1	-1																																														0	FEED INGREDIENT
P2PRO1	E					-1	-1	Z	Z																																										0	PROPORTIONS
P2PRO2	E					k	k	-1	-1																																										0	
CG1	E																										1				1																				F	
CG2	E																											1				1																			F	GROWER FEED
CG3	E																												1				1																		F	REQUIREMENTS
CG4	E																													1				1																	F	

		Assembling Feed Ingredients		Processing Feed		Processing Broilers		Delivering Feed		Assembling Birds	Assembling Meat		Delivering		
		Feed Mill 1	Feed Mill 2	Feed Mill 1	Feed Mill 2	Process-ing Plant 1	Process-ing Plant 2	Feed Mill 1	Feed Mill 2	Live Birds Supply From Growers	Meat Plant 1	Meat Plant 2	Processed Meat From Processing Plants		
C1	E							1	1	-1				0	FEED CONVERSION
C2	E							1	1	-1				0	AND LIVE BIRD
C3	E							1	1	-1				0	TRANSFER
C4	E							1	1	-1				0	
CHS1	E									d	-1	-1		0	
CHS2	E									d	-1	-1		0	LIVE BIRD
CHS3	E									d	-1	-1		0	TO DRESSED BROILER
CHS4	E									d	-1	-1		0	
MP1ROW	E					a a					-1-1-1-1			0	BROILER PROCESSING
MP2ROW	E						a a					-1-1-1-1		0	SEGMENTATION
MP1	E										b b b b		-1 -1	0	PROCESSED
MP2	E											b b b b	-1 -1	0	BROILER TRANSFER
MP1CHI	G												1	0	
MP2CHI	G												1	0	PROCESSED BROILER
MP1LOS	G												1	0	ACCOUNTING ROW
MP2LOS	G												1	0	
SH1PC	G												1 1	M	DEMAND POINT
SH1PL	G												1 1	M	REQUIREMENTS

Legend:

-1 = -1	c = cost coefficient	p = feed processing semgent	A = supply availabilities of feed ingredients
1 = 1	k = constant ≠ 0	a = meat processing segment	F = grower feed requirements
0 = 0	z = constant ≠ 0	b = live bird to processed meat coefficient	M = final demand requirements
		d = feed to live bird coefficient	

non-linear cost curves were incorporated simultaneously for feed mills and broiler processing plants.

Data related to assembly costs for feed ingredients, feed requirements, grower numbers, feed manufacturing and shipping costs were those estimated by Rodriguez. Potential feedmill and broiler processing plant locations relative to initial grower concentration and anticipated expansion areas are shown in Figure 1.

Feed ingredients considered were soybean meal, corn and remaining ingredients combined as other. Each ingredient was available in any quantity needed from each source. There were five soybean meal sources, three sources for corn and one source for other ingredients. The feed was distributed to 179 growers (152 broiler, 27 breeder). The quantity of feed shipped to each grower was determined outside the model and the cost of transporting feed was estimated as a function of distance. The formulas used to calculate the assembly cost for raw materials at the feed processing plants as well as transporting the feed from the feed mills to the growers were developed by Rodriguez.

The 152 broiler growers supplied birds for the two potential broiler processing plant sites. Trucking costs were estimated for assembling live birds based on distance and weight. A non-linear processing cost function based on the number of live birds processed per hour was estimated. Processed broilers were allocated to four markets by fixed amounts, 35 percent of the total processed meat was allocated to each of the Chicago and Los Angeles markets, 20 percent to New Orleans and the remaining 10 percent was allocated to the Jackson, Mississippi market area.

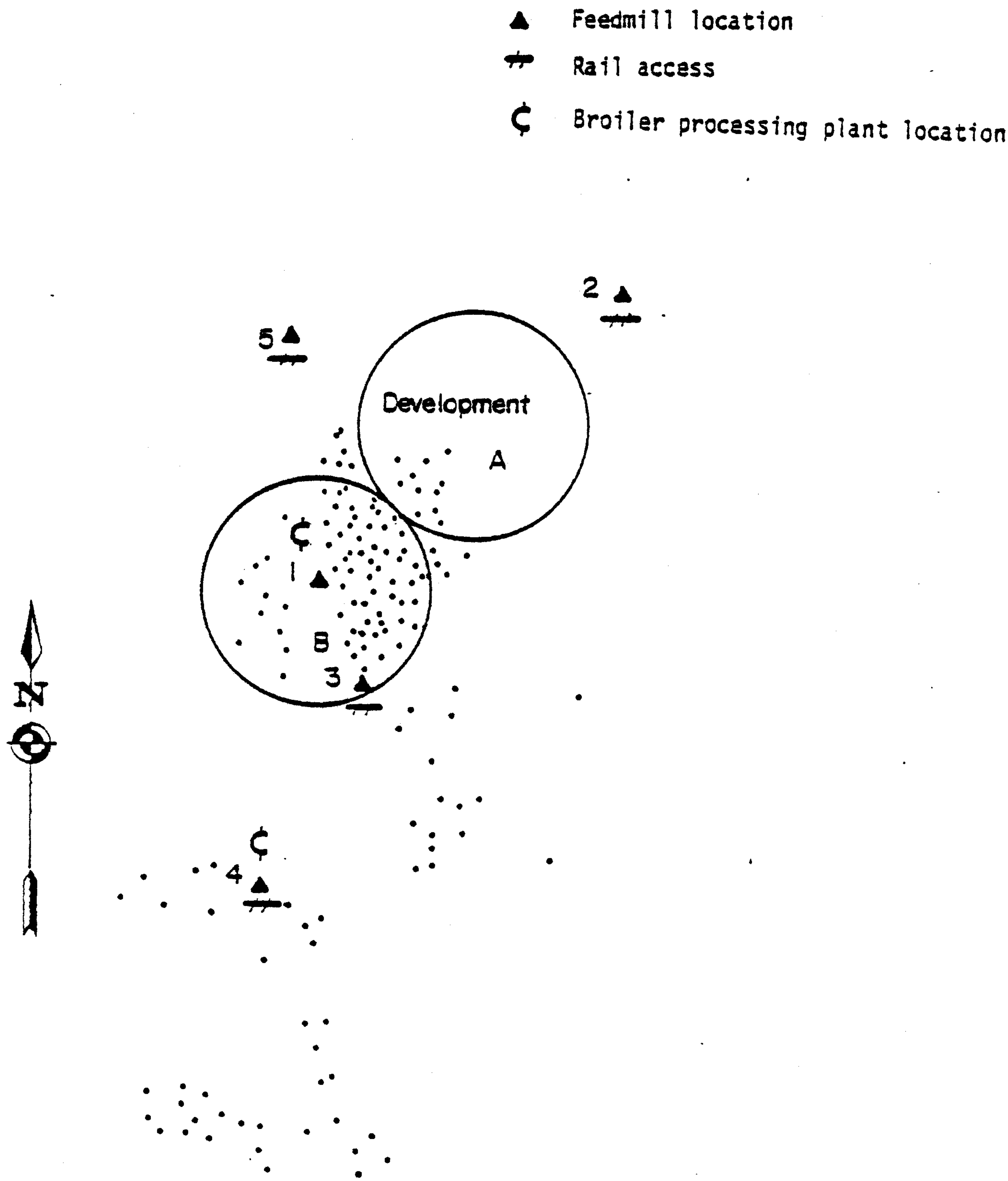

Figure 1. Relative locations of feedmills and growers.

Results

Simultaneous consideration of feed manufacturing and broiler processing costs resulted in the feed mill locations shown in Table 2. In the initial period only one feedmill entered the solution (Figure 1 and Table 2) at site 3. In the final period two feedmill sites entered the solution for this model.

Table 2. Least-cost feedmill location and size for two periods (grower locations).

Period	Number of feed mills	Optimum location	Feedmill size	Total volume
One	1	3	471.12	471.12
Ten	2	2	438.19	
		3	392.19	830.37

Both broiler processing plants entered the solution for all periods, thus only periods one and ten are shown for comparison.

Limitations and Implications

Since this analysis is a case study, conclusions about many economic questions were not answered. As used the model merely determined the least cost location and number of feed mills given different grower locations. It simultaneously considered the minimum cost size and location of broiler processing plants due to different grower locations. Grower location and volume of feed needed (therefore number of birds produced) were specified. Therefore, feed mill and broiler processing plant location were independent of each other. The problem could have been run as two separate problems. For this case, the only advantage of combining objective functions was in building the matrix.

The problem did indicate several important points. First, it did demonstrate that the separable routine can handle more than one separable function at one time. The usual convergence problems were encountered; however, the final solutions were stable and appeared consistent.

The problem could be made adaptable to a more general case quite readily. Allowing the model to select grower location, i.e. optimal grower location or expansion region would link milling and processing costs. Comparison of results for the two periods in this study indicates potential fruitful research.

Size economies in feed milling seem important compared to broiler processing as indicated by only one mill in period one. To what extent this would encourage concentration of growers and at what point one large processing plant might be feasible poses an interesting question. Conversely, transportation and utility needs of the processing sector could well influence feed mill and grower location.

Separable programming is effective for problems involving multistage processing. This study clearly demonstrates that this program can be used to evaluate the least cost organization for a firm where two distinct processing functions are employed and each process is subject to size economies.

In the future, with proper modification, this model could be used to determine the number of growers and their pattern of growth as well as the most economical feed and meat processing plants.

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