TOWARD A NEW OPTION VALUE

by

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INTRODUCTION

In the early literature on option price, option value and resource evaluation, the analysis emphasized demand uncertainty. Richard Bishop provided the first formal analysis of these concepts when supply is uncertain. Much of the discussion about the difference between supply-side and demand-side uncertainty has concentrated on the analytical distinctions, rather than the implications. There are, however, important implications for the analysis of resource policy which depend on the source of uncertainty.

In his classic essay, "Conservation Reconsidered," John Krutilla addresses one of the central problems in natural resources. What are the forces that lead to excessive and irreversible development of unique natural resources? Among these forces, according to Krutilla, is the inability of the market to appropriate the options that would be paid by economic agents with uncertain demands for the potential for the future use of the resource. Krutilla was especially interested in demand uncertainty of the sort present when future tastes are unknown. But he was also aware that the consumer making current decisions may not know his future demand for the resource because of uncertain future prices and income.

The concept of supply uncertainty arose in a context where researchers were still interested in the kinds of questions addressed by Krutilla, but were wrestling with problems of behavior towards risk and the distinctions between risk and variability. But some reflection will suggest that while supply risk or uncertainty is a useful concept in general, it is less relevant in the debate over the conservation of unique natural resources. In this debate, it is principally the availability of these natural resources which is at issue. In the notorious Hell's Canyon case, the damming of the Snake River would eliminate a unique natural gorge. There is no supply-side uncertainty. The uncertainty is strictly related to the tastes and the budget constraint: what will the future demand be, given uncertain preferences.

To treat this discussion analytically, suppose we have a decision about whether to preserve a unique natural resource when there is demand uncertainty. Let $V(m,p,\tilde{T})$ be the indirect utility function when p is non-random prices, m is income and T is a random vector representing tastes or prices. We use the convention of a tilde over a variable to represent its randomness. It is something which is random, influences demand, and is not subject to control by planners. The planning decision involves p. When p = $\hat{p}$, access to the resource is eliminated. When p = p^o, access is provided. The compensating option price associated with eliminating the resource is given by

$$EV(m,p^o,\tilde{T}) = EV(m - OP,\hat{p},\tilde{T}).$$

In this problem, p = $\{\hat{p}$ or $p^o\}$ is the choice variable and OP is the outcome. There is no uncertainty about p. If there is uncertainty about p, it cannot also be a choice variable. In the context of the classic problem of conserving a unique resource, the uncertainty concerns the preferences of users and potential users, not the decisions of planners.

Now change the story about what is random. Let T represent the quality of the resource. For example, T could be the quantity of fish caught per trip. Or it could be the effect of a pollutant on the resource user's health. Now the option price is the worth of access to a resource whose quality will not be known at the time the payment (OP) is made. In this case, several different kinds of policy questions are plausible. For example, instead of asking about access to the resource, as (1), we can solve for the indirect utility function as it depends on moments:

$$EV(m,p,\tilde{T}) = U(m,p,\mu)$$

where μ is a vector of the moments of the distribution of T. Then we can find the option price associated with the reduction in risk (the second moment):

$$U(m - OP,p,\mu_1,0) = U(m,p,\mu_1,\mu_2)$$

in the two-moment case.

There are some critical temporal differences between models with supply and demand uncertainty. In demand uncertainty, option price is the maximum sure payment that will be made, such that when the time comes, the true preferences are revealed to the individual. Consumers always face the problem of income allocation today based on uncertain preferences tomorrow. But in the classic problem, the uncertainty of future preferences has an impact only through the savings decision. In contrast, the decisions with supply uncertainty are risky decisions. During the current period, agents make decisions about resource use without having the uncertainty of supply resolved.

In sum, supply and demand uncertainty are both useful concepts, but their use pertains to different problems. Demand uncertainty is relevant in the long-run temporal sense in which Krutilla and others originally envisioned it. Supply uncertainty is relevant for management considerations when people are exposed to risky situations which can be controlled to some degree by public action. These distinctions are especially useful when viewed from the perspective of Hanoch's comments which are the basis of much current thinking about option prices: "ordinary demand functions, obtained under certainty, imply very little about an individual's behavior toward risk" (p. 414). There is a growing literature about behavior toward risk which is generated by conditions of supply, but practically no experience with risky decisions when the risk pertains to what the preference function will be in the following period.

For uncertain supply, we are able to estimate option prices in various ways. Smith (1985) shows how hedonic prices imply marginal option prices in the context of exposure to risks. Larson develops a moments approach to estimating option prices for recreational anglers uncertain of catch. But these measures do not help to address the question raised in the early literature: Can we determine when the expected value of surpluses exceeds option price? Thus, despite our apparently increasing ability to measure option price, we are not able to do so for some important cases of resource conservation.

AN EX ANTE PERSPECTIVE ON OPTION VALUE

Our interest in option value stems from its role in the temporal aspects of resource conservation. We are principally interested in demand uncertainty, for such uncertainty yields few natural experiments which would allow us to estimate option price. The arguments are more general, but are empirically most relevant for demand uncertainty.

The point of departure is the notion of risk premium. One of the initial puzzles of option value concerned its connection with risk aversion. It was generally argued that option value was a risk premium, and standard results from the theory of choices under uncertainty established the equivalence of positive risk premia and risk aversion. Why, it was asked, was option value not always positive for risk averse agents?

Several authors (Smith (1987), Chavas et al.), while analyzing option value within the conventional framework proposed by Cicchetti and Freeman, have argued that the problem of ambiguity in option value is with its definitions: option value is defined as the difference between an ex ante concept (option price) and an ex post concept (expected consumer's surpluses). Risk premiums, which are the primary motivation for option value, are on the other hand an entirely ex ante concept.

This paper proposes a set of consistent ex ante definitions for option price, expected surpluses, and option value. These definitions, based on certainty equivalence following Pratt, highlight the role of risk premiums in determination of the sign of option value. The desired option price can be formulated either as an ex ante compensating or equivalent variation measure with qualitatively similar results about the sign of option value. These definitions offer an intuitively clear explanation for the sign of option value while emphasizing the roles of ex ante welfare measurement and risk premiums in the theoretical determination of option value.

Cicchetti and Freeman defined option value (OV) for continued availability of a good as the difference between the consumer's "option price" (OP) and his or her expected consumer's surplus (E(CS)):

(1) $$OV = OP - E(CS).$$

A consumer's option price for a good is his or her ex ante willingness to pay for its availability, given that the consumption choice process is characterized by uncertainty. This good is often taken to be the services of a unique natural resource, since valuation of such resource is the context in which the idea of option value arose. The consumer will be able to react optimally once the values of the random variables are known, but ex ante only their joint distribution is known. A variety of specific cases are generated by different assumptions about the source of uncertainty, but the same general structure is applicable for the analysis. If good 1 is the good whose option value is being determined, then its option price can be written implicitly as

$$(2) \qquad EV(p_1^o, m^o, \tilde{T}^o) = EV(\hat{p}_1, m^o - OP, \tilde{T}^1)$$

where $V(\cdot)$ is the indirect utility function and the risk $\tilde{T}$ may be affected by the price change. For expositional convenience, p_1 will be taken to be non-random, though some or all other prices may be random, and (2) is written with non-random income, though random income could equally well be analyzed with appropriate modifications to the notation. Two cases where $\tilde{T}^o$ and $\tilde{T}^1$ would not be identical are where the quality of good 1 is random and where $\tilde{T}$ depends on p_1 (or, equivalently, T depends on consumption of good 1).

This formulation is somewhat more general than that of Plummer and Hartman, who assume $\tilde{T}$ is exogenous and is not affected by the price change. Here the option price is defined specifically as the consumer's willingness to pay for the availability of good 1 at price p_1^o, though the approach is easily generalized to determine the ex ante compensating variation of any parameter change (e.g., following Chavas et al.)

In contrast, the expected consumer's surplus (or more precisely, the expected compensating variation) is given in the discrete case by

$$(3) \qquad E(CS) = \sum_{i=1}^{m} \pi_i S_i$$

where π_i is the exogenous probability of state i (i.e., value T_i^j for the random variable $\tilde{T}^j$) occurring, and S_i is the surplus which results, defined implicitly by

$$(4) \qquad V(p_1^0, m^0, T_i^0) = V(\hat{p}_1, m^0 - S_i, T_i^1).$$

This is an ex post notion analogous to complete insurance, since surpluses are calculated after the uncertainty is resolved. They are defined as the payments which return the individual from a given level of utility ex post (after the outcome of the random vector is known), given good 1 is available, to the reference (without-good 1) expected utility level.

Plummer and Hartman used these definitions to derive conditions governing the sign of option value. With some simplifying assumptions about monotonicity in T, they showed the sign of option value depends on the covariance of the marginal utility of income and ex post (random) compensation. Chavas et al. used a two-period model to analyze the effects of changes in expected future prices on ex ante (period 1) willingness to pay or compensation. They found a difference between ex ante willingness to pay (i.e., option price) and expected consumer's surplus, a "correction factor" similar in concept to option value which depends on the covariance of the marginal utility of income and compensated demand for the good whose price changes. They refrain from equating their correction factor to option value, since their analysis concerns only ex ante willingness to pay and compensation, whereas they argue that option value compares option price to an ex post willingness to pay.

The approach here, like that of Chavas et al., is entirely ex ante in perspective, but uses the certainty equivalence framework of Pratt to highlight the role of risk premiums in the willingness to pay to exchange risks, which offers an intuitively clear explanation for the sign of option value. The definition of a consistent ex ante option value is premised on two ideas: (1) that under quite general and plausible conditions it is possible to use the utility function to define a change of variables which converts a given density function of a non-income variable (e.g., $\tilde{T}$) to an equivalent income risk (or to convert a joint density of income and other variables to an equivalent density function of income alone); (2) the option price or willingness to pay to trade one risk for another can be expressed as the difference in certainty equivalents of the two risks, which decomposes to two terms, a change in means (expected surplus) and a change in risk premiums (option value).

To develop the approach, consider the transformation

(5) $$V(p',m_{ji},T') \equiv V(p^j,m^o,T_i^j),$$

which is to be used to define an equivalent income risk m_{ji} corresponding to the outcome T_i^j of the random vector $\tilde{T}^j$, price p^j, and initial income m_o, with respect to reference levels p' for price and T' for the risk $\tilde{T}$.

The transformation given in (5) is quite general in that the income risk $\tilde{m}_j$ can, in principle, be defined for any reference levels p' and T'. Typically T' will be the mean $\bar{T}^j$ of a risk $\tilde{T}^j$ to facilitate the analysis of certainty equivalents. Provided the utility function is bounded and continuous and $\tilde{T}^j$ has finite range, the transformation in (5) is well defined and can be solved explicitly for the random variable $\tilde{m}_j$ as follows:

$$\tilde{m}_j = e(p',T',V(p^j,m^o,\tilde{T}^j)).$$

Naturally, by (5), the distribution of utility outcomes for the income risk $\tilde{m}_j$ conditional on T',p' is the same as the original distribution of utility outcomes and has the same mean:

(6) $$EV(p', \tilde{m}_j, T') = EV(p^j, m^o, \tilde{T}^j).$$

To make these definitions and the discussion more concrete, consider the definition of alternative income risks illustrated in Figure 1. The consumer of interest has income m^o, and faces prices p^j and the risk $\tilde{T}^j$. The utility outcomes corresponding to this situation are represented by the vertical shaded line in the graph, and have expectation $EV(p^j,m^o,\tilde{T}^j)$. The upper income contour $V(p^j,m,\bar{T}^j)$ corresponds to the risk $\tilde{T}^j$ held constant at its mean, with prices p^j. Note this contour does not intersect the set of utility outcomes at $EV(p^j,m^o,\tilde{T}^j)$ unless $V(\cdot)$ is linear in T^j. The equivalent income risk $\tilde{m}_1$ is defined for this contour; that is, $\tilde{m}_1 = e(p^j,\bar{T},V(p^j,m^o,\tilde{T}^j))$. The equivalent income risk $\tilde{m}_2$ is defined for another conditional indirect utility function where $p = p'$ and $T = T'$: $\tilde{m}_2 = e(p',T',V(p^j,m^o,\tilde{T}^j))$. Both these income risks have the same distributions of utility outcomes as the conditional T-risk with $p = p^j$, $m = m^o$.

The <u>certainty equivalent</u> of this income risk $\tilde{m}_j$ is given by $m_j^{CE} \equiv \bar{m}_j - \pi^j$, where π^j is a Pratt risk premium defined implicitly by

(7) $$V(p', \bar{m}_j - \pi^j, T') \equiv EV(p', \tilde{m}_j, T'),$$

where $\bar{m}_j \equiv E(\tilde{m}_j)$.

While the relationship between the income-risk premiums π^j corresponding to different situations confronting the consumer will be the primary focus of the analysis, there is another risk premium ϕ_j (a "T-risk" premium) which can be defined in terms of the original risk $\tilde{T}$ and m^o:

$$V(p', m^o - \phi^j, T') \equiv EV(p^j, m^o, \tilde{T}^j),$$

from which it is apparent, using (6) and (7), that

$$\phi^j = (m^0 - \bar{m}^j) + \pi^j.$$

In general, $\bar{m}^j \neq m^0$; that is, the mean of the equivalent income risk is not the consumer's income level m^0, even when $p' = p_j$ and $T' = E(\tilde{T}^j)$, due to the nonlinearity of preferences in income. Thus $\phi^j \neq \pi^j$ under risk aversion. While either risk premium can be used in the analysis of option price that follows, π^j is particularly revealing about the sign of option value.

The foregoing definitions are now applied to the specific case of measuring option price, represented by a price change from p^0 to $\hat{p}$. The approach will be to specify the initial and subsequent sets of utility outcomes, corresponding to prices p^0 and $\hat{p}$ respectively, as equivalent income risks for a single conditional indirect utility function. This allows one to compare the initial and subsequent situations in terms of differences in income risks, with all other arguments of indirect utility equal for both situations. While the choice of conditional indirect utility function used to make this comparison is arbitrary, the definitions of Hicksian welfare measures suggest suitable choices for reference conditional indirect utility functions.

The definition of option price given in equation (2) is an <u>ex ante</u> compensating variation measure; the reference situation is one where good 1 is available ($p = p^0$). The compensation is the amount of money which, when taken from income, leaves the individual at the initial utility level. Given the goal of defining suitable transformations so option price can be evaluated in terms of equivalent income risks, the appropriate conditional indirect utility function is $V(\hat{p},m,\bar{T}^1)$; i.e., indirect utility of income conditional on $\hat{p},\bar{T}^1$. This is the conditional indirect utility function corresponding to the certainty equivalent of the subsequent situation. Defined for this reference level of utility, the difference in equivalent income risks, a random variable, is the amount of money that moves the individual from the subsequent situation ($p = \hat{p}$) to the initial situation ($p = p^o$), and thus is a compensating variation measure. Expressed in terms of this conditional indirect utility function, the reference (initial) equivalent income risk is

$$(8) \qquad V(\hat{p},\tilde{m}_0,\bar{T}^1) \equiv V(p^o,m^o,\tilde{T}^o),$$

using (5). The subsequent equivalent income risk, expressed in terms of the same conditional indirect utility function, is

$$(9) \qquad V(\hat{p},\tilde{m}_1,\bar{T}^1) \equiv V(\hat{p},m^o,\tilde{T}^1),$$

which is also obtained using (5). Recall that the price change from p^o to $\hat{p}$ may, in general, cause the risk (i.e., the random vector $\tilde{T}$) to change from $\tilde{T}^o$ to $\tilde{T}^1$.

Figure 2 illustrates the analysis. The initial situtation is characterized by the upper shaded vertical line representing random utility outcomes conditional on consumer's income m^0 and initial price p^0. The subsequent situation, corresponding to removal of good 1 by a price change to $\hat{p}$, is given by the lower shaded vertical line. The equivalent income risk of the initial situation, conditional on $\hat{p}$ and $\bar{T}^1$, is $\tilde{m}_0$, with expectation $\bar{m}_0$. The equivalent income risk of the subsequent situation, also conditional on $\hat{p}$ and $\bar{T}^1$, is $\tilde{m}_1$, which has expectation $\bar{m}_1$; $\bar{m}_1$ is not equal to the consumer's initial income m^0 under risk aversion.

The utility outcomes for the initial situation, involving the risk $\tilde{T}^o$, given p^0 and m^o, have now been characterized as an equivalent income risk, $\tilde{m}_0$, conditional on $\hat{p}$ and $\bar{T}^1$. The utility outcomes for the subsequent situation, involving price $\hat{p}$, risk $\tilde{T}_1$ (induced by $\hat{p}$), and m^o, have been expressed as equivalent income risk $\tilde{m}_1$, also conditional on $\hat{p}$ and $\bar{T}^1$. The consideration of the consumer's willingness to pay (or need to be compensated) for a change from the initial to subsequent situation can, for qualitative purposes, be evaluated by a comparison of these two equivalent income risks.

To see this, note that by (6) and (7), one can write

(10a) $$EV(p^0, m^0, \tilde{T}^0) = EV(p^0, \tilde{m}_0, \bar{T}^0) = V(\hat{p}, \bar{m}_0 - \pi^0, \bar{T}^1)$$

(10b) $$EV(\hat{p}, m^0, \tilde{T}^1) = EV(p^0, \tilde{m}_1, \bar{T}^0) = V(\hat{p}, \bar{m}_1 - \pi^1, \bar{T}^1)$$

for the initial and subsequent situations, respectively. Compensating option price (equation (2)) involves a comparison of the left-hand terms in (10a) and (10b),

$$EV(\hat{p}, m^0 - OP, \tilde{T}^1) = EV(p^0, m^0, \tilde{T}^0),$$

and can equivalently be written as a comparison of the right-hand terms in (10a) and (10b):

$$V(\hat{p}, \bar{m}_1 - \pi^1 - OP, \bar{T}^1) = V(\hat{p}, \bar{m}_0 - \pi^0, \bar{T}^1),$$

from which it is clear that

$$\bar{m}_1 - \pi^1 - OP = \bar{m}_0 - \pi^0.$$

Rearranging to solve for option price yields

(11) $$OP = (\bar{m}_1 - \bar{m}_0) + (\pi^0 - \pi^1)$$

$$= E(S) + OV.$$

Equation (11) decomposes the compensating option price into the familiar expected surplus and option value. When the consumer exchanges a risk $V(p^0, m^0, \tilde{T}^0)$ for a risk $V(\hat{p}, m^0, \tilde{T}^1)$, the expected monetary equivalent of the utility change, evaluated with $V(p^0,m^0,\bar{T}^0)$, is $\bar{m}_1 - \bar{m}_0$. Each of the two risks is discounted, however, by their respective risk premiums, so that each risk has a certainty equivalent in the mind of the risk averse consumer. The consumer's actual willingness to pay is the difference in the certainty equivalents; this differs from the expected monetary gain by the difference in discounts for risk. Thus, the value of an option to have a risk $\tilde{m}_1$ is the "savings" due to its lower risk premium (or dissavings if its risk premium is higher); this is not accounted for by the difference in expected monetary gain (i.e., the expected surplus).[2]

While it is not obvious, the definition of option value in (11) is equivalent to the standard definition. This can be demonstrated by the following lemma.

<u>Lemma 1</u>. The difference in risk premiums $\pi^0 - \pi^1$ given in (11) is equivalent to option value, given by (1).

<u>Proof</u>. Note first that the equivalent income risk for $V(\hat{p},m^0,\tilde{T}^1)$, conditional on $\hat{p}$ and $\bar{T}^1$, is given by (9):

$$V(\hat{p},\tilde{m}_1,\bar{T}) \equiv V(\hat{p},m^0,\tilde{T}^1),$$

where $\tilde{m}_1 \equiv e(\hat{p},\bar{T}^1,V(\hat{p},m^0,\tilde{T}^1))$. Now recall that surpluses are defined as

$$V(p^0,m^0,\tilde{T}^0) \equiv V(\hat{p},m^0 - \tilde{S},\tilde{T}^1) \tag{12}$$

from equation (2) written as a continuous transformation. Using (9) and (12), one can express the definition of surpluses $\tilde{S}$ in terms of the equivalent income risk $\tilde{m}_1$:

$$\begin{aligned} V(p^0,m^0,\tilde{T}^0) &= V(\hat{p},m^0 - \tilde{S},T^1) \\ &= V(\hat{p},\tilde{m}_1 - \tilde{S},\bar{T}^1). \end{aligned} \tag{13}$$

Solving (13) explicitly for surpluses $\tilde{S}$ yields

$$\tilde{S} = \tilde{m}_1 - e(\hat{p},\bar{T}^1,V(p^o,m^o,\tilde{T}^o))$$

$$= \tilde{m}_1 - \tilde{m}_0,$$

by (8). Taking expectations,

$$E(\tilde{S}) = \bar{m}_1 - \bar{m}_0.$$

So by (11), $OV = \pi^0 - \pi^1$. QED.

Despite the similarity of the decomposition in (11) to the usual division of option price into two components, there are conceptual differences. As in the Chavas et al. analysis, this is a wholly ex ante treatment of the option value question, though unlike theirs, this model is static. Option price in this paper is the difference in certainty equivalent incomes of the subsequent and initial situations, instead of the payment that equates expected utilities, as in (2); however, from (10a) and (10b) it is seen that these are equivalent. In this analysis the expected-surpluses term is the difference in expectations of the equivalent monetary gains associated with the subsequent and initial situations, defined by the conditional indirect utility function for the certainty equivalent of the initial situation; expected surpluses in the option value literature (defined in equations (3) and (4)) is the expectation of consumers' surpluses from making the change from the initial to the subsequent situation, using initial expected utility for the reference level. Option value in this paper is a difference in risk premiums, whereas in the standard models it is simply the difference between option price and expected surpluses.

Formulating option value as a difference in risk premiums focuses on the riskiness of the prospects to the consumer. The preferences for these risks provides a guide to the sign of option value. As special cases, it is seen immediately from (11) that if the situation being analyzed involves a change from an uncertain situation to a certain one, option value is positive; conversely, for a change from certainty to uncertainty, the option value is negative. More generally, it seems reasonable to expect that analysts charged with benefit-cost analysis have information on the objective risks both with and without a proposed project, and it is probably reasonable to assume that information about these risks is (or can be) fully conveyed to consumers. Thus, it may be possible to combine information about comparative riskiness of the initial and subsequent situations with standard restrictions on preferences for risk (e.g., constant or decreasing absolute risk aversion) to make assertions about the sign of option value in different situations.

DETERMINING THE SIGN OF OPTION VALUE

The fact that option value can be determined as the difference in equivalent-income risk premiums makes it possible to draw upon results from models of decision making under risk. When option value is written

$$OV = \pi^0 - \pi^1, \tag{14}$$

it is tempting to equate the magnitude of the risk premium to the riskiness of the prospect, and say that

$$\pi^i \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \pi^j \quad \leftrightarrow \quad \text{"}\tilde{m}_i \text{ is } \begin{Bmatrix} \text{more} \\ \text{equally} \\ \text{less} \end{Bmatrix} \text{ risky than } \tilde{m}_j\text{"}$$

which leads to the conclusion that

$$\text{option value is} = \begin{Bmatrix} > \\ 0 \\ < \end{Bmatrix} \text{ as the initial situation is } \begin{Bmatrix} \text{less} \\ \text{equally} \\ \text{more} \end{Bmatrix} \text{ risky}$$

than (as) the subsequent situation.

While this may be a useful heuristic guide to the sign of option value, it suffers from two problems. First, it is only possible to equate unambiguously the riskiness of the prospect to the size of the risk premium when the two risks being compared have the same expectation (Rothschild and Stiglitz), and the way they are defined in this paper, usually $\bar{m}_0 \geq \bar{m}_1$; i.e., the expected equivalent income with good 1 available is typically at least as great as without good 1. Second, the equivalent-income risks are not observed, as they are the T-risks (which may in principle be observed) transformed through the preference function. Thus, the empirical usefulness of such a decision rule is limited without taking into account the structure of preferences.

To get more insight into the sign of option value, it is helpful to solve explicitly for the risk premiums at the initial and subsequent situations and compare them. Using (10a), one can write

$$(15) \qquad \pi^0 = \bar{m}_0 - e(\hat{p}, \bar{T}^1, EV(p^0, m^0, \tilde{T}^0))$$

$$= Ee(\hat{p}, \bar{T}^1, V(p^0, m^0, \tilde{T}^0)) - e(\hat{p}, \bar{T}^1, EV(p^0, m^0, \tilde{T}^0))$$

where the expression for $\bar{m}_0$, obtained using (8), has been substituted in. Note that under risk aversion the expenditure function is convex in utility, so by Jensen's inequality, $\pi^0 > 0$. Likewise, from (10b) the risk premium for the subsequent situation is

$$(16) \qquad \pi^1 = Ee(\hat{p}, \bar{T}^1, V(\hat{p}, m^0, \tilde{T}^1)) - e(\hat{p}, \bar{T}^1, EV(\hat{p}, m^0, \tilde{T}^1)).$$

Comparing (15) and (16), one can see how the curvature of the expenditure and utility functions influences the risk premia. A key to the sign of option value, their difference, is how the price change (the vehicle for analyzing removal of the good) affects this curvature. To address this question, we assume that T is a scalar and its distribution is exogenous; that is, $\tilde{T}^0 = \tilde{T}^1 = \tilde{T}_i$. This is the case analyzed by Plummer and Hartman.

The difference in risk premiums can be written

$$\pi^0 - \pi^1 = \int_{\hat{p}}^{p^0} [\partial\pi(\hat{p}, \bar{T}, V(p, m^0, \tilde{T}))/\partial p]dp.$$

When $\partial\pi/\partial p$ is monotonically decreasing in p over the price interval $[p^0, \hat{p}]$, option value will be positive. Looking at the conditions for $\partial\pi/\partial p$ positive helps us determine when option value is positive.

Letting $\hat{p} = p$ in (16) and differentiating with respect to p, noting that the change in risk premium is evaluated along $e(\hat{p}, \bar{T}, V(p, \cdot))$ yields

$$\partial\pi/\partial p = E\{[\partial e(\hat{p}, \bar{T}, V(p, m^0, \tilde{T}))/\partial V][\partial V(p, m^0, \tilde{T})/\partial p]\} - [\partial e(\hat{p}, \bar{T}, EV(p, m^0, \tilde{T}))/\partial V] \cdot E[\partial V(p, m^0, \tilde{T})/\partial p]. \tag{17}$$

Inspection of (17) suggests a plausible case where the sign of option value can be determined. If the price change does not affect preferences for T (i.e., if $V_{pT} = 0$) then the marginal utility of the price change is constant across all outcomes of $\tilde{T}$. If, in addition, the decisionmaker's preferences are not too risk averse, then option value is non-negative.

To explore this relation, we need the following definitions and lemma.

<u>Definition</u>. The coefficient of absolute risk aversion is $A \equiv -V_{mm}/V_m$, where $V(p,m,T)$ is the indirect utility function.

<u>Lemma 2</u>. If $dA/dm \geq -A^2$, then $e_{VVV}(p,T,V) \geq 0$.

Proof. Suppose that

$$dA/dm = (V_{mm}/V_m)^2 - V_{mmm}/V_m \leq -(V_{mm}/V_m)^2$$

$$= -A^2.$$

Adding $(V_{mm}/V_m)^2$ to both sides and multiplying by V_m^2,

(18) $$2V_{mm}^2 - V_{mmm}V_m \geq 0.$$

Now by duality, $e_v \equiv (V_m)^{-1}$, so $e_{vv} = -V_{mm}/V_m^2$ and $e_{vvv} = [2V_{mm}^2 - V_{mmm}V_m]/V_m^3$. By (18), $e_{vvv} > 0$. QED.

Note that Lemma 2 holds for risk preferences that are not too decreasingly risk averse; it holds for all preferences that exhibit constant or increasing risk aversion, and for many (though not all) preferences which have decreasing absolute risk aversion. Since this condition is fundamental to the sign of option value, it is worth considering what preferences are too decreasingly risk averse and which are not. Defining the coefficient of relative risk aversion as $R = A \cdot m$, the condition in Lemma 2 can be expressed

$$dA/dm \cdot (m/A) \geq -A \cdot m$$

or

(19) $$dA/dm(m/A) \geq -R$$

so that the elasticity of absolute risk aversion with income must exceed the negative of the coefficient of relative risk aversion for the condition in Lemma 2 to hold. Since a number of authors (e.g., Arrow) have argued that $R \doteq 1$, (19) suggests that preferences for which the coefficient of absolute risk aversion is inelastic with respect to income are "not too decreasingly risk averse."

It is now possible to address the case where $V_{pT} = 0$.

Theorem 1. If $V_{pT} = 0$ and preferences satisfy the condition of Lemma 1, then option value is non-negative.

Proof. $V_{pT} = 0$ implies that $\partial V(p,m,T)/\partial p = k$, a constant; thus $E[\partial V(p,m^o,T)/\partial p] = \partial V(p,m^o,T)/\partial p$ for all T. Substituting for $\partial V(p,m^o,T)/\partial p$ on the right side of (17) gives

$$\begin{aligned}\partial\pi/\partial p &= E\{[\partial e(\hat{p},\bar{T},V(p,m^o,\tilde{T}))/\partial V]E\{\partial V(p,m^o,\tilde{T})/\partial p]\} \\ &\quad -[\partial e(\hat{p},\bar{T},EV(p,m^o,\tilde{T}))/\partial V] \cdot E[\partial V(p,m^o,\tilde{T})/\partial p] \\ &= E[\partial V(p,m^o,\tilde{T})/\partial p]\{E[\partial e(\hat{p},\bar{T},V(p,m^o,\tilde{T}))/\partial V] - \partial e(\hat{p},\bar{T},EV(p,m^o,\tilde{T}))/\partial V\}.\end{aligned}$$

Now

$$E[\partial V(p,m^o,\tilde{T})/\partial p] = E[-V_m(p,m^o,\tilde{T})\cdot x(p,m^o,\tilde{T})] \leq 0$$

by Roy's Identity, where $x(\cdot)$ is the Marshallian demand for good 1. Also, if preferences satisfy Lemma 2, $e_{VVV} \geq 0$, which implies that the term in braces is non-negative. Therefore, $\partial\pi/\partial p \leq 0$ and

$$\pi^o - \pi^1 = \int_{\hat{p}}^{p^o} (\partial\pi/\partial p)dp \geq 0,$$

since $\hat{p} > p^o$, implying option value is non-negative. QED.

To interpret the implications of Theorem 1 for applied welfare analysis under uncertainty, the assumption that preferences for T are unaffected by the change in price from p^o to $\hat{p}$ is sufficient for $V_{pT} = 0$. This may be viewed as a strong restriction, as one implication of such an assumption is that the elasticity of Marshallian demand for good 1 with T equals the negative of the elasticity of marginal utility of income with T.

A set of weaker sufficient conditions for which option value is non-negative is given by the following theorem. For notational simplicity, $e(\hat{p},\bar{T},V(p,m^o,\tilde{T})) \equiv e(\tilde{V})$, $e(\hat{p},\bar{T},EV(p,m^o,\tilde{T})) \equiv e(EV)$, and $V(p,m^o,\tilde{T}) \equiv V(\tilde{T})$.

<u>Theorem 2</u>. A necessary and sufficient condition for the equivalent income risk premium to be monotonically decreasing in the price of good 1 is

$$\text{cov}[\partial e(\tilde{T})/\partial V, \partial V(\tilde{T})/\partial p] \leq -E[\partial V(\tilde{T})/\partial p]\{E[\partial e(\tilde{V})/\partial V] - \partial e(EV)/\partial V\}.$$

<u>Proof</u>. Adding and subtracting $E[\partial e(\tilde{V})/\partial V]$ from (17) gives

$$\begin{aligned}\partial \pi/\partial p &= E\{[\partial e(\tilde{V})/\partial V][\partial V(\tilde{T})/\partial p]\} - E[\partial V(\tilde{T})/\partial p]\{\partial e(EV)/\partial V \\ &\qquad + E[\partial e(\tilde{V})/\partial V] - E[\partial e(\tilde{V})/\partial V]\} \\ &= E\{[\partial e(\tilde{V})/\partial V][\partial V(\tilde{T})/\partial p]\} - E[\partial e(\tilde{V})/\partial V]E[\partial V(\tilde{T})/\partial p] \\ &\qquad + E[\partial V(\tilde{T})/\partial p]\{E[\partial e(\tilde{V})/\partial V] - \partial e(EV)\partial V\}\end{aligned}$$

so that

$$(18) \qquad \partial \pi/\partial p = \text{cov}[\partial e(\tilde{T})/\partial V, \partial V(\tilde{T})/\partial p] + E[\partial V(\tilde{T})/\partial p]\{E[\partial e(\tilde{V})/\partial V] - \partial e(EV)/\partial V\},$$

where in the last step the fact that $e(\tilde{V}) \equiv e(V(\tilde{T})) \equiv e(\tilde{T})$ is used. Necessity and sufficiency of the theorem follow immediately from (18). QED.

From (18) it is apparent that the covariance of $e_v(\tilde{T})$ and $V_p(\tilde{T})$ and $e_{vvv}(\tilde{V})$ are important to the sign of option value, where subscripts denote derivatives. If the conditions of Lemma 2 hold, the term in braces is positive, which combined with the fact that $E[V_p(\tilde{T})] \leq 0$ means that $\text{cov}(e_v(\tilde{T}),V_p(\tilde{T})) \leq 0$ is sufficient for non-negative option value. Note that Theorem 1 follows as a special case of (18) because when $V_{pT} = 0$, $\text{cov}(e_v(\tilde{T}),V_p(\tilde{T})) = 0$. Another potential special case where option value is zero is where $e_{vT} = 0$, but this is highly restrictive since $e_{vT} = e_v \cdot e_T$ for the conditional expenditure function $e(\hat{p},\bar{T},V(p,m^o,\tilde{T}))$, and $e_{vT} = 0$ implies $V_T = 0$.

The remainder of this section will take as given that preferences satisfy Lemma 2, which is quite plausible, and attempt to determine what additional restrictions on preferences are sufficient for $cov(e_v(\tilde{T}),V_p(\tilde{T}))$ to be non-positive.

One basic result is the following Lemma. Sufficient conditions for $cov(e_v(\tilde{T}),V_p(\tilde{T})) \leq 0$ are

(a) e_v and V_p convex in T; and

(b) $e_v \cdot V_p$ concave in T.

<u>Proof</u>. e_v and V_p convex in T implies that $E[e_v(\tilde{T})] \geq e_v(\bar{T})$ and $E[V_p(\tilde{T})] \geq V_p(\bar{T})$, hence $V_p(\bar{T}) \cdot e_v(\bar{T}) \leq E[e_v(\tilde{T})] \cdot E[V_p(\tilde{T})]$. Also, $e_v \cdot V_p$ concave in T implies $e_v(\bar{T}) \cdot V_p(\bar{T}) \geq E[e_v(\tilde{T}) \cdot V_p(\tilde{T})]$. Combining these relationships,

$$E[e_v(\tilde{T}) \cdot V_p(\tilde{T})] \leq e_v(\bar{T}) \cdot V_p(\bar{T}) \leq E[e_v(\tilde{T})]E[V_p(\tilde{T})]$$

so that

$$cov[e_v(\tilde{T}),V_p(\tilde{T})] = E[e_v(\tilde{T}) \cdot V_p(\tilde{T})] - E[e_v(\tilde{T})]E[V_p(\tilde{T})] \leq 0.$$

QED.

CONCLUSION

There is a growing consensus that the correct measure for valuing resources under uncertainty is option price. Further, economists are finding various methods for observing behavior in risky settings to estimate option price. However, it is difficult to envision such settings when the uncertainty is of a particular temporal sort--today's decisions are risky because tomorrow's preferences are uncertain. Further, this demand uncertainty is precisely the kind of uncertainty that makes the preservation of future natural resources an open policy question. Consequently, methods for determining the sign of option value, especially in the case of demand uncertainty, are still relevant.

In this paper, we have reconstructed option value using the notion of risk premium. The idea of risk premium is implicit in the original debate about option value. By constructing option value from definitions of risk premia, we are able to show the circumstances when option value is positive and relate these circumstances to the traditional measures of risk aversion.

Footnotes

[1] This assertion is warranted on two grounds. First, Helms (1985) has shown that expected surpluses is not always a valid welfare indicator and the same is true for the "fair bet" point of Graham (1981). Second, since most cost-benefit analysis is performed before the true state is known, the ex ante perspective implied by the option price measure is perhaps more appealing.

[2] In a recent paper, Wilman used the same decomposition of option price into a difference in means of risks and a difference between risk premiums.

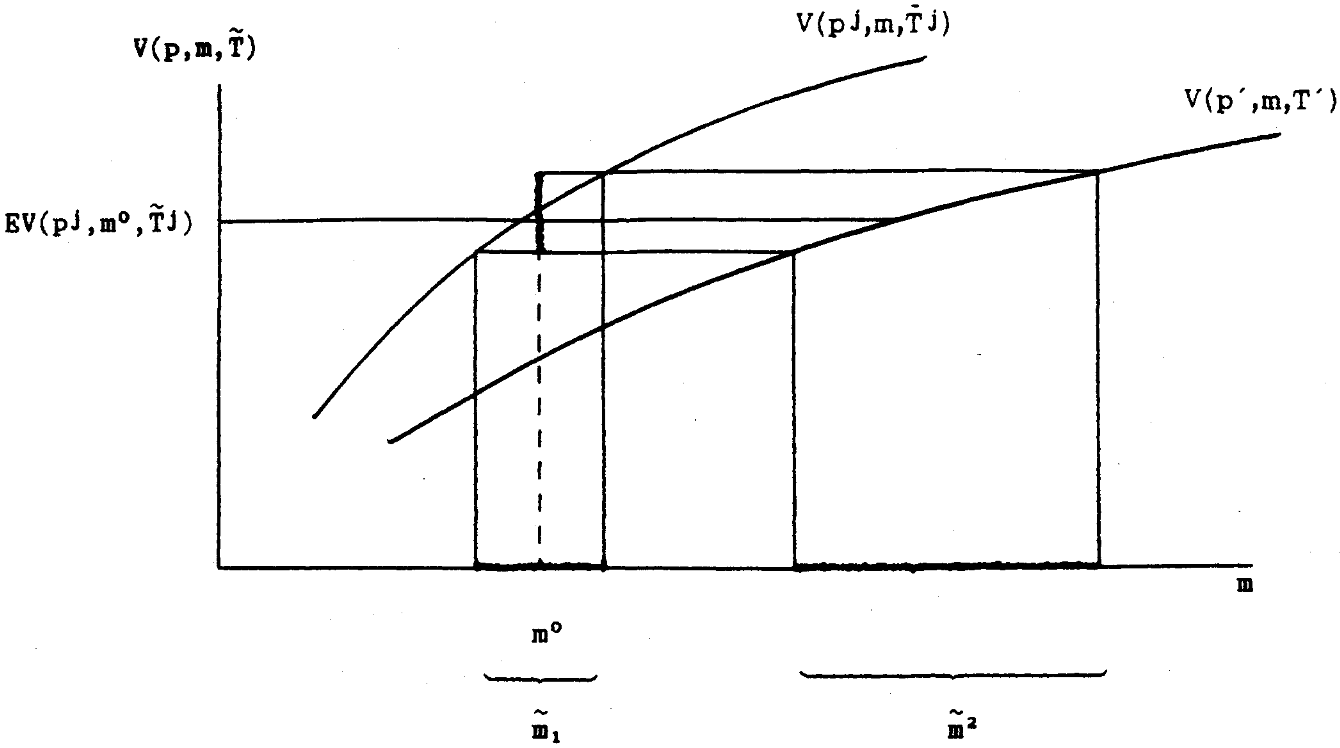

Figure 1

Defining Equivalent Income Risks

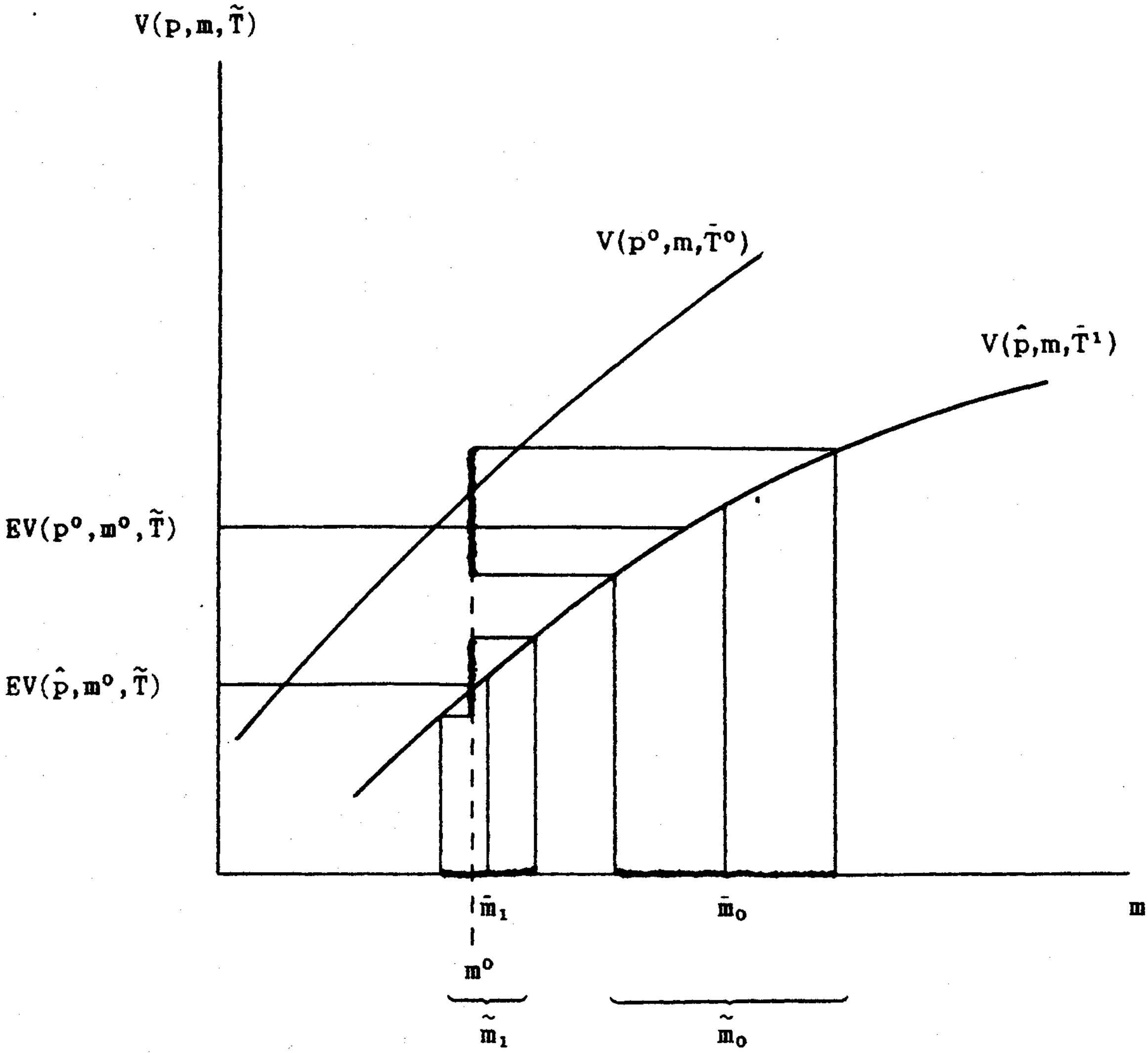

Figure 2

Equivalent Income Risks for the Compensating Variation Measure of Option Price; Price Changes from p^o to $\hat{p}$.

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