Abstract for

"Measuring Technical Efficiency Under Stochastic Production: An Application to Dairy Production"

by

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This paper shows that the output distribution moments can be used to measure technical efficiency under stochastic production and that the moments can be ordered according to the derivatives of the utility function for a broad class of utility functions. Applying the moment-based approach to California dairy production, it was found that scheduled veterinary services and management quality both affect technical efficiency, with management increasing in importance relative to veterinary services as risk aversion increases. Emergency veterinary services were found not to have a significant effect on technical efficiency.

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MEASURING TECHNICAL EFFICIENCY UNDER STOCHASTIC PRODUCTION: AN APPLICATION TO DAIRY PRODUCTION

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The modern technical efficiency literature, from Farrell's (1954) seminal work to more recent statistical models of frontier production functions (Forsund, Lovell, and Schmidt, 1982), is based on the neoclassical production theory. The neoclassical model of production is static and deterministic: firms choose inputs to maximize profit in a single production period. Technical efficiency, represented by the production function, is important because profit is increasing in output.

The modern literature on firm decision making under uncertainty recognizes that production may be better described as a stochastic phenomenon. Decision makers can be expected to take the stochastic nature of production into account. However, when output is stochastic, a neoclassical production function does not exist. Therefore, the frontier concept of technical efficiency is not meaningful, and the frontier rate of output is not a relevant benchmark by which technical efficiency can be measured. To define and measure technical efficiency under stochastic production, and more generally to model firm behavior, an alternative definition of technical efficiency is needed.

The aim of this paper is to present a framework for the definition and measurement of technical efficiency under stochastic production. This framework is based on the concept of an efficient probability distribution of output, or output distribution. It is shown that the output distribution's moments can be defined as functions of inputs and technology. This system of moment functions can be used to define technical efficiency and to develop empirical methods to quantify it. This approach nests the noeclassical model as a limiting case.

The paper is organized as follows. Section 1 develops a definition of technical efficiency under stochastic production, and relates it to a moment-based approach to stochastic technology measurement. Section 2 describes how the moment-based approach can be empirically implemented and illustrates it with an application to milk production.

1. DEFINING TECHNICAL EFFICIENCY UNDER STOCHASTIC PRODUCTION

The neoclassical definition of technical efficiency is justified by the fact that profit is increasing in output, *ceteris paribus*, so an increase in technical efficiency represents an increase in the firm's objective function. The following definition of technical efficiency is proposed which includes the neoclassical one as a limiting case.

Definition: A production process is technically efficient if and only if it gives the maximum feasible value of the firm's objective function for a given technology and input set.

This definition makes technical efficiency a meaningful concept relative to the firm's objective function.

In the neoclassical theory, the basic theoretical construct is the production function. To develop a theory of stochastic production, the logical analytical foundation is the probability distribution of output, or output distribution. Thus, define $f(Q|x)$ as the output distribution for a given input vector x. Q is defined over an interval $[0, Q^*]$ of the real line. The following general objective function is used to operationalize the above definition of technical efficiency.

$$(1) \quad J(x, P) = \int G(Q, P, x)\ f(Q|x)dQ$$

G is assumed to be monotonic increasing in Q, and P is a vector of variables related to the firm's objectives. The elements of P can be prices, preference indicators, or any other factors affecting the firm's production decisions. If the price of output p is random, then revenue pQ can replace Q in (1) without loss of generality.

According to the above definition, a technically efficient process is represented by that output distribution which gives the highest value of J(x, P). Note that an efficient distribution depends on the pair (x, P). As in the neoclassical theory, the above definition shows that technical efficiency is a function of the input quantities which are employed. One technology may be more efficient at a high capital intensity, another at a lower capital intensity, for example. The above definition also implies that efficiency generally depends on the vector P. This is not true in the neoclassical model, i.e., technical efficiency is independent of prices. Thus, a relevant question is under what conditions technical efficiency is independent of P under stochastic production. Elsewhere (Antle 1983a) it is established that technical efficiency under stochastic production is valid for all P when the objective function has the form (1).

It should be noted that the neoclassical concept of technical efficiency satisfies the above definition. The objective function (1) admits the neoclassical model as a limiting case in which the output distribution is degenerate (at the frontier rate Q^* if the firm is technically efficient) and G(Q, P, x) is defined as profit.

In the following analysis, I specialize the general objective function in (1) to represent expected utility of profit. Thus, let

$$(2) \quad G(Q, P, x) = U(\pi), \qquad \pi = Q - Px.$$

where P is a nonstochastic input price vector normalized by the output price and $U(\pi)$ is a regular utility function.

The Moment-Based Approach

The above definition of technical efficiency, combined with the output distribution concept, shows that the problem of technical efficiency under stochastic production is the problem of ordering output distributions according to the firm's objective function. The problem of ordering distributions is, of course, not a new one. However, for the distribution-based approach to be theoretically and empirically useful, a general means of representing output distributions is needed. One approach is to assume a parametric system of probability distribution, such as the Pearson system. However, this would be mathematically difficult and somewhat restrictive. The approach pursued here is to use the moments of the output distribution. This moment-based approach will prove to be quite general for theoretical analysis and also amenable to empirical application. Define the moment functions:

$$(3)\quad \mu_1(x) = \int Qf(Q|x)dQ$$

$$\mu_i(x) = \int (Q - \mu_1)^i f(Q|x)dQ, \ i \geq 2.$$

The following result is a major theoretical justification for the moment-based approach.

Proposition 1: The moment sequence $\mu_1, \mu_2, \ldots,$ uniquely determines the output distribution $f(Q|x)$.

Proof: Rao (1973, p. 106) shows that a sufficient condition for a moment sequence to uniquely determine a distribution is that the range of the random variable is finite. This condition is satisfied by Q. Q.E.D.

Proposition 1 suggests that an ordering of moments induces an ordering of distributions for technical efficiency. Therefore, it is convenient to exploit a Taylor series expansion of the utility function about expected profit $\bar{\pi}$. Such an approach has been discussed extensively in the portfolio theory literature. This approach also has been used in the production economics literature to approximate decision rules (e.g., Anderson, Dillon, and Hardaker, 1977). The Taylor series approach has been criticized by Loistl (1976). However, neither Loistl nor the proponents of this approach exploited the fact that random variables in economics, such as a firm's output or profits, <u>can be defined over a finite interval of the real line</u>. Without this condition, the Taylor series approach may be of limited usefulness, as Loistl showed, because a Taylor series expansion of the utility function may not converge. The following result (for proof see Antle 1983a) shows that the Taylor series approach is valid for a very broad class of utility functions with desirable properties.

<u>Proposition 2</u>: Consider the class of real-valued utility functions $U(\pi)$ defined on any finite interval (a, b) with radius of convergence R satisfying

$$\inf_{\{\pi\}} R = B > 0$$

where B is a constant not depending on π. Also assume $U' > 0$. Then

$$(4) \quad EU(\pi) = U(\bar{\pi}) + \sum_{i=2}^{\infty} \frac{U_i(\bar{\pi})}{i!} \mu_i$$

where $\bar{\pi} = E(\pi)$, π is defined in (2), the μ_i are defined in (3), and $U_i \equiv \partial^i U/\partial \pi^i$.

Propositions 1 and 2 provide the theoretical foundations for the moment-based approach to technical efficiency measurement. Using (4), the effects each moment has on the objective function is

(5) $$\frac{\partial EU(\pi)}{\partial \mu_1} = U_1(\bar{\pi}) + \sum_{i=2}^{\infty} \frac{U_{i+1}(\bar{\pi})}{i!} \mu_i = EU_1(\pi) > 0$$

(6) $$\frac{\partial EU(\pi)}{\partial \mu_i} = \frac{U_i(\bar{\pi})}{i!}, \; i \geq 2.$$

Equation (5) follows from the assumption that $U_1(\pi) > 0$ and, hence, $EU_1(\pi) > 0$. These results suggest that technical efficiency under stochastic production is generally a multi-dimensional concept. If the firm is risk-neutral, then $EU(\pi) = \mu_1$, and according to the definition of efficiency, the mean of the output distribution characterizes technical efficiency. If the firm is risk averse, (4), (5), and (6), show that in addition to mean efficiency, the higher moments of output contribute to the determination of technical efficiency. Within the class of utility functions defined in Proposition 2, the efficiency of the firm's technology in terms of higher moments is shown by equation (6) to depend on the signs of the corresponding derivatives of the utility function. Ceteris paribus, as $U_i > 0$ (< 0), an increase (decrease) in μ_i increases the value of the objective function and, hence, represents an increase in technical efficiency.

An important question, raised by Loistl (1976), is how general are these results based on the Taylor series expansion? Loistl showed that the negative exponential utility function

(7) $$U(\pi) = \alpha - \beta e^{-\gamma\pi}, \; \alpha, \beta, \gamma > 0$$

has an infinite radious of convergence and thus satisfies the conditions of Proposition 2. In addition, it is easily verified that the power and log functions

$$U(\pi) = \alpha(\beta + \pi)^{\gamma}, \quad \alpha,\ \beta \geq 0, \quad 0 < \gamma < 1$$

$$U(\pi) = \alpha \log(\beta + \pi), \ \alpha,\ \beta \geq 0$$

satisfy Proposition 2 for $\beta > 0$. However, Loistl showed not all utility functions satisfy Proposition 2. In particular, for $\beta = 0$ the power and log functions fail the conditions of Proposition 2. Thus, although some utility functions exist whose Taylor series do not converge, Proposition 2 shows that the results based on Taylor series expansions are valid for a broad class of functions which encompasses these three important utility functions for $\beta > 0$.

2. AN APPLICATION TO CALIFORNIA DAIRY PRODUCTION.

The moment-based approach to technical efficiency measurement described in the previous section can be implemented using the flexible moment-based approach to measuring stochastic technology developed in Antle (1983b). In this section the moment-based approach is used to evaluate the relative contributions of veterinary services and management to technical efficiency of large-scale Tulare County, California, dairies.

The data represent nine large-scale dairies that participated in the electronic data processing system of Agri-Tech Analytics for Dairy Improvement production records from 1976 to 1978. These nine dairies had 5,052 Holstein cows, all received some veterinary services during the sample period, and were above average in capital investment, production, and management quality compared to other dairies in Tulare County. The data were originally collected for Goodger's study of the economic contribution of veterinary services to large scale dairy production. A detailed description of the data can be found in that study, and also in Goodger et al. The specific subset of the data and variable descriptions used in the results reported here, except for the veterinary service data, are described in Antle and Goodger. The output variable was average monthly pounds of milk per dairy. Input variables included feed, a quality adjusted measure of the herd size, a physical capital variable that measured the dairy's milking capacity, an index of management performance, and expenditures on both emergency and scheduled veterinary services.

The index of management performance was based on expert opinion and is described in detail in Goodger et al. The veterinary service variables measure the average expenditure by each dairy for each service type over the period the service was employed. Some dairies employed both emergency and scheduled services during a transition period from purely emergency to purely scheduled services. These dairies were judged to be closest to the scheduled dairies in operation and therefore, were treated as dairies with scheduled services.

A third-order Taylor series expansion of the negative exponential utility function was used to evaluate the effects of the first, second, and third moments on expected utility. Using (7) gives

$$EU(\pi) = \alpha - \beta e^{-\bar{\gamma}\pi} - \beta e^{-\bar{\gamma}\pi} \sum_{i=2}^{\infty} \frac{(-\gamma)^i}{i!} \mu_i$$

The effect of a fixed factor z_k on expected utility using a third-degree expansion of $EU(\pi)$ is

$$\frac{\partial EU(\pi)}{\partial z_k} = \beta e^{-\bar{\gamma}\pi} \left[\left(\gamma + \frac{\gamma^3}{2} \mu_2 - \frac{\gamma^4}{6} \mu_3\right) \frac{\partial \mu_1}{\partial z_k} - \frac{\gamma^2}{2} \frac{\partial \mu_2}{\partial z_k} + \frac{\gamma^3}{6} \frac{\partial \mu_3}{\partial z_k}\right]$$

To eliminate the units of z_k in the analysis we rewrite the above expression as follows:

$$(8)\quad \frac{\partial EU(\pi)}{\partial z_k}\ \frac{1}{z_k} = \beta e^{-\gamma\bar{\pi}} \left[\left(\gamma + \frac{\gamma^3}{2}\mu_2 - \frac{\gamma^4}{6}\mu_3\right)\mu_1\ \eta_{1k} - \frac{\gamma^2}{2}\mu_2\eta_{2k} + \frac{\gamma^3}{6}\mu_3\eta_{3k}\right]$$

where

$$\eta_{ik} = \frac{\partial \mu_i}{\partial z_k}\frac{z_k}{\mu_i}, \quad i = 1, 2, 3.$$

Note that these η_{ik} are the elasticities of the moment functions with respect to the fixed factors. Equation (8) can be interpreted as an "absolute" efficiency measure depending on the utility function's scaling. A "relative" efficiency scale can be obtained by measuring the effect of the ℓth factor as a percent of the kth factor. Thus, define

$$(9)\quad RE_{\ell,k} \equiv \frac{\partial EU(\pi)}{\partial z_\ell}\, z_\ell\ \ \frac{\partial EU(\pi)}{\partial z_k}\, z_k.$$

The results of the empirical analysis showed that feed, herd size, milking capacity, management quality, and scheduled veterinary services all had statistically significant effects on the probability distribution of milk output. One major finding was that emergency veterinary services did not have statistically significant effects on milk output.

The results of the analysis, using the efficiency index defined above, are presented in Table 1. The table shows values of the index (9) calculated at the sample averages of the data, that is, for the "typical" large scale dairy. The table shows results for representative risk neutral, moderately risk averse, and strongly risk averse decision makers. The index is calculated relative to the management variable, hence its value for management is always 1.00 in the table.

Examination of Table 1 reveals that the relative importance of the variables is the same for both risk neutral and mildly risk averse managers, but different for the strongly risk averse case, where management becomes the most important factor in place of scheduled veterinary services. Thus, a major implication of the study is that management quality becomes more valuable relative to veterinary services as risk becomes more important. This finding thus provides objective support for the common sense notion that the ability of the dairy manager is increasingly important as production risk becomes more important.

Table 1. Productivity Indices for Large Scale Dairy Production

Factor	Risk Attitude		
	Risk Neutral	Moderate Risk Aversion[1]	Strong Risk Aversion[2]
Emergency Vet Services	- .01[3]	- .01[3]	- .01[3]
Scheduled Vet Services	1.70	1.55	.90
Management Quality	1.00	1.00	1.00

[1]Defined as an individual who would be willing to pay up to 2 percent of expected returns for insurance against production risk.

[2]Defined as an individual who would be willing to pay up to 20 percent of expected returns for insurance against production risk.

[3]Statistically insignificant.

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