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E,V FRONTIER ANALYSIS USING TOTAL AND RANDOM VARIANCE AS MEASURES OF RISK*

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Abstract

Expected income-variance (E,V) frontiers in crop production are derived using total and random (variate difference) variance. The results show that the farm organization is influenced by the variance measure used. The authors argue that random variance is a better measure of risk; consequently random E,V frontiers provide better estimates of risk-income tradeoffs.

Introduction

Agricultural production is subject to constant fluctuations forcing producers to make planning decisions with limited information of future conditions. This variability framework under imperfect knowledge leads to a distinction between risk and uncertainty. Risk refers to events whose probability distribution can be empirically evaluated. In contrast, uncertainty refers to events whose parameters cannot be measured statistically (Knight). Consequently, any empirical study measuring variability is an attempt to remove the farmer from an uncertain into a risky environment.

The primary source of risk in returns[1] from crop production stems from variability in prices and yields. Yields are subject to random variations due to weather conditions as well as systematic variations caused by technological improvement. Likewise, prices tend to move systematically with general economic conditions and randomly due to a variety of unpredictable events.

Diversification has been widely discussed in the literature as a strategy to reduce risk (measured by income variability) in agricultural activities (Carter and Dean, Heady). The primary idea behind diversification is that managers attempt to maximize profits and minimize income variability simultaneously. Given these goals, "a fundamental management problem is to determine what combination of crop alternatives can best satisfy the two objectives" (Johnson and Terfertiller, p. 3). One approach to the risk aspect is searching for a mix of crops for which net returns over time move in opposite directions so that high and low incomes from different crops can be evened out.

Diversification can be pursued by adding resources or by redistributing existing inputs among more enterprises. Only the second approach is considered here. Specifically, we assume that the quantity of land is fixed and that all other resources required for the various cropping systems can be readily obtained.

The objective of this paper is to evaluate the effects of total and random variance as measures of risk on efficient farm plans. An efficient farm plan or organization is defined as that combination of crops resulting in the lowest level of risk for a given level of income.

The variate difference method (Tintner) is used to separate the random from the systematic component in a time series in order to estimate total and random variance for net returns, and their corresponding variance-covariance matrices. Quadratic Programming is presented as an approach to determine expected income-variance tradeoffs or E,V frontiers. E,V frontiers are efficient sets of crop combinations where variance is minimum for a given level of income.

Two E,V frontiers are generated: one using the total variance-covariance matrix and the second using its random counterpart. We argue that the second method is more desirable, because it provides a better measurement of risk.

Methodology

The Variate Difference Method

The variate difference method, developed by Tintner, is the statistical procedure employed in this study. This approach has been used by other researchers in estimating crop variability indexes (Carter and Dean, Mathia, Yahya and Adams).

An essential assumption of this statistical methodology is that economic time series data consist of two additive parts: 1) a mathematical expectation or systematic component and 2) a random element. The systematic component of an economic time series corresponds to technological changes and long run trends such as price cycles and inflation. The random element is a consequence of purely random or unpredictable events. An advantage of this method is that it does not require any specific assumptions about the functional form of the systematic component.

The variate difference method eliminates the systematic component of a time series by successive finite differencing leaving an estimate of the random element (Tintner).

We contend that farm managers are aware of those factors accounting for systematic variations in an economic time series since they can be predicted with a fair degree of accuracy. Consequently, the variance of the random portion of the time series is a better measure of risk than total variance. If we assume that there is a close relationship between past and future variability, then an empirical estimate of random variability based on historical data should provide a good indication of risk and thus aid in the decision making process.

Quadratic Programming

Risk aversion can be traced back to the Bernoullian theory of expected utility maximization. Under this framework, the goal of decision makers is to maximize expected utility.

Quadratic Programming, following work by Markowitz, is an optimizing technique which allows the derivation of an efficient set of mean income-variance tradeoffs. Variance is used as the measure of risk.

These efficient sets, typically called E,V frontiers, are determined by minimizing variance (risk) for varying income levels.

Some studies have supported the use of an expected utility objective function over a profit maximization objective function (Lin, Dean and Moore, Officer and Halter). However, Brink and McCarl found that a profit maximizing function performed well for Indiana farmers.

Quadratic Programming studies have differed in their use of risk aversion coefficients. Some studies have estimated such coefficients to obtain utility maximizing solutions. Other researchers have stopped short of deriving unique solutions preferring to derive E,V frontiers from parametric solutions (Scott and Baker). The use of E,V frontiers compared to the derivation of unique solutions can be supported for various reasons. One is the wide range of risk aversion coefficients that can be found among farm producers. Another, is the difficulty in estimating such coefficients (Young, et al). Finally, the specification of an a priori risk aversion coefficient bothers some researchers who suggest that risk aversion coefficients rapidly change and are very time specific. Hence, it may be of more general value to have a physical choice relationship available for specific decisions.

Data

Two Nebraska counties, Butler and Sherman, were selected to empirically derive E,V frontiers. The analysis was conducted on six of the major field crops grown in these counties: alfalfa, corn, grain sorghum, soybeans, oats, and wheat. Net returns were calculated for irrigated and non-irrigated land for the first four crops and only for dryland in the last two cases.

Twenty years were selected for the analysis covering the period

1957 to 1976. The data for prices and yields for all of the years con-
sidered were obtained from Nebraska Agricultural Statistics, Annual
Reports. The price data corresponds to the yearly average of prices
received by Nebraska farmers, and the yield information is based only
on harvested acreage in both counties.[2]

Net returns were computed by subtracting from gross returns
(yields per acre x price per unit) the variable costs of production.
These net figures represent a return to land, management, and other
fixed resources. We assumed that constant unit costs prevail regard-
less of farm size. The variable costs of production were estimated
from budgets developed at the Department of Agricultural Economics in
the University of Nebraska (Bitney, et al). In order to account for
technological changes in agricultural production, these budgets were
modified using farm production and efficiency index numbers reported
by the United States Department of Agriculture (U.S.D.A. 1977).

Net returns were converted to real terms by deflating the various
nominal dollar figures by the Consumer Price Index based on 1967 = 100.
A major effect of deflating the series is that of reducing the sys-
tematic variation caused by the general price level trend.

Results

In this section we present the graphical E,V frontiers with the
corresponding farm organizations obtained for Butler and Sherman
Counties.[3] For both counties we assumed that a total of 100 acres of
land were available: 50 irrigated and 50 dryland.

To derive all four frontiers, we first obtained the Linear Pro-
gramming solution which was then used as the initial level of income

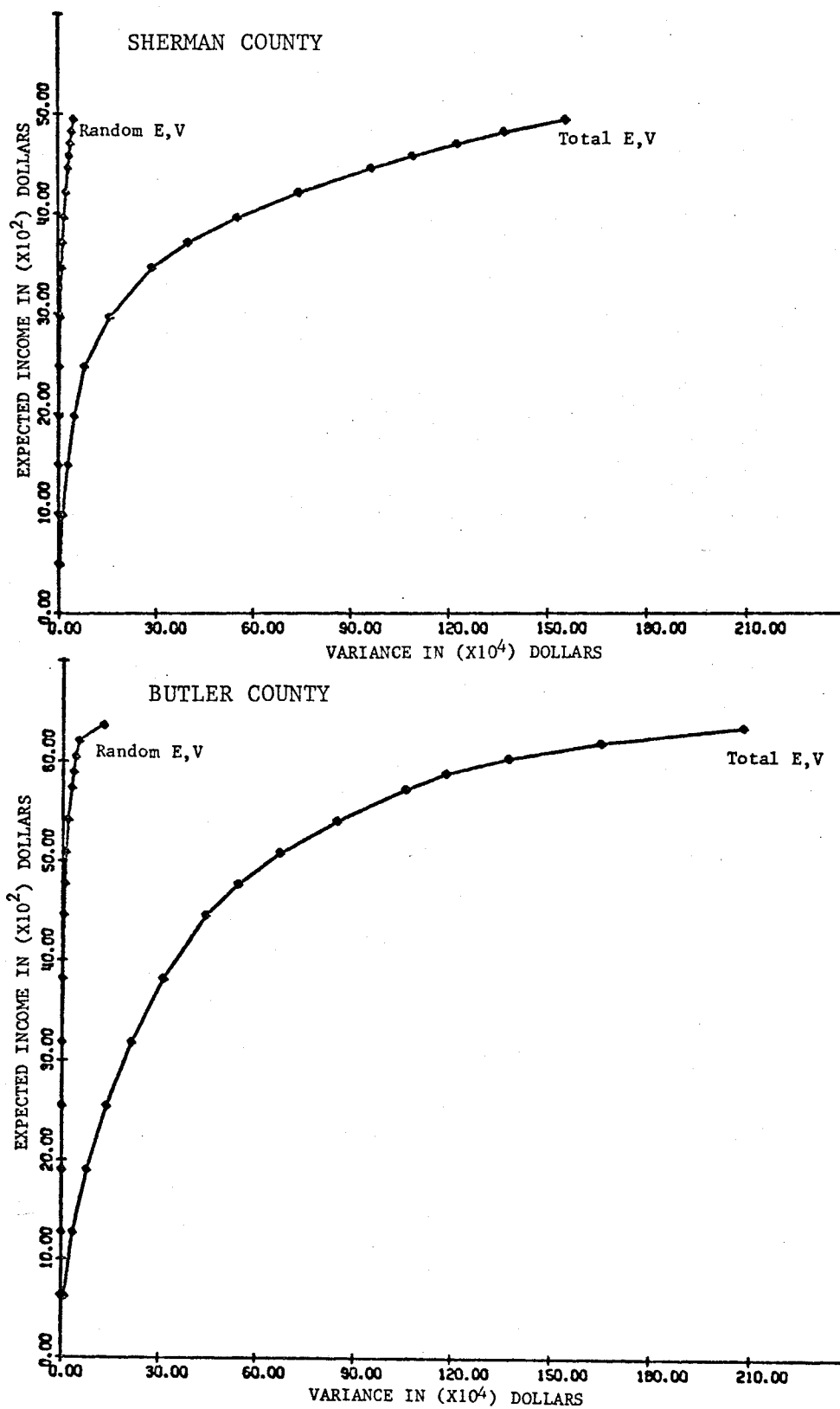FIGURE 1: TOTAL AND RANDOM E,V FRONTIERS FOR SHERMAN AND BUTLER COUNTIES

Table 1. Farm Organizations in Butler County, Resulting From the Use of Total and Random Variance in E,V Frontier Analysis

Income	Total Variance	Organization		Random Variance	Organization	
$6,370.50	(L.P. Solution)	I.C.[a]	50.00 acres	---	I.C.	50.00 acres
		D.G.	50.00 acres		D.G.	50.00 acres
6.369.22	$2,075,788.43	I.C.	50.00 acres	$126,417.02	I.C.	49.64 acres
		D.G.	49.82 acres		I.G.	.36 acres
		D.S.	.18 acres		D.G.	50.00 acres
6,211.23	1,650,717.84	I.C.	50.00 acres	49,074.45	I.G.	45.12 acres
		D.G.	27.94 acres		I.C.	4.88 acres
		D.S.	22.06 acres		D.G.	50.00 acres
6,051.97	1,371,129.26	I.C.	44.35 acres	38,787.65	I.G.	50.00 acres
		I.G.	5.65 acres		D.G.	43.35 acres
		D.S.	37.80 acres		D.A.	6.65 acres
		D.G.	9.29 acres			
		Wh.	2.91 acres			
5,892.71	1,181,798.89	I.C.	33.46 acres	32,384.90	I.G.	50.00 acres
		I.G.	14.92 acres		D.G.	35.89 acres
		I.S.	1.63 acres		D.A.	14.11 acres
		D.S.	43.08 acres			
		Wh.	6.92 acres			
5,733.45	1,059,175.37	I.C.	30.97 acres	26,607.49	I.G.	50.00 acres
		I.G.	12.34 acres		D.G.	28.43 acres
		I.S.	6.70 acres		D.A.	21.57 acres
		D.S.	41.18 acres			
		Wh.	8.82 acres			
5,414.92	845,587.62	I.C.	25.99 acres	16,928.56	I.G.	50.00 acres
		I.S.	16.84 acres		D.A.	36.49 acres
		I.G.	7.17 acres		D.G.	13.51 acres
		D.S.	37.39 acres			
		Wh.	12.61 acres			
5,096.40	674,218.28	I.S.	26.98 acres	10,038.94	I.G.	49.26 acres
		I.C.	21.07 acres		D.A.	50.74 acres
		I.G.	2.00 acres			
		D.S.	33.60 acres			
		Wh.	16.40 acres			
4,777.88	$43,997.99	I.S.	35.04 acres	7,284.29	I.G.	41.48 acres
		I.C.	14.96 acres		D.A.	58.52 acres
		D.S.	27.73 acres			
		Wh.	18.00 acres			
		D.A.	4.27 acres			
4,459.35	442,575.09	I.S.	40.76 acres	4,976.42	I.G.	33.70 acres
		I.C.	9.24 acres		D.A.	66.30 acres
		D.S.	18.90 acres			
		Wh.	17.41 acres			
		D.A.	13.69 acres			
3,822.30	311,464.53	I.S.	43.41 acres	1,701.22	I.G.	18.14 acres
		D.A.	26.83 acres		D.A.	81.86 acres
		Wh.	16.66 acres			
		D.S.	9.40 acres			
3,185.25	216,294.81	I.S.	36.17 acres	213.29	I.G.	2.57 acres
		D.A.	22.36 acres		D.A.	97.43 acres
		Wh.	13.88 acres			
		D.S.	7.83 acres			
2,548.20	138,428.70	I.S.	28.94 acres	95.49	D.A.	82.73 acres
		D.A.	17.89 acres			
		Wh.	11.10 acres			
		D.S.	6.27 acres			
1,911.15	77,866.12	I.S.	21.70 acres	53.71	D.A.	62.05 acres
		D.A.	13.42 acres			
		Wh.	8.33 acres			
		D.S.	4.70 acres			
1,274.10	34,607.17	I.S.	14.47 acres	23.87	D.A.	41.37 acres
		D.A.	8.94 acres			
		Wh.	5.55 acres			
		D.S.	3.13 acres			
637.05	8,651.79	I.S.	7.23 acres	5.97	D.A.	20.68 acres
		D.A.	4.47 acres			
		Wh.	2.78 acres			
		D.S.	1.57 acres			

a: The meaning of the abbreviations in the order they appear are as follows: I.C. = irrigated corn; D.G. = dryland grain sorghum; I.G. = irrigated grain sorghum; D.S. = dryland soybeans; D.A. = dryland alflafa; Wh. = dryland wheat; I.D. = irrigated soybeans.

Table 2. Crop Combinations in Sherman County, Resulting From the Use of Total and Random Variance in E,V Frontier Analysis

Income	Total Variance	Organization		Random Variance	Organization	
$ 4,950.50	---	I.C.[a]	50.00 acres	---	I.C.	50.00 acres
		D.G.	50.00 acres		D.G.	50.00 acres
4,949.51	$1,558,019.18	I.C.	50.00 acres	$49,063.27	I.C.	48.79 acres
		D.G.	48.76 acres		I.G.	1.21 acres
		Wh.	1.24 acres		D.G.	50.00 acres
4,826.73	1,371,833.12	I.C.	50.00 acres	43,779.89	I.C.	41.51 acres
		D.G.	28.60 acres		I.G.	8.49 acres
		Wh.	12.07 acres		D.G.	41.60 acres
		D.S.	9.32 acres		D.A.	5.61 acres
					Wh.	2.79 acres
4,702.97	1,227,907.02	I.C.	47.30 acres	39,192.20	I.C.	40.01 acres
		I.S.	2.70 acres		I.G.	9.99 acres
		D.G.	24.71 acres		D.G.	36.18 acres
		D.S.	12.90 acres		D.A.	11.62 acres
		Wh.	12.39 acres		Wh.	2.20 acres
4,579.21	1,093,310.10	I.C.	44.10 acres	34,897.05	I.C.	38.50 acres
		I.S.	5.90 acres		I.G.	11.50 acres
		D.G.	21.81 acres		D.G.	30.75 acres
		D.S.	15.26 acres		D.A.	17.64 acres
		Wh.	12.93 acres		Wh.	1.61 acres
4,455.45	967,697.95	I.C.	40.89 acres	30,894.44	I.C.	37.00 acres
		I.S.	9.11 acres		I.G.	13.00 acres
		D.G.	18.92 acres		D.G.	25.32 acres
		D.S.	17.60 acres		D.A.	23.66 acres
		Wh.	13.48 acres		Wh.	1.02 acres
4,207.92	743,634.72	I.C.	34.48 acres	23,766.98	I.C.	33.98 acres
		I.S.	15.52 acres		I.G.	16.02 acres
		D.S.	22.30 acres		D.A.	35.69 acres
		Wh.	14.57 acres		D.G.	14.31 acres
		D.G.	13.13 acres			
3,960.40	555,795.39	I.C.	28.07 acres	17,827.15	I.C.	30.89 acres
		I.S.	21.93 acres		I.G.	19.11 acres
		D.S.	26.99 acres		D.A.	47.67 acres
		Wh.	15.67 acres		D.G.	2.33 acres
		D.G.	7.34 acres			
3,712.88	404,173.13	I.S.	28.34 acres	14,126.01	I.C.	27.43 acres
		I.C.	21.66 acres		I.G.	17.73 acres
		D.S.	31.69 acres		I.A.	3.74 acres
		Wh.	16.76 acres		I.S.	1.10 acres
		D.G.	1.55 acres		D.A.	50.00 acres
3,465.35	290,288.84	I.S.	36.11 acres	11,143.11	I.C.	24.17 acres
		I.C.	13.89 acres		I.G.	15.46 acres
		D.S.	33.19 acres		I.A.	9.30 acres
		Wh.	16.81 acres		I.S.	1.07 acres
					D.A.	50.00 acres
2,970.30	158,653.37	I.S.	42.77 acres	6,251.40	I.A.	20.42 acres
		I.C.	7.23 acres		I.C.	17.66 acres
		D.S.	21.70 acres		I.G.	10.92 acres
		O.	15.67 acres		I.S.	1.00 acres
		Wh.	12.54 acres		D.A.	50.00 acres
2,475.25	82,082.87	I.S.	45.26 acres	2,791.83	I.A.	31.53 acres
		I.C.	4.74 acres		I.C.	11.15 acres
		O.	37.95 acres		I.G.	6.39 acres
		Wh.	6.74 acres		I.S.	.93 acres
		D.S.	5.31 acres		D.A.	50.00 acres
1,980.20	49,526.54	I.S.	40.39 acres	764.41	I.A.	42.64 acres
		I.C.	3.10 acres		I.C.	4.63 acres
		O.	37.27 acres		I.G.	1.86 acres
		Wh.	3.71 acres		I.S.	.86 acres
					D.A.	50.00 acres
1,485.15	27,858.68	I.S.	30.29 acres	18,637.02	I.A.	49.61 acres
		I.C.	2.33 acres		I.S.	.39 acres
		O.	27.95 acres		D.A.	33.57 acres
		Wh.	2.78 acres			
990.10	12,381.64	I.S.	20.20 acres	82.28	I.A.	37.77 acres
		I.C.	1.55 acres		I.S.	.25 acres
		O.	18.64 acres		D.A.	13.97 acres
		Wh.	1.85 acres			
495.05	3,095.41	I.S.	10.10 acres	20.57	I.A.	18.88 acres
		I.C.	.78 acres		I.S.	.13 acres
		O.	9.32 acres		D.A.	6.98 acres
		Wh.	.93 acres			

a: The meaning of the abbreviations in the order they appear are as follows: I.C. = irrigated corn; D.G. = dryland grain sorghum; I.G. = irrigated grain sorghum; Wh. = dryland wheat; D.S. = dryland soybeans; D.A. = dryland alfalfa; I.S. = irrigated soybeans; I.A. = irrigated alfalfa; O. = oats.

in generating the efficient sets. In all cases, as it would be ex-
pected, the L.P. solution yields the highest level of income. In both
counties, the organization resulting from the L.P. solution was 50 acres
of irrigated corn and 50 acres of dryland grain sorghum.

In Figure 1, the various points on each E,V frontier show arbi-
trarily selected income intervals with corresponding variance measures.
In both counties we observe a pronounced difference between the random
and total E,V sets. The random E,V curves show a significantly lower
variance for each income level, implying that these net income series
have a noted systematic component which was largely extracted by the use
of the variate difference method.

With regard to the farm organizations, marked differences are found
among the counties and frontiers. In Table 1 we find that Butler County
shows a high degree of diversification when total variance is used as
the measure of net income variability. Farm plans showing up to five
crops are observed for various income levels. Farm plans stemming
from the use of random variance as the variability measure show much
less diversification, reaching a maximum of three crops. Moreover, for
the lowest four income levels we observe complete specialization in
dryland alfalfa.

By contrast, as shown in Table 2 we find that in Sherman County
the two variability measures lead to a high degree of diversification.
In both cases the farm plans contain up to five different crops. How-
ever, we observe that at the various income levels the farm organizations
resulting from the use of total and random variance are quite different.

In summary, our results clearly indicate that the measure of variance

used in E,V analysis has major consequences in the specific farm organization.

Concluding Remarks

The work presented here could be criticized given the level of aggregation in the data used. Some argue that aggregate data has a tendency to underestimate individual farm varibility (Eisgruber and Schuman). Others sustain that variability measures derived from historical yields at the farm level "may overestimate the true variability" (Carter and Dean, p. 178). We claim that the data used here provide reasonable relative estimates of variability for the various crops analyzed.

Another limitation is that the variate difference method "can never completely eliminate the systematic components from the error terms (of an income time series). Consequently (this method) provides only an approximation to the random variance for...income" (Carter and Dean, p. 218)

We conclude that if random variance is a better measure of risk than total variance, the former should be considered in E,V, frontier work. In addition, we support the use of Quadratic Programming as a method to generate efficient sets of crop combinations when analyzing diversification.

Footnotes

[1]Returns, net returns and income are terms used synonymously in this paper. A precise definition is given in page 5.

[2]Using yields based on harvested acreage might lead to a downward bias in variability estimates. Distortion between relative variability of dryland and irrigated yields can be introduced if a higher proportion of planted dryland acres are not harvested compared to irrigated acres. A correction for this potential deficiency was not possible due to a lack of data.

[3]The E,V frontiers are generated using Minos (Murtagh and Saunders). Due to limitations on this algorithm, the variance measures presented in figure 1 are only one-half the actual variance.

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