A THEORY OF PRODUCTION, INVESTMENT AND DISINVESTMENT

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Abstract

Changes in a farm's economic environment often times necessitates changes in the combination of durable assets owned by the firm. A model which links durable asset acquisition and disposal decisions to production decisions is presented. The model allows for variable service extraction rates. The optimal economic lifetime for the durable is also determined internally.

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A THEORY OF PRODUCTION, INVESTMENT AND DISINVESTMENT

Often times changes in a farm firm's economic environment will cause the farm manager to alter the combination of durable assets owned by the farm. These alterations may involve the acquisition of additional durables, the disposal of current durables, and/or using retained durables in a different manner. The current theory of production, investment and disinvestment in durable assets does not handle accurately the issues relating to using durable assets at varying rates, nor does it specify completely the related issue of the optimal length of life for durable assets. In this paper we consider a production process which has both durable assets and the flow of services from the durables as inputs. We allow for a varying extraction rate and determine internally both the optimal amount of services to extract from the durable in each production period as well as the optimal life for the durable. We relate the optimal production activities associated with the durable to investments and disinvestments in the durable.

Theoretical Model

Our specification of the production, investment and disinvestment process conceives the production process to be vertically integrated. The determination of the flow of services from durables will be specified at one level. This service flow is then fed into the production function to determine output. The expected future time pattern of utilization will determine in part the investment/disinvestment decision.

A diagrammatic representation of this process for a production process using one durable is presented in figure 1.

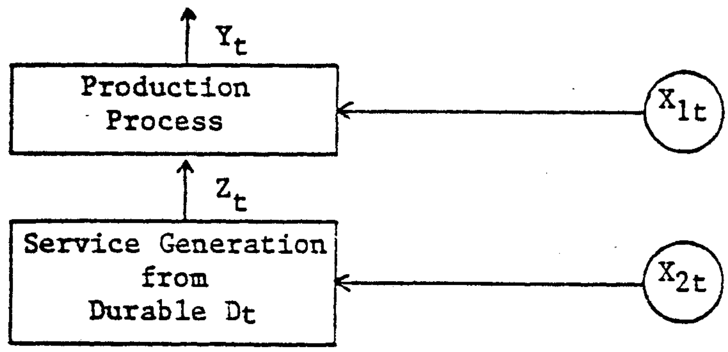

Figure 1. Two tiered vertically integrated production process

The mathematical characterization of the physical production process represented in figure 1 is contained in equations (1) through (3).

(1) $Y_t = F(X_{1t}, Z_t)$

(2) $Z_t = G(X_{2t}/D_t)$

where Y_t = quantity of product Y produced and sold in time period t.
X_{1t} = quantity of nondurable inputs X_1 used in production of Y_t in time period t.
Z_t = quantity of services generated from D_t used in production of Y in time period t.
D_t = the stock of the durable asset D in time period t.

Equation (1) is a standard representation of a production process with flow variables as inputs. Equation (2) is a production relationship which indicates that service flows from a durable asset are generated or produced according to the function G (·) by using one nondurable input (a flow variable) with a given stock of the durable asset. Thus at this level of integration we need both stocks and flows in the production of services.

Specification of the production process in this manner allows us to vary the rate of use for durable assets. It also permits us to determine investment and disinvestment in durables simultaneously with the production activities associated with the durable. Finally, the optimal length of life for the durable is also determined internally.

The physical life of a durable asset is related to both the services extracted and the maintenance performed during each year of its life. In our model we express this physical relationship in equation (3).

$$(3)\quad T_D = h(Z_1, \ldots, Z_t, \ldots, Z_{TH}, X_{31}, \ldots, X_{3t}, \ldots, X_{3TH})$$

where T_D = physical life of durable.
X_{3t} = aggregated maintenance variable in time period t.
T_H = planning horizon for the firm. T_H is chosen such that costs and returns beyond T_H would be discounted essentially at zero for any positive discount rate $T_H \geq T_D$.

We assume that the firm operates in each time period to maximize current profits plus the change in the net present value of the durable asset. This objective function is consistent with the gain function used by Edwards and with Boulding's writings.

$$(4)\quad G_t = P_{yt} \cdot Y_t - P_{x1t} X_{1t} - P_{x2t} X_{2t} - P_{x3t} X_{3t} - TUC_N(Z_t) - FC_t + \alpha (D_t - D_t^o)$$

where P_{yt} = price received for Y in time period t.
P_{xjt} = price paid for nondurable X_j in time period t j=1,2,3.
$TUC_N(Z_t)$ = total use cost of extracting services Z_t in time period t.
FC_t^o = fixed cost associated with the durable in time period t.[1/]
α = gain in net present value of a unit of the durable.
For $D_t > D_t^o$, α will equal the difference between the durable's value in use, NRD, and its acquisition price, P_{Dt}^a.

[1/] The "o" rotation refers to initial levels while the "*" refers to optimal levels.

For $D_t < D_t^o$, α will equal the difference between the durable's value in use and its salvage price, P_{DT}^S.

For $D_t = D_t^o$, α will equal the durable's value in use.

The total user cost variable (TUC_N) in (4) deserves special explanation. The concept of user cost as recognized by Keynes and subsequently modified by Neal and Lewis considers the cost of using the asset as opposed to not using it. Equations (5) and (6) express the Neal and Lewis versions, respectively.

(5) $TUC_N(Z_t) = S(t|Z_t = 0) - S(t,Z_t)$

where $S(t|Z_t = 0)$ = salvage value at time t given no services are extracted.

$S(t,Z_t)$ = salvage value at time t with Z_t services extracted.

(6) $TUC_L(Z_t) = NPV_{T+dt}$

where $TUC_L(Z_t)$ = Lewis' formulation of user cost.

NPV_{T+dt} = Net present value of asset in time period T+dt. This is the time period excluded by current use of the asset.

The Neal version is an "off-firm" opportunity cost while the Lewis version is a "within-firm" opportunity cost. The former is important for service extraction decisions, while the latter is relevant for investment/disinvestment decisions.

Maximizing (4) subject to (1) through (3) involves determining the optimal production, service generation and investment/disinvestment activities. We separate the determination of the production and service generation activities from the investment/disinvestment activities to ease the presentation. Determining the optimal production and service generation activities involves maximizing the following Lagrangian expression.

$$(7)\ L = P_{yt} Y_t - P_{x1t} X_{1t} - P_{x2t} X_{2t} - P_{x3t} X_{3t} - TUC_N(Z_t) - FC.$$
$$- \lambda_{1t}(Y_t - F(X_{1t}, Z_t)) - \lambda_{2t}(Z_t - G(X_{2t}|D_t))$$
$$- \lambda_{3t}(T_D - h(Z_1, \ldots, Z_{TH}, X_{31}, \ldots, X_{3TH}))$$

Upon taking the required partial derivatives, equating them with zero, and making appropriate substitutions, the following necessary conditions are derived.

$$(8)\ P_{yt} \frac{\partial Y_t}{\partial X_{1t}} = P_{x1t}$$

$$(9)\ P_{yt} \frac{\partial Y_t}{\partial Z_t} \frac{\partial Z_t}{\partial X_{2t}} = P_{x2t} + MUC_N (Z_t) \frac{\partial Z_t}{\partial X_{2t}} - \frac{P_{X3t}}{\frac{\partial h}{\partial X_{3t}}} \frac{\partial h}{\partial Z_t} \frac{\partial Z_t}{\partial X_{2t}}$$

$$(10)\ \left[\frac{MUC_N(Z_t) + \frac{P_{x2t}}{\frac{\partial Z_t}{\partial X_{2t}}} - P_{Yt} \frac{\partial Y_t}{\partial Z_t}}{\frac{\partial h}{\partial Z_t}} \right] \frac{\partial h}{\partial X_{3t}} = P_{x3t}$$

$$(11)\ P_{yt} \frac{\partial Y_t}{\partial Z_t} = MUC_N(Z_t) + \frac{P_{x2t}}{\frac{\partial Z_t}{\partial X_{2t}}} - \frac{P_{x3t}}{\frac{\partial h}{\partial X_{3t}}} \frac{\partial h}{\partial Z_t}$$

Equation (8) indicates that the optimal quantity of X_{1t} to use is determined by equating the value of its marginal product to its price. Equation (9) states that the optimal quantity of X_{2t} to use involves having the instrumental marginal value product equal to the marginal cost of using X_{2t}. The marginal cost of X_{2t} is respectively the price of X_{2t} plus the marginal user cost of the services generated by using X_{2t} plus the increase maintenance costs which must be incurred as a result of using the durable.

For X_{3t}, equation (10) indicates that the net marginal value of maintenance should be equated to the marginal factor cost of maintenance. The net value of a unit of maintenance is given in the square brackets in (10).

Equations (8) through (10) state the marginal conditions for the optimal levels of X_{1t}, X_{2t} and X_{3t}, respectively. For services from the durable, equation (11) indicates that the value of the marginal product of services should be equated with the marginal cost of acquiring services. This marginal cost is composed of the marginal user cost, the weighted cost of acquiring X_{2t} and the weighted cost of increased maintenance.

The simultaneous solution of equations (8) through (11) for each t, t=1, ..., T_H will yield the optimal production activities for the firm with its initial endowment of D_t. The following section specifies the optimality conditions for acquiring additional durables and/or disposing of currently held durables.

Investment, Disinvestment Decisions

In making adjustments to its initial quantities, the firm will want to acquire units of a durable when its value in use exceeds its acquisition price. It will want to dispose of units of an existing durable when its value in use is less than its salvage price. A durable's value in use is derived from the services generated over its lifetime. Both the services generated and the lifetime of the durable are variables whose optimal levels are determined internally. The optimal quantity of services to generate in each time period was specified above. Determining the optimal lifetime for a durable, in essence, determines the point in time when the firm should disinvest in the durable.

The durable's value in use can be represented as:

$$(12)\quad NRD(Z^*, T_D) = PVS(Z^*, T_D) + \frac{1}{(1+r)^{T_D}} S(Z^*, T_D)$$

where $NRD(Z^*, T_D)$ represents the net return to the durable as a function of the optimal services generated in each time period, Z^*, and the length of time the durable is used, T_D.

$PVS(Z^*, T_D)$ = present value of services generated which depends on, Z^*, and T_D.

r = discount rate.

$S(Z^*, T_D)$ = salvage value of durable in time period T_D after Z^* services have been extracted.

With Z^* determined according to equations (8) through (11), T_D is determined so as to maximize $NRD(Z^*, T_D)$.

If we treated time as a continuous variable, we would differentiate (12) with respect to t and equate with zero. However, our model treats time as a discrete variable; thus, we cannot take derivatives. We can only state approximate marginal rules for determining T_D^*. Our approximate rule is to equate the additions to $PVS(T_D)$ with the reductions in $S(T_D)$, T_D^* is the point in time when the additions to $PVS(T_D^* + 1)$ are less than the reductions in $S(T_D^* + 1)$. In other words, $PVS(T_D^*) > S(T_D^*)$, but $PVS(T_D^* + 1) < S(T_D^* + 1)$. This procedure determines when to disinvest in a durable. It is based on comparing the durable's value in use with its salvage value.

As indicated above, the firm will acquire units of a durable when $NRD(Z^*, T_D^*)$ exceeds the acquisition price. Note that the investment decision requires the determination of both the optimal production activities and the disinvestment activities. The optimal quantity of a particular durable is determined in an iterative manner, since we consider them to be available in discrete units only. For each unit the firm considers, the potential value in use is calculated and compared with the acquisition price. If the potential value in use exceeds the acquisition price, the firm acquires that unit and repeats the calculations for another unit. It continues until it finds the unit whose value in use does not cover its acquisition price. A similar process is followed for disinvesting. The firm disinvests in units of durables until either (a) the value in use for a particular unit exceeds its salvage price, or (b) the initial endowment of durables is entirely disposed of.

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