MARKETING RULES FOR CALIFORNIA FRESH NEWTOWN APPLE SALES: AN APPLICATION OF DYNAMIC PROGRAMMING

by

David E. Kenyon

and

Hoy F. Carman

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An important decision made each year at harvest by the firms comprising the California apple industry involves the allocation of fresh sales between current and future time periods. This paper indicates how dynamic programming can be used to determine the optimum intraseasonal allocation of California fresh Newtown apple sales to maximize discounted net revenue to growers. The following formulation uses only demand for regular storage Newtown apples, whereas the complete study [2] includes demand equations for regular and Controlled Atmosphere storage for both Newtown and Delicious variety apples. The objective here is to present a simplified example and the logic of the procedure without cluttering the exposition. Studies of the optimal intraseasonal allocation of sales during the marketing season for several commodities have been conducted previously [1, 3, 4, 6, 7]. However, there has been no previous attempt to apply dynamic programming to this particular type of allocation problem. Quadratic programming and calculus with LaGrangian multipliers are the methods typically used for solving the revenue maximization problem.

David E. Kenyon and Hoy F. Carman are Assistant Professors of Agricultural Economics at Virginia Polytechnic Institute and State University and University of California, Davis, respectively.

The Data

The marketing season for regular storage Newtown apples was divided into five time periods.[1/] Data were obtained from several secondary sources, and a demand equation was estimated at the wholesale level by ordinary least squares regression using dummy variables to permit intercept and slope changes in each period.[2/] The final equation explained 63 percent of the variation in Newtown fresh apple prices and contained dummy variables permitting an intercept change in period five and a change in slope between period 1 and the remaining periods. The equation related Newtown fresh apple prices to Newtown apple sales, Washington apple sales in California, fresh orange sales in California, and consumer income. The coefficients of all these variables were significantly different from zero at the five percent level.

Packing and wholesaling costs were assumed to remain constant during the marketing season, thus not affecting the allocation procedure. However, storage costs (c_i) increase with time in storage; therefore, this cost must be considered. Current storage rates in California were used.

The Model

The determination of the optimum allocation of sales among periods was based upon the theory of monopolist price discrimination. The problem was to maximize discounted net total revenue (DNTR) by equating discounted net marginal revenue (DNMR) in each time period. The total revenue (TR) functions in each period were derived from the estimated demand functions. Net total revenue functions (NTR) in each period were obtained by subtracting the appropriate storage costs from each TR function. Maximum discounted net total revenue was then determined by equating discounted net marginal revenue in each time period using dynamic programming (DP).

The dynamic programming problem was formulated as a backward multistage problem in which stages were counted from the end of the planning horizon instead of the beginning. The relationship between time periods and stages of the decision process are illustrated in Figure 1. It also contains the functional relationships and data used in solving the DP and the resultant optimal marketing decision rules and stage return functions. Note that stage 5 is the beginning of the first period in the marketing season, stage 1 is the beginning of the last period in which sales may occur, and stage 0 is the end of the marketing season. In order to have the subscripts on the time periods agree with the order in which the problem was solved, the time periods were renumbered in reverse order.

The following definitions and notations were used in formulating the DP problem:

X_n = quantity (in 1000 box units) of Newtown apple sales during the nth period from the end of the planning horizon.

S_n = quantity (in 1000 box units) of apples available for sale at the beginning of the nth period.

$r_n(S_n, X_n)$ = net total revenue resulting from sale of X_n units

S_5 S_4 S_3 S_2 S_1 S_0

Period 1	Period 2	Period 3	Period 4	Period 5
$P_5=5.16-.00043X_5$	$P_4=5.28-.00229X_4$	$P_3=5.30-.00229X_3$	$P_2=5.36-.00229X_2$	$P_1=5.74-.000229X_1$
$d_0=1$	$d_1=.992$	$d_2=.984$	$d_3=.977$	$d_4=.969$
$c_0=0$	$c_1=.16$	$c_2=.17$	$c_3=.18$	$c_4=.19$
$DNTR_5=5.16X_5-.00043X_5^2$	$DNTR_4=5.08X_4-.00227X_4^2$	$DNTR_3=5.05X_3-.00225X_3^2$	$DNTR_2=5.25X_2-.000224X_2^2$	$DNTR_1=5.38X_1-.00222X_1^2$
$DNMR_5=5.16-.00086X_5$	$DNMR_4=5.08-.00454X_4$	$DNMR_3=5.05-.00450X_3$	$DNMR_2=5.25-.000448X_2$	$DNMR_1=5.38-.00444X_1$
$X_5^*=-15.60163$ $+.56622179S_5$	$X_4^*=-24.43443$ $+.24726511S_4$	$X_3^*=-39.41608$ $+.33140580S_3$	$X_2^*=-14.57400$ $+.497757S_2$	$X_1^*=S_1$
$f_5(S_5)=8.21746$ $+5.17341741S_5$ $-.00024347S_5^2$	$f_4(S_4)=7.97617$ $+5.19093148S_4$ $-.00056129S_4^2$	$f_3(S_3)=6.17565$ $+5.22737234S_3$ $-.00074567S_3^2$	$f_2(S_2)=0.94731$ $+5.31529148S_2$ $-.00111527S_2^2$	$f_1(S_1)=5.38S_1$ $-.00222S_1^2$

Figure 1. Chronological time contrasted with stages of the decision process and functional relationships used and calculated in the dynamic programming model

of apples with storage stocks at level S_n, during the nth period from the end of the planning horizon.

d_i = relevant discount rate to adjust the stream of benefits to present value.3/

c_i = cost per unit for i periods of storage.

t_n = stage transformation, expressing each component of the output of the nth stage as a function of the input state and decision in the nth stage, i.e., $S_{n-1} = S_n - X_n$.

The DP problem was formulated as:

$$f_n(S_n) = \max_{X_n \leq S_n} d_i[Q_n(S_n, X_n)], \qquad n = 1,\ldots,N \qquad (1)$$

$$Q_n(S_n, X_n) = d_i[r_n(S_n, X_n) - c_i X_n], \qquad n = 1 \qquad (2)$$

$$= d_i[r_n(S_n, X_n) - c_i X_n] + d_i[f_{n-1}(t_n(S_n, X_n))] \qquad n = 2,\ldots,N \qquad (3)$$

These equations represent the usual recursion equations of dynamic programming [5]. Equation (3) can be interpreted as the maximization, with respect to apple sales at stage n, of current discounted net total revenue plus the discounted net total revenue in the (n-1) remaining stages, given that the optimal policy will be used during the remaining (n-1) stages. The recursive solution of equations (2) and (3), starting with n = 1 and continuing through n = N, yields the optimal N- stage return, $f_N(S_N)$, the optimal decision $X_N^* = X_N^*(S_N)$, and the decision rules $X_n = X_n(S_n)$, n = 1,...,N - 1.

The solution procedure is as follows. Start with a one-stage process (the last unit in the planning period, n = 1). Regular storage apples cannot generally be stored successfully for more than eight months; therefore, the ending storage stock, S_0, must equal zero. Recalling the stage transformation function, $S_0 = S_1 - X_1$, the optimal policy for a one-stage process is $X_1 = S_1$. The value of a one-stage process, as measured by discounted net total revenue, is computed using the net average revenue function, $p = b_{01} - b_{11}X_1$, where b_{01} is that value that would have been needed in conjunction with b_{11} to predict actual wholesale Newtown apple prices in 1968-69 without error.[4/] Since the optimal policy for a one-stage process is $S_1 = X_1$, the value of a one-stage process is $r_1 = d_i[b_{01}S_1 - b_{11}S_1^2 - c_4S_1]$. For a two-stage process, using the results from a one-stage process, we get $f_2(S_2) = \max_{X_2} d_i[r_2(S_2, X_2] + f_1\ (S_1)$. To maximize $f_2(S_2)$, set the partial derivative of $f_2(S_2)$ with respect to X_2 equal to zero:

$$\frac{\partial f_2(S_2)}{\partial X_2} = d_i[r_2'\ (S_2, X_2)] + f_1'(S_1) = 0. \qquad (4)$$

Since the individual returns functions are quadratic equations (see r_1 above), equation (4) gives a linear decision rule of the form $X_2 = a + bS_2$.[5/] This process is carried out for each stage, always remembering that $S_{n-1} = S_n - X_n$. The N stage process leaves us at the current period looking N periods into the future with the problem of managing movement from storage in each period. We know the quantity of apples in storage at the beginning of the current period and $f_n(S_n)$ gives a value X_n which is optimal. Upon reaching the (N - 1) stage we find ourselves in another stage and $f_{N-1}(S_{N-1})$ gives us a X_{N-1} which is optimum, etc.

The quantity of apples that will be sold each period is not known until that period is reached; the information on the quantity of apples in storage is used sequentially as it becomes available. The functional equation (3), however, gives the discounted net revenue for the entire planning period given that an optimal policy is followed with respect to periodic apple sales.

Two important economic measures are contained in equation (4). First, $f_1'(S_1)$ is the marginal value of stored apples in stage 1. This marginal value is an indicator of the value from increased storage in stage 1. Second, from equation (4), we see that:

$$d_i[r_2'(S_2, X_2)] = f_1'(S_1), \tag{5}$$

or that discounted net marginal revenue in stage 2 equals discounted net marginal revenue in stage 1 - the necessary criterion for revenue maximization indicated by price discrimination theory.

The Results

The solution of the set of recursive equations of the dynamic programming problem yielded the set of marketing decision rules and stage return functions shown on the last two lines of Figure 1.[6/] Application of these decision rules to 1968-69 California fresh Newtown sales produced the optimal allocation, prices and total revenue presented in Table 1.

A comparison of actual and optimal sales as presented in Table 1 indicates that returns per box could have been increased by 4 cents

Table 1. Actual and computed optimal allocation, prices, and total revenue for California fresh Newtown apple sales 1968-69

	Actual			Optimal		
Month(s)	Quantity	Price	Total Revenue	Quantity	Price	Total Revenue
	boxes	$/box	$1,000	boxes	$/box	$1,000
Sept.-Oct.-Nov.	1414702	4.55	6437	1244130	4.63	5760
Dec.	159206	4.92	783	218052	4.78	1042
Jan.	131434	5.00	657	213321	4.81	1026
Feb.	204185	5.09	1039	258843	4.97	1286
Mar.-Apr.	315275	5.02	1583	290456	5.07	1473
TOTAL	2224802	4.72	10499	2224802	4.76	10587

per box in 1968-69 by reallocating sales between the five periods. The optimal allocation indicates sales should have been reduced in September, October, November, March, and April and increased in December, January, and February. The increase in total returns of four cents a box in 1968-69 is quite small, but similar calculations in other years indicates returns in some years could be improved by as much as 15 cents per box [2]. Comparison of non-harvest month sales with available storage capacity indicates current capacity is sufficient. A check of discounted net marginal revenue in each period verifies the fact that the decision rules generate quantities which equate DNMR among periods, thus maximizing total revenue.

In practice, actual sales in a given period may not equal the computed optimal sales quantity. However, the decision rules are constructed such that whatever the first decision is the remaining decisions will be optimal with respect to the outcome which results from the first decision. This is the strong point of the dynamic programming decision rules in comparison to the usual quadratic programming or calculus solutions. In the latter two cases, non-optimal sales in period 1 would require resolving the problem to determine the optimal allocation of remaining stocks.

Conclusions

The results indicate that dynamic programming is a useful tool for solving intraseasonal allocation problems. The calculated decision rules are such that whatever value the initial stage variable, discounted net total revenue is maximized by equating discounted net marginal revenue in each time period. A simplified problem was solved in order to present the logic of dynamic programming without cluttering the exposition. However, the dynamic programming procedure is capable of handling larger problems with two stage and decision variables and storage capacity restrictions.

Footnotes

* The authors are indebted to Bill Hardy for his helpful comments on an earlier draft.

1. The five time periods were: period 1, Sept.-Oct.-Nov.; period 2, Dec.; period 3, Jan.; period 4, Feb.; and period 5, Mar.-Apr.

2. Farm level price data by variety were not available. Maximizing returns at the wholesale level will not necessarily maximize grower returns. This problem is discussed in more detail in [2].

3. $d_i = 1/(1 + ri)$ where r = monthly interest rate (.008) and i = N-n.

4. These adjusted demand equations are shown on the first line of Figure 1. For a more detailed explanation of this approach see Sosnick [7].

5. The function $f_1'(S_1)$ is transformed into a function in S_2 and X_2 using the stage transformation $S_1 = S_2 - X_2$.

6. $F_5(S_5)$ does not equal total revenue under the optimum allocation in Table 1 since $F_5(S_5)$ is discounted net total revenue instead of total revenue.

References

[1] Shaul Ben-David and William G. Tomek, Storing and Marketing New York State Apples Based on Intraseasonal Demand Relationships, Cornell University Agricultural Experiment Station Bulletin 1007, 1965.

[2] David E. Kenyon, "Optimum Utilization and Intraseasonal Allocation of California Apple Production," unpublished Ph.D. dissertation, University of California, Davis, 1971.

[3] Richard M. A. Loyns, "An Economic Analysis of the Marketing Order for California Almonds," unpublished Ph.D. dissertation, University of California, Berkeley, 1968.

[4] R. E. Moffett, J. P. H. Brand, L. H. Myers and S. K. Seaver, Economic Benefits of Optimum Intraseasonal Allocation of New England-New York McIntosh Apples, University of Connecticut Agricultural Experiment Station Bulletin 394, June 1966.

[5] George L. Nemhauser, Introduction to Dynamic Programming, John Wiley and Sons, Inc., 1967.

[6] E. C. Pasour, Jr. and Robert L. Gustafson, Intraseasonal Supply and Demand Functions for Apples, Research Bulletin 10, Michigan State University, 1966.

[7] Stephen H. Sosnick, "Orderly Marketing for California Avocados," Hilgardia, Vol. 33, No. 14, December 1962.