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AN OPTIMAL LOCATION PATTERN OF PROCESSING PLANTS A CASE STUDY*

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I. Introduction

This paper intends to suggest a ~~competitive~~-equilibrium locational pattern for Colorado milk processing plants as an efficient form of spatial organization for dairying. The ~~competitive~~-equilibrium locational pattern is envisaged as one which simultaneously minimizes the total costs of assembling and processing raw milk and distributing finished milk for a self-contained region, given the regional distributions of the raw milk supply and the demand for finished milk.

Conceptually, equilibrium locations and the size of market areas for the competitive producers of a particular industry have, to a certain extent, been analyzed in the past. For instance, assuming elastic consumer demand and ubiquity of raw materials for production (thus zero assembling cost), Lösch considered economies of scale of production against the costs of distributing finished commodities in determining the monopolistically competitive market areas (of hexagonal shape) for profit-maximizing producers in a regional economy with uniformly distributed settlements.[1] French estimated assembly costs in conjunction with processing costs in analyzing the efficient scale of agricultural processing operations; while Williamson incorporated these two types of costs into his considerations in working out a methematical function which shows how the equilibrium size of marketing plants depends on market demand density (volume of business per unit of market area) under certain conditions.[2] The work of both researchers ~~Both their work~~ also involved the simplifying assumption of uniformly distributed consumers and producers as Lösch did. Consequently, for a region which is not in accord with this prescribed feature, as is true in most real situations,

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[1] Lösch, August, The Economics of Location, 2nd edition. Translated by William H. Woglan, Yale University Press, New Haven, Conn. 1954, pp. 105-123.

[2] French, B. C., "Some Considerations in Estimating Assembly Cost Functions for Agricultural Processing Operations," Journal of Farm Economics, Vol. XLII, Nov. 1960, pp. 767-78.

Williamson, J. C., Jr., "The Equilibrium Size of Marketing Plants in a Spatial Market," Journal of Farm Economics, Vol. XLIV, Nov. 1962, p. 953.

these theorists could, at most, explain the optimal size of a certain market area in terms of the average radius from a processing (producing) center on the basis of average densities of resources and demand of that region. The network of regular-shape (such as hexagonal, circular) market areas would no longer be able to define the actual marketing boundaries of competing processing centers even in a region of homogeneous topography. In other words, for a region dissenting from uniform distribution of resources and economic units, they did not provide an answer to such questions as: (1) what is the optimal size (and boundary) of the market area for a particular processing center? (2) where does the supply of a particular commodity to a particular consuming center come from? (3) if a consuming center is supplied by two or more producing (processing) centers, how much is supplied by each?

This paper will present an analytical scheme by which the above questions may be answered conceptually. Then, this scheme will be put into empirical implementation in a case study with the aid of an established programming algorithm. The empirical results thus achieved will hopefully fulfill the original intention of this paper--to suggest an ~~competitive~~ equilibrium locational pattern of Colorado milk processing plants.

II. Analytical Scheme

Consider a set of a finite number of raw material deposits (resources), a set of a finite number of processing centers, and a set of a finite number of consuming (marketing) centers in a given self-contained region. These three sets of discrete points may or may not spatially coincide with one another; and there are no restrictions on their spatial distribution patterns. Assuming that the supply of raw materials at each deposit and the demand at each consuming center are given, the economies of scale in processing exist at each processing center so that the unit processing cost varies with the amount processed, and that economies of scale in assembling raw materials and commodity distribution over longer distances (miles) also exist. The problem then is to discover ~~in the~~ an efficient spatial organization of the particular processing industry sense of minimizing the total costs of operation of the industry instead of maximizing individual producers' benefit or profit.

An attempt is now made to answer the first question under the specifications made above. Take a processing center, A. In Figure 1 the points on the right side of A along the horizontal axis, 1, 2, 3, etc. represent the nearest consuming center, the next nearest consuming center, the still next nearest consuming center, etc., with their locations on the axis measuring the distance that separates them from the processing center A. The points on the left side of A along the same axis, a, b, c, etc. represent the nearest raw material deposit, the next nearest deposit, the still next nearest deposit, etc., with their location relative to A being measured by the horizontal distances along the axis. The vertical axis is designed to measure cost figures. Admittedly, for regional

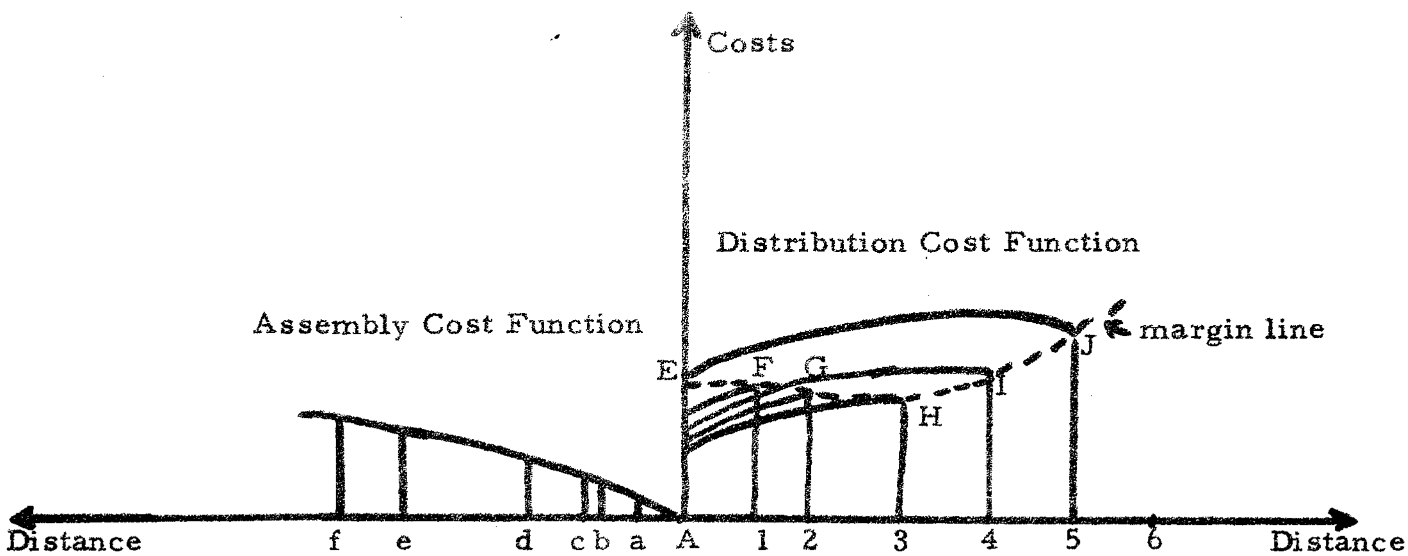


Figure 1

systems with an irregular distribution pattern like this, a unique function which relates incremental total costs to incremental market supply does not exist. This function would, in each individual case, depend on the raw material supply density and commodity (market) demand density around the processing center. Nonetheless, it is true, in general, that as a processing center supplies more and more distant consuming centers, the raw materials to be processed have to be assembled from more and more distant deposits. If the processing center could operate more economically by processing to supply beyond the local market, the economies of scale in processing would outweigh the additional assembly and distribution costs for a while before it is outweighed by the latter two. This is illustrated in Figure 1 wherein raw materials at a, b, c, etc. are to be assembled in that order as A increases processing to supply 1, 2, 3, etc., respectively; and E, F, G, H, I, J represent the total unit costs (of assembly, processing, and distribution) at the boundary of A's market as it extends from local market A to 1, 2, 3, 4, and 5 respectively.[3] These points, when connected together, form the "margin line", the lowest point of which H would correspond to the limit (3 in this diagram) of the optimal size of A's market.[4]

Suppose, now, a competitive processing center B is introduced. The problem of competition does not arise if the optimal-size markets of A and B are independent of each other as is shown in Figure 2a. The competition occurs only if the two optimal-size markets overlap, such as in the case shown in Figure 2b. Then, the question (2) posed in the last section arises: which of the two processing centers will supply the consuming centers located in both markets (such as 3) if they have to be served by one processing center or another.

The answer to this question is obvious in the margin-line diagram. Processing center B would supply consuming center 3 simply because the total unit costs of supplying at the market front 3 are lower. The difference between the two processing centers' total unit costs at 3 MN would measure

[3]If two or more deposits have to be used to assemble raw materials in order to supply an additional consuming center, the weighted average unit assembly costs must be computed to obtain total unit costs.

[4]The concept of "margin line" is originally Hoover's. See Hoover, E. M. Location Theory and the Shoe and Leather Industries, Harvard Economic Studies, Vol. LV, Harvard University Press. It is redefined here to accommodate explanations of the present problem.

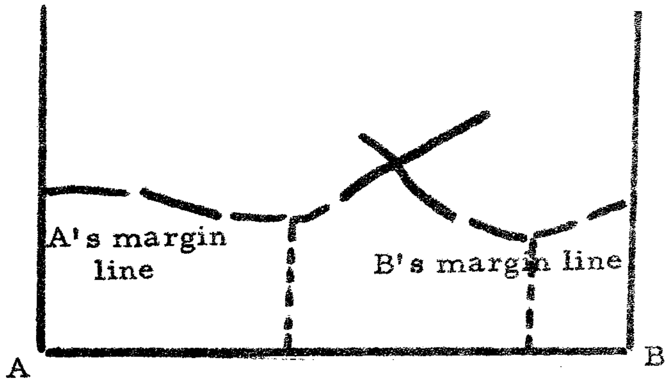


Figure 2a

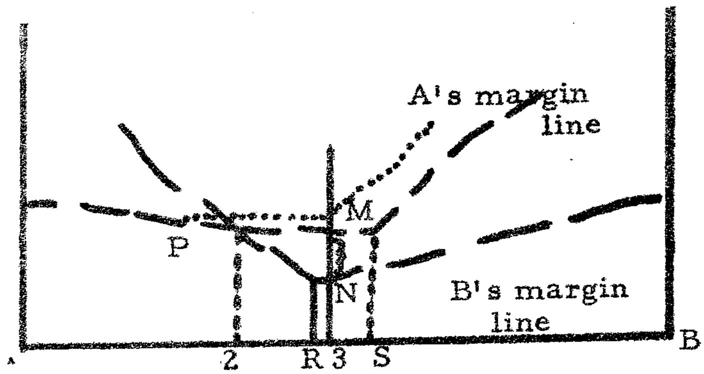


Figure 2b

the relative locational advantage or the "location rent" of B in serving 3. Therefore, the market within the range RS would be taken away by B if B were to retain its own optimal size market. However, the present scheme does not aim at individual processors' interests. Rather, it is concerned with an efficient spatial organization of a processing industry which implicitly contemplates the existence of a centralized decision maker who would, in this case, dictate that B continue to compete with A for its share of the market until it succeeds in taking away the consuming centers located somewhere from R to 2 for supply.[5]

It should be noted that it may sometimes be possible for two or more competing processing centers to assemble a limited supply of raw materials at common deposits. In such cases, the processing center where the greatest savings is incurred in the total unit costs of processing to supply (as a result of assembling raw materials from these deposits) would have the highest bidding ability and thus get the raw materials there.

[5]That the amount of production at different producing centers varies according to the levels of decision making was analyzed by Tung. See Tung, T. H., "Optimal Spatial Patterns of Production and Decision Makint," Papers of the Regional Science Association, Vol. 15, 1965, pp. 143-157.

Then the processing center which would have the next greatest cost savings and bidding ability would assemble raw materials at those deposits if there are certain available amounts left over, and so on. Those processing centers that fail in vying for the most desirable locations of deposits for assembling would be forced to assemble from less desirable locations. Consequently, their original margin lines would shift upward from there on (as shown by the dotted line from P for the processing center A in Figure 2b.)

Now, the question (3) posed above--concerning the problem of joint supply--remains to be answered. The contemplated centralized decision maker would consider, in his reckoning, all possible substitutions of supply patterns in the process of minimizing total costs of assembly, processing and distribution for the processing industry concerned. He would substitute joint-supply patterns for single-supply patterns if the former should incur less total costs than the latter. To illustrate this, consider a consuming center X with three units of demand for the commodity processed. Assuming the rest of the regional system has been efficiently supplied except X which may be supplied by either A or B or both. In Figure 3, it is seen that, according to the previous analysis, A would supply X if X is to be supplied by one processing center. However, it may be that A's margin line would drop to the level T from V if A supplies two units to X due to the fact that the third unit of supply requires assembling raw materials from a much more distant deposit and thus incurs a significantly higher assembly cost. In such case, the joint-supply pattern (i.e., A supplies 2 units and B supplies 1 unit to X) would be cheaper than the single-supply pattern (i.e. A alone supplies 3 units to X) if $\frac{2TX + UX}{3} < VX$.

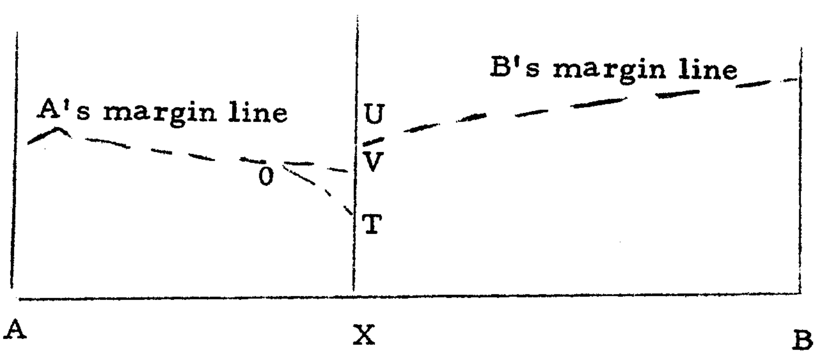


Figure 3

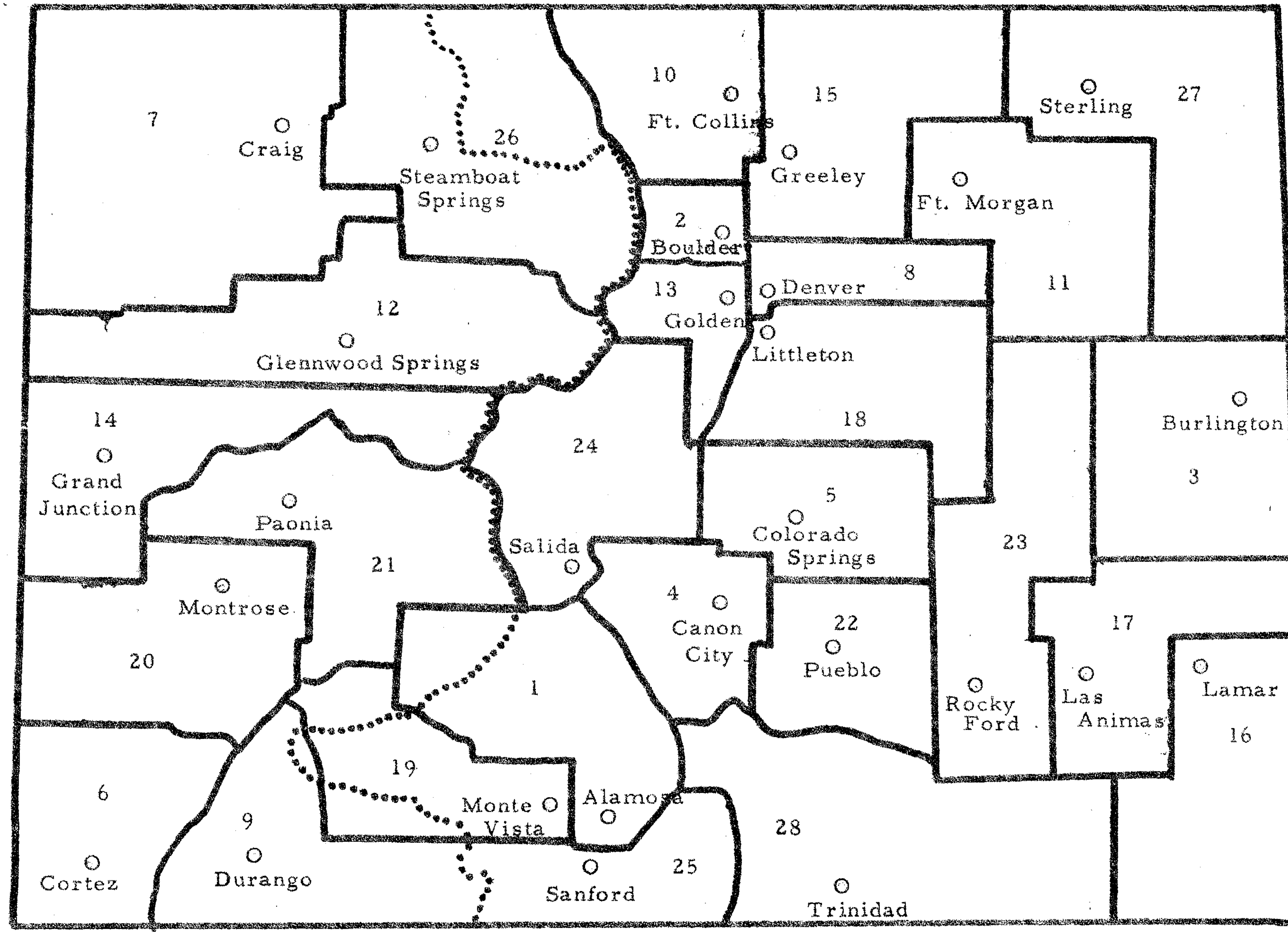
The analyses carried on in the preceding paragraphs can readily be generalized for any finite number of processing centers, raw material deposits, and consuming centers. The generalization would essentially invoke a standardized programming model which is to minimize the total costs of assembly, processing, and distribution subject to the constraints that the demand of all consuming centers must be satisfied and that the assembly of raw materials at each deposit should not be beyond its capacity.

III. Empirical Implementation

Empirical application of the above analytical scheme is carried out for the Colorado dairy industry. The State of Colorado is operationally divided into twenty-eight sub-regions, the location of each being designated by a representative point which, in most cases, coincides with the city carrying the greatest weight of demand for milk and supply of raw milk in that region. This is shown in the map of Figure 4. Assume that each of these sub-regions are potential locations of milk processing plants. The given demand for finished milk and capacity of raw milk supply in each sub-region are shown in Table 1. A transshipment type programming model is adopted. Each of the twenty-eight sub-regions may serve as transshipping points as well as destinations of processing and/or consumption and, thus, there would be (2 x 28) x (2 x 28) possible variables in this model. The objective function of this model then becomes:

$$\text{Min} \cdot \sum_{j=1}^{56} \sum_{i=1}^{56} C_{ij} X_{ij}$$

wherein the unit assembly or distribution costs $C_{ij} = 0$ for $i = j$ $(i, j = 1, \ldots, 28, 29 \ldots, 56)$; the unit assembly (transshipment) costs $C_{ij} = Y_{ij} > 0$ for $i \neq j$ $(i, j = 1, \ldots 28)$; the unit assembly and processing costs $C_{ij} = Y_{ij} + Y_p > 0$ for $i \neq j$ $(i = 1, \ldots 28, j = 29, \ldots, 56)$; the unit processing costs $C_{ij} = Y_p > 0$ for $(i = 1, \ldots, 28; j = 29, \ldots 56)$; the unit distribution (transshipment) costs $C_{ij} = \infty$ $(i = 29, \ldots, 56;$



1. Mountain Dividing Line
2. Subregional Boundaries
3. Subregional Representative Points

Fig. 4

Table 1 *

Subregion Index	Name	Demand for finished milk products by subregion, (homogenized milk equivalent quarts / per day)	Capacity of raw milk supply by subregion (quarts/per day)
1	Alamosa	7710	9620
2	Boulder	39560	40360
3	Burlington	5190	22410
4	Canon City	11460	21030
5	Colo. Springs	77920	44010
6	Cortez	8640	19840
7	Craig	5970	22770
8	Denver	263140	56190
9	Durango	12090	20110
10	Ft. Collins	28420	63710
11	Ft. Morgan	14820	44150
12	Glennwood Springs	11170	13410
13	Golden	69800	20520
14	Grand Junction	27020	31620
15	Greeley	38550	195190
16	Lamar	10450	10600
17	Las Animas	5240	6640
18	Littleton	64980	47800
19	Monte Vista	6280	2580
20	Montrose	12170	23500
21	Paonia	11230	23630
22	Pueblo	63250	27020
23	Rocky Ford	17810	20920
24	Salida	9180	6100
25	Sanford	6740	6570
26	Steamboat Springs	5950	15850
27	Sterling	20190	38930
28	Trinidad	14840	14690

* Figures shown in this table are estimated from 1962 statistics of state average per capita consumption of milk and the number of cows.

$j = 1, \ldots, 28$);[6] the unit distribution costs $C_{ij} = Z_{ij}$ for $i \neq j$ ($i, j = 29, \ldots, 56$); and X_{ij} are the number of quarts per day of raw milk shipped for $i = 1, \ldots, 28$; $j = 1, \ldots 56$ and X_{ij} are the number of quarts per day of finished milk shipped for $i = 29, \ldots 56$; $j = 1, \ldots 56$.

The constraints required to be satisfied in the model are ($i, j = 1, \ldots 56$):

A. $-\sum_j X_{ij} = -S_i - K$

where S_i is the capacity of available raw milk at i and $S_i = 0$ for $i = 29, \ldots, 56$; K being an artificial constant, 700,000 quarts, which is used to allow for the possibility that milk-processing could be heavily concentrated in one sub-region.

B. $\sum X_{ij} = V_j + K$

where V_j is the consumption requirement at j and $V_j = 0$ for $j = 1, \ldots, 28$.

The unit processing costs Y_p (cents/quarts), unit assembly costs Y_{ij} (cents/quart-mile), and unit distribution costs Z_{ij} (cents/quart-mile) may be computed from the following estimated regression equations:

$$Y_p = 34{,}817\, X_i^{-0.2141} \quad (7400 \leqq X_i \leqq 120{,}000 \text{ quarts per day})$$

where X_i is the number of quarts processed per day in plant i.

$$Y_{ij} = 0.00663307 - 0.00000240\, D_{ij} \quad (D_{ij} < 425 \text{ miles})$$

where D_{ij} is the distance between the two locations i and j. (the representative points of subregions i and j)

$$Y_{ij} = 0.005581 + 0.00000064D \quad (510 \geqq D_{ij} \geqq 425 \text{ miles}).$$

$$Z_{ij} = 0.01125 - 0.000056192\, D_{ij} \quad (D_{ij} \leqq 100 \text{ miles}).$$

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[6] The ∞ costs are imposed to preclude the inefficient distribution of transshipments.

$Z_{ij} = 0.0075111 - 0.000018803\ D_{ij}$ (100 miles $\leqq D_{ij} \leqq$ 200 miles).

$Z_{ij} = 0.0045572 - 0.0000040335\ D_{ij}$ (510 $\geqq D_{ij} \geqq$ 200 miles.)[7]

The transportation program 10.1.005 written up for IBM 1620 is selected to solve the above problem.[8] As required by this program for solution, total supply of resource deposits should be equal to the total demand. Since the homogenized milk production function is approximately a 1:1 conversion of raw milk into finished milk, the 15,929 quarts per day of raw milk which is the amount Colorado production is short of total demand (which is primarily imported from Utah) is now assumed to come from Craig of Colorado (the sub-region closest to Utah). Because of the uneven topography of Colorado, cross-mountain shipments (both assembling and distribution) are very expensive and thus doubled in costs, as a simplest procedure of approximation. The costs of assembling raw milk are obtained for round trips in conformity with the actual situation.

On account of the non-linear nature of the unit processing costs function, the optimal solution of the above model may be obtained by procedures of iterative approximation. In the first run, 2.97 cents

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[7] These equations are estimated on the basis of sample surveys of large milk processing plants, interviews with experienced haulers, and a previous study (Paul O'Connel and W. E. Snyder, Cost Analysis of Fluid Milk Processing and Distribution in Colorado, Technical Bulletin 86. Agricultural Experiment Station, Colorado State University, Fort Collins, Colorado, 1963). The assembly costs exclude the costs of raw milk itself. The units of all dairy products are converted into equivalent quarts of homogenized milk, e.g. 1 pint of whipping cream is equivalent to two quarts of homogenized milk; 1 pint of half and half is equivalent to two quarts of homogenized, etc.

[8] Application of this program to solving locational programming model has been done for cattle slaughtering industry in California. See King, Gordon A. and Logan, Samuel H., "Optimal Location, Number and Size of Processing Plants with Raw Product and Final Product Shipments," Journal of Farm Economics, Vol. 46, No. 1, Feb. 1964, pp. 94-108.

(which is the unit processing cost corresponding to average size of biggest existing processing plants, processing approximately 118,000 quarts per day) is used for all sub-regions to allow the chance that each of them could maintain the largest plant size. Inconsistencies between unit processing costs and processing amounts in many sub-regions would show up in the solution if the largest size plant in those sub-regions would not be justified due to lack of local and nearby outside demand and/or raw milk supply. Thus, in the second run, the unit processing costs have to be adjusted to correspond to the processing amount given by the first run solution for each sub-region in an attempt to eliminate the inconsistencies. This adjusting process repeats until overall consistency is reached. In addition, during the adjusting process, attention should be directed to those sub-regions whose locational advantage in processing may be concealed by allowing unreasonably low unit processing costs in neighboring processing sub-regions. In the present study, the sub-region 15 (Greeley) is found to be such a case. This subregion has more than the amount of raw milk supporting an efficient biggest processing plant and yet was processing little or nothing in the locational competition. In such a situation, the low unit processing costs of the biggest plant should retain with the sub-region 15 in some of the later runs when those neighboring sub-region's unit processing costs have been duly adjusted. This procedure is applied and reveals the capability of sub-region 15 to maintain an efficient biggest size of plant. As a result, it shares with sub-region 8 (Denver) in supplying finished milk to other neighboring sub-regions in the eastern slope of Colorado.

The overall optimal solution finally obtained after some six iterative runs are summarized in the graph of Figure 5 with total costs being $30,667.41083. It is seen that Colorado's milk processing can be efficiently carried out at seven locations, sub-regions 5, 6, 8, 9, 14, 15 and 22, with their processing capacity being 77,920; 19,840; 397,920; 20,110; 82,690; 172,650 and 98,640 quarts per day respectively. Three of them (6, 9, and 14) are located in the western slope while the other four (5, 8, 15 and 22) are located in the eastern slope of Colorado.

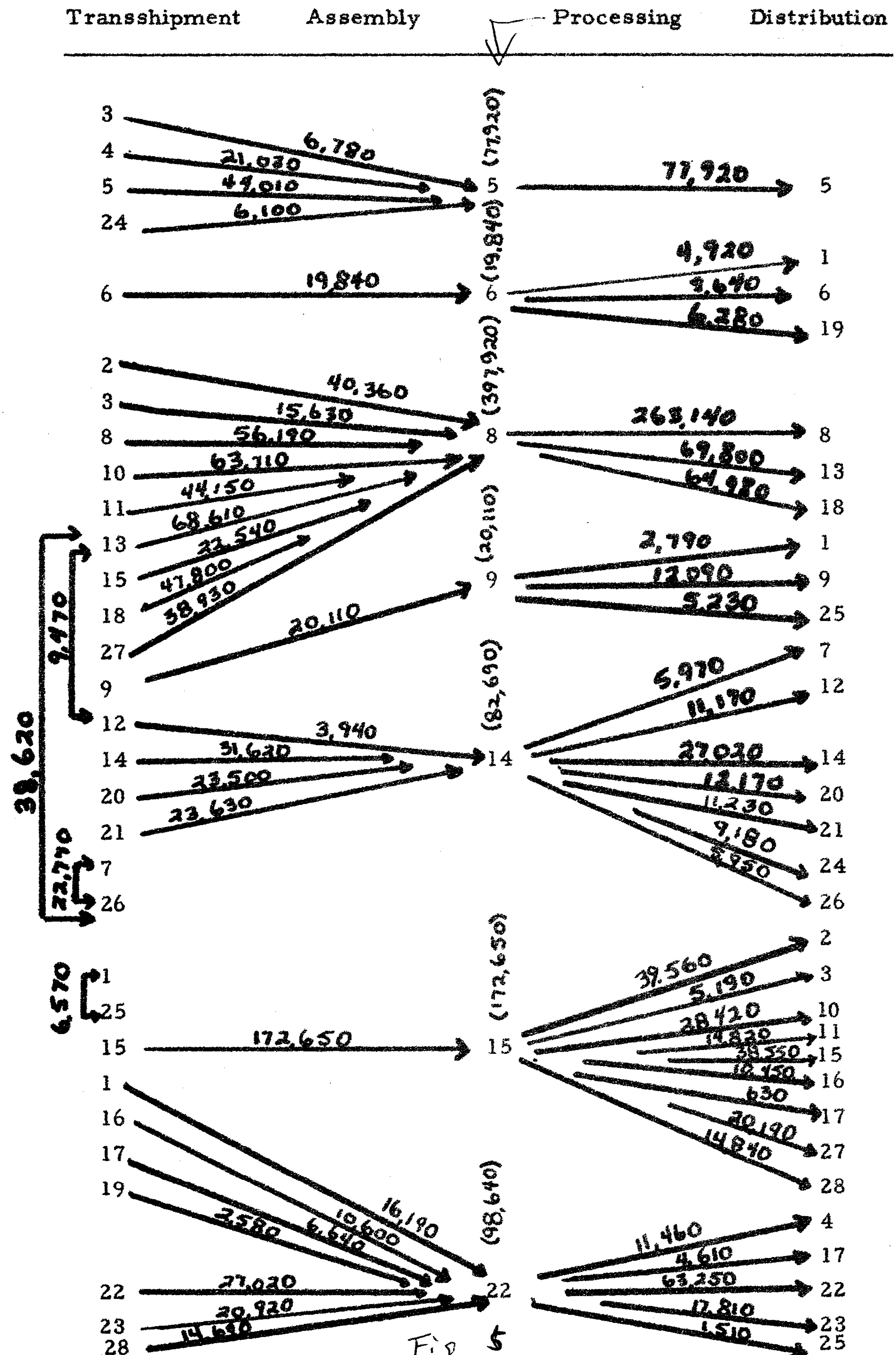

Fig 5

* Figure are in quarts per day.

The size of processing plant corresponding to any processing capacity falling within the range of values that X_i may assume in the unit processing costs function is implicitly assumed to be feasible. The processing capacity of Greeley (sub-region 15) and Denver (sub-region 8) are way over this range. If the maximum limit of plant size is considered strictly according to the existing biggest size in Colorado, then Greeley would have a biggest size of plant plus one of a smaller size (processing 54, 650 quarts per day) and Denver would have three biggest size of plant plus one of a smaller size (processing 43, 920 quarts per day). Consequently, their locational advantage in processing milk relative to other sub-regions would be somewhat modified and the total costs of the objective function solved should be correspondingly adjusted upward. If certain plant sizes even larger than the biggest that exists are deemed feasible, then Denver could establish three plants of real large size while Greeley could establish an even larger size of plant. However, the economies of scale in processing ought not optimistically be extrapolated beyond the amount of 120, 000 quarts per day without any empirical basis.[9] The transshipments as indicated in Figure 5 are those cross-mountain assembling of raw milk through some intermediate sub-regions located closely to the edge of mountains. This is so owing to the simplifying procedure that was made in doubling cross-mountain assembly costs and, therefore, they are actually unnecessary. The solution given above also has to be qualified in light of many simplications made, such as the use of demand and capacity of raw milk figures of a given year, the simple ways of estimating them by sub-region, and the treatment of certain amount of imported raw milk as supply from within the region considered.

The dual of the above minimizing programming-model would be a problem of pricing the raw milk and finished milk at each of the sub-regions so as to maximize the total returns. These (relative) prices given in the dual solution are shown in Table 2. By arbitrarily setting the price of raw milk

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[9]According to O'Connel and Snyder's study, there are still certain amounts of unused capacity in the biggest size of plants. Thus, some arbitration is made to extend the effective range of economies of scale in processing to 120, 000 quarts per day to take care of this fact. See O'Connel and Snyder, op. cit., p. 15.

TABLE 2

Subregions (relative) Prices of	1	2	3	4	5	6	7	8	9	10	11	12	13	14
raw milk cents (per quart)	0000	.8827	.0424	.6834	.9835	-1.2893	-1.2597	1.0600	-1.0713	.6516	.5509	-.9802	.9610	-.4088
finished milk (cents per quart)	4.3391	4.1057	4.2779	4.1651	4.1035	2.8907	3.2716	3.9100	3.1087	3.8913	4.0725	3.3568	4.0911	2.6712

Subregions (relative) Prices of	15	16	17	18	19	20	21	22	23	24	25	26	27	28
raw milk cents (per quart)	.7152	.3181	.2456	1.0004	-.1021	-.8109	-.8677	.7796	.4348	.3318	-.1252	-.9918	.2743	.2269
finished milk (cents per quart)	3.5652	4.4328	4.4183	4.0218	4.2399	3.2476	3.2936	3.7496	4.2705	4.0966	4.3911	3.3896	4.2650	4.4257

of sub-region 1 at zero level, the other prices are solved relative to it. Two important relationships between the relative prices should be noted: (1) for either assembly or distribution, the price of the sub-region of origin plus the shipping costs involved is equal to the price of the sub-region of destination. For example, Denver (sub-region 8) supplies finished milk to Golden (sub-region 13), their relative price difference is 4.0911 - 3.9100 = .1811 cents which is the unit distribution cost between these two sub-regions. (2) The difference in relative price of competing processing sub-regions supplying a common consuming sub-region is equal to costs savings on the part of the higher-price sub-region and, thus, reflects the locational advantage or "rent" of that sub-region. For example, Durango (sub-region 9) and Cortez (sub-region 6) both are processing to supply Alamosa (sub-region 1). The difference in relative prices of the two processing sub-regions 3.1087 - 2.8907 = .2180 cents measures Durango's unit distribution costs saving since it is 1.4484 - 1.2304 = .2180 cents per quart cheaper from Durango than from Cortez to distribute finished milk to Alamosa.

The minimum costs solution and its dual prices presented above provide a basis against which the efficiency of operation and marketing of existing milk-processing industry in Colorado may be evaluated. They also provide guidelines for making location decisions of individual milk processors and formulating milk pricing policies.