Regional and Farm-Level Risk Analyses with the Single-Index Model

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This paper estimates single-index model (SIM) beta coefficients for the major cash crops of Ontario's agricultural-producing counties. Beta coefficients are estimated using per acre gross revenues weighted by (1) the proportion of each crop planted in each county and (2) equal weights. The results show that sometimes substantial differences arise from these procedures. Implications for government policy and farm-level risk-management strategies are discussed. A practical approach for using SIM for farm management purposes is presented and the effects of systematic and nonsystemic risk are discussed.

One of the most important areas in agricultural finance and farm management is risk management. The management of risk posits a substantial problem since few simple tools are available. Many academics work with mean-variance optimization or stochastic dominance models, while some extension offices provide publications that include risk measures such as the coefficient of variation. An approach to measuring risk in agriculture, which may prove fruitful for both academics and farm managers or extension agents, is the single-index model (SIM).

The single-index model (Sharpe) has been used to derive optimal mean-variance-efficient portfolios (Collins and Barry; Turvey, Driver, and Baker), to examine the relative riskiness of farm enterprises (Turvey and Driver; Gempesaw, Tambe, Nayga, and Toensmeyer), and to estimate the marginal costs of diversification (Sharpe and Baker; Blank). It provides a method of risk analysis that reduces substantially the amount of probability information required for making single-period portfolio-choice decisions. Moreover, it is conceptually simple to understand and is consistent with the mean-variance rule.

The purpose of this paper is to present SIM concepts within a risk-management framework that can benefit farmers and policy makers. Farmers benefit because SIM can be used to measure and compare enterprise risk as well as portfolio risk using relatively simple arithmetic. Policy makers benefit because the approach readily lends itself to evaluating regional risk differences and diversification.

The outline of the paper is as follows. The theoretical framework of SIM is first presented. The risk measures are then applied to a cross section of counties in Ontario, and the results pertaining to alternative crop portfolios are presented. The last section provides a discussion of some of the issues involved as well as the conclusions of the paper.

The Single-Index Model

The single-index model assumes that revenues associated with various farm enterprises are related only through their covariance with some basic underlying factor or index. The risk associated with this index is called nondiversifiable, or systematic, risk. Specifically, systematic risk measures the proportionate contribution of an individual enterprise's risk to the variance of the underlying index. The second risk component, called nonsystematic risk, is the portion of enterprise returns uncorrelated with the index. That is, nonsystematic risk is the commodity's specific risk. Diversification can potentially reduce nonsystematic risk.

In a portfolio context, a measure of systematic risk can be determined by defining an index com-
prised of the stochastic revenues of a reference portfolio, $\tilde{R}_p$:

$$\tilde{R}_p = \sum_{i=1}^{n} w_i \tilde{R}_i,$$

where $w_i$ are the weights of enterprise $i$, and $\tilde{R}_i$ are stochastic enterprise revenues. The expected value of (1) is

$$E[R_p] = \sum_{i=1}^{n} w_i E[R_i]$$

and its variance is

$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij},$$

where $\sigma_{ij}$ represents the enterprise variance and covariance relationships. An important aspect of portfolio theory is the relationships between portfolio risk, the relative proportions of crops held in the portfolio, and the contribution that each crop makes to portfolio variance. For example, differentiating $\sigma_p^2$ with respect to $w_i$ yields (Berndt)

$$\frac{\partial \sigma_p^2}{\partial w_i} = 2 \sum_{j=1}^{n} w_j \sigma_{ij} = 2 \sigma_{ip}.$$

Thus, the change in portfolio variance due to a change in the weight of the commodity depends simply on the covariance between the two. The insight which SIM gives to this problem is that it provides an exact measure of this marginal risk response in a simple and intuitive way.

Least squares regressions of $R_i$ on the underlying reference portfolio, $R_p$, are the characteristic equations that determine systematic and nonsystematic risk; i.e.,

$$R_i = \alpha_i + \beta_i R_p + e_{it},$$

where $\alpha_i$ is the intercept, $\beta_i$ is the regression (beta) coefficient, and $e_{it}$ is the error term.

The beta coefficients measure the anticipated response of a particular commodity to changes in portfolio returns. By definition, $\beta_i = \sigma_{ip}/\sigma_p^2$, so that rearranging in terms of $\sigma_{ip} = \beta_i \sigma_p^2$ and substituting this into equation (4) yields

$$\frac{\partial \sigma_p^2}{\partial w_i} = 2 \sigma_p^2 \beta_i.$$

Since $2 \sigma_p^2$ is a constant, knowledge of enterprise beta coefficients provides a sufficient measure of marginal risk.

SIM parameters can also be used to provide measures of portfolio risk. Dropping the time subscript, the expected value of (5) is

$$E[R_i] = \alpha_i + \beta_i E[R_p],$$

and its variance is

$$\sigma_i^2 = \beta_i^2 \sigma_p^2 + \sigma_{ei}^2,$$

where $\beta_i^2 \sigma_p^2$ is the systematic risk for enterprise $i$ and $\sigma_{ei}^2$ is nonsystematic risk. The covariance between two enterprises is $\beta_i \beta_j \sigma_p^2$. Substituting these variance and covariance measures into (3) yields the portfolio variance in terms of the single-index model parameters:

$$\sigma_p^2 = \left( \sum_{i=1}^{n} w_i \beta_i \right)^2 \sigma_p^2 + \sum_{i=1}^{n} \sigma_{ei}^2.$$
tematic risk for the gross-revenue measure and low systematic risk for the net income measure. The appropriateness of which measure should be used is discussed in the following sections.

Methodology

This section discusses some of the major issues confronting the adoption and use by farmers of SIM in general and the portfolio beta measure in particular. Data issues, measurement issues, and application are discussed.

There are two considerations with respect to data issues. The first consideration concerns the data to be used, that is, net income or gross revenue and real or nominal values. This issue is of great concern since the accuracy of the risk measure is contingent on the measure used. Adams, Menkhaus, and Woolery, and Lin, Dean, and Moore find substantial differences in portfolio solutions depending on how risk and returns were defined. Gempesaw et al. examined this issue with explicit reference to SIM and found conflicting results. Collins and Barry, and Blank use real net income in their SIM, while Turvey and Driver, and Turvey, Driver, and Baker use nominal gross revenues.

The results of these studies show that using net income decreases systematic risk relative to the gross-revenue measure. While arguments for using real data are compelling, there is little guidance in the literature as to whether or not gross or net revenues should be used. Turvey and Driver (p. 399) argue that the main issue is the timing of inputs. Since most of the inputs for production agriculture have been purchased or negotiated prior to planting, their costs are generally known and deterministic. The only relevant stochastic variables are enterprise prices and yields, which can conveniently be combined into a single univariate random variable. An appropriate variance measure, therefore, would be based on real gross revenues rather than net income, although under the assumption of deterministic costs, the variance of both are equal.

This study used county-level price and yield data for Ontario from 1973 to 1988 (Ontario Ministry of Agriculture and Food). Prices were converted to real 1988 values using the implicit price deflator of the gross national product. All revenues are in Canadian dollars. Using the characteristic regression (equation 5), enterprise beta coefficients were estimated for grain corn, silage corn, soybeans, white beans, winter wheat, hay, barley, oats, and mixed grain. These coefficients were estimated using ordinary least squares (OLS) for each of Ontario's 50 agriculture-producing counties. In general, OLS estimates of equation (5) are biased and inconsistent since the dependent variable, $R_{it}$, is a component of the independent variable, $R_{pt}$. The order of bias will be larger for beta estimates of crops that comprise a substantial proportion of the portfolio, but as the number of crops in the portfolio increases, the overall bias is diminished.

Beta coefficients were estimated using two different weighting techniques. The first applied equal weights to all crops in the county portfolios, while the second applied weights based on the proportion of crop acreage planted in each county in each year. The first method is consistent with the approach used in Collins and Barry, Turvey and Driver, and Turvey, Driver, and Baker for farm-level decision making, whereas the second was used by Gempesaw et al., and Blank. The intent of this approach is to choose appropriate portfolio weights using enterprise beta coefficients as the risk measure and then evaluate the risk relative to expected portfolio revenues.

The second approach is a more precise measure for county-level analyses since the weighting considers fully the contribution that each enterprise makes to the variance of the county portfolio. The marginal risk criterion (equation 6) can be applied to assess the change in county-level risks in response to an incremental increase or decrease in acres planted to a particular crop. Hence, differences in beta coefficients between counties can be attributed not only to inherent systematic risk, but also to the proportion of acreage in the county relative to other crops. However, it should be noted that these beta coefficients are only relevant for small increments of risk. A substantial change in the counties' crop mix would require the beta coefficient and, therefore, the incremental variance to be reestimated.

Results

Results of the empirical model are presented in this section. County-specific beta coefficients are presented and discussed. The use of these beta coefficients is illustrated with a farm management

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2 The characteristic equations were estimated assuming a naive expectations framework whereby the expected value is equal to the means and the variance is measured as the deviations from the means. Other measures could be used that measure deviations from expectations other than means. For example, expected enterprise and portfolio revenues could be specified as an n year moving average with no loss of generality. The beta coefficient would have the same interpretation, but the values may be different. For a general discussion on estimating procedures, see Gempesaw et al.
Table 1. Equally and Proportionally Weighted Beta Coefficients for Selected Ontario Counties

<table>
<thead>
<tr>
<th>County</th>
<th>Wheat</th>
<th>Oats</th>
<th>Barley</th>
<th>Mixed Grain</th>
<th>Hay</th>
<th>Grain</th>
<th>Silage</th>
<th>Soybean</th>
<th>White Beans</th>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Essex</td>
<td>1.587</td>
<td>0.807</td>
<td>0.848</td>
<td>0.793</td>
<td>0.467</td>
<td>1.513</td>
<td>0.974</td>
<td>1.011</td>
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<td>Middlesex</td>
<td>0.879</td>
<td>0.562</td>
<td>0.743</td>
<td>0.768</td>
<td>0.453</td>
<td>1.392</td>
<td>0.720</td>
<td>0.681</td>
<td>2.801</td>
</tr>
<tr>
<td>Oxford</td>
<td>1.197</td>
<td>0.915</td>
<td>0.838</td>
<td>1.192</td>
<td>0.383</td>
<td>1.682</td>
<td>0.794</td>
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<td>—</td>
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<tr>
<td>Peel</td>
<td>1.182</td>
<td>0.736</td>
<td>1.149</td>
<td>0.926</td>
<td>0.459</td>
<td>1.430</td>
<td>1.119</td>
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<tr>
<td>Waterloo</td>
<td>1.243</td>
<td>0.782</td>
<td>0.959</td>
<td>1.043</td>
<td>0.274</td>
<td>1.681</td>
<td>1.017</td>
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<tr>
<td>Wellington</td>
<td>1.146</td>
<td>0.915</td>
<td>1.292</td>
<td>1.250</td>
<td>0.268</td>
<td>1.576</td>
<td>0.552</td>
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<tr>
<td>Northumberland</td>
<td>1.118</td>
<td>0.842</td>
<td>0.808</td>
<td>1.007</td>
<td>0.413</td>
<td>1.950</td>
<td>0.862</td>
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<tr>
<td>Lanark</td>
<td>—</td>
<td>0.392</td>
<td>0.549</td>
<td>0.806</td>
<td>0.435</td>
<td>2.463</td>
<td>1.355</td>
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<tr>
<td>Ott.-Carleton</td>
<td>—</td>
<td>0.746</td>
<td>0.954</td>
<td>0.876</td>
<td>0.497</td>
<td>1.814</td>
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<tr>
<td>Essex</td>
<td>1.128</td>
<td>0.580</td>
<td>0.595</td>
<td>0.562</td>
<td>0.399</td>
<td>1.253</td>
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<tr>
<td>Middlesex</td>
<td>0.839</td>
<td>0.541</td>
<td>0.711</td>
<td>0.741</td>
<td>0.439</td>
<td>1.405</td>
<td>0.726</td>
<td>0.683</td>
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<tr>
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<td>0.944</td>
<td>0.758</td>
<td>0.623</td>
<td>0.997</td>
<td>0.302</td>
<td>1.613</td>
<td>0.720</td>
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</tr>
<tr>
<td>Peel</td>
<td>1.426</td>
<td>0.832</td>
<td>1.332</td>
<td>1.100</td>
<td>0.691</td>
<td>1.582</td>
<td>1.422</td>
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<tr>
<td>Waterloo</td>
<td>1.148</td>
<td>0.743</td>
<td>0.936</td>
<td>1.016</td>
<td>0.345</td>
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<td>0.985</td>
<td>1.380</td>
<td>1.339</td>
<td>0.422</td>
<td>1.738</td>
<td>0.769</td>
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</tr>
<tr>
<td>Northumberland</td>
<td>1.090</td>
<td>0.757</td>
<td>0.791</td>
<td>0.973</td>
<td>0.657</td>
<td>1.935</td>
<td>1.124</td>
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<tr>
<td>Lanark</td>
<td>—</td>
<td>0.308</td>
<td>0.441</td>
<td>0.802</td>
<td>0.842</td>
<td>1.844</td>
<td>1.352</td>
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</tr>
<tr>
<td>Ott.-Carleton</td>
<td>—</td>
<td>0.338</td>
<td>0.494</td>
<td>0.423</td>
<td>0.717</td>
<td>1.275</td>
<td>1.053</td>
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</tr>
</tbody>
</table>

example. Finally, the effect of nonsystematic risk is investigated.

Table 1 presents real, equally and proportionally weighted betas for 10 of the 50 agricultural-producing counties in Ontario. The reported counties span the province from west to east, with Essex being the most western county and Ottawa-Carleton the most eastern county. An immediate observation is that there are differences in enterprise beta coefficients within and between the two tables. These differences occur because the proportionally weighted beta coefficients measure the variance contribution of individual crops to the county portfolio rather than to a farm portfolio.

There are substantial regional differences in beta coefficients. For example, equally weighted barley betas range from 0.549 in Lanark County to 1.292 in Wellington. These numbers suggest that a $1.00 increase in expected revenues from the Lanark County portfolio implies a $0.55 increase in expected barley revenues, whereas a similar increase in Wellington implies an increase of $1.29. Similar differences are found for the proportionately weighted beta coefficients.

The results indicate that within the portfolio context, fodder corn, grain corn, and wheat have, in general, beta coefficients greater than 1.0. This implies that the revenues of these crops have proportionately more variance than the revenues for the portfolio as a whole. For example, in Table 1 expected per acre grain corn revenues in Wellington County increase (decrease) by $1.58/acre for every $1.00 increase (decrease) in revenues from the equally weighted portfolio. Barley, mixed grain, oats, and hay have, in contrast, a more stabilizing effect. For example, a $1.00/acre increase (decrease) in the equally weighted portfolio revenues for Middlesex County implies a per acre increase (decrease) in expected barley revenues of only $0.74/acre.

Systematic Risk

As previously defined, systematic risk reflects that portion of an enterprise’s total risk which contributes to the variance of the farm portfolio. Diversification cannot reduce this risk. The residual enterprise risk is defined as nonsystematic risk. This risk could potentially be reduced should alternative enterprises or opportunities present themselves.

Systematic and nonsystematic risk measures for Middlesex, Northumberland, and Ottawa-Carleton counties are calculated according to equation (7).
and presented in Table 2. These counties were selected since they are located at the approximate center of southwestern, central, and eastern Ontario, respectively, and are therefore representative of regional risks. For example, the standard deviations of the proportionally weighted reference portfolios are $79.77/acre, $48.93/acre, and $43.45/acre for Middlesex, Northumberland, and Ottawa-Carleton, respectively. The expected per acre gross revenues are also different: $362.62/acre, $254.40/acre, and $239.40/acre, respectively.

At the county level, systematic risk is dependent upon the risk profile of the weighted portfolio and, for this reason, differences in the beta coefficients are observed. For example, the proportionally weighted beta coefficients for grain corn are 1.405, 1.935, and 1.275, respectively, for each of the three counties, whereas betas for the equally weighted portfolio are 1.392, 1.950, and 1.814, respectively. Coincidental with this are differences in the absolute values of systematic and nonsystematic risk. For example, using an equally weighted portfolio, the systematic risk for grain corn is $108.20/acre, $108.05/acre, and $79.14/acre for Middlesex, Northumberland, and Ottawa-Carleton, respectively, but for the proportionally weighted portfolios, systematic risk values are $112.08/acre, $108.20/acre, $108.05/acre, and $79.14/acre for Middlesex, Northumberland, and Ottawa-Carleton, respectively.
reflect the relative contribution of an individual crop to the total variance of county crop portfolios. The betas are conditional on the relative proportions of crops grown as well as their variances. Both of these may differ by county. Recognition of heterogeneity in county risk profiles is, therefore, a necessary component of targeted extension activities. For example, according to the proportionately weighted betas in Table 2, it is useful to recognize that a major contribution to revenue risk in Middlesex County is white beans, and the revenue potential of the county varies accordingly.

At the farm level, equally weighted betas can be used to analyze portfolio risks. This can be done using the portfolio beta concept introduced in equation (10). Essentially, varying the portfolio weights will result in a portfolio beta greater than or less than 1.0. The change in a portfolio beta reflects marginal risks. A value greater (less) than 1.0 implies that the new portfolio is more (less) risky than the reference (equally weighted) portfolio. For example, suppose that an individual farmer in Middlesex County faced, without restriction, a crop-opportunity set described in Table 2. Selecting equal weights of each crop would result in an expected gross revenue of $395.20/acre and a systematic risk of $77.73/acre. Alternatively, the farmer could select an equally proportioned portfolio of wheat, grain corn, and white beans with a portfolio beta of 1.69 ([.879 + 1.392 + 2.801]/3) with an expected gross revenue of $395.20/acre and a systematic risk of $131.36/acre. The risk of this portfolio, 0.738, is less than 1.0, indicating less risk than the reference portfolio; however, in order to reduce risk to $57.36/acre, expected gross revenues are reduced to $220.38/acre.

Nonsystematic Risk

The previous section served two purposes: first to illustrate how the single-index model can be used to compare the relative gross-revenue risk among enterprises, and second to extend the risk measure to a portfolio basis. Much of the analysis thus far has assumed that nonsystematic risk is inconsequential, yet in some cases it might be substantial. The purpose of this section is to investigate the extent to which ignoring nonsystematic risk may affect the portfolio-choice decision rule presented.

It is important to recognize that with the portfolio index chosen, expected portfolio revenues are the same regardless of whether or not total risk or systematic risk is used. This is because the expected value of the error term in the characteristic equation (5) is zero. The fact that the enterprise and portfolio probability distributions, with and without nonsystematic risk, are mean-preserving implies that any bias occurs in the tails of the probability distribution, and the cumulative distribution functions (CDF) will always intersect at the 50th percentile.

Monte Carlo simulations were used to investigate the distribution of outcomes in the tails of portfolio CDFs. Separate Monte Carlo draws were obtained with and without nonsystematic risk using the characteristic regression equation, with portfolio revenues being drawn from $\bar{R}_p \sim N(E[R_p], \sigma^2_p)$ and residual errors being drawn from $\varepsilon \sim N(0, \sigma^2_{ns})$, where $\sigma^2_{ns}$ is the nonsystematic variance and $\text{cov}(\bar{R}_p, \varepsilon) = 0$.

Comparing the CDFs obtained with and without nonsystematic risk indicates that excluding nonsystematic risk would not unduly affect the ranking of crops (portfolios) by first- or second-order stochastic dominance. In most cases, the maximum difference in either the upper or lower tails amounted to less than 4 percentage points on the cumulative probability scale (vertical axis of the CDF). For example, in Figure 1, the CDFs for a portfolio comprised only of white beans in Middlesex County are compared. The beta for white beans is 2.801, its standard deviation is $258.54/acre$, which is made up of $217.72/acre$ systematic risk and $139.43/acre$ nonsystematic risk (Table 2).4 The area between the distribution functions represents nonsystematic risk.

Discussion and Conclusions

The overall objective of this paper was to develop single-index model concepts into a workable risk-management framework. It was suggested that farmers and policy makers could use beta coefficients in their respective decision-making capacities. The empirical analyses were based on the major cash crops of Ontario and focused on a county-level analysis. Beta coefficients were estimated for each crop in each county, and for some representative counties, measures of systematic and nonsystematic risk were presented.

The concept of systematic and nonsystematic risks can be incorporated into the assessment of risk-management strategies at both the policy and farm level.
level. For policy makers, identifying regional risk profiles is important not only for farm extension applications, but in defining risk for farm policies. Proportionally weighted betas reflect the marginal risk of an individual enterprise to the total risk of a county portfolio. Proportionally weighted indices can, therefore, be used to target regionally based risk-management strategies and to establish regional risk profiles. One possibility of future research may be to devise an area revenue insurance policy congruent with the recent propositions on area-yield crop insurance proposed by Miranda.

At the farm level, the use of enterprise beta coefficients and the portfolio beta concept is markedly simpler than many other techniques used to assess risk in agriculture. For purposes of risk management, the equally weighted portfolio may prove fruitful. By varying portfolio weights, farmers can identify marginal risk increases or decreases relative to a portfolio with an index value of 1.0. The approach is a simplified one, which is consistent with the mean-variance rule. One of the greatest impediments to the acceptance of the single-index model for risk-management purposes is information. Information with regards to SIM concepts would necessarily evolve through extension efforts. The second information need is with respect to the empirical application of beta coefficient and county-risk measures. While some farmers have substantial knowledge of statistics, this cannot be said of farmers in general. Thus, empirical data would likely have to be provided through extension publications. The format of such publications, however, need not be extensive or overly complex.

References


