Invited Presentation

Recent Applications of Nonparametric Programming Methods

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Nonparametric techniques have recently come into vogue in agricultural economics: Applications abound in both consumer and producer models of the agricultural economy. Moreover, several distinct approaches to nonparametric analysis exist. There are nonparametric statistical techniques, semiparametric estimation techniques, nonparametric revealed-preference analysis of consumption data, and nonparametric analysis of production data. Both revealed-preference analysis and nonparametric analysis of production data rely on the basic fact, which provides the foundation for much of modern duality theory, that convex sets can be completely characterized by their supporting hyperplanes. This observation allows one to apply simple mathematical programming (in particular, linear programming) methods to analyze production and consumption data. My task today is to provide an overview of nonparametric programming approaches to production data. Thus, I will not address any of the other topics cited above. However, I would be remiss if I did not mention the close connection between these subject areas and what I intend to survey today. Moreover, one should also recognize that very closely related to the literature on nonparametric programming analysis of production data are the fields of estimation of efficiency frontier via statistical methods. (A useful survey here is Lovell and Schmidt).

Nonparametric programming methods are most widely used to evaluate productive efficiency at the firm level. The basic approach (due originally to Farrell, and later to Farrell and Fieldhouse, Hanoch and Rothschild, and Afriat) is to compare observations on individual firm practices to a best-practice technology constructed from the convex hull of the observations on all firms' production practices. Färe and Lovell showed that Farrell's approach is equivalent to the calculation of Shephard's input-distance function. These original contributions have been greatly extended by Färe et al. (1985), and Charnes, Cooper, and Rhodes. Charnes et al. refer to the techniques discussed below as "data envelopment analysis." Although cast in a somewhat different context, their approach is virtually equivalent mathematically to what is discussed below.

While much of the focus has been on relative productivity efficiency measurement, a new series of applications has emerged: the use of nonparametric programming methods to analyze the consequences of economic, regulatory, and contractual constraints on firm practices. I intend to use this paper to present an overview of some of the applications and further to suggest and formulate several novel applications of nonparametric programming methods.

The plan of the paper is as follows. The next section provides a brief overview of applications of the nonparametric programming approach to production problems: the Farrell method and its extension to nonconstant returns to scale are first discussed. Then a brief discussion of the general maximization problem associated with nonparametric programming methods is presented, with an emphasis on the dual interpretation of this general problem. A number of studies using these methods are briefly reviewed. The paper then turns to a consideration of alternative applications of nonparametric programming methods to firm-level problems. These include the use of nonparametric programming methods to investigate the effects of different economic environments, market regulations, market structures, and absence of markets upon individual firms.

Overview

Nonparametric programming techniques have previously been used mostly within a frontier framework to calculate the relative efficiency of individual firms. The fact that one even attempts to measure relative efficiency reflects a basic presumption that firms may operate at different efficiencies; that is, using the same input mix, two

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firms can produce different output levels. This raises an interesting conceptual point not usually addressed in neoclassical models of the firm. If firms operate at different efficiency levels, one must explain why they do so. Obvious explanations are (1) different firms in the same industry have different technologies or (2) different firms have the same technology but some firms use inputs inefficiently. The potential problem with the first explanation is that it seemingly violates one of the postulates of neoclassical theory, that of perfect information. In a world where information is perfect and symmetric, no one firm should have access to a technology that is not available to all firms. A similar problem arises with the second explanation: It violates the assumption of rationality. Why should a rational firm or individual throw away productive resources? These observations underscore an important point. Outcomes of nonparametric programming production studies always offer several possible alternative explanations. Thus, their robustness is always open to question. Of course, they are not unique in this aspect. Similar comments can be applied to virtually any empirical study in economics, even those usually thought to be "robust."

In his seminal paper, Farrell addressed these issues by decomposing efficiency into two components, technical and allocative efficiency, for a single-product constant-returns-to-scale technology. Farrell's approach is nonparametric in the sense that he uses a series of linear inequalities to construct the free-disposal, convex hull of the observed input-output ratios. Linear programming techniques are then used to construct efficiency measures. The properties that are imposed on the technology are convexity (perfect divisibility), input-free disposability, and free disposability of outputs. A nonparametric piecewise linear technology having these properties can be constructed from any data set on inputs and outputs. To illustrate the construction of such a technology, suppose there are $k = 1, \ldots, K$ producers using inputs $x^k \in R^n$ to produce outputs $y^k \in R^m$. The piecewise linear technology consistent with these data is given by

$$(1)\quad T = \{(x,y): x \geq \sum_{k=1}^{K} \lambda^k x^k,\ y \leq \sum_{k=1}^{K} \lambda^k y^k,\ \sum_{k=1}^{K} \lambda^k = 1,\ \lambda^k > 0 \text{ for all } k\},$$

where λ is an intensity vector. Formally, T is the convex hull of the observed data and input-output pairs that can be inferred from free disposability of inputs and outputs. The construction of T is illustrated pictorially in Figure 1. Suppose $K = 4$, with the observations labelled (a, b, c, d). The intensity vector λ ensures that all convex combinations of the observed inputs and outputs belong to T. Thus, any "average" of two observed production plans must belong to T. Hence, the presumption of divisibility of the technology is inherent to the Farrell approach. Moreover, the facets connecting the "extreme" points (a, b, c) also must belong to T. The inequality, $x \geq \Sigma\lambda^k x^k$, allows for inclusion of all horizontal extensions of the data, reflecting free disposability of the input. Pictorially this is reflected by the inclusion of all points directly to the right of c in the technology set. Likewise, the inequality $y \leq \Sigma\lambda^k y^k$ allows for vertical extensions, reflecting free disposability of the output. Pictorially, this is reflected by the inclusion of all points directly below a in the technology set. The fact that the interior points like d in Figure 1 do not contribute to the construction of T indicates that the resulting technology is best interpreted as a "best-practice" or frontier technology. For any input bundle, it gives the best output combination possible that is consistent with real-world observations and perfect divisibility.

Two useful scalar-valued representations of the technology described by T have been defined by Shephard. These are the input- and output-distance functions. These functions generalize the notion of a production function to encompass multi-output technologies. The output-distance function is defined by

$$(2)\quad \Gamma(x,y) = \text{Min}\ \{\Lambda:(x, y/\Lambda)\in T\},$$

and the input-distance function is defined by

$$(3)\quad \Omega(x,y) = \text{Max}\ \{\Lambda:(x/\Lambda, y)\in T\}.$$

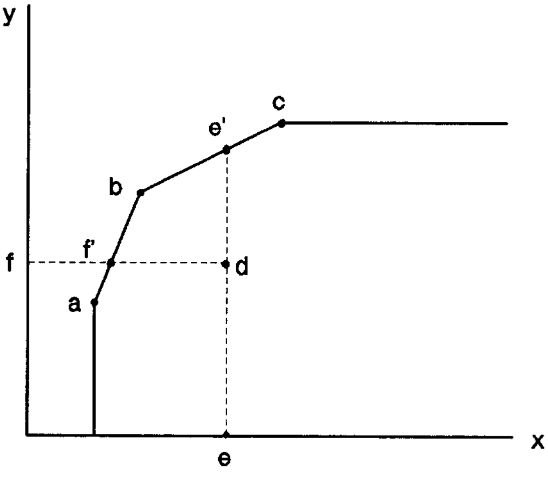

Figure 1. Production Possibility Set

The output-distance function is positively linearly homogeneous, convex, and nondecreasing in y, and quasi-convex and nonincreasing in x. The input-distance function is positively linearly homogeneous, concave and nonincreasing in x, and quasi-concave in y. To illustrate the calculation of the distance functions, consider point d in Figure 1. The output-distance function associated with that input-output combination is given by the vertical distance ed/ee'. The input-distance function is given by the horizontal distance $fd/f'd$. Notice that for points (a, b, c), both the input- and the output-distance functions equal one. However, for points on the horizontal extension emanating from c, the output-distance function is always one, while the input-distance function is always greater than one. Similarly, for points on the frontier of T below a, the input-distance function is always one, while the output-distance function is always less than one.

Obviously, therefore, the input- and output-distance functions offer natural measures of the relative efficiency of the input combination for a given level of output and the relative efficiency of the output combination for a given bundle of inputs. Farrell's measure of technical efficiency corresponds exactly to the input-distance function.

Both the input- and output-distance functions for a given observation relative to the frontier technology T can be calculated as the solution to a simple linear program. The input-distance function is calculated as

$$\text{(4)} \quad \Gamma(x,y) = \text{Max } \Gamma \text{ s.t. } \{x/\Gamma \geq \sum_{k=1}^{K} \lambda^k x^k, \quad y \leq \sum_{k=1}^{K} \lambda^k y^k, \; \lambda^k \in \Re_+ \quad (k = 1, \ldots, K), \; \sum_{k=1}^{K} \lambda^k = 1\},$$

while the output-distance function is calculated by

$$\text{(5)} \quad \Omega(x,y) = \text{Min } \Omega \text{ s.t. } \{x \geq \sum_{k=1}^{K} \lambda^k x^k, \quad y/\Omega \leq \sum_{k=1}^{K} \lambda^k y^k, \; \lambda^k \in \Re_+ \quad (k = 1, \ldots, K), \; \sum_{k=1}^{K} \lambda^k = 1\}.$$

This representation of the calculation of the input- and output-distance functions relies solely on the primal representation of the technology. Chambers, however, has shown that the above has an interesting dual formulation. To see this, consider the dual linear program associated with the calculation of the input-distance function:

$$\text{(6)} \quad \text{Max } \{py + \phi\} \text{ s.t. } \sum wx \leq 1, \quad py^k - wx^k + \phi \leq 0, \quad \text{for all } k = 1, \ldots, K.$$

Here ϕ is an unrestricted dual scalar, and $w \in R^n$ and $p \in R^m$ are non-negative dual vectors that can be thought of as shadow input prices and shadow output prices. With some slight manipulation, it can be shown that this dual problem is equivalent to finding the input and output price vectors that make input-output combinations located on the boundary of T profit maximizing relative to the given technology. A similar interpretation is available for the calculation of the output-distance function.

The recognition that the dual linear program to the calculation of the input- and output-distance functions is itself a shadow profit-maximization problem suggests that the frontier technology can be usefully applied when more than just observations on inputs and outputs are available. For example, if one has data on input prices, input levels, and outputs, the presumption of cost minimization (profit maximization for a fixed output vector) can be used to construct a representation of the cost function dual to the frontier technology T. Inefficiency measures can then be based upon monetary units. For example, continuing with the cost-minimization example, a measure of a given input-output bundle's inefficiency can be obtained by comparing the observed cost of producing that output bundle with the minimum cost of the output bundle relative to the frontier technology T. The cost difference then offers a cost-based measure of relative inefficiency. Similar calculations could be made when output prices, output levels, and input levels are available (revenue function for T), and when both input and output prices, and input and output levels are available (profit function for T). Again, however, the robustness of the approach depends upon the presumption of profit-maximizing behavior. In what follows, we shall see that this general philosophy proves useful in the construction of various tests based on the nonparametric programming approach to production data.

Economic and Regulatory Constraints

To motivate the general topic of how the effects of economic constraints on individual decision-makers can be evaluated using nonparametric programming techniques, one might consider the topic of expenditure constraints on agricultural producers. The notion that agricultural producers face constraints on the amount that they can use to finance input expenditures is not new. (Ferguson provided a treatment of neoclassical theory of expenditure-contrained production for a single output, and Schultz explicitly recognized the importance of credit rationing in agriculture.) However, the analysis of such constraints was rigorously formalized by Lee and Chambers, who introduced the notion of an expenditure-constrained profit function and derived its properties. Färe, Grosskopf, and Lee later used this theoretical framework to examine the existence of expenditure constraints using nonparametric programming techniques. Recall the piecewise linear representation of the production technology T, described above. The possible existence of expenditure constraints can be introduced into the model by including yet a further constraint—that input expenditures not exceed a given level of cost:

$$wx \leq E, \tag{7}$$

where E is the predetermined level of expenditure and w is a vector of input prices.

A nonparametric representation of the expenditure-constrained profit function can be expressed given data on (x, y, w, p, E), where p is the output price:

$$P(w, p, E) = \text{Max } \{py - wx\colon wx \leq E, (x, y) \in T\}. \tag{8}$$

The traditional profit-maximization problem can be calculated from the above problem (8), ignoring the expenditure constraint. If the expenditure constraint in (8) were nonbinding, the solution to (8) is identical to the solution of the traditional profit-maximization problem. This situation is illustrated in Figure 2. The expenditure constraint is depicted as a vertical line, EE'. The effect of the vertical expenditure constraint is to reduce the feasible production set to the shaded area in Figure 2. If the expenditure constraint is binding, the profit-maximizing input/output choice will be at point E^o. Thus, one can examine whether the expenditure constraint binds by comparing the solution of the two problems. The shadow price of the expenditure constraint, that is, the loss in profit due to the expenditure constraint, can be calculated as the difference between constrained and unconstrained

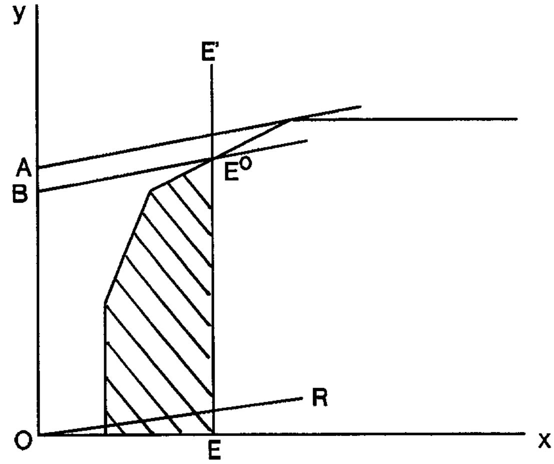

Figure 2. Expenditure-Constrained Profit Maximization

profit. Suppose relative input price normalized by the output price is given by the slope of the ray, OR, in Figure 2, and the shadow price of the expenditure constraint in output-numeraire units is given by the vertical distance AB. The data are taken to be consistent with the maintained hypothesis if observed profit equals the optimal value of the unconstrained objective function at every observation.

The expenditure-constrained profit-maximization example again illustrates a point raised in the previous section. One can always construct a convex technology that is consistent with a given set of production data by constructing the free-disposal convex hull of the input and output combinations. Taking this technology as given, one can then test various behavioral hypotheses (such as constraints on input expenditures) by comparing the solution to the constrained-optimization problem, subject to the technological restrictions imposed by this representation of the technology. However, this procedure is not very powerful in detecting departures from the maintained hypothesis (e.g., profit maximization) that emerge from considerations other than whether the constraints imposed are effective or not. For example, in the expenditure-constrained profit-maximization problem, a failure for the data to exhibit behavior consistent with the traditional profit-maximization model could be due either to allocative inefficiency by the firm or to the existence of expenditure constraints. Thus, different explanations for the data must always be confounded with one another.

The fact that expenditure-constrained profit maximization has been analyzed using nonparametric programming methods implies that the same basic approach can be used to analyze other constraints to simple profit maximization: These might include rate-of-return regulation, quantitative restrictions to international trade, and their domestic implications.

For example, if firm profits are restricted to be no more than α percent of variable costs, the rate-of-return problem can be formulated as

$$(9) \quad P(w, p, \alpha) = \text{Max}\,\{py - wx: py \leq (\alpha + 1)wx,\ (x, y) \in T\}.$$

Formally, therefore, the rate-of-return problem is virtually identical to the expenditure-constrained profit-maximization problem in that it involves the addition of a single linear inequality constraint to the original profit-maximization problem. To determine whether the rate-of-return regulation actually impinges upon firm behavior, one can pursue an approach similar to that described for the expenditure-constrained maximization problem: maximize profit subject to the linear inequalities that describe the free-disposal convex hull of the data points (i.e., the frontier technology), and compare this solution with what is obtained from maximization in the presence of the constraint. Pictorially, this is represented in Figure 3 by noting that the presence of rate-of-return regulation reduces the feasible set from the free-disposal convex hull of the points (a, b, c) to the shaded area that represents the intersection of the free-disposal convex hull of (a, b, c), with the halfspace defined by $y \leq \{\alpha + 1)w/p\}x$. The shadow value of the rate-of-return regulation is now given by the vertical distance AB. Alternatively, the dual shadow value to the inequality $py \leq (\alpha + 1)wx$ in the

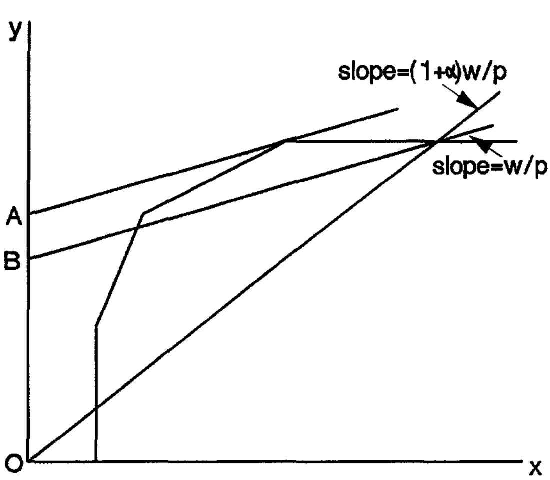

Figure 3. Profit Maximization under the Rate of Return Regulation

linear program can be checked to see whether the rate-of-return constraint is actually binding. As with the expenditure-constrained problem, however, such tests are not very robust because a failure for the data to exhibit behavior consistent with the maintained hypothesis could be due to other than the rate-of-return regulation.

Contract Structure

Contracts are important elements of both developed and developing economies. Why a certain contractual form emerges from a certain type of economy or whether some contracts are inherently inefficient has been the subject of much discussion. For example, the Marshallian efficiency of share contracts has been the focus of economic controversy since the days of Adam Smith. It is often argued that share contracts provide tenants with inappropriate incentives. Any inefficiency arising from share contracts is related to price inefficiency, often called "allocative inefficiency." Marshall (and many others who followed) argued that because output sharing does not provide the share tenant with the full marginal return from his or her input utilization, the tenant will have a tendency to underutilize inputs (presuming inputs are normal). Johnson, and later Cheung, countered that share relations need not be inefficient if other contract provisions correct for this perceived inefficiency. (One obvious way is to have full variable cost sharing as well as output sharing). Since the late 1960s, the Johnson-Cheung hypothesis has been the subject of a number of empirical tests. At best, the results are mixed. Virtually all of these tests have been econometric in nature. However, a nonparametric approach is apparent.

Consider a simple output-sharing contract where a tenant receives α percent of his or her crop. As before, one again can construct a convex technology, specify a linear objective function consistent with output sharing, and identify if there is any observation for which the resulting optimal solution value is greater than observed profit. If there is, then the hypothesis that the data were generated by a competitive profit-maximizing entity is not supported by the nonparametric test. Lee and Somwaru used precisely this approach to examine the alleged Marshallian inefficiency of share relations. To understand their approach, consider the following modification of the optimization programs considered above.

Consider a tenant farmer operating on share-rented land, L, using a production-possibility set $T = \{(y, L, x): (L, x) \text{ can produce } y\}$, where y is a vector of outputs and $x \in R^n_+$ is a vector of purchased inputs. Assuming y is a scalar, the landlord specifies the tenant's output share α, $0 < \alpha < 1$, and, to recognize the possibility of cost sharing, the tenant's cost shares for each input, $\beta = (\beta_1, \ldots, \beta_i, \ldots, \beta_N)$, $0 < \beta_i \leq 1$, $\forall i$. Under the Marshallian hypothesis, the tenant chooses x and y to maximize his or her own share of profit:

$$(10)\quad \underset{x,y}{\mathrm{Max}}\ \{\alpha py - \sum_{i=1}^{N} \beta_i w_i x_i\text{: } (y,x,L) \in T\}.$$

The "effective" output and input prices faced by the tenant are αp and βw, rather than p and w. If relative input prices are given by the ray OR in Figure 4, these new "effective" prices are reflected by the slope of OR' in Figure 4. So long as $\alpha < \beta$, one expects there to be an insufficient incentive for input use because the tenant equates the marginal value product of the ith input to $\beta_i w_i/\alpha$, instead of w_i. In other words, price-related allocative inefficiency may exist. Pictorially, this can be illustrated by Figure 4 where the profit-maximizing solution for the original prices is at point A, while the profit-maximizing solution for the effective prices is at B. As drawn, the tenant underutilizes inputs at point B.

According to the Johnson-Cheung monitoring hypothesis, efficient input use, which maximizes $\{py - wx\}$, can be stipulated contractually for share contracts and enforced through appropriate landlord monitoring. If the Johnson-Cheung hypothesis is true, share tenancy can then be as price-efficient as cash renting or owner farming. To examine which hypothesis describes reality, one can

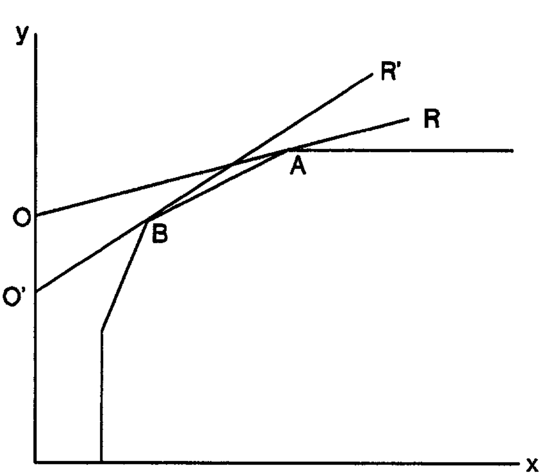

Figure 4. Profit Maximization under a Share Contract

look for the presence of allocative inefficiency in share-tenant farms relative to that of the cash-tenant or owner farmers. If share-rented farms are found to be allocatively inefficient relative to the other tenancy forms, we may conjecture that share tenancy does indeed create allocative inefficiency. Using data for California rice farmers, Lee and Somwaru identified, using the nonparametric programming technology detailed above, inefficiencies in input use for farmers producing under output sharing. Thus, their results support the Marshallian view of share relations rather than the Johnson-Cheung view (at least for this data set).

Shadow-Value Calculation

Another novel application of nonparametric programming models of production relations involves measurement of goods and bads not exchanged through organized markets. For example, Färe et al. (1989) recently measured the productivity of firms producing multiple outputs, some of which were undesirable pollution, using a modification of the nonparametric approach to efficiency measurement. Formally, they relaxed the assumption of strong output free disposability to allow for the fact that undesirable outputs may not be freely disposable. In other words, disposing of undesirable outputs, such as controlling pollution, may incur a positive marginal cost to the producer.

Although their study focuses on productivity measurement rather than the calculation of the shadow values of bads, they provide a nonparametric framework that incorporates pollution as a joint output into conventional production analysis. Writing the output vector $y = (v, w)$, where the subvector v denotes the goods and the subvector w denotes the bads, they used weak disposability of y while maintaining strong disposability of goods, v.[1]

The weak disposal production possibility set T satisfying the above properties is

$$(11)\ T = \{(x,v,w)\text{: } x \geq \sum_{k=1}^{K} \lambda^k x^k,\ v \leq \sum_{k=1}^{K} \lambda^k v^k,$$
$$w = \sum_{k=1}^{K} \lambda^k w^k,\ \lambda^k > 0 \text{ for all } k\}.$$

[1] Outputs are weakly disposable if $y \in P(x)$ implies $\alpha y \in P(x)$ for all $0 \leq \alpha \leq 1$, and strongly disposable if $y' \leq y \in P(x)$ implies $y' \in P(x)$, where $P(x)$ is the set of all output vectors producible by the input vector x.

Figure 5 illustrates the construction of the above T in output space. $P(x)$ in Figure 5 denotes the set of feasible outputs, given x. The equality $w = \Sigma\lambda^k w^k$ creates a region of the backward-bending curve, implying that undesirable outputs are not freely disposable. (Note that the conventional treatment of free disposal output is described by $P^c(x)$.) In fact, in their subsequent study, Färe et al. (forthcoming) derived the shadow value of pollution based on the notion of output weak disposability using a parametric programming technique. However, the nonparametric approach of shadow-value calculation should be apparent. As alluded to earlier, once the primal technology is constructed, we can derive the shadow value of pollution from the dual formulation of the primal technology.

The same basic approach can be used to calculate the shadow value of other nonmarket goods. A prime example from agriculture is the shadow value of grazing rights on federally owned land.

Some Novel Applications

Over the last decade, one of the fastest-growing areas in the theoretical literature on international trade has been on effects of external economies of scale on trade patterns and international comparative advantage. As summarized by Helpman, and later Markusen and Schweinberger, the basic idea is that at the firm level, there exist single product production functions of the form (each firm has the same function)

$$(12) \qquad y^i = f(x^i, \Sigma y^i),$$

where y^i denotes the ith firm's output and x^i denotes the ith firm's input vector. Hence, a firm's

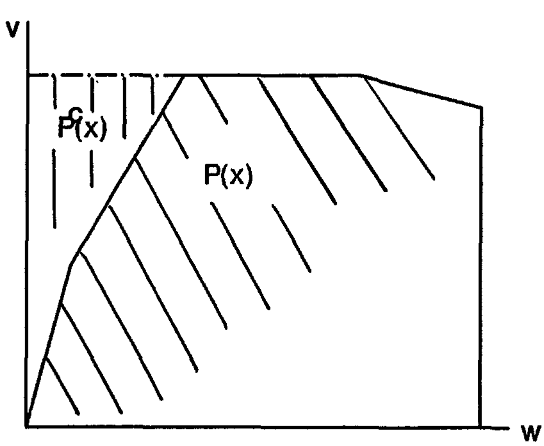

Figure 5. Production of Pollution as a Joint Output

output depends not only upon its inputs, but also upon industry output because of the presence of what are referred to as "external economies of scale." Firms are assumed to maximize myopically, taking one industry output as given; that is, they perceive themselves as so small relative to the overall market that they cannot affect total output. Thus, the firm-level cost-minimization problem becomes

$$(13) \quad c(w, y^i, \Sigma y^i) = \text{Min}\,\{wx^i\text{: } y^i = f(x^i, \Sigma y^i)\}.$$

Markusen and Schweinberger have developed the properties of a general version of this type of cost function with the presence of external economies of scale. However, the basic presumption of external economies has not yet been checked by nonparametric means, but an obvious procedure for doing so exists; that is, simply define a scalar fixed input, $Y = \Sigma y^i$, common to all firms and construct the free-disposal convex hull of the technology set characterized by this production function using cross-sectional data on firms from a given industry. Then, the marginal product of the industry output on firm costs and production can be evaluated by using sensitivity analysis.

References

Afriat, S.N. "Efficiency Estimation of Production Functions." *International Economic Review* 13(1972):568–98.

Aigner, D.J., and S.F. Chu. "On Estimating the Industry Production Function." *American Economic Review* 58, no. 4 (September 1968):826–39.

Chambers, R.G. "Recent Developments in Production Economics." Paper presented at the Australian Economics Congress, Canberra, August 1988.

Charnes, A., W.W. Cooper, and E. Rhodes. "Measuring Efficiency of Decision Making Units." *European Journal of Operational Research* 2(1978):429–44.

Cheung, S.N.S. *The Theory of Share Tenancy*. Chicago: University of Chicago Press, 1969.

Färe, R., S. Grosskopf, and C.A.K. Lovell. *The Measurement of Efficiency of Production*. Boston: Kluwer-Nijhoff, 1985.

Färe, R., S. Grosskopf, C.A.K. Lovell, and C. Pasurka. "Multilateral Productivity Comparisons When Some Outputs Are Undesirable: A Nonparametric Approach." *Review of Economics and Statistics* 71, no. 1 (1989):90–98.

Färe, R., S. Grosskopf, C.A.K. Lovell, and S. Yaisawarng. "Derivation of Virtual Prices for Undesirable Outputs: A Distance Function Approach." *Review of Economic and Statistics*. Forthcoming.

Färe, R., S. Grosskopf, and H. Lee. "A Nonparametric Approach to Expenditure-Constrained Profit Maximization." *American Journal of Agricultural Economics* 12, no. 3 (August 1990):574–81.

Farrell, M.J. "Measurement of Productive Efficiency." *Jour-*

nal of the Royal Statistical Society, Series A, General (1957):253–81.

Farrell, M.J., and M. Fieldhouse. "Estimating Efficient Production Functions under Increasing Returns to Scale." *Journal of the Royal Statistical Society* Series A-125, no. 2 (1962):252–67.

Ferguson, C.E. *The Neoclassical Theory of Production and Distribution*. Cambridge: Cambridge University Press, 1971.

Hanoch, H., and M. Rothschild. "Testing the Assumptions of Production Theory: A Nonparametric Approach." *Journal of Political Economy* 80(1972):256–75.

Helpman, E. "Increasing Returns, Imperfect Markets and Trade Theory." In *Handbook of International Economics*, edited by R. Jones and P. Kenen. Amsterdam: North-Holland.

Johnson, D.G. "Resource Allocation Under Share Contracts." *Journal of Political Economy* 58, no. 2 (April 1950):111–23.

Lee, H., and R. Chambers. "Expenditure Constraints and Profit Maximization in U.S. Agriculture." *American Journal of Agricultural Economics* 68, no. 4 (November 1986):857–65.

Lee, Hyunok, and Agapi Somwaru. "The Efficiency of Share Tenancy." In *The Measurement of Productive Efficiency: Techniques and Applications*, edited by K. Lovell, S. Schmidt, and H. Fried. Oxford University Press. Forthcoming.

Lovell, C.A.K., and P. Schmidt. "A Comparison of Alternative Approaches to the Measurement of Productive Efficiency." In *Applications of Modern Production Theory: Efficiency and Productivity*, edited by A. Dogramaci and R. Färe, 3–32. Boston: Kluwer Academic Publishers, 1988.

Markusen, J.R., and A.G. Schweinberger. "The Positive Theory of Production Externalities Under Perfect Competition." *Journal of International Economics* 29(1990).

Schultz, Theodore W. *The Economic Organization of Agriculture*. New York: McGraw-Hill Book Co., 1953.

Shephard, Ronald W. *Indirect Production Functions*. Meisenheim Am Glan: Verlag Anton Hain, 1974.

Timmer, C.P. "Using a Probabilistic Frontier Production Function to Measure Technical Efficiency." *Journal of Political Economy* 79(1971):776–94.

Varian, H.R. "The Nonparametric Approach to Production Analysis." *Econometrica* 52(1984):579–97.