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A water economy model

by

Yacov Tsur

P.O. Box 12, Rehovot 76100, Israel

ת.ד. 12, רחובות 76100

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Yacov Tsur◇

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Abstract

We study water management in the context of a prototypical water economy. The optimal water policy is specified in terms of water allocations from each source to each user sector at each point of time and the investment in capital infrastructure needed to carry out these allocations. We find that the optimal policy evolves along three stages: a transition stage, where the water capital stocks are brought as rapidly as possible to their respective turnpikes (singular paths); a singular stage, where the water capital stocks evolve along their turnpikes while the natural water stock advances toward a steady state; and a steady-state stage. Optimal regulation by means of water pricing and quotas are discussed. Extensions to non-stationary situations involving growing water scarcity (due to population and climate change trends) and improved desalination technology are considered.

Keywords: Water resources; intertemporal management, scarcity, optimal policy, turnpike property.

JEL classification: C61, Q25, Q28

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◇Department of Environmental Economics and Management and the Center for Agricultural Economic Research, The Hebrew University of Jerusalem, POB 12, Rehovot 7610001, Israel (yacov.tsur@mail.huji.ac.il).

1 Introduction

We study the salient features of water management in the context of a prototypical water economy, consisting of the main sectors of water users and water sources. The user sectors are domestic, agriculture, industry and environment; the water sources are natural, recycling and desalination. The sectors and sources are elaborately entwined via physical (equipment, infrastructure) and social (institutions, norms, laws) capital, and the term water economy refers to the whole setup.

Water economies vary with respect to their hydrology and climate (water sources, precipitation, evapotranspiration) as well as social situation (demography, water rights and laws, institutions), and these features affect the range of feasible policies (see the diversed examples in Saleth and Dinar 2004, Tsur et al. 2004). Without committing to a particular setting, we characterize the optimal water policy in terms of the water allocation from the different sources to the different suer sectors and the investments in the capital stocks needed to carry out these allocations over time. We then discuss possible regulation schemes to implement desirable policies.

The primary source of water is nature (rainfall, lakes, stream flows, aquifers). In regions where the (sustainable) supply of natural water suffices to meet human and environmental needs, water is not a scarce resource and its management may not be high on the priority list. Such regions become fewer over time due to demographic and climatic trends. In many populated regions, water scarcity has become critical (see Dinar and Tsur 2015), stressing the need for proper management. There are two basic approaches for water management: demand management and supply management. The former seeks to do more with a given supply of water; the latter seeks to increase the water supply.

Two sources of produced water can be added to natural sources: recycling and desalination. Recycled water is the outcome of collecting and treating domestic and industrial sewage. As such, its supply is determined by the allocation of water to these sectors. Sewage treatment is required primarily due to health and environmental considerations, disregarding whether the treated water is reused later on. The level of treatment (secondary, tertiary) determines the range of feasible uses of the recycled water. These considerations bear important implications for the allocation of water in general as well as for the level of treatment and who should pay for the different stages of the recycling process. The model developed herein addresses these considerations.

Desalinization is, for all practical purposes, an unlimited source of water, hence can be considered as a backstop technology. However, at the current state of technology, it is an expensive source. This raises the issues of when to begin desalination (if at all) and the extent of desalination over time. The framework developed herein addresses these concerns.

We build on the framework of Tsur (2009) and extend it in a number of ways. While Tsur (2009) simplified the dynamic policy aspects by considering steady states, the water policy characterized herein is fully intertemporal, covering both the water allocation from each source to each user sector at each point of time and the investment in capital stocks (equipments, infrastructure) needed to carry out these allocations. We show that the optimal water policy possesses a turnpike property, in that each of the water capital stocks is brought as rapidly as possible to its respective turnpike and is kept along this path thereafter. The turnpikes (also called singularly trajectories) depend on the natural water stock and are shown to converge to a steady state. The optimal water policy, thus, is shown to evolve along three stages: a transition stage, where the water infrastructure is built as rapidly as possible; a turnpike stage, where the water infrastructure evolves along well-specified turnpikes; and a steady-state stage.

We extend the canonical (stationary) model to account for growing (non-stationary) water demands and allow for (exogenous) technical change in desalination technology. We also show (in the appendix) how the model can be extended to accommodate arbitrary number of user sectors and water sources, thereby allowing to refine the the water economy as needed.

The next section specifies the stylized water economy and defines water policies. The optimal policy is derived in Section 3. Section 4 discusses regulation policies that implement the optimal policy. Extensions to non-stationary situations are presented in Section 5 and section 6 concludes. The appendix presents extensions to more general cost structures and to arbitrary number of sources and sectors.

2 A stylized water economy

The stationary water economy specified in Tsur (2009) provides a convenient starting point. On the whole, water can be derived from three main sources and is allocated to four main user sectors. The sources are nature (rainfall, aquifers, lakes, reservoirs, stream flows), indexed n, recycling facilities, indexed r, and desalination plants, indexed d. The user sectors include domestic, indexed D, agriculture (irrigation), indexed A, industry, indexed I, and environment (instream, river restoration), indexed E (the case of many water sources and user sectors is outlined in appendix B).

Let $q_{ij}(t)$ denote the supply of water from source $i = n, r, d$, to sector $j = A, D, I, E$, in year t. The annual water supply from source i is

$$q_{i\circ}(t) = \sum_{j=A,D,I,E} q_{ij}(t),\ i = n, d, r, \tag{2.1a}$$

and the annual allocation to sector j is

$$q_{\circ j}(t) = \sum_{i=n,r,d} q_{ij}(t),\ j = D, A, I, E. \tag{2.1b}$$

2.1 Water sources

We discuss each of the water sources in turn.

Natural sources: Natural water is derived from limited (finite) water stocks, such as aquifers, lakes and stream flows. Accordingly, let $Q(t)$ represent the stock of natural water at time (year) t, which evolves in time according to

$$\dot{Q}(t) = R(Q(t)) - q_{n\circ}(t), \tag{2.2}$$

where $R(\cdot)$ is a recharge function. The recharge function is defined over $[0, \bar{Q}]$ and assumed decreasing and concave. The upper bound $\bar{Q}$ satisfies $R(\bar{Q}) = \underline{R}$, where $\underline{R} \geq 0$ can be interpreted as average precipitation. The zero lower bound,

$$Q(t) \geq 0, \tag{2.3}$$

is a normalization.[1] The allocation of natural water requires equipment and infrastructure capital (pumps, pipelines, filters), denoted $K_n(t)$.

[1] If irrigation and environmental water contribute to the recharge of underlying aquifers, the recharge function takes the form $R(Q, q_{\circ A}, q_{\circ E})$, where R decreases in Q and increases in both $q_{\circ A}$ and $q_{\circ E}$. In the interest of simplicity, the latter effects are ignored.

Recycling: Recycled water is the outcome of collecting and treating residential and industrial sewage. The annual flow of treated sewage equals

$$q_{s\circ}(t) = \beta(q_{\circ D}(t) + q_{\circ I}(t)), \tag{2.4}$$

where $\beta \leq 1$ accounts for water loss during sewage collection and treatment.[2] The capital needed to collect and treat residential and industrial sewage is denoted $K_s(t)$.

The treated sewage, $q_{s\circ}(t)$, can be disposed of or reused, where the latter constitutes the supply of recycled water $q_{r\circ}(t)$. Thus,

$$q_{r\circ}(t) \leq q_{s\circ}(t). \tag{2.5}$$

Dumping the treated sewage is assumed costless, but reusing it requires capital (pipelines, pumps) to convey the treated water from the recycling plants to potential users. This recycled capital is denoted $K_r(t)$.

The distinction between $q_{s\circ}(t)$ and $q_{r\circ}(t)$, and the associated capital stocks $K_s(t)$ and $K_r(t)$, is needed because sewage collection and treatment, on the one hand, and allocating the treated water to potential users, on the other hand, are two separate activities. The former is (often) required by health and environmental regulation, disregarding whether the treated water is reused later on. Reusing the treated water, on the other hand, is a policy decision that depends on the cost of conveying the recycled water from the treatment facilities to potential users and on the demand for the recycled water. The treatment level (secondary, tertiary) entails restrictions on potential uses of the recycled water. For example, secondary-treated water may not be allowed to irrigate certain crops and health regulations may prohibit the allocation of

[2]Under current technology and practice, $\beta \approx 0.65$ (see Tsur 2015).

any recycled water to households, i.e.,

$$q_{rD}(t) = 0. \tag{2.6}$$

Desalination: The supply of desalinated water at time t is restricted only by the capacity of existing desalination plants, i.e., by the available desalination capital, denoted $K_d(t)$.

2.2 Supply cost

The cost of water supply includes variable and fixed costs. The former entails costs of variable inputs, such as labor, energy and material; the latter includes mainly the cost of capital. Both of these components vary spatially and temporally (see examples in Renzetti 1999, Harou et al. 2009, Allen et al. 2014). We discuss below the role of these cost components in the present framework.

Capital (fixed) cost: Source i's annual supply at time t, $q_{i\circ}(t)$, is restricted by source i's capital stock, $K_i(t)$, according to

$$q_{i\circ}(t) \leq \gamma_i K_i(t),\ i = n, s, r, d, \tag{2.7}$$

where γ_i is capital utilization parameter (the K_i's are defined above). The capital stocks evolve in time according to

$$\dot{K}_i(t) = x_i(t) - \delta_i K_i(t),\ i = n, s, r, d, \tag{2.8}$$

where $x_i(t)$ represents investment and δ_i is a depreciation rate. We assume that $0 \leq x_i(t) \leq \bar{x}_i$, where $\bar{x}_i$ is a finite upper bound on investment in K_i.

The capital cost of water supply from source i comes from the investments $x_i(t)$ and the (shadow cost of the) supply restrictions (2.7).

Variable costs: The variable costs associated with the supply of $q_{i\circ}(t)$ is represented by the function $C_i(q_{i\circ})$, $i = n, s, r, d$, assumed increasing and convex.[3] These variable cost functions account for the costs of variable inputs such as temporary labor, energy and material.

2.3 Annual allocation

An annual water allocation is represented by

$$q(t) = \{q_{ij}(t),\ i = n, d, r,\ j = D, A, I, E\}.$$

An allocation is feasible if it satisfies (2.5), (2.7), $q(t) \geq 0$ and possibly other restrictions such as (2.6). The sectoral allocations and supplies from each source associated with $q(t)$ are specified in (2.1).

2.4 Sectoral demands and surpluses

The annual (inverse) demand for water of sector j is denoted $D_j(\cdot)$, $j = D, A, I, E$. This curve measures the quantity of water demanded at any water price and can be interpreted as the price sector j's users are willing to pay for the last unit of water.[4]

The annual gross surplus of sector j generated by $q_{\circ j}$ (not including the cost of water supply) is the area underneath the demand curve to the left of

[3] $C_n(\cdot)$ may also depend on the natural water stock Q, in which case it is non-increasing in Q as pumping and extraction costs usually decrease with Q.

[4] There is a large literature on sectoral water demands. Examples of agricultural water demand include Just et al. (1983), Moore et al. (1994), Howitt (1995), Mundlak (2001), Tsur et al. (2004), Schoengold et al. (2006), Scheierling et al. (2006); examples of urban and industrial demands include Baumann et al. (1997), Renzetti (2002, 2015), Worthington and Hoffmann (2006), Olmstead et al. (2007), House-Peters and Chang (2011), Baerenklau et al. (2014), Smith and Zhao (2015); examples of environmental water demand include Dudley and Scott (1997), Loomis et al. (2000), Pimentel et al. (2004), Thiene and Tsur (2013), Koundouri and Davila (2015).

$q_{\circ j}$:

$$B_j(q_{\circ j}) = \int_0^{q_{\circ j}} D_j(v)dv, \; j = D, A, I, E. \tag{2.9}$$

2.5 Annual Benefit

The annual benefit generated by an allocation $q(t)$, not including the cost of capital, is the total surplus minus the cost of supply:

$$B(Q(t), q(t)) = \sum_{j=A,D,I,E} B_j(q_{\circ j}(t)) - C_n(Q(t), q_{n\circ}(t)) - \sum_{i=s,r,d} C_i(q_{i\circ}(t)). \tag{2.10}$$

2.6 Water policy and welfare

A water policy consists of $q(t) = \{q_{ij}(t), \, i = n, r, d, \, j = A, D, I, E\}$ and $x(t) = \{x_i(t), \, i = n, s, r, d\}$, $t \geq 0$, where $x(t)$ determines $K(t) = \{K_i(t), \, i = n, s, r, d\}$ via (2.8) and $q_{n\circ}(t) = \sum_j q_{nj}(t)$ determines $Q(t)$ via (2.2). A water policy generates the payoff (welfare)

$$\int_0^\infty \left(B(Q(t), q(t)) - \sum_{i=n,s,r,d} x_i(t) \right) e^{-\rho t} dt, \tag{2.11}$$

where ρ is the time rate of discount.

2.7 Summary

The figure below provides a graphical overview of the water economy described above. Note in particular the links leading to the supply of recycled water: from domestic and industrial allocation to sewage treatment plants and from there to recycled water users or to natural outlets (assuming costless dumping of treated sewage). The first link is mandatory and requires the capital K_s to provide the treated sewage $q_{s\circ}$. The second link – supplying the recycled water $q_{r\circ}$ to potential users – is subject to policy decisions and requires the capital K_r. The unused (residual) treated sewage is dumped to outlets such as rivers, aquifers or directly to a nearby sea.

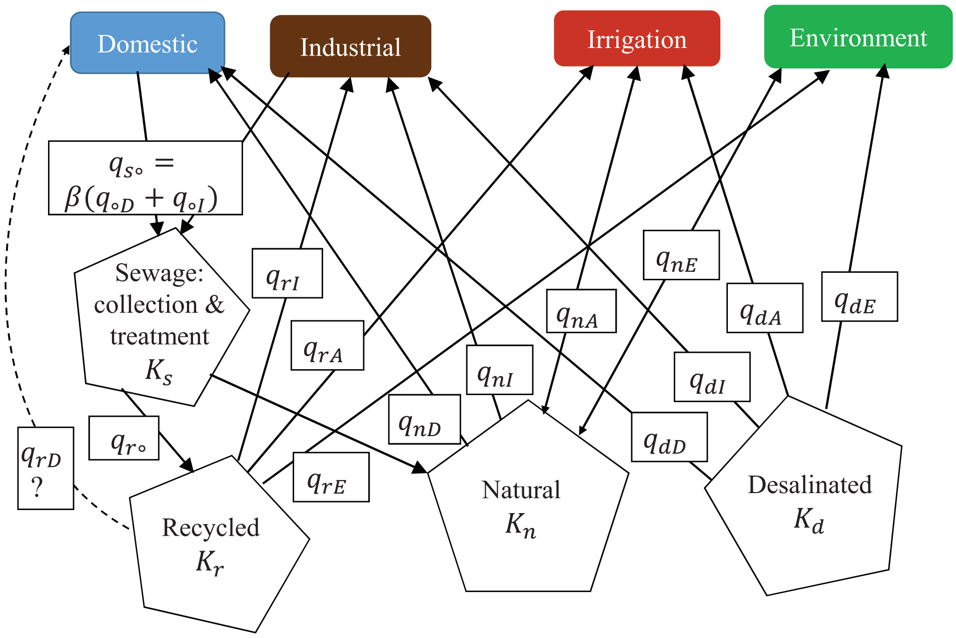

Figure 1: A graphical view of the water economy.

The quality of the treated sewage (secondary, tertiary), hence also the sewage capital K_s, is determined exogenously by health and environmental standards. The capital cost associated with any improvement beyond this required quality level, requested by potential users, is attributed to the recycled capital K_r, which includes also the equipment and infrastructure needed to convey the treated water to potential users.

Notice also that the links between natural sources and agricultural and environmental sectors have arrows on both ends. The backward arrows represent return flow of irrigated water and the recharge role of environmental water (see footnote 1).

3 Optimal policy

The optimal policy is the feasible policy that maximizes (2.11) subject to (2.2) and (2.8) given $K(0)$ and $Q(0)$, where feasibility entails conditions (2.3), (2.5), (2.7), $q(t) \geq 0$, $x_i(t) \in [0, \bar{x}_i]$, $i = n, s, r, d$ and other feasibility restrictions, e.g., (2.6), when imposed.

The current-value Hamiltonian corresponding to this problem is

$$H(t) = B(Q(t), q(t)) - \sum_{i=n,s,r,d} x_i(t) + \sum_{i=n,s,r,d} \lambda_i(t)[x_i(t) - \delta_i K_i(t)] + \theta(t)[R(Q(t)) - q_{n\circ}(t)], \quad (3.1)$$

where $\lambda_i(t)$, $i = n, s, r, d$, and $\theta(t)$ are the costates of $K_i(t)$, $i = n, s, r, d$, and $Q(t)$, respectively. The corresponding Lagrangian, which accounts for the feasibility constraints (2.3), (2.5) and (2.7), is

$$\mathcal{L}(t) = H(t) + \sum_{i=n,s,r,d} \mu_i(t)[\gamma_i K_i(t) - q_{i\circ}(t)] + \xi(t)[\beta(q_{\circ D}(t) + q_{\circ I}(t)) - q_{r\circ}(t)] + \vartheta(t)Q(t), \quad (3.2)$$

where $\vartheta(t)$, $\xi(t)$ and $\mu_i(t)$, $i = n, s, r, d$, are the lagrange multipliers of (2.3), (2.5) and (2.7), respectively.

Necessary conditions for an optimal allocation include (see, e.g., Leonard and Long 1992):

(a) Natural water allocation to domestic and industrial sectors (q_{nD}, q_{nI}):

$$D_j(q_{\circ j}(t)) \leq C_n'(Q(t), q_{n\circ}(t)) + \mu_n(t) + C_s'(q_{s\circ})\beta + \mu_s(t)\beta + \theta(t) - \xi(t)\beta, \quad (3.3a)$$

equality holding if $q_{nj} > 0$, $j = D, I$, where $C_i' \equiv \partial C_i / \partial q_{i\circ}$.

(b) Natural water allocation to agriculture and environmental sectors (q_{nA}, q_{nE}):

$$D_j(q_{\circ j}(t)) \leq C_n'(Q(t), q_{n\circ}(t)) + \mu_n(t) + \theta(t), \quad (3.3b)$$

equality holding if $q_{nj} > 0$, $j = A, E$.

(c) Recycled water allocation to households and industrial sectors $(q_{rD},\, q_{rI})$:

$$D_I(q_{\circ I}(t)) \leq C_r'(q_{r\circ}(t)) + \mu_r(t) + C_s'(q_{s\circ})\beta + \mu_s(t)\beta + \xi(t)(1-\beta), \qquad (3.4a)$$

equality holding if $q_{rj}(t) > 0$, $j = D, I$. If (2.6) is imposed, then (3.4a) applies only for $j = I$.

(d) Recycled water allocation to agriculture and environmental sectors $(q_{rA},\, q_{rE})$:

$$D_j(q_{\circ j}(t)) \leq C_r'(q_{r\circ}(t)) + \mu_r(t) + \xi(t), \qquad (3.4b)$$

equality holding if $q_{rj}(t) > 0$, $j = A, E$.

(e) Desalinated water allocation to domestic and industrial sectors $(q_{dD},\, q_{dI})$:

$$D_j(q_{\circ j}(t)) \leq C_d'(q_{d\circ}(t)) + \mu_d(t) + C_s'(q_{s\circ}(t))\beta + \mu_s(t)\beta - \xi(t)\beta, \qquad (3.5a)$$

equality holding if $q_{dj}(t) > 0$, $j = D, I$.

(f) Desalinated water allocation to agriculture and environmental uses $(q_{dA},\, q_{dE})$:

$$D_j(q_{\circ j}(t)) \leq C_d'(q_{d\circ}(t)) + \mu_d(t), \qquad (3.5b)$$

equality holding if $q_{dj}(t) > 0$, $j = A, E$.

(g) Investment in water capital stocks:

$$x_i(t) = \begin{cases} 0 & \text{if } \lambda_i(t) < 1 \\ \bar{x}_i & \text{if } \lambda_i(t) > 1 \\ x_i^a(t) & \text{if } \lambda_i(t) = 1 \end{cases}, \; i = n, s, r, d. \qquad (3.6)$$

(h) Costate dynamics:

$$\dot{\lambda}_i(t) - \rho\lambda_i(t) = -\partial\mathcal{L}/\partial K_i = \lambda_i(t)\delta_i - \mu_i(t)\gamma_i, \; i = n, s, r, d, \qquad (3.7a)$$

$$\dot{\theta}(t) - \rho\theta(t) = -\partial\mathcal{L}/\partial Q = C_{nQ}(Q(t), q_{n\circ}(t)) - \theta(t)R'(Q(t)) - \vartheta(t), \qquad (3.7b)$$

where $C_{nQ} \equiv \partial C_n/\partial Q$ and $R'(Q) \equiv \partial R/\partial Q$.

(i) Complementary slackness:

$$\mu_i(t)[\gamma_i K_i(t) - q_{i\circ}(t)] = 0,\ i = n, s, r, d, \tag{3.8a}$$

$$\xi(t)[q_{s\circ}(t) - q_{r\circ}(t)] = 0, \tag{3.8b}$$

$$\vartheta(t)Q(t) = 0. \tag{3.8c}$$

(j) Transversality:

$$\lim_{t\to\infty} H(t)e^{-\rho t} = 0. \tag{3.9}$$

Conditions (3.3)-(3.5) are essentially demand-equals-supply, with demand and supply on the left and right sides, respectively. Note that the C_i' represent marginal costs, the $\mu_i(t)$ represent capital costs, and $\theta(t)$ and $\xi(t)$ are natural and recycled water scarcity costs, all expressed water price units (e.g., $\$/m^3$).

The optimal investment rule (3.6) implies a most-rapid-approach to a singular policy, explained next.

3.1 Singular policy

In condition (3.6), $x_i^a(t)$ is the singular investment policy and we use the superscript "a" to signify processes under the singular policy. In particular, $K_i^a(t)$ is the K_i trajectory that solves (2.8) under the singular investment policy and is referred to as the singular trajectory (or turnpike). Along the singular policy, $\lambda_i(t) = 1$ and (3.7a) gives

$$\mu_i^a(t) = (\rho + \delta_i)/\gamma_i,\ i = n, s, r, d. \tag{3.10}$$

The complementary slackness conditions (3.8a), then, imply

$$q_{i\circ}^a(t) = \gamma_i K_i^a(t),\ i = n, s, r, d. \tag{3.11}$$

Conditions (3.10) and (3.11) give

$$\mu_i^a(t) = (\rho + \delta_i)K_i^a(t)/q_{i\circ}^a(t), \ i = n, s, r, d.$$

As $(\rho+\delta_i)K_i$ represents the annual cost of K_i,[5] the μ_i^a's are the cost of capital per unit water or the capital component of the water price.

It turns out that the singular trajectories $K_i^a(t)$, $i = n, s, r, d$, serve as turnpikes to which the optimal $K_i(t)$ processes approach as rapidly as possible, as stated in:

Property 1. *The optimal $K_i(t)$ trajectory approaches as rapidly as possible the singular path $K_i^a(t)$ and proceeds along it thereafter, $i = n, s, r, d$.*

Proof. Consider first $i = n$ (natural water) and suppose that $q_{n\circ}(t) > 0$ under the optimal policy. To show that $x_n(t) = 0$ when $K_n(t) > K_n^a(t)$, suppose otherwise – that $\lambda_n(t) > 1$ and $x_n(t) = \bar{x}_n$. In view of (3.8a) and (3.10), the optimal $q_{n\circ}^a(t) = \gamma_n K_n^a(t) < \gamma_n K_n(t)$ is feasible, hence (using (3.8a) again) $\mu_n(t) = 0$. Condition (3.7a), then, implies that $\lambda_n(t)$ grows at the rate $\rho + \delta_n$, so $\lambda_n(t) > 1$ for all $t > 0$, which in turn implies that $\lambda_n(t)$ continuous to grow at the rate $\rho + \delta_n$ indefinitely, violating the transversality condition (3.9). To see this, note that when $x_n(t) = \bar{x}_n$, (2.8) implies that $\bar{x}_n - \delta_n K_n(t)$ behaves like $e^{-\delta_n t}$. Thus, $\lambda_n(t)(\bar{x}_n - \delta_n K_n(t))$ behaves like $e^{\rho t}$, so $e^{-\rho t}\lambda_n(t)(\bar{x}_n - \delta_n K_n(t))$ does not vanish asymptotically, violating (3.9). We conclude that $K_n(t) > K_n^a(t)$ implies $\lambda_n(t) < 1$ and, by virtue of (3.6), $x_n(t) = 0$.

To show that $K_n(t) < K_n^a(t)$ implies $\lambda_n(t) > 1$ and $x_n(t) = \bar{x}_n$, suppose otherwise – that $\lambda_n(t) < 1$ and $x_n(t) = 0$. Then, condition (3.7a) gives

$$\dot{\lambda}_n(t) = [\lambda_n(t) - 1](\rho + \delta_n) - [\mu_n(t) - \mu_n^a(t)]\gamma_n.$$

[5]The capital K_i could have yielded the interest ρK_i if deposited in the bank and in addition inflicts the cost $\delta_i K_i$ due to depreciation. The annual cost of K_i is therefore $(\rho + \delta_i)K_i$.

Now, $K_n(t) < K_n^a(t)$ implies $q_{n\circ}(t) \leq \gamma_n K_n(t) < \gamma_n K_n^a(t) = q_{n\circ}^a(t)$. Thus, conditions (3.3) imply (noting that D_j decreases in $q_{\circ j}$ and C_n' is non-decreasing in $q_{n\circ}$) that $\mu_n(t) > \mu_n^a(t)$. It follows from the above equation that if $\lambda_n(t) \leq 1$ then $\lambda_n(t)$ always decreases and remains below one forever, which in turn implies that $x_n(t) = 0$ for all $t \geq 0$ and that $K_n(t) \to 0$, violating the $q_{n\circ}(t) > 0$ condition. We conclude that $x_n(t) = \bar{x}_n$ whenever $K_n(t) < K_n^a(t)$. The same arguments can be used to verify the property for $i = s, r, d$. □

Property 1 is useful because it allows focusing attention on the singular policy, along which $\mu_i^a(t) = (\rho + \delta_i)/\gamma_i$ and the corresponding water allocation processes $q_{ij}^a(t)$ depend solely on the natural water stock process $Q(t)$ (or its scarcity price $\theta(t)$) via conditions (3.3)-(3.5). Now, rewriting (3.11) as

$$K_i^a(t) = q_{i\circ}^a(t)/\gamma_i, \; i = n, s, r, d, \tag{3.12}$$

reveals that the turnpikes $K_i^a(t)$ also depend on the $Q(t)$ process. Moreover, noting (2.8), the singular investment processes satisfy $x_i^a(t) = \dot{K}_i^a(t) + \delta_i K_i^a(t)$, $i = n, s, r, d$, hence are also determined by the $Q(t)$ process. We conclude that:

Property 2. *Along the singular trajectories, the water allocation processes* $\{q_{ij}^a(t),\; i = n, r, d,\; j = A, D, I, E\}$*, the capital processes* $K_i^a(t)$ *and the corresponding investment processes* $x_i^a(t)$*,* $i = n, s, r, d$*, depend solely on the natural water stock process* $Q(t)$*.*

The property states that, along the singular trajectories, $q^a(t)$, $K^a(t)$ and $x^a(t)$ are not independent states and controls, but rather depend on the natural water stock process $Q(t)$. It thus follows that, after the transition to the singular trajectory, the optimal policy is driven by a single state, namely the natural water stock $Q(t)$. We can thus apply results of Tsur and Zemel (2014) to conclude that:

Property 3. *Under the optimal policy, the* $Q(t)$ *and* $K_i(t)$, $i = n, s, r, d$, *processes approach a steady state.*

We denote steady states by a "hat" over a variable and assume that the steady state is feasible, i.e., $\hat{Q} \in (0, \bar{Q})$ and $\hat{K}_i \in (0, \bar{x}_i/\delta_i)$, $i = n, s, r, d$.

3.2 Steady state

In a steady state, $K(t)$ and $Q(t)$, hence also $q(t)$ and $x(t)$, remain constant. Noting (2.2) and (2.8),

$$\hat{q}_{n\circ} = R(\hat{Q}) \tag{3.13}$$

and

$$\hat{x}_i = \delta_i \hat{K}_i,\ i = n, s, r, d. \tag{3.14}$$

From (3.7b) and (3.13) we obtain

$$\hat{\theta} = \frac{\hat{\vartheta} - C_{nQ}(\hat{Q}, R(\hat{Q}))}{\rho - R'(\hat{Q})}\,, \tag{3.15}$$

where $C_{nQ} \equiv \partial C_n/\partial Q$ and ϑ is the shadow price of the $Q(t) \geq 0$ constraint. From $\hat{\vartheta} \geq 0$, $C_{nQ} \leq 0$ and $R'(Q) \leq 0$, it follows that $\hat{\theta} \geq 0$, equality holding when $C_{nQ} = 0$ and $\hat{Q} > 0$ (the latter condition implies, noting (3.8c), $\hat{\vartheta} = 0$).

As the steady state is attained under the singular policy, all the singular-policy properties discussed above remain valid. In particular

$$\hat{\mu}_i = (\rho + \delta_i)/\gamma_i,\ i = n, s, r, d, \tag{3.16}$$

and

$$\hat{K}_i = \hat{q}_{i\circ}/\gamma_i,\ i = n, s, r, d. \tag{3.17}$$

The optimal steady state policy $(\hat{q}, \hat{x})$ and the associated natural water and capital stocks $\hat{Q}$ and $\hat{K}$ are characterized in:

Property 4. *The steady state water allocations $\hat{q}_{ij}$, $i = n, r, d$, $j = A, D, I, E$, are solved from equations (3.3)-(3.5) with $\hat{\mu}_i = (\rho + \delta_i)/\gamma_i$, $i = n, s, r, d$, and $\hat{Q}$, $\hat{\theta}$, $\hat{\vartheta}$ and $\hat{\xi}$ are solved using (3.13), (3.15), (3.8c) and (3.8b). The $\hat{q}_{i\circ}$, $i = n, s, r, d$, are then calculated from (2.1) and (2.4), and $\hat{K}_i$, $\hat{x}_i$, $i = n, s, r, d$, are obtained from (3.17) and (3.14).*

3.3 Summary of the optimal policy

The optimal policy proceeds along three stages: most-rapid-approach transition to the singular trajectories (turnpikes); evolution along the singular trajectories; and steady state. During the transition period, investment in water capital is maximal or minimal as $K_i(t)$ lies below or above its turnpike, respectively, and the length of this period is inversely related to the investments upper bounds $\bar{x}_i$, $i = n, s, r, d$. The transition period ends when all capital stocks reach their singular trajectories. The singular trajectories (or turnpikes) depend solely on the natural water stock and the latter eventually enters a steady state, at which time the entire system enters a steady state (the third and final stage).

4 Regulation

Regulation entails implementing a policy and optimal regulation is concerned with implementing the optimal policy. In the present context a policy consists of water allocation $q(t)$ and investment in water infrastructure $x(t)$, and both components need to be regulated. We refer to the former as q-regulation and to the latter as x-regulation.

The above analysis provides straightforward rules for regulating investment in water infrastructure (x-regulation), namely, a most-rapid-approach to the singular capital trajectories $K_i^a(t)$, $i = n, s, r, d$. This requires solving for the singular capital trajectories, which, noting (3.12) and (3.17), depend on the singular water allocations. Due to their high capital intensity and because they serve many suppliers and users, ownership of water supply systems is often centralized (locally, regionally or state-wide) and this feature facilitates the regulation task. The remainder of this section considers the regulation of water allocation (q-regulation).

Regulation tools vary from case to case based on social, cultural, institutional, legal, political as well as (in the case of water) climatic and hydrological conditions, but rely in one way or another on quotas (quantities) and prices. In the present context quota regulation entails setting the water allocation $q_{ij}(t)$, or bounds thereof, directly while price regulation consists of setting water prices and letting suppliers and users choose the water allocation. In between lies a range of regulation schemes that combine quotas and pricing (see Dinar 2000, Tsur et al. 2004, Tsur 2009, Booker et al. 2012, and works they cite). The above framework lends itself directly to such regulation schemes.

To simplify the exposition, it is convenient to consider the following linear water economy:

$$D_j(q_{\circ j}) = d_{j0} - d_j q_{\circ j},\ d_{j0}, d_j > 0,\ j = D, A, I, E; \tag{4.1a}$$

$$C_i(q_{i\circ}) = c_{i0} + c_i q_{i\circ},\ c_{i0}, c_i > 0,\ i = s, r, d; \tag{4.1b}$$

$$C_n(Q, q_{n\circ}) = [c_{n0} - c_n Q]\, q_{n\circ},\ c_{n0} > 0,\ c_n \geq 0; \tag{4.1c}$$

$$R(Q) = r_{n0} - r_n Q,\ r_{n0}, r_n > 0. \tag{4.1d}$$

Under these specifications (3.15) specializes to

$$\hat{\theta} = \left[\hat{\vartheta} + c_n(r_{n0} - r_n\hat{Q})\right]/(\rho + r_n), \tag{4.2}$$

where $\hat{\vartheta} \geq 0$, equality holding if $\hat{Q} > 0$ (cf. (3.8c)).

It is convenient to consider the case where the natural water stock is at its steady state $\hat{Q}$. In this case, Properties 1 and 3 imply that the optimal policy is to bring the capital stock K_i to its steady state value $\hat{K}_i$ as rapidly as possible and maintain the steady state thereafter. Except for a short transition period, the optimal policy is the steady state policy characterized in Property 4. We discuss regulation schemes that use prices and quotas.

4.1 Optimal pricing

A pricing policy involves setting $p = \{p_{ij},\, i = n, r, d,\, j = A, D, I, E\}$, where p_{ij} represents the price users in sector j pay for water supplied from source i. Under prices p, users consume water up to the point where their demand (i.e., their willingness to pay for the last water unit) just equals the water price. Let $q_{ij}(p)$ denote sector j's demand for source i's water at prices p. Then, $q_{ij}(p)$ satisfies

$$D_j(q_{\circ j}(p)) \leq p_{ij},\ i = n, r, d,\ j = A, D, I, E, \tag{4.3}$$

equality holding if $q_{ij}(p) > 0$, where $q_{\circ j}(p) = \sum_{i=n,r,d} q_{ij}(p)$, $j = A, D, I, E$, $q_{i\circ}(p) = \sum_{j=n,r,d} q_{ij}(p)$, $i = n, r, d$ and $q_{s\circ}(p) = \beta(q_{\circ D}(p) + q_{\circ I}(p))$ are as defined in (2.1) and (2.4). A feasible p satisfies $q_{n\circ}(p) \leq R(\hat{Q})$ and $q_{r\circ}(p) \leq q_{s\circ}(p)$.

Following (3.12), let

$$K_i(p) = q_{i\circ}(p)/\gamma_i,\ i = n, s, r, d, \tag{4.4}$$

and define the following policy:

Definition 1 (p-policy). *Water is priced according to* p *and investments follow a most-rapid-approach to* $K_i(p)$, $i = n, s, r, d$.

Define the following water prices:

$$\hat{p}_{nj} = \underbrace{c_n}_{\text{marg cost } n} + \underbrace{(\rho+\delta_n)/\gamma_n}_{\text{cap cost } K_n} + \underbrace{\beta c_s}_{\text{mar cost } s} + \underbrace{\beta(\rho+\delta_s)/\gamma_s}_{\text{cap cost } K_s} + \underbrace{\hat{\theta}}_{\text{nat scarcity}} + \underbrace{(-\hat{\xi}\beta)}_{\text{rec scarcity}}, \; j = D, I; \quad (4.5a)$$

$$\hat{p}_{nj} = \underbrace{c_n}_{\text{mar cost}} + \underbrace{(\rho+\delta_n)/\gamma_n}_{\text{cap cost } K_n} + \underbrace{\hat{\theta}}_{\text{nat scarcity}}, \; j = A, E; \quad (4.5b)$$

$$\hat{p}_{rI} = \underbrace{c_r}_{\text{mar cost } r} + \underbrace{(\rho+\delta_r)/\gamma_r}_{\text{cap cost } K_r} + \underbrace{\beta c_s}_{\text{mar cost s}} + \underbrace{\beta(\rho+\delta_s)/\gamma_s}_{\text{cap cost } K_s} + \underbrace{\hat{\xi}(1-\beta)}_{\text{rec scarcity}}; \quad (4.5c)$$

$$\hat{p}_{rI} = \underbrace{c_r}_{\text{mar cost } r} + \underbrace{(\rho+\delta_r)/\gamma_r}_{\text{cap cost } K_r} + \underbrace{\beta c_s}_{\text{mar cost s}} + \underbrace{\beta(\rho+\delta_s)/\gamma_s}_{\text{cap cost } K_s} + \underbrace{\hat{\xi}(1-\beta)}_{\text{rec scarcity}}; \quad (4.5d)$$

$$\hat{p}_{rj} = \underbrace{c_r}_{\text{mar cost } r} + \underbrace{(\rho+\delta_r)/\gamma_r}_{\text{cap cost } K_r} + \underbrace{\hat{\xi}}_{\text{rec scarcity}}, \; j = A, E; \quad (4.5e)$$

$$\hat{p}_{dj} = \underbrace{c_d}_{\text{mar cost } d} + \underbrace{(\rho+\delta_d)/\gamma_d}_{\text{cap cost } K_d} + \underbrace{\beta c_s}_{\text{mar cost } s} + \underbrace{\beta(\rho+\delta_s)/\gamma_s]}_{\text{cap cost } K_s} + \underbrace{(-\hat{\xi}\beta)}_{\text{rec scarcity}}, \; j = D, I; \quad (4.5f)$$

and

$$\hat{p}_{dj} = \underbrace{c_d}_{\text{mar cost}} + \underbrace{(\rho+\delta_d)/\gamma_d}_{\text{cap cost } K_d}, \; j = A, E. \quad (4.5g)$$

Under (4.1), the $\hat{p}_{ij}$ defined in (4.5) are the right-hand sides of the corresponding optimality conditions (3.3)-(3.5), evaluated at a steady state. Calculating these prices requires the scarcity prices $\hat{\theta}$ and $\hat{\xi}$ of natural and recycled water, respectively. The former, $\hat{\theta}$, is specified in (4.2), where $\hat{\vartheta} = 0$ if $\hat{Q} > 0$ and $\hat{\vartheta} \geq 0$ otherwise (cf. (3.8c)). When $\hat{Q} = 0$, $\hat{\vartheta}$ is set in order to satisfy (3.13).

The scarcity price of recycled water satisfies, noting (3.8b),

$$\hat{\xi}[\hat{q}_{s\circ} - \hat{q}_{r\circ}] = 0. \tag{4.6}$$

Thus, $\hat{\xi} = 0$ if the demand for recycled water ($\hat{q}_{r\circ}$) does not exceed the supply ($\hat{q}_{s\circ}$) and $\hat{\xi} \geq 0$ otherwise. In the latter case, $\hat{\xi}$ is set so as to equate demand and supply of recycled water. Notice that $\hat{\xi}$ acts as a subsidy on natural water allocated to households and industrial users and as a tax on recycled water allocated to agricultural and environmental purposes. Thus, for example, it encourages reallocation of natural water from agricultural and environmental users to domestic and industrial users. The reason is that the water allocated to domestic and industria users can be reused, thereby increasing the overall supply of water.

We now verify that, given $Q = \hat{Q}$ and (4.1),

Property 5. *The $\hat{p}$-policy (i.e., the p-policy with $p = \hat{p}$) is optimal.*

Proof. Under (4.1), the $\hat{p}_{ij}$ defined in (4.5) are the right-hand sides of the corresponding optimality conditions (3.3)-(3.5). When the price of q_{ij} is $\hat{p}_{ij}$, sector j users will demand water from source i such that $D_j(q_{\circ j}) = \hat{p}_{ij}$ (if $D_j(0) \leq \hat{p}_{ij}$ they will demand zero). Clearly, the optimal steady state allocations $\hat{q}_{ij}$, $i = n, r, d$, $j = A, D, I, E$, satisfy these conditions. Moreover, under decreasing water demands, the $\hat{q}_{ij}$'s are the only allocations that satisfy these conditions. It follows that $q_{ij}(\hat{p}) = \hat{q}_{ij}$, $i = n, r, d$, $j = A, D, I, E$, and (4.4) implies $K_i(\hat{p}) = \hat{K}_i$, $i = n, s, r, d$. Thus, the investment policy associated with the $\hat{p}$-policy is a most rapid approach to $\hat{K}_i$. □

The $\hat{p}_{ij}$ prices bear important properties regarding cost allocation and cost recovery. The marginal cost components, c_n, c_s, c_r and c_d, are obvious. Consider the capital cost component associated with K_n, $(\rho+\delta_n)/\gamma_n$, that appears in the prices of natural water allocations $\hat{p}_{nj}$, $j = A, D, I, E$. The water proceeds associated with this component are

$$(\hat{q}_{nD} + \hat{q}_{nI} + \hat{q}_{nA} + \hat{q}_{nE})(\rho + \delta_n)/\gamma_n = \hat{q}_{n\circ}(\rho + \delta_n)/\gamma_n = \hat{K}_n(\rho + \delta_n),$$

where the right-most equality follows from (3.17). These proceeds exactly cover the annual cost of K_n (see footnote 5).

Consider now the price component $\beta(\rho+\delta_s)/\gamma_s$ associated with the cost of K_s (sewage collection and treatment capital), which appears in the prices of the water allocations to domestic and industrial sectors. The water proceeds associated with this component are

$$(\hat{q}_{\circ D} + \hat{q}_{\circ I})\beta(\rho + \delta_s)/\gamma_s = \hat{q}_{s\circ}(\rho + \delta_s)/\gamma_s = \hat{K}_s(\rho + \delta_s),$$

where the first equality uses (2.4) and the second uses (3.17). These proceeds exactly cover the (annual) cost of sewage capital $(\rho+\delta_s)\hat{K}_s$. Because only the domestic and industrial sectors generate sewage, which must be collected and treated, these sectors are required to pay for these activities.

In a similar manner it can be shown that the water proceeds associated with $(\rho + \delta_d)/\gamma_d$ and $(\rho + \delta_r)/\gamma_r$ cover the annual cost of the desalination capital $(\rho + \delta_d)\hat{K}_d$ and recycling conveyance capital $(\rho + \delta_r)\hat{K}_r$, respectively. Note that desalination is often more capital intensive than recycling or natural water supply, in which case $\gamma_d \ll \gamma_i$, $i = n, r$.[6] When the depreciation rates are the same (or similar), this implies $(\rho + \delta_d)/\gamma_d \ll (\rho + \delta_i)\gamma_i$, $i = n, r$, so the capital cost component of K_d exceeds that of K_n and K_r. This implies that prices of desalinated water allocated to the various sectors, which include

[6]Recall that $\gamma_i K_i$ is source i's supply constraint. If desalination is more capital intensive than the other two sources, it requires more capital to supply the same annual flow, hence γ_d is smaller than both γ_n and γ_r.

the $(\rho + \delta_d)/\gamma_d$ component, are higher than prices of water allocated from natural or recycled sources. As a result, it is likely that $D_A(0) \leq \hat{p}_{dA}$ and $D_E(0) \leq \hat{p}_{dE}$, in which case (4.3) implies $\hat{q}_{dA} = \hat{q}_{dE} = 0$, i.e., desalinated water is too expensive for agricultural and environmental uses. If the same holds also for the domestic and industrial sectors, i.e., $D_D(0) \leq \hat{p}_{dD}$ and $D_I(0) \leq \hat{p}_{dI}$, then desalination is not desirable.

The water proceeds generated by the marginal cost components cover, up to a fixed amount, the variable costs of water supply, and we saw above that the water proceeds generated by the capital cost components cover the capital costs. The (net) water proceeds generated by the scarcity components $\hat{\theta}$ and $\hat{\xi}$ have no counterpart costs to cover. These proceeds can be returned to water users in an undistorted (e.g., lump sum) fashion. Alternatively, optimality can be maintained without these scarcity components by introducing water quotas.

4.2 Optimal quotas

Quota regulation entails imposing bounds on water supply from various sources to various sectors. In an elaborate scheme, the quotas are the $\hat{q}_{ij}$, $i = n, r, d$, $j = A, D, I, E$, i.e., the water allocation from each source to each sector. In a less elaborate scenario, the regulator imposes the quota $\hat{q}_{i\circ}$ on source i suppliers, where the $\hat{q}_{i\circ}$ are calculated from the $\hat{q}_{ij}$ (i.e., $\hat{q}_{i\circ} = \sum_j \hat{q}_{ij}$). To prevent undersupply the regulator can use lower bound quotas or impose fines on supplies below the quota. The option to oversupply is usually avoided by the available infrastructure, which does not allow supplying above the stated quota.

A pure quota regulation raises the problem of how to pay for the cost (variable and capital) of water supply. Without water proceeds that are directly related to water use, the cost of water supply must be covered from other sources, such as tax revenues (local, state or national).

4.3 Combining prices and quotas

Quotas and prices can be combined in many ways; we discuss two examples. In the first, the stock of natural water is required not to decline below $\hat{Q}$ (or any other exogenously set stock). This can be accomplished in a pure pricing scheme by adding the scarcity rent $\hat{\theta}$, defined in (4.2), to the price of natural water allocated to any sector. Alternatively, the restriction can be imposed as a quota, i.e., $q_{n\circ} = \sum_j q_{ij} \leq R(\hat{Q})$. In the latter case, the quota restriction entails a shadow price which replaces the scarcity rent $\hat{\theta}$ and the latter should not be included in the price of natural water.

The second example is concerned with the allocation of water for environmental purposes. Unlike the domestic, agricultural and industrial sectors, where individual users can be identified and required to pay for the water they use, environmental water serves all (or many) households, i.e., is essentially a public good. It is thus complicated to charge individual users for environmental water. In such cases, the optimal allocation of environmental water can be imposed as a (lower bound) quota: $\sum_i q_{iE} \geq \hat{q}_{\circ E}$. To motivate suppliers to allocate the right amount of environmental water, source i suppliers can be required to supply $\hat{q}_{iE}$, for which they will be reimbursed according to the cost of supply. The amount needed to reimburse suppliers can be raised by appropriate taxes (the public good natura of environmental water justifies such funding).

5 Extensions

The above stylized water economy can be extended in a number of ways to better resemble ubiquitous real-world situations. Extensions to situations with more general cost structure and arbitrary number of sources and sectors are presented in the appendix. Below we present two non-stationary extensions: the first incorporates technical change in desalination technology; the second accounts for growing water demands.

5.1 Technical change in desalination technology

Recall that a desalination capital worth (in monetary terms) K_d can produce at most $\gamma_d K_d$ cubic meter per year of desalinated water. Advances in desalination technologies can be represented by letting $\gamma_d(t)$ increase over time, reaching (perhaps only asymptotically) the upper bound $\bar{\gamma}_d$. Thus, as time goes by, the same (monetary value of) desalination capital can produce larger flows of desalinated water.[7] Initially $\gamma_d(0)$ is small and the ensuing capital cost component of desalination $(\rho + \delta_d)/\gamma_d(0)$ is large. If under the steady state policy, conditions (3.5) hold as strict inequality, then no desalination is initially desirable.

As time goes by, $\gamma_d(t)$ increases and the capital cost component $(\rho + \delta_d)/\gamma_d(t)$ decreases, eventually reaching the lowest cost of desalination capital $(\rho + \delta_d)/\bar{\gamma}_d$. If under the steady state policy with the most advanced desalination technology $\bar{\gamma}_d$, conditions (3.5) still hold as strict inequalities, then desalination will never be desirable. If some desalination is desirable under $\bar{\gamma}_d$ (i.e., in the steady state with $\bar{\gamma}_d$ one or more of conditions (3.5) hold

[7]The declining cost of desalination in the last decade (see Tsur 2015) attests to such technical change process. Tsur and Zemel (2000) analyzed technical change in desalination technology induced by intentional R&D activities. This case is beyond the current scope.

as equality), then there must exist some finite time $t_d \geq 0$ following which desalination becomes desirable. In the latter case, for $t > t_d$, the scale of desalination increases over time, eventually approaching

$$\hat{K}_d(\bar{\gamma}_d) = \hat{q}_{d\circ}(\bar{\gamma}_d)/\bar{\gamma}_d,$$

where $\hat{q}_{d\circ}(\bar{\gamma}_d)$ is the steady state allocation of desalinated water under the most advanced technology $\bar{\gamma}_d$.

The optimal desalination trajectory is obtained by solving the optimal policy with the time-dependent $\gamma_d(t)$ replacing the constant γ_d. The necessary conditions (3.3)-(3.8) hold as specified above with $\gamma_d(t)$ instead of γ_d. Thus, in view of (3.6), the optimal capital stocks evolve along their most-rapid-approach pathes with the singular trajectories K_i^a corresponding to $x_i^a(t)$, $i = n, s, r, d$. Noting (3.6), $\lambda_i(t) = 1$ along the singular policy and (3.7a) implies

$$\mu_i^a = (\rho + \delta_i)/\gamma_i, \; i = n, s, r, \tag{5.1a}$$

and

$$\mu_d^a(t) = (\rho + \delta_d)/\gamma_d(t). \tag{5.1b}$$

The μ_i^a, $i = n, s, r, d$, are the capital cost components in conditions (3.3)-(3.5), according to which the allocations along the singular policy, $q_{ij}^a(t)$, are determine and give rise to $q_{i\circ}^a(t) = \sum_j q_{ij}^a(t)$, $i = n, s, r, d$. The complementary slackness conditions (3.8), then, imply

$$K_i^a(t) = q_{i\circ}^a(t)/\gamma_i(t), \; i = n, s, r, d. \tag{5.2}$$

Notice that the allocation of desalinated water (along the singular policy) increases as time goes by because the capital cost component of desalinated water, $\mu_d^a(t)$, decreases as the desalination technology is improved, i.e., $\gamma_d(t)$ increases.

To summarize, if $\hat{K}_d = 0$ when $\gamma_d = \bar{\gamma}_d$, then it is not desirable to desalinate even under the most advanced technology $\bar{\gamma}_d$ and desalination should never be performed. If, on the other hand, $\hat{K}_d > 0$ for some $\gamma_d \in [\gamma_d(0), \bar{\gamma}_d]$, then there exists some finite time $t_d \geq 0$ following which desalination is desirable and the optimal desalination policy is characterized by t_d and the singular trajectory $K_d^a(t)$, $t \geq t_d$. The steady state is attained only after the technical change process $\gamma_d(t)$ has reached its upper bound $\bar{\gamma}_d$ (which may happens asymptotically), but the singular policy is easily characterized and includes the optimal desalination policy $K_d^a(t)$.

5.2 Growing water demand

We consider an economy that grows at the rate $g < \rho$. Suppose that initially all water demands are satisfied from natural sources. However, the supply of natural water cannot grow indefinitely as it is limited by the natural recharge (derived from precipitation) and the finite natural water stock Q. As time goes by, the economy grows and the supply of water must increase to meet the growing demand. Improved water management, including recycling, may suffice to meet the growing water needs for a while, but eventually these possibilities are exhausted or become more expensive than desalination and the latter is the only source that, for all practical purposes, can be considered unlimited. We extend the above model to accommodate this situation.

To focus on growth effects, we consider the situation where natural water sources have been fully exploited, i.e., the natural water stock has reached its lower bound $Q = 0$,[8] the supply of natural water is

$$q_{n\circ}(t) = R(0) \tag{5.3}$$

and the associated capital stock is

$$\hat{K}_n = R(0)/\gamma_n. \tag{5.4}$$

The planning challenge, then, is to set the time profile of desalination and recycling and allocate water to the different sectors.

Each sector grows at the same rate g, such that at time t it consists of of $N(t) = e^{gt}$ identical sub-sectors. Thus, allocating $q_{\circ j}$ to each sub-sector generates the surplus

$$e^{gt}B_j(q_{\circ j}) = e^{gt}\int_0^{q_{\circ j}} D_j(s)ds,\ j = A, D, I, E, \tag{5.5}$$

for sector j during year t, where $D_j(\cdot)$ is the (inverse) demand for water of each sub-sector (or the inverse demand of sector j at $t = 0$).

With $q_{\circ j}(t)$ representing the water allocated to each of its e^{gt} sub-sectors, the total water allocation to sector j during year t is $e^{gt}q_{\circ j}(t)$. This must equal the water supply to sector j from all sources. Let $\tilde{q}_{ij}(t)$ represent the water supply from source i to sector j during year t, so that

$$q_{ij}(t) = e^{-gt}\tilde{q}_{ij}(t) \tag{5.6}$$

is the water allocated to each of the e^{gt} sub-sectors of sector j during year t. In view of (5.6), the identity $e^{gt}q_{\circ j} \equiv \tilde{q}_{\circ j}(t) = \sum_{i=n,r,d} \tilde{q}_{ij}(t)$ implies

$$q_{\circ j} = \sum_{i=n,r,d} q_{ij}(t),\ j = A, D, I, E. \tag{5.7}$$

[8]The zero lower bound is a normalization that represents the water stock below which natural water cannot or should not (e.g., the stock at which the pumping cost exceeds the cost of desalination) be exploited. Notice that, because $R(Q)$ is non-increasing, $R(0)$ is the maximal sustainable supply of natural water.

In addition, total water allocated from source $i = r, d$, satisfies

$$\tilde{q}_{i\circ}(t) = \sum_{j=A,D,I,E} \tilde{q}_{ij}(t),\ i = r, d,$$

or, using (5.6),

$$q_{i\circ}(t) = \sum_{j=A,D,I,E} q_{ij}(t),\ i = r, d, \tag{5.8}$$

and total natural water supply in year t is

$$q_{n\circ}(t) = \sum_{j=A,D,I,E} q_{nj}(t) = R(0). \tag{5.9}$$

The supply of recycled water is restricted according to

$$\tilde{q}_{r\circ}(t) \leq \beta \left(e^{gt} q_{\circ D}(t) + e^{gt} q_{\circ I}(t) \right),$$

which, invoking (5.6) again, reproduces (2.5).

$$q_{r\circ}(t) \leq \beta \left(q_{\circ D}(t) + q_{\circ I}(t) \right). \tag{5.10}$$

Let $\tilde{K}_i(t)$ and $\tilde{x}_i(t)$ denote, respectively, source i's capital stock and investment rate, satisfying (as in (2.8))

$$\dot{\tilde{K}}_i(t) = \tilde{x}_i(t) - \delta_i \tilde{K}_i(t),\ i = s, r, d. \tag{5.11}$$

Let

$$K_i(t) = e^{-gt} \tilde{K}_i(t) \text{ and } x_i(t) = e^{-gt} \tilde{K}_i(t), \tag{5.12}$$

so, in view of (5.11),

$$\dot{K}_i(t) = x_i(t) - (\delta_i + g) K_i(t),\ i = s, r, d, \tag{5.13}$$

and the capacity constraints (2.7) remain intact.

The optimal policy maximizes

$$\int_0^\infty \left[\sum_{j=A,D,I,E} e^{gt} B_j(q_{\circ j}(t)) - \sum_{i=s,r,d} C_i(e^{gt} q_{i\circ}(t)) - \sum_{i=s,r,d} e^{gt} x_i(t) \right] e^{-\rho t} dt =$$
$$\int_0^\infty \left[\sum_{j=A,D,I,E} B_j(q_{\circ j}(t)) - \sum_{i=s,r,d} e^{-gt} C_i(e^{gt} q_{i\circ}(t)) - \sum_{i=s,r,d} x_i(t) \right] e^{-(\rho-g)t} dt \tag{5.14}$$

subject to (5.13) and feasibility constraints, given $K_s(0)$, $K_r(0)$ and $K_d(0)$, where the costs of natural water supply, $C_n(q_{n\circ}(t)) = C_n(R(0))$, and the investment in natural capital, $x_n(t) = \hat{K}_n/\delta_n = (R(0)/\gamma_n)/\delta_n$, are given, hence ignored.

The current-value Hamiltonian is

$$H(t) = \sum_{j=A,D,I,E} B_j(q_{\circ j}(t)) - \sum_{i=s,r,d} e^{-gt} C_i(e^{gt} q_{i\circ}(t)) - \sum_{i=s,r,d} x_i(t) +$$
$$\sum_{i=s,r,d} \lambda_i(t)[x_i(t) - (\delta_i + g)K_i(t)] \tag{5.15}$$

and the Lagrangian, which accounts for the feasibility constraints (2.7), (5.9) and (5.10), is

$$\mathcal{L}(t) = H(t) + \sum_{i=s,r,d} \mu_i(t)[\gamma_i K_i(t) - q_{i\circ}(t)] +$$
$$\xi(t)[\beta(q_{\circ D}(t) + q_{\circ I}(t)) - q_{r\circ}(t)] + \theta(t)(R(0) - q_{n\circ}(t)). \tag{5.16}$$

The necessary conditions for optimal allocation of desalinated and recycled water are identical to conditions (3.5) and (3.4) of the stationary economy; the necessary conditions for optimal investments are identical to conditions (3.6) of the stationary economy; condition (3.7a) remain the same (noting that g cancel out) and so are the complementary slackness conditions (3.8a) and (3.8b).

We thus conclude that the optimal investment policy is to bring $K_i(t) = e^{-gt}\tilde{K}_i(t)$ to its (above determined) stationary-economy steady state $\hat{K}_i$, $i = s, r, d$, as rapidly as possible and maintain the steady state thereafter. The water economy, thus, approaches as rapidly as possible the steady-state growth path $\hat{\tilde{K}}_i(t) = e^{gt}\hat{K}_i$, $i = s, r, d$, with desalination driving the growth in water supply.

6 Concluding comments

Water economies are complex constructs, each with its own physical and social environments. Yet, they all share common features and their management therefore is based on common principles. This work elucidates these common principles in the context of a prototypical water economy containing the features common to most water economies. A water policy consists of water allocation from each source to each user sector at each point of time and the investment in capital infrastructure needed to carry out these allocations. We find that the optimal policy evolves along three stages: a transition stage, where the water capital stocks are brought to their respective turnpikes as rapidly as possible; a turnpike (singular) stage, where the water capital stocks are kept along their turnpikes and move in tandem with the natural water stock; and a steady-state stage. The turnpike (singular) trajectories depend solely on the natural water stock. Depending on the functional forms underlying the water economy, the steady state stage may be entered at a finite time or asymptotically.

Implementing the optimal policy by means of pricing and quotas is discussed. The optimal water prices (the prices that implements the optimal policy) consist of three components: marginal cost, capital cost and scarcity cost – all expressed in water price (e.g., dollar per cubic meter) units. As these components vary across users and sources, so do the optimal water prices. The scarcity prices are associate with natural water and recycled water. The former is obvious in regions where natural water sources are insufficient to meet water demand. The latter follows from the constraint imposed on the supply of recycled water due to its dependence on the water allocation to the residential and industrial sectors.

Desalination is an unlimited but expensive source. Its use, therefore, is justified only under severe water shortage. However, demographic and climatic trends imply that the number of regions undergoing severe water shortage increases with time. The model presented herein can be used to determine when to begin desalinization activities and the extent of desalination over time.

The model is extended to situations involving arbitrary number of sources and user sectors, as well as to non-stationary economies with growing water demands and improved desalination technology. It is found that the turnpike property of the optimal policy persists in all cases. For growing economy, the optimal policy is a most-rapid-approach to a balance growth path driven by investment in desalination and recycling capital. The model can be used to detect the time at which desalination should be initiated and the scale of desalination activities thereafter.

The broad view undertaken in this work facilitates the presentation and allows a sharp characterization of optimal policy rules, but inevitably leads to simplifications and abstractions. A notable abstraction is the assumption of deterministic water supplies and demands. In actual practice, natural water supplies often fluctuate randomly with precipitation and the latter affects some (e.g., agricultural) water demands as well (see, e.g., Tsur 1990, Tsur and Graham-Tomasi 1991, Provencher and Burt 1994, Knapp and Olson 1995, Leizarowitz and Tsur 2012). This aspect can have profound effects on optimal policies and should, when relevant, be incorporated in empirical applications.

Appendix

The appendix shows how to account for source-and-sector specific costs and to include arbitrary number of sources and sectors.

A Source-and-sector specific cost

So far the supply costs (variable and fixed) are assumed to be specific to the water source but not to the user sectors. Some costs may apply only to water allocated from a specific source to a specific sector. A common example is when natural water allocated to households must be treated to a drinking quality, whereas natural water allocated to industrial users, irrigators or environmental restoration can remain at its raw state. In this case, q_{nD} entails further treatment activities that require capital and variable costs.

Considering this example, let $C_{nD}(\cdot)$ represent the variable cost associated with the drinking-quality treatment of natural water and K_{nD} represent the (cost of) capital (infrastructure, equipment) needed to perform the treatment. The capital constraint

$$q_{nD}(t) \leq \gamma_{nD} K_{nD}(t) \qquad \text{(A.1)}$$

is added to constraints (2.7) and the investment rate $x_{nD}(t)$, driving the dynamics of $K_{nD}(t)$ according to

$$\dot{K}_{nD}(t) = x_{nD}(t) - \delta_{nD} K_{nD}(t), \tag{A.2}$$

is added to the investment decisions.

The term $\lambda_{nD}(t)[x_{nD}(t) - \delta_{nD} K_{nD}(t)] - x_{nD}(t)$ is added to the Hamiltonian, defined in (3.1), where $\lambda_{nD}(t)$ is the costate of $K_{nD}(t)$, and the term $\mu_{nD}(t)[\gamma_{nD} K_{nD}(t) - q_{nD}(t)]$ is added to the Lagrangian, specified in (3.2), where $\mu_{nD}(t)$ is the Lagrange multiplier of (A.1). Condition (3.3a) holds only for $j = I$ (natural water allocated to industrial use), and for $j = D$ (natural water allocated to households) the condition changes to

$$D_D(q_{\circ D}(t)) \le C'_n(Q(t), q_{n\circ}(t)) + \mu_n(t) + C'_s(q_{s\circ})\beta_s + \mu_s(t)\beta_s + \theta(t) - \xi(t)\beta + C'_{nD}(q_{nD}(t)) + \mu_{nD}(t), \tag{A.3}$$

equality holding if $q_{nD}(t) > 0$, where $C'_{nD}(\cdot)$ is the marginal cost of the treatment activity. In addition,

$$x_{nD}(t) = \begin{cases} 0 & \text{if } \lambda_{nD}(t) < 1 \\ \bar{x}_{nD} & \text{if } \lambda_{nD}(t) > 1 \\ x^a_{nD}(t) & \text{if } \lambda_{nD}(t) = 1 \end{cases}. \tag{A.4}$$

is added to (3.6) and

$$\dot{\lambda}_{nD}(t) - \rho\lambda_{nD}(t) = \lambda_{nD}(t)\delta_{nD} - \mu_{nD}(t)\gamma_{nD} \tag{A.5}$$

is added to (3.7a).

The analysis of subsection 3.1 can be repeated to show that all the singular policy properties apply also to $x^a_{nD}(t)$, $K^a_{nD}(t)$, $q^a_{nD}(t)$ and $\mu_{nD}(t)$.

B Many sources and sectors

In actual practice there may be a number (say N_n) sources of natural water (aquifers, lakes and stream flows in different locations with varying water quality), a few (say N_r) recycling facilities (in different locations and producing recycled water of different quality) and multiple (say N_d) desalination plants (geographically dispersed). Likewise, there are $M_D > 1$ domestic sectors (municipalities), M_I industrial users (that vary geographically and with respect to the water quality they require), M_A agricultural users and M_E environmental sites.

Let $M = \sum_{j=D,A,I,E} M_j$ represent the total number of user sectors. The natural water allocation is represented by $q^n(t) = \{q^n_{mj}(t), m = 1, 2, \ldots, N_n,\ j = 1, 2, \ldots, M\}$, where q^n_{mj} is the water allocated from natural source m to sector j in year t. Similarly, $q^r(t) = \{q^r_{mj}(t), m = 1, 2, \ldots, N_r,\ j = 1, 2, \ldots, M\}$ represents the recycled water allocation and $q^d(t) = \{q^d_{mj}(t),\ m = 1, 2, \ldots N_d,\ j = 1, 2, \ldots, M\}$ represents the allocation of desalinated water. The annual allocation from source type i to sector j is

$$q^i_{\circ j}(t) = \sum_{m=1}^{N_i} q^i_{mj}(t),\ j = 1, 2, \ldots, M,\ i = n, r, d, \tag{B.1}$$

and the annual water allocation to sector j is

$$q^\circ_{\circ j}(t) = \sum_{i=n,r,d} q^i_{\circ j}(t),\ j = 1, 2, \ldots, M. \tag{B.2}$$

The supply of recycled water from any of the N_r facilities is restricted by the sewage collected and treated in this facility. We assume that each domestic user (a municipality, say) is served by only one recycling facility and the same holds for industrial users.[9] Consequently, let J^D_m and J^I_m be the (index) sets of, respectively, domestic and industrial users served by recycling facility $m = 1, 2, \ldots, N_r$. Then, extending (2.4), the (annual) flow of recycled

[9]Economies of scale in recycling support the design of regional recycling plants, such that each user is served by only one facility while each facility can serve multiple users.

water produced by facility m is

$$q^s_{m\circ}(t) = \beta \left(\sum_{j \in J^D_m} q^\circ_{\circ j}(t) + \sum_{j \in J^I_m} q^\circ_{\circ j}(t) \right), \; m = 1, 2, \ldots, N_r. \tag{B.3}$$

The supply of recycled water from facility m is thus restricted by

$$q^r_{m\circ}(t) \le q^s_{m\circ}(t), \; m = 1, 2, \ldots, N_r. \tag{B.4}$$

The N_n natural water stocks $Q_m(t)$, $m = 1, 2, \ldots, N_n$, evolve in time according to

$$\dot{Q}_m(t) = R_m(Q_m(t)) - q^n_{m\circ}(t), \; m = 1, 2, \ldots, N_n, \tag{B.5}$$

where $R_m(\cdot)$ is the recharge and

$$q^n_{m\circ}(t) = \sum_{j=1}^{M} q^n_{mj}(t), \; m = 1, 2, \ldots, N_n \tag{B.6}$$

is the (annual) water withdrawal associated with natural stock m.

The capital stocks $K^i = (K^i_1, K^i_2, \ldots, K^n_{N_i})$, $i = n, s, r, d$ restrict water allocation according to

$$q^i_{m\circ}(t) \le \gamma^i_m K^i_m(t), \; i = n, s, r, d, \; m = 1, 2, \ldots, N_i, \tag{B.7}$$

and evolve in time according to

$$\dot{K}^i_m(t) = x^i_m(t) - \delta^i_m K^i_m(t), \; i = n, s, r, d, \; m = 1, 2, \ldots, N_i. \tag{B.8}$$

The annual benefit function (2.10) extends to

$$B(Q(t), q(t)) = \sum_{j=1}^{M} B_j(q^\circ_{\circ j}(t)) - \sum_{m=1}^{N_n} C^n_m(Q_m(t), q^n_{m\circ}(t)) - \sum_{i=s,r,d} \sum_{m=1}^{N_i} C^i_m(q^i_{m\circ}(t)) \tag{B.9}$$

and the payoff (2.11) becomes

$$\int_0^\infty \left(B(Q(t), q(t)) - \sum_{i=n,s,r,d} \sum_{m=1}^{N_i} x^i_m(t) \right) e^{-\rho t} dt. \tag{B.10}$$

The optimal policy consists of the feasible $q(t)$ and $x(t)$, $t \geq 0$, that maximize (B.10) subject to (B.5) and (B.8), given the initial water and capital stocks, where feasibility entails (B.4), (B.7), $q(t) \geq 0$, $x(t) \in [0, \bar{x}]$, $Q_m(t) \geq 0$, $m = 1, 2, \ldots, N_n$, and possibly other restrictions (e.g., no allocation of recycled water to domestic sectors).

The Hamiltonian corresponding to this problem extends (3.1) to

$$H(t) = B(Q(t), q(t)) - \sum_{i=n,s,r,d} \sum_{m=1}^{N_i} x_m^i(t) + \sum_{i=n,s,r,d} \sum_{m=1}^{N_i} \lambda_m^i(t)[x_m^i(t) - \delta_m^i K_m^i(t)] + \sum_{m=1}^{N_n} \theta_m(t)[R_m(Q_m(t)) - q_{m\circ}^n(t)], \quad \text{(B.11)}$$

where $\lambda_m^i(t)$ is the costate of $K_m^i(t)$, $i = n, s, r, d$, $m = 1, 2, \ldots, N_i$, and $\theta_m(t)$ is the costate of $Q_m(t)$, $m = 1, 2, \ldots, N_n$. The Lagrangian extends (3.2) to

$$\mathcal{L}(t) = H(t) + \sum_{i=n,s,r,d} \sum_{m=1}^{N_i} \mu_m^i(t)[\gamma_m^i K_m^i(t) - q_{m\circ}^i(t)] + \sum_{m=1}^{N_r} \xi_m(t)\,[q_{m\circ}^s(t) - q_{m\circ}^r(t)] + \sum_{m=1}^{N_n} \vartheta_m(t) Q_m(t) \quad \text{(B.12)}$$

where μ_m^i, $i = n, s, r, d$, $m = 1, 2, \ldots, N_i$, are the Lagrange multipliers of (B.7), $\xi_m(t)$, $m = 1, 2, \ldots, N_r$, are the Lagrange multipliers of (B.4) and $\vartheta_m(t)$ are the Lagrange multipliers of $Q_m(t) \geq 0$, $m = 1, 2, \ldots, N_n$.

The optimality conditions extend (3.3)-(3.8) in a straightforward manner. For example, the condition regarding water allocation from natural source m to domestic sector j extends (3.3a) to

$$D_j(q_{\circ j}^{\circ}(t)) \leq C_m^{n\prime}(Q_m(t), q_{m\circ}^n(t)) + \mu_m^n(t) + C_{j^s}^{s\prime}(q_{j^s\circ}^m)\beta + \mu_{j^s}^s(t)\beta + \theta_m(t) - \xi_{j^s}(t)\beta, \quad \text{(B.13)}$$

equality holding if $q_{mj}^n > 0$, where j^s is the index of the sewage treatment facility serving domestic sector j. The most-rapid-approach investment rule is now

$$x_m^i(t) = \begin{cases} 0 & \text{if } \lambda_m^i(t) < 1 \\ \bar{x}_m^i & \text{if } \lambda_m^i(t) > 1 \\ x_m^{ia}(t) & \text{if } \lambda_m^i(t) = 1 \end{cases}, \; i = n, s, r, d, \; m = 1, 2, \ldots, N_i, \tag{B.14}$$

the costates dynamics extend to

$$\dot{\lambda}_m^i(t) - \rho\lambda_m^i(t) = \lambda_m^i(t)\delta_m^i - \mu_m^i(t)\gamma_m^i, \; i = n, s, r, d, \; m = 1, 2, \ldots, N_i, \tag{B.15a}$$

$$\dot{\theta}_m(t) - \rho\theta_m(t) = C_{mQ_m}^n(Q_m(t), q_{m\circ}^n(t)) - \theta_m(t)R_m'(Q_m(t)) - \vartheta_m(t), \; m = 1, 2, \ldots, N_n \tag{B.15b}$$

and the complementary slackness conditions are

$$\mu_m^i(t)[\gamma_m^i K_m^i(t) - q_{m\circ}^i(t)] = 0, \; i = n, s, r, d, \; m = 1, 2, \ldots, N_i, \tag{B.16a}$$

$$\xi_m(t)[q_{m\circ}^s(t) - q_{m\circ}^r(t)] = 0, \; m = 1, 2, \ldots, N_r, \tag{B.16b}$$

$$\vartheta_m(t)Q_m(t) = 0, \; m = 1, 2, \ldots, N_n. \tag{B.16c}$$

Although more elaborate, the optimal policy is similar in structure to the above simpler situation. In particular, the most-rapid-approach property is retained and optimal $K_m^i(t)$ processes approach as rapidly as possible their respective singular pathes and proceed along them thereafter. Along the singular trajectories, $\mu_m^{ia}(t) = (\rho + \delta_m^i)/\gamma_m^i$ and $K_m^{ia}(t) = q_{m\circ}^{ai}(t)/\gamma_m^i$, $i = n, s, r, d$, $m = 1, 2, \ldots, N_i$. Provided a steady state is eventually approached under the optimal policy, the following conditions hold at a steady state:

$$\hat{q}_{m\circ}^n = R_m(\hat{Q}_m), \; m = 1, 2, \ldots, N_n, \tag{B.17a}$$

$$\hat{\theta}_m = \frac{\hat{\vartheta}_m - C_{mQ}(\hat{Q}_m, R_m(\hat{Q}_m))}{\rho - R_m'(\hat{Q}_m)}, \; m = 1, 2, \ldots, N_n, \tag{B.17b}$$

$$\hat{\mu}_m^i = (\rho + \delta_m^i)/\gamma_m^i, \; i = n, s, r, d, \; m = 1, 2, \ldots, N_i, \tag{B.17c}$$

$$\hat{K}^i_m(t) = \hat{q}^i_{m\circ}(t)/\gamma^i_m,\ i = n, s, r, d,\ m = 1, 2, \ldots, N_i, \tag{B.17d}$$

$$\hat{x}^i_m = \delta^i_m \hat{K}^i_m,\ i = n, s, r, d,\ m = 1, 2, \ldots, N_i. \tag{B.17e}$$

and (B.16b). As in the previous case, these conditions, together with the optimality conditions determining $\hat{q}^i_{mj}$, solve for the optimal steady state policy.

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