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The management of fragile resources: A long term perspective

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Abstract

Excessive exploitation diminishes the capacity of natural resources to withstand environmental stress, increasing their vulnerability to extreme conditions that may trigger abrupt changes. The onset of such events depends on the coincidence of random environmental conditions and the resource state (determining its resilience). Examples include species extinction, ecosystem collapse, disease outburst and climate change induced calamities. The policy response to the catastrophic threat is measured in terms of its effect on the long-term behavior of the resource state. To that end, the L-methodology, developed originally to study autonomous systems, is extended to non-autonomous problems involving catastrophic threats.

Keywords: Catastrophic threats, extreme events, endogenous discounting, steady state, stability.

JEL classification: C61, C62, Q54

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1 Introduction

Many resource situations reach the stage where further exploitation threatens to trigger an abrupt event that, once occurred, changes the underlying regime for the worst. When the conditions that trigger such events are well-understood and predictable, the occurrence time T can, in principle, be controlled. Often, however, these conditions are not well-understood or involve genuine stochastic elements or both, in which case T can be determined only up to a probability distribution that depends on the exploitation policy. An early example is the exploitation of a stock of unknown size, studied by Kemp (1976), where T stands for the depletion time. A slight extension of the term "depletion" to include situations in which the resource can no longer be exploited or becomes obsolete allows one to associate T with an uncertain date of nationalization (Long 1975) or with the occurrence date of various environmental catastrophes (Cropper 1976). The same framework can be used to consider advantageous events, such as the arrival of a backstop substitute (Dasgupta and Heal 1974, Dasgupta and Stiglitz 1981).

While the uncertainty in the cake-eating problem of Kemp (1976) is solely due to ignorance, the uncertainty in political (nationalization) or environmental events involves genuine stochastic elements. The distinction between the two types of uncertainty plays out most pronouncedly via the specification of the hazard rate function, measuring the probability density of the event occurrence (the realization of T) in the next time instant. In all of these variants, the optimal policy maximizes the expected payoff, where the expectation is taken with respect to the distribution of T. This distribution, in turn, depends on the nature of the event (see the survey by Tsur and Zemel

2014a).

This work considers the type of catastrophic events that are triggered by a confluence of conditions involving the resource state and genuinely stochastic elements. Such events show up in a variety of resource situations, including exploitation and exploration of nonrenewable resources (Deshmukh and Pliska 1985), biological resources vulnerable to a catastrophic collapse (Reed and Heras 1992), forest fires (Reed 1984), pollution control (Clarke and Reed 1994, Tsur and Zemel 1998), nuclear accidents (Cropper 1976, Aronsson et al. 1998), ecological regime shift (Mäler 2000, Dasgupta and Mäler 2003, Mäler et al. 2003, Polasky et al. 2011, de Zeeuw and Zemel 2012), and climate change induced calamities (Tsur and Zemel 1996, 2008, 2009, Gjerde et al. 1999, Nævdal 2006, Bahn et al. 2008).

When a full analytic solution to the management problem is not available (a common situation), the analysis is focused on long-term behavior as summarized by the optimal steady state to which the system converges. A careful examination of optimal steady states, then, allows discerning the impacts of the catastrophic threat on optimal management policies. Recently, Tsur and Zemel (2014b) generalized the L-method developed in Tsur and Zemel (2001) to characterize the location, stability and approach time of optimal steady states by means of a simple function of the resource state. The analysis in Tsur and Zemel (2014b) is confined to single-state, infinite-horizon, autonomous models. Here we extend the results to the situation of resource management under risk of abrupt change. The difficulty is that the introduction of catastrophic threats renders the underlying model non-autonomous because the accumulated hazard depends explicitly on time. The problem can be recast in an autonomous form at the cost of introducing a second state variable, but the two-states formulation also does not fall into the category considered by Tsur and Zemel (2014b). Extending the L-method to problems involving catastrophic threats allows deriving properties analogous to those of risk-free models regarding the location and stability of the optimal steady states.

The next section lays out the general framework and specifies the catastrophic threat. In Section 3, properties of optimal steady states under catastrophic threats are derived. Section 4 illustrates numerically the potential effects of catastrophic threats on optimal resource management policies. Section 5 concludes and the appendix contains technical derivations.

2 Setup

Let $X(t)$ represent a resource or environmental state at time t, e.g., the stock of mineral, freshwater, biomass or the concentration of some pollutants in the soil, water or atmosphere. The state $X(t)$ evolves in time according to

$$\dot{X}(t) = g(X(t), c(t)), \tag{2.1}$$

where the control variable $c(t)$ represents the exploitation rate at time t. Given the initial state $X(0)$, an exploitation policy $\{c(t),\, t \geq 0\}$ generates the state process $\{X(t),\, t \geq 0\}$ according to (2.1) and gives rise to the utility flow $\{u(X(t), c(t)),\, t \geq 0\}$.

The functions $g(\cdot,\cdot)$ and $u(\cdot,\cdot)$ are assumed to be twice continuously differentiable and to satisfy

$$|g_c| \geq a > 0, \quad u_{cc} < 0, \quad \text{and} \quad (g_{cc}/g_c)u_c \geq 0 \tag{2.2}$$

for all $X \in (\underline{X}, \bar{X})$ and $c \in (\underline{c}, \bar{c})$, where the subscripts denote partial derivatives with respect to the corresponding arguments. The feasible domains of the state and the control, $[\underline{X}, \bar{X}]$ and $[\underline{c}, \bar{c}]$, represent physical or regulatory constraints (see discussion in Tsur and Zemel 2014b), and a is a given positive constant. The bound on g_c implies that the action chosen is effective in controlling the evolution of the stock, while the curvature assumptions on u and g ensure that the Hamiltonian is strictly concave in c (see Appendix B). Notice that we impose no constraints on the signs of u_X or g_X, as the state X can be beneficial (e.g., a biomass stock) or damaging (e.g., a pollution stock).

In addition to its contribution to the instantaneous utility $u(\cdot, \cdot)$, the state X also affects the occurrence probability of a detrimental event of catastrophic consequences. The catastrophic threat is characterized by the occurrence probability and by what happens after occurrence. The consequences of occurrence are represented by the post-event value $\varphi(X)$. Examples of various specifications of the post-event value are presented in Section 4.

Denote the event occurrence time by T and let $F(t) = Pr\{T \leq t)$ and $f(t) = F'(t)$ be the associated probability distribution and density functions, as perceived at the initial time ($t = 0$). The stock-dependent hazard rate $h(X)$ is related to $F(t)$ and $f(t)$ according to

$$h(X(t))\Delta = Pr\{T \in (t, t + \Delta] \,|\, T > t\} = \frac{f(t)\Delta}{1 - F(t)} \,,$$

where Δ is an infinitesimal time interval. Thus, $h(X(t)) = -d\ln(1-F(t))/dt$, implying

$$F(t) = 1 - \exp\left(-\int_0^t h(X(s))ds\right) \quad \text{and} \quad f(t) = h(X(t))[1 - F(t)]. \qquad (2.3)$$

For beneficial states (e.g., when X is a productive stock), $h(\cdot)$ is non-increasing (a higher stock entails a smaller occurrence probability), whereas for harmful states (e.g., pollution), $h(\cdot)$ is non-decreasing.

Given the occurrence time T, the exploitation policy $\{c(t),\ t \geq 0\}$ generates the payoff

$$\int_0^T u(X(t), c(t))e^{-\rho t}dt + e^{-\rho T}\varphi(X(T)),$$

where ρ is the time rate of discount. Taking expectation with respect to the distribution of T, noting (2.3), yields the expected payoff

$$\int_0^\infty U(X(t), c(t)) \exp\left(-\int_0^t [\rho + h(X(s))]ds\right) dt, \tag{2.4}$$

where

$$U(X, c) \equiv u(X, c) + h(X)\varphi(X) \tag{2.5}$$

is the catastrophic-threat inclusive instantaneous benefit. Note that the exponential term in the expected payoff (the hazard-inclusive discount factor) renders the problem non-autonomous. Therefore, the problem falls outside the class of models considered in Tsur and Zemel (2014b). We turn now to extend the methodology of Tsur and Zemel (2014b) in order to characterize the steady state properties of problems involving catastrophic threats.

3 Steady state properties

The optimal policy is the feasible policy that maximizes (2.4) subject to (2.1) given $X(0) = X_0$. We assume that an optimal policy exists and denote the corresponding value (the expected payoff under the optimal policy) by $v(X_0)$. An important feature of optimal trajectories of autonomous single state problems carries over to the problem at hand:

Property 1. *When the optimal state trajectory is unique, it must be monotonic in time. If multiple optimal trajectories exist, at least one of them is monotonic.*

To verify the claim, consider first the case where the optimal state trajectory is unique. Notice that the exponential factor in the objective is similar to a simple discount factor in that a manager reaching the state X at some time t faces, at that time, the same optimization problem he would have to solve at $t = 0$ if the initial stock were the same state X. This is so because the value of $\exp\left(-\int_0^t [\rho + h(X(s))]ds\right)$ serves at t merely as an overall normalization constant for the objective which cannot affect future decisions. Consider now a non-monotonic optimal state trajectory: there exist two distinct times $t_1 < t_2$ around a local extremum of the trajectory such that $X(t_1) = X(t_2) = X$ while $\dot{X}(t_1) = g(X, c(t_1)) \neq \dot{X}(t_2) = g(X, c(t_2))$ so that $c(t_1) \neq c(t_2)$. But the optimization problem at time t_2 is identical to that at time t_1, as both state processes are initiated at the same stock X. Since $c(t_1)$ is optimal at t_1, setting $c(t_2) = c(t_1)$ must also be optimal at t_2. This contradicts the assumption of a unique optimal trajectory. When the problem admits multiple optimal solutions we can apply a consistent selection rule (e.g., always choose the maximal optimal c) to obtain a monotonic optimal state trajectory. In such cases, we shall always refer to the monotonic optimal process.

As the state space is bounded, the monotonic optimal state process must converge to a steady state:

Property 2. *The optimal state trajectory converges monotonically to a steady state.*

Let $M(X)$ represent the (not necessarily optimal) steady state exploitation policy satisfying

$$g(X, M(X)) = 0. \tag{3.1}$$

It is assumed that $M(X)$ exists and is feasible for all $X \in [\underline{X}, \bar{X}]$. Noting

(2.2), the derivative

$$M'(X) = -g_X(X, M(X))/g_c(X, M(X)) \tag{3.2}$$

is well defined. Adopting the policy $c = M(X)$ indefinitely leaves the state fixed at X and yields the expected payoff

$$W(X) \equiv \frac{U(X, M(X))}{\rho + h(X)} \leq v(X), \tag{3.3}$$

equality holding at the optimal steady state $\hat{X}$.

Define the function $L : [\underline{X}, \bar{X}] \mapsto \mathbb{R}$ by

$$L(X) \equiv [\rho + h(X)] \left[\frac{U_c(X, M(X))}{g_c(X, M(X))} + W'(X) \right], \tag{3.4a}$$

which, using (3.2) and (3.3), can be expressed as[1]

$$L(X) = \frac{U_c(X, M(X))}{g_c(X, M(X))} [\rho + h(X) - g_X(X, M(X))] + U_X(X, M(X)) - W(X)h'(X). \tag{3.4b}$$

The latter form shows how $L(\cdot)$ can be obtained from the model's primitives $u(\cdot, \cdot)$, $g(\cdot, \cdot)$, $\varphi(\cdot)$ and $h(\cdot)$.

Let $\hat{X}$ denote an optimal steady state. The function $L(\cdot)$ is used to identify candidates for such states as follows:

Property 3. *(i)* $L(\hat{X}) = 0$ *is necessary for* $\hat{X} \in (\underline{X}, \bar{X})$*; (ii)* $L(\underline{X}) \leq 0$ *is necessary for* $\hat{X} = \underline{X}$*; (iii)* $L(\bar{X}) \geq 0$ *is necessary for* $\hat{X} = \bar{X}$*.*

Property 3 extends Proposition 1 of Tsur and Zemel (2014b) to the present, non-autonomous model. The proof is presented in Appendix A.

[1]The factor $\rho + h(\cdot)$ in (3.4a) might appear redundant, as it affects neither the roots of $L(\cdot)$ nor the sign of its derivative at these roots. However, in actual applications this factor often simplifies the expression for $L(\cdot)$ which includes the $W'(\cdot)$ term, while $W(\cdot)$ has the factor $\rho + h(\cdot)$ in its denominator.

The property identifies two types of potential steady states: unconstrained steady states, where L vanishes, and constrained (corner) steady states, where $L \neq 0$. Property 3 provides necessary conditions. In fact, not every root of L qualifies as a stable steady state. The following result, which extends Proposition 2 of Tsur and Zemel (2014b) to the present model, serves to narrow down the list of candidates for a stable steady state:

Property 4. *A root X of $L(\cdot)$ cannot be a stable steady state if $L'(X) > 0$.*

The proof is presented in Appendix B.

The catastrophic threat affects the resource management problem via the hazard rate, which enters the objective (2.4) both in the discount rate and in the instantaneous benefit U. The running discount rate increases from ρ to $\rho + h(X(t))$, with two conflicting effects. First, the increased impatience promotes aggressive exploitation (less conservation) because it reduces the importance of future outcomes, thereby depresses motives to give up current utility in favor of future benefits.

Second, the discount rate $\rho + h(X)$ turns endogenous through its dependence on the stock X. When the event is damaging (i.e., $\varphi(X) < W(X)$) and the state is beneficial (e.g., a biomass stock), the endogeneity effect encourages conservation because it calls for efforts to reduce the hazard. To see this, note that the terms involving $h'(\cdot)$ in (3.4b) can be combined together to form the positive expression $h'(X)[\varphi(X) - W(X)]$.[2] Recalling that $L(\cdot)$ is decreasing at a stable steady state, this positive contribution acts to increase the value of the root, representing a higher steady state stock and more conservative extraction. Which of these conflicting effects dominates depends on

[2] The $h'(X)\varphi(X)$ term is obtained from U_X.

the magnitude of $h'(X)/[\rho + h(X)]$ and varies from case to case.[3]

Properties 3 and 4 extend the results of Tsur and Zemel (2014b) to the present case of catastrophic threat. The current situation is more complicated because the introduction of catastrophic threats renders the problem non-autonomous, and this violates a requirement in Tsur and Zemel (2014b). It turns out, however, that this difficulty can be overcome because in a small vicinity of the steady state the variations in hazard are very small, hence the non-autonomous term $\exp\left(-\int_0^t h(X(s))ds\right)$ is close to a simple exponential, similar to the standard discount factor. Thus, with some modification, the arguments of Tsur and Zemel (2014b) follow through, even though the corresponding L function obtains additional terms that account for the effects of the catastrophic threat on the optimal policy (see Appendix A).

Figure 1 illustrates the use of $L(\cdot)$ for identifying candidates for optimal steady states according to Properties 3 and 4.

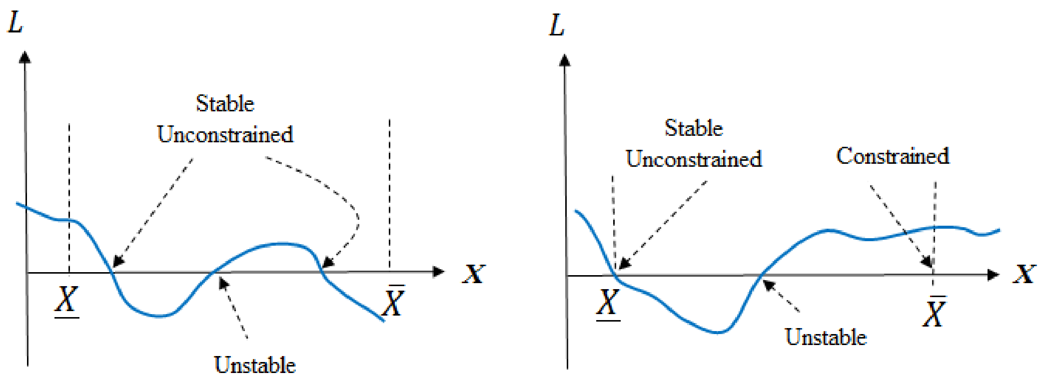

Figure 1: Possible L functions and the corresponding optimal stable steady states.

It follows from Properties 3 and 4 that $\hat{X}$ is unique in the following cases:

Property 5. *(i) If $L(\cdot)$ crosses zero once from above in $[\underline{X}, \bar{X}]$, then the steady*

[3] See the numerical illustration in Section 4 as well as the examples in Tsur and Zemel (1998) and de Zeeuw and Zemel (2012).

state $\hat{X}$ falls on the unique root of $L(\cdot)$; (ii) If $L(X) > 0$ for all $X \in [\underline{X}\,, \bar{X}]$, then $\hat{X} = \bar{X}$; (iii) If $L(X) < 0$ for all $X \in [\underline{X}\,, \bar{X}]$, then $\hat{X} = \underline{X}$.

In the next section we solve for the optimal steady states in several concrete examples with different catastrophic threats. The examples illustrate how characteristic features of catastrophic events affect optimal management policies in the long run.

4 Applications to various types of events

Catastrophic events are characterized by the corresponding post-event values and hazard rate functions. Events that impact ecosystems often entail abrupt changes in the system dynamics. The post-event value in such cases is the outcome of the management problem proceeding under the post-event regime. The discrete regime shift is in many cases a simplified description of the actual complex, non-convex behavior that underlies the ecosystem dynamics (see Polasky et al. 2011, and references they cite). A slightly more general formulation, offered by Tsur and Zemel (1998), describes the post-event value $\varphi(\cdot)$ in terms of a penalty ψ inflicted upon occurrence. This formulation distinguishes between single occurrence and recurrent events. The latter allow for multiple penalties inflicted each time the event occurs. Examples of single occurrence events include disease outbursts, affecting fish, plants or animal (including human) populations, following which there is no risk of another outburst (because the disease has led to extinction or because the remaining population became immune). As another example, consider the abrupt and massive intrusion of saline or polluted water into a freshwater lake or aquifer, which is rendered thereafter obsolete.

Recurrent events inflict a penalty ψ upon occurrence but otherwise do not change the underlying resource dynamics and the post-event problem continues under the same occurrence risk as before. The post-event value in such cases equals the pre-event value at the state of occurrence minus the penalty. Various climate change induced calamities are of this nature, e.g., category five hurricanes or forest fires (Reed 1984) with occurrence hazards that depend on climate parameters which in turn may vary with the atmospheric concentration of greenhouse gases.

In the examples below X represents a pollution stock, accumulated due to emissions from production activities. The latter generate a constant income stream which is allocated between consumption $c(t)$ and abatement $x(t)$. Normalizing the income rate to unity, c is restricted to the interval $[0, 1]$. Abatement activities, $x = 1 - c \in [0, 1]$, reduce emissions via the emission function $E(\cdot)$ given by

$$E(x) \equiv \alpha - (\alpha - \beta)x, \quad \alpha > \beta \geq 0, \tag{4.1}$$

such that the pollution dynamics take the form

$$\dot{X} = g(X, c) = E(1 - c) - \delta X, \tag{4.2}$$

where δ is a natural pollution decay parameter. The constants α and β represent maximal (no abatement) and minimal (all income is allocated to abatement) emissions, yielding $\bar{X} = \alpha/\delta$, $\underline{X} = \beta/\delta$ and $X \in [\underline{X}, \bar{X}]$. The tradeoffs between consumption and pollution are manifest in (4.2): consumption comes at the expense of abatement, increasing emissions and the associated pollution stock.

The instantaneous utility takes the iso-elastic form

$$u(X, c) = u(c) \equiv \frac{c^{1-\eta} - c_{min}^{1-\eta}}{1 - \eta}, \ \eta > 0, \tag{4.3}$$

where $c_{min} > 0$ is a given small constant (see below). Note that the pollution stock X does not enter directly in the utility function. In this example the detrimental role of pollution is introduced only via its effect on the occurrence probability. In particular, the hazard rate function is assumed linear in the stock,

$$h(X) = bX, \tag{4.4}$$

so that $h'(X) = b > 0$. By keeping a clean environment ($X = 0$), the occurrence risk can be avoided altogether.

Using (4.1)-(4.2), the steady state policy $M(X) = 1 - E^{-1}(\delta X)$ specializes to

$$M(X) = \frac{\delta X - \beta}{\alpha - \beta} = \frac{X - \underline{X}}{\bar{X} - \underline{X}}, \tag{4.5}$$

hence $M(\underline{X}) = 0$ and $M(\bar{X}) = 1$.

4.1 Single occurrence events

We consider two types of single-occurrence events. Both entail an immediate penalty ψ upon occurrence. Events of the first type damage the environment irreversibly to the extent that the post-event income flow ($t > T$) reduces to the small trickle c_{min} which must be allocated entirely for essential consumption.[4] Such consequences might follow when the event destroys some major factor of production that cannot be restored. Single-occurrence events of the second type inflict a penalty ψ upon occurrence and in addition initiate a regulation that restricts the pollution level not to exceed $X(T)$ anytime in the future.[5]

[4]Note that the utility (4.3) is normalized such that $u(c_{min}) = 0$.

[5]Such a regulation might come as a political response to appease public outrage associated with the occurrence.

Using the superscripts *so1* and *so2* to denote the first and second types of single occurrence events, the corresponding post-event values are

$$\varphi^{so1}(X) = \int_0^\infty u(c_{min}) e^{-\rho t} dt - \psi = -\psi \tag{4.6}$$

and

$$\varphi^{so2}(X) = \int_0^\infty u(M(X)) e^{-\rho t} dt - \psi = u(M(X))/\rho - \psi, \tag{4.7}$$

where $u(\cdot)$ and $M(\cdot)$ are specified in (4.3) and (4.5), respectively. These post-event values can be used to derive the corresponding functions $L^j(X)$, $j = so1, so2$. Setting the utility parameters (c_{min} and η), the discount rate (ρ), the emission parameters (α and β), the pollution decay rate (δ) and hazard sensitivity (b) equal to the values given in Table 1, leaves the penalty ψ as the only free parameter for the numerical experiments below.

Table 1: Parameter values.

Parameter	Value
c_{min}	0.05
ρ	0.03
η	2
α	0.5
β	0.01
δ	0.025
b	0.01

Using these specifications, the L-function for $so1$ events becomes

$$L^{so1}(X) = \frac{\bar{X} - \underline{X}}{\delta(X - \underline{X})^2} \left[\rho + \delta + bX + \frac{b\delta(X - \underline{X})}{\rho + bX} \right] - \frac{\rho b[\psi + 1/(\rho c_{min})]}{\rho + bX}. \tag{4.8}$$

The first term is positive and diverges at $X = \underline{X}$. Thus, $L(\cdot)$ is positive near the lower bound, which is excluded, therefore, from the list of candidate steady states (Property 3). The second term comes with a minus sign, so that L vanishes when these two terms cancel out. Note that the penalty ψ appears in the second term in combination with $(-c_{min}^{1-\eta}/(1-\eta))/\rho = 1/(\rho\, c_{min})$, that is, with the present value associated with the constant term of the utility function. This can be understood by noting that, over and above the one-time penalty ψ, $so1$ events inflict a cost in the form of reduction in consumption to the subsistence level c_{min}. Indeed, even when $\psi = 0$, the value of c_{min} (5% of income – see Table 1) is sufficiently small (hence the latter cost is high) to ensure that the second (negative) term of L^{so1} dominates the first (positive) term at $X = \bar{X}$, hence $L(\bar{X}) < 0$. Thus, L^{so1} must have a root in $(\underline{X}, \bar{X})$ and some abatement activities should take place for all $\psi \geq 0$ (see lower panel of Fig. 2).

As the penalty ψ increases, the second (negative) term of (4.8) increases (in absolute value) decreasing $L^{so1}(\cdot)$ and pushing its root to lower X values, in accordance with the extra precaution called for by the increase in damage (see Fig. 2). Note also that the coefficient of ψ in the second term of L^{so1} can be written as $h'(X)\rho/[\rho + h(X)]$, indicating the tradeoffs between the hazard endogeneity $h'(\cdot)$, which acts to reduce emissions, and the hazard inclusive discount rate $\rho+h(\cdot)$, representing impatience and acting to increase pollution. In particular, an exogenous (constant) hazard, with $h'(\cdot) = 0$, eliminates the penalty term and pushes the steady state to $\bar{X}$ no matter how large the hazard and penalty are. It is the *option to decrease the hazard*, rather than the hazard value per se, that drives the abatement policy.

These considerations are illustrated in Figure 2, where the upper panel depicts $L^{so1}(X)$ for various values of ψ. One notes that although the function is not monotonic, it possesses a unique root (where L decreases), which identifies

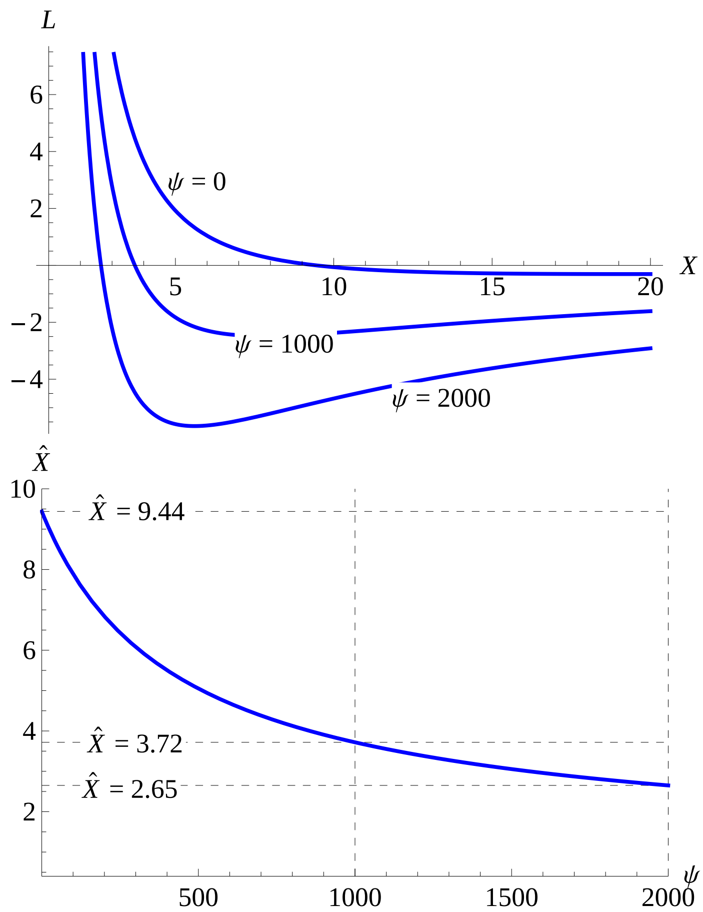

Figure 2: Upper panel: $L^{so1}(X)$ for various ψ values. Lower panel: Optimal steady state $\hat{X}$ as a function of the penalty ψ for $so1$ events.

uniquely the optimal steady state (Property 5) for each ψ value. The lower panel displays the ensuing optimal steady states $\hat{X}$ as a function of ψ. The effect of the penalty on decreasing the steady state pollution is evident.

Single occurrence events of the second type add the term $u(M(X))/\rho$ to the post event value (see equation (4.7)). Except for exceedingly small stocks (where $M(X) < c_{min}$) this term is positive hence the event is not as damaging as those of the first type. Adding the contribution of this term gives

$$L^{so2}(X) = L^{so1}(X) + \frac{h'(X)u(M(X))}{\rho + h(X)} + \frac{h(X)u'(M(X))M'(X)}{\rho}$$

hence $L^{so2}(X) > L^{so1}(X)$ for all X such that $M(X) > c_{min}$, which implies higher steady state values under this type of events. Indeed, one finds

$$L^{so2}(X) = \frac{\bar{X} - \underline{X}}{\delta(X - \underline{X})^2} \frac{(\rho + \delta)(\rho + bX)}{\rho} - \frac{\rho b \psi}{\rho + bX} \, . \tag{4.9}$$

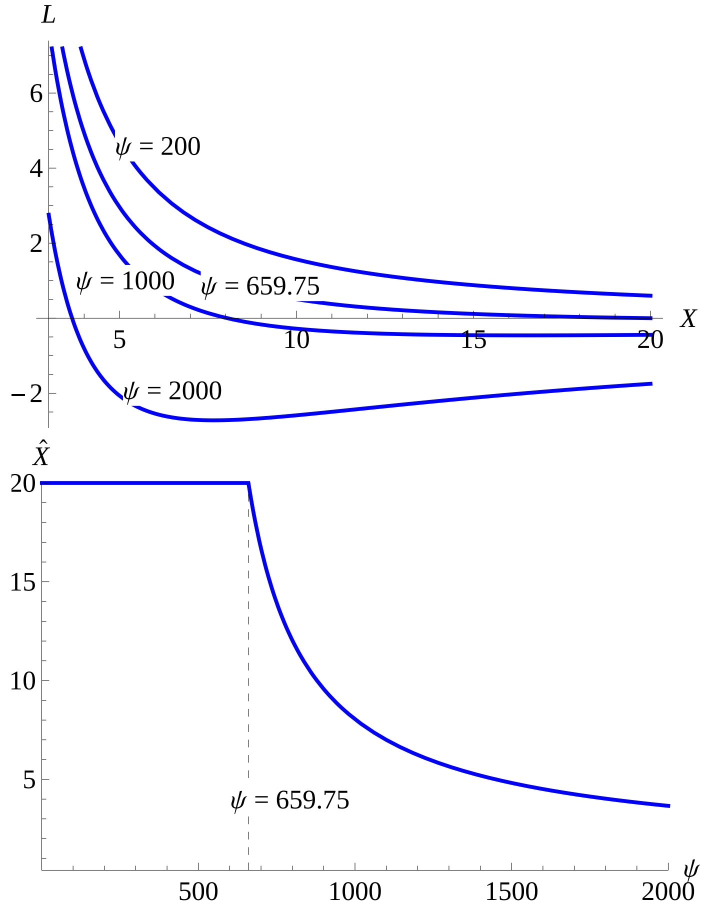

Figure 3: Upper panel: $L^{so2}(X)$ for various ψ values. Lower panel: Optimal steady state $\hat{X}$ as a function of the penalty ψ for $so2$ events.

The first term of (4.9) is again positive and tends to infinity at the lower bound $\underline{X}$. One notes that the penalty in the second term is not accompanied by $1/(\rho c_{min})$, as was the case for $so1$ events. This is so because the post-occurrence restriction to the essential consumption rate c_{min} is not imposed here. As a result, the term vanishes for $\psi = 0$ and is small for small penalties. This implies that $L^{so2}(\cdot)$ does not have a feasible root with small penalties, and the corner state $\bar{X}$ is the unique steady state in these cases (Property 5). The intuition here is that when the damage is small, abatement expenses are not justified. The penalty coefficient $h'(X)\rho/[\rho+h(X)]$ in front of ψ remains as for the $so1$ events, demonstrating that the tradeoffs discussed above hold also for this type of events.

Examples of $L^{so2}(\cdot)$ functions are displayed in the upper panel of Figure 3, where they take only positive values over the feasible domain $[\underline{X}, \bar{X}] = [0.4, 20]$ for all $\psi < 659.75$, implying that no abatement is desirable at the steady state (i.e., $\hat{X} = \bar{X} = 20$). For larger penalties, $L^{so2}(\cdot)$ admits a unique, stable root $\hat{X} \in [\underline{X}, \bar{X}]$ which defines the optimal steady state. Again, the root decreases with ψ, as shown in the lower panel of Figure 3.

4.2 Recurrent events

Recurrent events also inflict a damage ψ upon occurrence but the problem continues under the same risk of more events occurring later on. The post-event value, thus, is given by

$$\varphi(X) = v(X) - \psi, \tag{4.10}$$

where $v(X)$ is the value function, defined by

$$v(X) = \max_{c(t)} \int_0^\infty [u(c(t)) + h(X(t))\varphi(X(t))] \exp\left(-\int_0^t [\rho + h(X(s))]ds\right) dt$$

subject to (2.1), given $X(0) = X$ and $c(t) \in [0, 1]$. Since $\varphi(\cdot)$ of (4.10) contains $v(\cdot)$ and at the same time appears in the objective defining it, both functions are only implicitly defined. Nevertheless, the corresponding $L-$function can be obtained and used to characterize optimal steady state candidates in much the same way as in the previous, single-occurrence events.

According to (3.3), $W(X) \leq v(X)$, equality holding at an optimal steady state. It follows that at such a state both $v(X) = W(X)$ and $v'(X) = W'(X)$ must hold. Thus, we can use (4.10) and express $W(\cdot)$ at an optimal steady state as

$$W(X) = \frac{u(M(X)) + h(X)[W(X) - \psi]}{\rho + h(X)}.$$

Solving for $W(X)$ yields

$$W(X) = [u(M(X)) - h(X)\psi]/\rho.$$

The first term $u(M(X))/\rho$ is the steady state value *without catastrophic risk* (the relevant discount rate is the riskless rate ρ because, in this recurrent event example, occurrence does not interrupt the utility flow).

The second term measures the expected damage from an infinite series of Poisson inflicted penalties, when each penalty is discounted at the factor corresponding to the respective random occurrence time. Thus,

$$W'(X) = [u'(M(X))M'(X) - h'(X)\psi]/\rho,$$

which gives, upon substituting (4.3)-(4.5) in (3.4a),

$$L^{re}(X) = \frac{\bar{X} - \underline{X}}{\delta(X - \underline{X})^2} \frac{(\rho + \delta)(\rho + bX)}{\rho} - \frac{(\rho + bX)b\psi}{\rho}. \tag{4.11}$$

Comparing with (4.9) we see that the positive term remains unchanged, while the negative penalty term is multiplied by the factor $[(\rho + h(X))/\rho]^2 > 1$ hence $L^{re}(X)$ falls short of $L^{so2}(X)$ and its respective root is obtained at a lower value of X. Recurrent events imply more prudence than their *so2* counterparts. In fact, (4.11) can be recast in the form

$$L^{re}(X) = \frac{\rho + h(X)}{\rho} \left[\frac{\bar{X} - \underline{X}}{\delta(X - \underline{X})^2}(\rho + \delta) - h'(X)\psi \right].$$

The coefficient $(\rho + h(X))/\rho$ serves merely as a normalization factor that cannot modify the roots of $L^{re}(\cdot)$ which are determined by the expression in the square brackets. Thus, the tradeoffs depend on the hazard endogeneity $h'(\cdot)$, but not on $h(\cdot)$ per se. As explained above, the hazard-inclusive discount rate is not the relevant rate for recurrent events, hence the incentive it provides to increase emissions does not apply in this case.

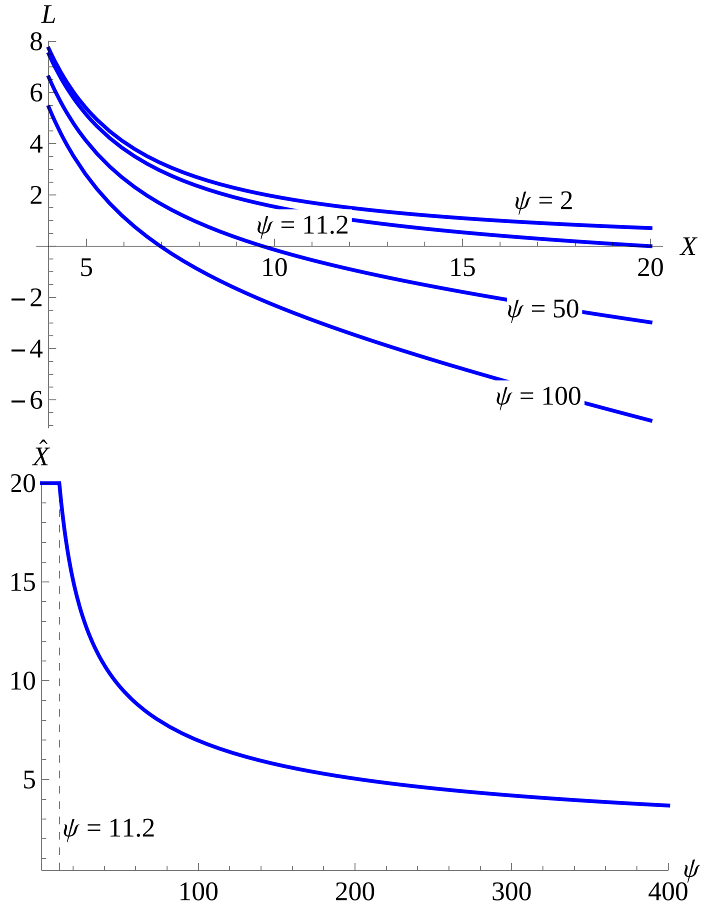

Figure 4: Upper panel: $L^{re}(X)$ for various ψ values. Lower panel: Optimal steady state $\hat{X}$ as a function of the penalty ψ for recurrent events.

Figure 4 presents $L^{re}(X)$ for different values of ψ (upper panel) and the optimal steady state as a function of ψ (lower panel). For small penalties ($\psi < 11.2$) $L^{re}(X) > 0$ for all $X \in [\underline{X}, \bar{X}]$, implying that $\hat{X} = \bar{X} = 20$

(Property 3). For $\psi \geq 11.2$, $L^{re}(X)$ admits a unique root $\hat{X} \in [\underline{X}, \bar{X}]$, where $L'(\hat{X}) < 0$, and this root is the unique optimal steady state (Property 5). Again, the root of $L(\cdot)$ decreases with ψ, as the lower panel of Figure 4 reveals.

4.3 The three events compared

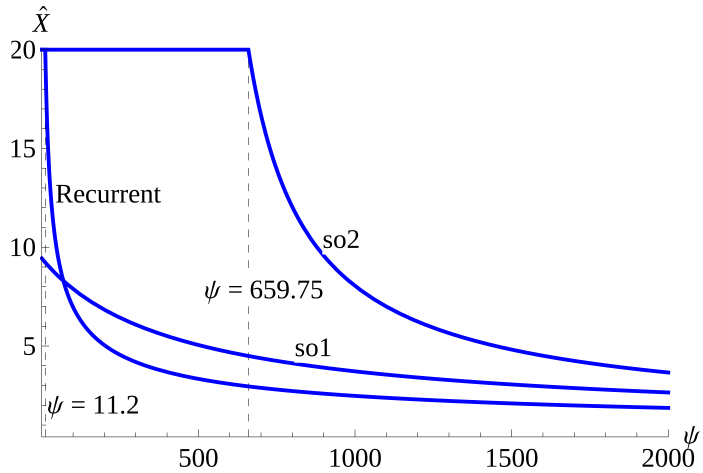

Figure 5: Optimal steady states for the three event types vs. the penalty ψ.

Figure 5 compares the optimal steady states for the three types of events. It shows that the response to catastrophic threats, in terms of abatements efforts to reduce the associated hazard, varies considerably across the three event types. Evaluated at the same one-off occurrence penalty ψ, *so2* and recurrent events are similar in that both leave no room for abatement when the penalty is small. Otherwise, recurrent events are more harmful, and the long-run pollution levels are smaller. *so1* events incorporate another component into the damage term, hence imply significant abatement even when $\psi = 0$. The crossing of the *so1* and recurrent curves shows how the relative weights of the various tradeoff considerations change with ψ.

5 Concluding comments

Situations where resource exploitation threatens to trigger abrupt catastrophic changes have become prevalent as more of our planetary resources dwindle in quantity and/or quality due to prolonged exploitation over and above their capacity to renew. In many examples, the catastrophic threats enter the resource management problem via the hazard function with two conflicting effects. First, the hazard increases the discount rate thereby reduces the importance of future utility and discourages conservation. Second, the hazard endogeneity encourages conservation. The long-run overall effect can be evaluated in terms of the steady states. It turns out that the details of the specifications of the hazard and damage associated with the events are of great importance. To study these effects, we characterize the steady states by extending the L-method of Tsur and Zemel (2001, 2014b) to non-autonomous models involving catastrophic threats.

The L-method is implemented by means of a simple function of the resource state, denoted $L(\cdot)$ and specified in terms of the model's primitives, such that an internal optimal steady state must be a root where L crosses zero from above. If only one such root exists, this root is the unique optimal steady state. When multiple roots exist, the one corresponding to the optimal steady state may depend on the initial state. The overall effect of a catastrophic threat is then identified by investigating how the details of the event specifications modify the appropriate roots of L. Examples of three prototypical events illustrate the application of the L-method and illuminate the tradeoffs discussed above.

Appendix: Proofs

The proofs in this Appendix extend the arguments of Tsur and Zemel (2014b) to non-autonomous problems involving catastrophic threats.

A Proof of Property 3

Proof. For any feasible state X we compare the value $W(X)$ obtained by the policy $c = M(X)$ with the value obtained from a small feasible variation of this policy. If the value under the variation policy exceeds $W(X)$, then X does not qualify as an optimal steady state.

For arbitrarily small constants $\epsilon > 0$ and δ, consider the variation policy

$$c^{\epsilon\delta}(t) = \begin{cases} M(X) + \delta/g_c(X, M(X)) & \text{if } t < \epsilon \\ M(X(\epsilon)) & \text{if } t \geq \epsilon \end{cases}.$$

For the short period $t < \epsilon$, this policy deviates slightly from the constant state policy, then it enters a steady state at $X(\epsilon)$. During the initial period $t < \epsilon$, $\dot{X} = g\left(X(t), M(X) + \delta/g_c(X, M(X))\right) = \delta + o(\delta)$,[6] hence

$$X(\epsilon) - X = \epsilon\delta + o(\epsilon\delta).$$

Let $\Gamma(t) \equiv \int_0^t [\rho + h(X(s))]ds$ and $g_c = g_c(X, M(X))$. The contribution of $c^{\epsilon\delta}$ to the objective during $t \in [0, \epsilon)$ is evaluated as

$$\int_0^\epsilon U\left(X(t), M(X) + \delta/g_c\right) e^{-\Gamma(t)} dt = \int_0^\epsilon U\left(X(t), M(X) + \delta/g_c\right) e^{-[\rho + h(X)]t} dt + \int_0^\epsilon U\left(X(t), M(X) + \delta/g_2\right) \left[e^{-\Gamma(t)} - e^{-(\rho + h(X))t}\right] dt\,,$$

The first integral in the right can be expressed, recalling (3.3), as

$$\int_0^\epsilon U(X, M(X)) e^{-[\rho + h(X)]t} dt + \frac{U_c(X, M(X))}{g_c(X, M(X))} \epsilon\delta + o(\epsilon\delta) = W(X)\left[1 - e^{-[\rho + h(X)]\epsilon}\right] + \frac{U_c(X, M(X))}{g_c(X, M(X))} \epsilon\delta + o(\epsilon\delta),$$

[6] The notation $o(x)$ denotes small terms such that $o(x)/x \to 0$ when $x \to 0$

and the second integral is $o(\epsilon\delta)$.

The contribution of $c^{\epsilon\delta}$ during the infinite period $t \geq \epsilon$ is

$$\int_\epsilon^\infty U(X(\epsilon), M(X(\epsilon)))e^{-[\rho+h(X(\epsilon))]t}dt = \int_\epsilon^\infty [\rho+h(X(\epsilon))]W(X(\epsilon))e^{-[\rho+h(X(\epsilon))]t}dt =$$
$$\int_\epsilon^\infty [\rho+h(X(\epsilon))]W(X)e^{-[\rho+h(X(\epsilon))]t}dt + \int_\epsilon^\infty [\rho+h(X(\epsilon))]W'(X)\epsilon\delta e^{-[\rho+h(X(\epsilon))]t}dt + o(\epsilon\delta).$$

The first integral on the second line can be expressed as

$$W(X)\int_\epsilon^\infty [\rho+h(X(\epsilon))]e^{-[\rho+h(X(\epsilon))]t}dt = W(X)e^{-[\rho+h(X(\epsilon))]\epsilon} = W(X)e^{-[\rho+h(X)]\epsilon} + o(\epsilon\delta)$$

and the second integral is approximated by $W'(X)\epsilon\delta + o(\epsilon\delta)$.

Summing the contributions of both periods gives

$$v^{\epsilon\delta}(X) = W(X) + \left(\frac{U_c(X, M(X))}{g_c(X, M(X))} + W'(X)\right)\epsilon\delta + o(\epsilon\delta),$$

or

$$v^{\epsilon\delta}(X) - W(X) = L(X)\epsilon\delta/[\rho + h(X)] + o(\epsilon\delta) \tag{A.1}$$

where $L(X)$ is defined in (3.4a).

While $\epsilon > 0$, the sign of δ can be freely chosen. Thus, if $L(X) \neq 0$ we can set $\text{sign}(\delta) = \text{sign}(L(X))$ to ensure that $v^{\epsilon\delta}(X) > W(X)$ hence X is not an optimal steady state. It follows that only the roots of $L(\cdot)$ qualify as candidates for optimal steady states. The only exceptions are the bounds $\underline{X}$ and $\bar{X}$. Choosing $\delta > 0$ is not feasible at $\bar{X}$ because this policy would lead the process outside the feasible domain. Therefore, $\bar{X}$ cannot be excluded as an optimal steady state if $L(\bar{X}) > 0$. A similar argument holds for the lower bound $\underline{X}$ if $L(\underline{X}) < 0$. □

B Proof of Property 4

Proof. Consider $S(t) = \exp\left(-\int_0^t h(X(s))ds\right)$ as a second state variable and let λ and μ denote the current-value co-states corresponding to $X(\cdot)$ and $S(\cdot)$, respectively. The current-value Hamiltonian corresponding to the problem of maximizing the objective (2.4) subject to the dynamic constraint (2.1) is

$$\mathcal{H} = U(X,c)S + \lambda g(X,c) - \mu h(X)S. \tag{B.1}$$

The necessary conditions for (an interior) optimum include:

$$U_c(X,c)S + \lambda g_c(X,c) = 0, \tag{B.2}$$

$$\dot{\lambda} - \rho\lambda = -[U_X(X,c)S + \lambda g_X(X,c)] + \mu h'(X)S. \tag{B.3}$$

and

$$\dot{\mu} - \rho\mu = -U(X,c) + \mu h(X). \tag{B.4}$$

The last equation is integrated from t to ∞, yielding

$$\mu(t) = v(X(t)),$$

where $v(X)$ is the value obtained for the maximal objective when the initial stock is X. Denoting the normalized shadow price by

$$\Lambda \equiv \lambda/S,$$

the necessary conditions take the form

$$U_c(X,c) + \Lambda g_c(X,c) = 0, \tag{B.5}$$

$$\dot{\Lambda} = [g_X(X,c) - (\rho + h(X))]\frac{U_c(X,c)}{g_c(X,c)} - U_X(X,c) + h'(X)v(X) \equiv \zeta(X,c). \tag{B.6}$$

At an optimal interior steady state $\hat{X}$, where $c = M(\hat{X})$ and $v(\hat{X}) = W(\hat{X})$, we find

$$\zeta(\hat{X}, M(\hat{X})) = -L(\hat{X}) = 0, \tag{B.7}$$

which agrees with $\Lambda(\cdot)$ being stationary at the steady state.

Next, we express the optimal control c as a function of the state variable X, say $c(t) = C(X(t))$[7] where

$$C(\hat{X}) = M(\hat{X}). \tag{B.8}$$

Define the functions

$$A(X) = g_c(X, C(X))U_{cc}(X, C(X)) - U_c(X, C(X))g_{cc}(X, C(X)), \tag{B.9}$$

$$B(X) = g_c(X, C(X))U_{cX}(X, C(X)) - U_c(X, C(X))g_{cX}(X, C(X)). \tag{B.10}$$

According to assumption (2.2), the expression $A(X)/g_c(X, C(X))$ is strictly negative, which ensures that $\mathcal{H}$ is concave in c. Taking the time derivative of (B.5) and using (B.6) to eliminate $\dot{\Lambda}$, we find

$$C'(X)\frac{A(X)}{g_c^2(X, C(X))} + \frac{B(X)}{g_c^2(X, C(X))} + \frac{\zeta(X, C(X))}{g(X, C(X))} = 0. \tag{B.11}$$

Equation (B.11) is a first order differential equation, which together with (B.8) defines $C(X)$ for all X in the relevant neighborhood. Indeed, for $X \neq \hat{X}$ the coefficient of $C'(X)$ is non vanishing while the other two terms of (B.11) are finite, hence the derivative $C'(X)$ is well defined. A difficulty with its evaluation at $\hat{X}$ arises because the function $g(\cdot,\cdot)$, appearing at the denominator of the last term, vanishes at $\hat{X}$. However, in an unconstrained steady state, $L(\hat{X}) = 0$ and the singularity is removed because $\zeta(\hat{X}, C(\hat{X})) = \zeta(\hat{X}, M(\hat{X})) = -L(\hat{X}) = 0$ (cf. (B.7)) This term, then, can be evaluated using l'Hôpital's rule. Using (B.7), we find

$$\frac{d\zeta(\hat{X}, C(\hat{X}))}{dX} = -L'(\hat{X}) + \zeta_c(\hat{X}, C(\hat{X}))[C'(\hat{X}) - M'(\hat{X})],$$

[7] For the existence and continuity of $C(\cdot)$ near the steady state, see Tsur and Zemel (2014b).

while (3.2) implies

$$\frac{dg(X,C(X))}{dX} = g_X(X,C(X)) + g_c(X,C(X))C'(X) = g_c(X,C(X))[C'(X) - M'(X)].$$

It follows that

$$\lim_{X\to\hat{X}} \left\{ \frac{\zeta(X,C(X))}{g(X,C(X))} \right\} = \frac{1}{g_c(\hat{X},C(\hat{X}))} \left(\frac{-L'(\hat{X})}{C'(\hat{X}) - M'(\hat{X})} + \zeta_c(\hat{X},C(\hat{X})) \right).$$

The last term on the right and side is obtained by taking the derivative of (B.6) with respect to c,

$$\zeta_c(X,C(X)) = -A(X)\frac{\rho + h(X) - g_X(X,C(X))}{g_c^2(X,C(X))} - \frac{B(X)}{g_c(X,C(X))},$$

which, using (3.2), reduces (B.11) in the limit $X \to \hat{X}$ to

$$\frac{A(\hat{X})}{g_c(\hat{X},C(\hat{X}))} \left(C'(\hat{X}) - M'(\hat{X}) - \frac{\rho + h(X)}{g_c(\hat{X},C(\hat{X}))} \right) + \frac{-L'(\hat{X})}{C'(\hat{X}) - M'(\hat{X})} = 0.$$

Denoting

$$\Delta(X) \equiv C'(X) - M'(X), \tag{B.12}$$

we obtain the quadratic equation

$$\Delta^2(\hat{X}) - \frac{\rho + h(X)}{g_c(\hat{X},C(\hat{X}))}\Delta(\hat{X}) - \frac{g_c(\hat{X},C(\hat{X}))L'(\hat{X})}{A(\hat{X})} = 0. \tag{B.13}$$

To determine which of the solutions of (B.13) corresponds to the stable steady-state slope-difference $\Delta(\hat{X})$, observe that the state $\hat{X}$ is attractive only if $g_c(\hat{X},C(\hat{X}))\Delta(\hat{X}) \le 0$. To see this, consider a state just below the steady state, say $X_\varepsilon = \hat{X} - \varepsilon$. To approach $\hat{X}$ from below requires $\dot{X} = g(X_\varepsilon, C(X_\varepsilon)) > 0$. Recalling that $g(X_\varepsilon, M(X_\varepsilon)) = 0$, this implies $g_c[C(X_\varepsilon) - M(X_\varepsilon)] > 0$, while $g_c[C(\hat{X}) - M(\hat{X})] = 0$. Recalling that g_c is bounded away from 0, we confirm that $g_c\Delta(\hat{X}) \le 0$.

Next, we write the solutions of (B.13) as

$$g_c(\hat{X}, C(\hat{X}))\Delta(\hat{X}) = \frac{\rho + h(X)}{2}\left(1 \pm \sqrt{1 + \frac{4L'(\hat{X})g_c^3(\hat{X}, C(\hat{X}))}{[\rho + h(X)]^2 A(\hat{X})}}\right). \quad \text{(B.14)}$$

Since $A(\hat{X})/g_c^3 < 0$, the argument of the square-root operator above does not fall short of unity only if $L'(\hat{X}) \leq 0$. In this case, we have one non-positive solution for $g_c\Delta(\hat{X})$ which can provide the boundary value $C'(\hat{X}) = M'(\hat{X}) + \Delta(\hat{X})$ for the differential equation (B.11). In contrast, if $L'(\hat{X}) > 0$, the argument falls short of unity and the two solutions in (B.14) are either positive or complex, hence (B.11) does not yield a solution that converges to $\hat{X}$. This rules out the possibility that $L'(\hat{X}) > 0$ at a stable steady state, verifying Property 4. □

References

Aronsson, T., Backlund, K. and Löfgren, K.-G.: 1998, Nuclear power, externalities and non-standard pigouvian taxes: a dynamic analysis under uncertainty, *Environmental and Resource Economics* **11**, 177–195.

Bahn, O., Haurie, A. and Malhamé, R.: 2008, A stochastic control model for optimal timing of climate policies, *Automatica* **44**, 1545–1558.

Clarke, H. R. and Reed, W. J.: 1994, Consumption/pollution tradeoffs in an environment vulnerable to pollution-related catastrophic collapse, *Journal of Economic Dynamics and Control* **18**, 991–1010.

Cropper, M. L.: 1976, Regulating activities with catastrophic environmental effects, *Journal of Environmental Economics and Management* **3**, 1–15.

Dasgupta, P. and Heal, G.: 1974, The optimal depletion of exhaustible resources, *The Review of Economic Studies* **41**, 3–28.

Dasgupta, P. and Mäler, K.-G.: 2003, The economics of non-convex ecosystems: Introduction, *Environmental and Resource Economics* **26**, 499–525.

Dasgupta, P. and Stiglitz, J.: 1981, Resource depletion under technological uncertainty, *Econometrica* **49**, 85–104.

de Zeeuw, A. and Zemel, A.: 2012, Regime shifts and uncertainty in pollution control, *Journal of Economic Dynamics and Control* **36**, 939–950.

Deshmukh, S. D. and Pliska, S. R.: 1985, A martingale characterization of the price of a nonrenewable resource with decisions involving uncertainty, *Journal of Economic Theory* **35**, 322–342.

Gjerde, J., Grepperud, S. and Kverndokk, S.: 1999, Optimal climate policy under the possibility of a catastrophe, *Resource and Energy Economics* **21**, 289–317.

Kemp, M. C.: 1976, How to eat a cake of unknown size, *in* M. C. Kemp (ed.), *Three Topics in the Theory of International Trade*, North-Holland.

Long, N. V.: 1975, Resource extraction under the uncertainty about possible nationalization, *Journal of Economic Theory* **10**, 42–53.

Mäler, K.-G.: 2000, Development, ecological resources and their management: A study of complex dynamic systems, *European Economic Review* **44**, 645–665.

Mäler, K.-G., Xepapadeas, A. and de Zeeuw, A.: 2003, The economics of shallow lakes, *Environmental and Resource Economics* **26**, 603–624.

Nævdal, E.: 2006, Dynamic optimization in the presence of threshold effects when the location of the threshold is uncertain – with an application to a possible disintegration of the western antarctic ice sheet, *Journal of Economic Dynamics and Control* **30**, 1131–1158.

Polasky, S., de Zeeuw, A. and Wagener, F.: 2011, Optimal management with potential regime shifts, *Journal of Environmental Economics and Management* **62**, 229–240.

Reed, W. J and Heras, H. E.: 1992, The conservation and exploitation of vulnerable resources, *Bulletin of Mathematical Biology* **54**, 185–207.

Reed, W. J.: 1984, The effect of the risk of fire on the optimal rotation of a forest, *Journal of Environmental Economics and Management* **11**, 180–190.

Tsur, Y. and Zemel, A.: 1996, Accounting for global warming risks: Resource management under event uncertainty, *Journal of Economic Dynamics and Control* **20**, 1289–1305.

Tsur, Y. and Zemel, A.: 1998, Pollution control in an uncertain environment, *Journal of Economic Dynamics and Control* **22**, 967–975.

Tsur, Y. and Zemel, A.: 2001, The infinite horizon dynamic optimization problem revisited: a simple method to determine equilibrium states, *European Journal of Operational Research* **131**, 482–490.

Tsur, Y. and Zemel, A.: 2008, Regulating environmental threats, *Environmental and Resource Economics* **39**, 297–310.

Tsur, Y. and Zemel, A.: 2009, Endogenous discounting and climate policy, *Environmental and Resource Economics* **44**, 507–520.

Tsur, Y. and Zemel, A.: 2014a, Dynamic and stochastic analysis of environmental and natural resources, *in* M. M. Fischer and P. Nijkamp (eds), *Handbook of Regional Science*, Springer Berlin Heidelberg, pp. 929–949.

Tsur, Y. and Zemel, A.: 2014b, Steady-state properties in a class of dynamic models, *Journal of Economic Dynamics and Control* **39**, 165–177.