Invited Presentation

# Consumer Demand Analysis According to GARP

**Julian M. Alston and James A. Chalfant**

The nonparametric approach to consumer-demand analysis—based on revealed-preference axioms—is reviewed. Particular attention is paid to questions of size and power of tests for consistency of data with the existence of a stable, well-behaved utility function that could have generated the data. An application to Australian meat demand is used to show how these notions can be quantified and how prior information about elasticities, following Sakong and Hayes, may be used to increase the power of the approach.

Consumer demand analysis has become increasingly difficult for the applied economist over the past 25 years or so. Twenty-five years ago, a single-equation linear or double-log demand equation, estimated using OLS, would have been regarded as generally acceptable (and publishable), so long as the coefficients were plausible and there were no obvious statistical problems with the model. Nowadays, such models are mostly scorned in applied work. It is virtually obligatory to estimate demand parameters in the context of a system of demand equations based on some flexible functional form. The use of flexible forms, of course, is meant to avoid the joint hypothesis that preferences are of a particular assumed form, say CES, that is inherent in more traditional specifications. These requirements thus represent a movement in the direction of avoiding the imposition of restrictions that are not desirable, and not implied by theory, while permitting the imposition or testing of restrictions that are implied by theory.

Even when flexible forms are used, there are important econometric specification issues remaining to be dealt with. For instance, how do we know that the results would not be different with an alternative functional form? Should the equations be estimated with SUR, or should the possibility of simultaneity bias be recognized? Is a price-dependent or quantity-dependent system more appropriate? How should the structure of dynamics in the error term be specified? Is a trend, a Chow test, or some other technique most appropriate for detecting structural change? Should the model be first-differenced? The implication is that results are conditional on such specification choices and must be heavily qualified.

Nonparametric demand analysis—as Varian has termed it—holds the potential to allow conclusions to be drawn with fewer qualifications. The method uses only the conditions imposed on a data set by utility maximization, through revealed-preference axioms. Simply put, the method does not involve the use of any assumed (parametric) functional forms for demands; hence, it is *nonparametric* and avoids altogether the problem of functional form. In addition, since the method does not involve regression, it avoids all of the specification concerns that arise in any regression analysis. Nonparametric demand analysis is thus free from *econometric* specification errors—those that involve regression problems—although it still must contend with what might be called *economic* specification errors—those such as erroneous assumptions concerning weak separability or aggregation over goods or consumers. However, it is fair to say that there are many unresolved questions concerning how useful the method will prove to be in applied work. Nonparametric methods are attractive in part because they avoid the fragility of inferences arising from whimsical specification choices in parametric models. The counterpoint to this apparent advantage is a concern about the properties of inferences obtained using the nonparametric approach, and a reasonable question to ask is whether the results from nonparametric methods would withstand any better an intense scrutiny such as we now impose on parametric results.

Below we illustrate how nonparametric demand analysis might be used to check consistency with the underlying theory, test the stability of prefer-

Julian M. Alston and James A. Chalfant are associate professors in the Department of Agricultural Economics, University of California, Davis.
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ences, and test weak separability. We review approaches that have been proposed to increase the power of the tests and report on results from applying these methods using data on Australia's quarterly per capita consumption of beef, lamb, pork, and chicken from 1970:2 to 1988:4. These data (from Alston and Chalfant 1991b) are similar to those we analyzed earlier (Chalfant and Alston) except that mutton is excluded, we have additional years, and some observations are slightly revised.

## Progress in Applications of Nonparametric Methods in Demand Analysis

The nonparametric approach to demand analysis uses the results of revealed-preference theory, first established by Samuelson (1938, 1948) and Houthakker, and more recently advanced by Koo (1963, 1971), Afriat, Varian (1982, 1983), and others. The early papers were aimed at establishing the properties of demands and preferences from observable decisions, rather than from axioms of behavior or arbitrarily assumed utility functions. More recently, the tendency in this literature has been to recognize revealed preference not as a substitute for utility theory, but as a set of observable implications of utility theory, making it possible to draw inferences from utility theory directly and to test whether those inferences are supported by particular data sets. Nonparametric methods have been developed to test data for consistency with utility maximization (i.e., the tests for consistency of data with the weak, strong, or generalized axioms of revealed preference—WARP, SARP, and GARP), homotheticity (HARP), weak separability, and a rationing model (Varian 1983), and to recover the properties of the underlying preferences and forecast demand behavior (Varian 1982). Varian (1992) summarizes much of the relevant theory.[1]

Although it would seem reasonable to expect researchers to apply the tests for consistency (at a minimum, WARP) as a matter of course in any demand analysis, the total number of studies that have applied the nonparametric approach remains small, and only a very small fraction of journal articles that report parametric demand systems also report whether the data were tested for consistency with GARP.[2] The low popularity of the nonparametric approach is surprising, given its potential to complement a parametric approach; for instance, a check for consistency with revealed preference seems useful as a pretest for any demand analysis. The explanation appears to be primarily that people lack confidence in the capacity of the nonparametric methods to provide definitive results. This has been described by some as an issue of power. Is a failure to reject consistency of the data with GARP strong evidence of the absence of structural change, or could the data be consistent with revealed preference even in the presence of substantial structural change?

As discussed by Landsburg, Varian (1982), Thurman, and Chalfant and Alston, when income growth is large compared to relative price variation, as in the case of aggregate consumption series, one is unlikely to find a violation of revealed-preference axioms. Each bundle of goods will be revealed preferred to all previous ones, and effectively there will be no comparable data points. Chalfant and Alston (and more recently Alston and Chalfant 1991a) suggested that when the data are such that nonparametric tests are low-powered, parametric tests are likely also to be low-powered. This discussion raises two further questions. First, what is the power of the nonparametric test, what are its determinants in a particular application, and how does it compare to the power of a corresponding parametric test? Second, what can be done to enhance the power of the nonparametric tests? In relation to the first question, there is little evidence available in the literature.[3] In relation to the second question, there has been some progress with methods for improving the power of the nonparametric test. Progress in this direction seems to require imposing nonsample evidence on the data. For example, Chalfant and Alston adjusted their data for expenditure growth in order to increase the number of comparable data points, but to do so, it was necessary to assume values for income elasticities.[4] More recently, Sakong and Hayes have bor-

[1] In particular, Varian (1992) or Varian (1982) should be consulted for a clarification of the differences between the various revealed-preference axioms. When the distinction is unimportant, we refer to GARP throughout the paper, although the statements made usually will not require that any practical distinction be made between SARP and GARP. We refer specifically to WARP in cases where the statement involves only pairwise comparisons of bundles.

[2] Empirical studies that have used nonparametric methods in consumer demand analysis include Landsburg; Swofford and Whitney (1986, 1987, 1988); Chavas and Cox (1987); Chalfant and Alston (1988); Cox and Chavas (1988, 1990); Belongia and Chalfant; Alston, Carter, Green, and Pick; Belongia and Chrystal; Burton and Young; Jensen and Bevins; Sakong and Hayes; and Fisher. Examples of parametric studies that checked for consistency with GARP include Alston and Chalfant (1987), Thurman (1987), Chalfant (1987), Barnhart and Whitney, Alston and Chalfant (1991), Piggott, Piggott and Griffith, and Wellman.

[3] Exceptions are the papers by Bronars, Aizcorbe, and Alston and Chalfant (1991a).

[4] They assumed that expenditure elasticities were all one in transforming the data, but this is not the same as testing GARP subject to a

rowed from the literature on applications of nonparametric methods to producer problems (Chavas and Cox 1990) to develop a test for consistency of data with maximization of utility, subject to restrictions on the utility function. Their method yields the implied set of expenditure elasticities that will minimize a measure of the set of "taste changes" necessary to make the data consistent with GARP, and it can be applied subject to restrictions on those expenditure elasticities.

This paper explores the issue of power of nonparametric tests for consistency of data with utility maximization. Particular attention is paid to the application of developments, such as that by Sakong and Hayes, that are designed to impose restrictions in the nonparametric approach, so as to increase the ability to glean results from the data, while avoiding the imposition of restrictions (such as functional form) that we cannot believe with confidence to be true and whose effects are often unknown.

## Nonparametric Approaches to Demand Analysis

### *Tests for Consistency*

The weak axiom of revealed preference (WARP) states that a bundle of goods, $q_1$, is revealed to be preferred to any other bundle, $q_2$, (denoted $q_1Rq_2$) that could have been purchased instead (i.e., $q_1$ is revealed to be preferred to all other bundles within the budget line when $q_1$ is purchased). The weak axiom is violated if any such bundle $q_2$ is also revealed to be preferred to bundle $q_1$ (i.e., $q_1$ lies within the budget line that applies when $q_2$ is purchased). Such a result implies that both $q_1Rq_2$ and $q_2Rq_1$, which could occur only if indifference curves had shifted, given our maintained hypotheses of utility maximization and an absence of measurement errors in the data.

A WARP violation in a two-good case is illustrated in Figure 1. On the vertical axis is shown consumption of good 2 (say chicken), and on the horizontal axis is shown the consumption of good 1 (say beef). Two budget constraints are shown, one with a relatively high chicken price (time 1, say 1960) and one with a relatively low chicken price (time 2, say 1990). In time 1 the consumption bundle is $q_1$ at price vector $p_1$ with expenditure of $E_1 = p_1'q_1$, and in time 2 the consumption bundle is $q_2$ at price vector $p_2$ with expenditure of $E_2 = p_2'q_2$. In time 1, $q_1Rq_2$ while in time 2, $q_2Rq_1$, which is a violation of WARP.[5]

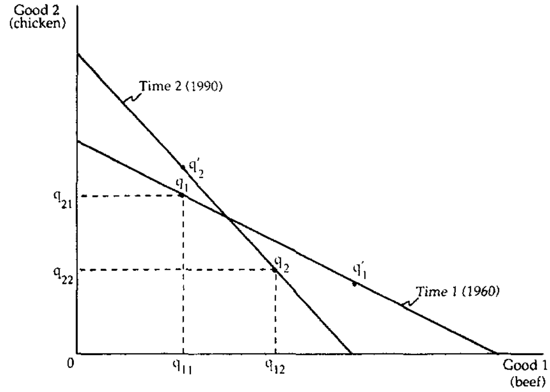


**Figure 1. A WARP Violation**

Some other possibilities also are illustrated in Figure 1. For example, if the bundle in time 1 had been $q_1'$ instead of $q_1$, there would be no violation—$q_1'Rq_2$; however, the converse is not true. Similarly, if the bundle in time 2 had been $q_2'$ instead of $q_2$, there would be no violation—$q_2'Rq_1$—but the converse is not true. Alternatively, if the two bundles had been $q_1'$ (instead of $q_1$) and $q_2'$ (instead of $q_2$), neither would be revealed preferred to the other—the bundles are not comparable in such a case.

Let us now generalize this discussion to $N$ goods. If the $T$ by $N$ matrix $\mathbf{P}$ consists of $T$ observed price vectors of length $N$, and the $T$ by $N$ matrix $\mathbf{Q}$ consists of the corresponding bundles of quantities consumed, then each element $C_{ij}$ of the matrix $\mathbf{C} = \mathbf{PQ}'$ gives the cost, at time $i$ prices, of buying time $j$'s bundle of goods. Thus, the elements in column $j$ give the cost, at various price vectors, of obtaining the consumption bundle $q_j$, while the elements in any row $i$ allow a comparison

restriction on the income elasticities, because there is no restriction on the elasticities in the test with the transformed data. The same procedure was also used by Burton and Young, and Jensen and Bevins.

[5] Note that in order to generate a WARP violation, we have contrived a counterfactual situation in which a chicken-intensive bundle was consumed in period 1 when chicken was relatively expensive. With only two goods, holding expenditure constant, it appears that a WARP violation can be generated only by one price rising and the other one falling, so that budget lines cross, with an increase in consumption of the good whose price has risen and a decrease in consumption of the good whose price has fallen. Thus, the only way to have a WARP violation and have the trends in data that have occurred in most countries (a downward trend in the price and increasing consumption of chicken) is to include more than two goods. If the pork price, say, also moves with the prices of chicken and beef, a WARP violation that does not require chicken prices to rise could be generated. The only practical importance of this result is that it indicates that testing for stable preferences using WARP and only two goods against an alternative hypothesis of increased preference for chicken is possible only for certain patterns of relative prices, but this does illustrate the importance of adequately defining the relevant group of goods for the analysis.

of the costs of various bundles at the fixed set of prices $p_i$.

A time series of prices and quantities of $N$ goods can be checked easily for consistency with the weak axiom of revealed preference.[6] It is convenient to summarize this information using a matrix $\mathbf{\Phi}$, which is defined by dividing every element of $\mathbf{C}$, $C_{ij}$, by the corresponding diagonal element, $C_{ii}$. A WARP violation corresponds to finding both $\mathbf{\Phi}_{ij}$ and $\mathbf{\Phi}_{ji}$ less than one. The absence of any such violations is consistent with stable, well-behaved preferences. However, finding that the data are consistent with the weak axiom does not rule out the problem of intransitivity. Even when there is no WARP violation with $q_i$ and $q_j$, it is necessary also to check for consistency with the strong axiom (SARP). This involves a search for intransitivity in the data to see if a combination of bundles $q_i$, $q_j$, and $q_k$ can be found that together imply $q_iRq_k$, $q_kRq_j$, and $q_jRq_i$. The number of bundles of goods that can come between $q_i$ and $q_j$ is limited only by the size of the data set. The data are consistent with SARP if no such intransitivities are found using $\mathbf{\Phi}$. When no such violations are found, it is possible to "rationalize" the data—the data could have been generated by the maximization of a stable, well-behaved utility function.

The Australian meat consumption data are consistent with WARP, SARP, and GARP. Thus, the data could have been generated by the maximization of utility, with the goods comprising a weakly separable group, and it follows that the imposition of symmetry, homogeneity, and adding-up conditions in a parametric system could therefore be justified (although they might not hold in a particular form). The only distinction between GARP and SARP is that GARP allows multiple solutions to the consumer's optimization problem for a given price vector and total expenditure (i.e., a given budget constraint). Since, as is likely in such data sets, we never observed an exact repeat observation of the budget constraint in this data set, there were no opportunities to observe a violation of SARP that was not also a violation of GARP (as can arise only when two different consumption bundles are chosen for two different observations with an identical budget constraint). Both GARP and SARP, of course, imply WARP.

*Separability*

Varian (1983) considers testing separability using GARP. The method, briefly, consists of checking whether subgroups of goods satisfy GARP separately and whether aggregates then satisfy GARP in a "first-stage utility function." The latter step is checked, in Varian's software, by attempting to construct the so-called Afriat numbers (Varian 1983).

Barnett and Choi have shown convincingly that this latter requirement is too strong; that is, Varian's test will reject separability too often. On the other hand, since satisfying GARP in each subgroup is necessary but not sufficient, finding that a subset of goods violates GARP is sufficient to reject weak separability. If a subset of goods passes GARP, one may still want to worry about the sufficient conditions related to the first stage.

Several studies have used Varian's test for separability, including Alston, Carter, Green, and Pick; Belongia and Chalfant; Belongia and Chrystal; and Swofford and Whitney (1987, 1988). Although there are some questions about the interpretation of the results from either the necessary condition alone (i.e., GARP applied to subgroups of goods) or the necessary and sufficient conditions together (i.e., GARP plus the Afriat inequalities), the results of both tests are of interest.

We noted that the Australian data are consistent with GARP. Thus, at least one of the conditions for separability has been met. The results from testing various subgroups of the Australian meat consumption data indicate that none of the subgroups satisfy both the necessary and sufficient conditions, and only one subgrouping—{beef and chicken} and {lamb and pork}—satisfied the necessary conditions for separability. Prior beliefs alone are probably sufficient to convince most agricultural economists that chicken and beef are not weakly separable from lamb and pork, so it is not surprising that the Afriat conditions do not hold in this case.

## Interpretation–Size and Power of the Nonparametric Tests

The results above may be interpreted as supporting the view that the Australian meat consumption data

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[6] To illustrate the simplicity of these computations using SHAZAM (White et al. 1990), checking WARP consists of the following steps, assuming that variables $P_1 - P_N$ and $Q_1 - Q_N$ are available:

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COPY P1-PN PRICES
COPY Q1-QN QUANTS
MATRIX C = PRICES*(QUANTS');
MATRIX D = INV(DIAG(DIAG(C)))
MATRIX PHI = D*C
DO # = 1,N
GEN1 I = # + 1
DO % = I,N
MATRIX PHI#% = PHI(#,%)
MATRIX PHI%# = PHI(%,#)
GEN1 WARP = (PHI#%.LT.1). AND. (PHI%#.LT.1)
ENDIF(WARP.GT.0)
ENDO
ENDO
```

set represents a weakly separable group of goods in which all of the restrictions from consumer theory can be assumed to hold. However, questions have been raised in the literature about the appropriate interpretation of such findings. Can the results be taken with any confidence, or must there be a very large structural change in demand before it can be detected by GARP, as was suggested by the results of Alston and Chalfant (1991a)—that is, what is the so-called *power* of the test? Suppose that a violation was detected, as occurred with Chalfant and Alston. Does that mean that utility theory must be abandoned, or is there a measurement-error interpretation—that is, what is the *size* of the test? The nonparametric tests are capable of yielding both false positives and false negatives, and relatively little is known about either, especially the latter.

### *"Size" of the Tests*

The nonparametric tests based on revealed preference include no provision for the fact that prices and quantities are likely to be measured with error. It is possible to think of each observed quantity as being made up of two components, the true quantity and a measurement error.[7] It is the true quantities that are of interest and that should be tested for consistency with utility maximization, not those measured with some error. The "size" question then becomes whether there can be constructed a set of "small" measurement errors to be subtracted from the observed quantities in such a way so as to satisfy GARP. Varian (1985) has shown how a comparison of the variance implied by the constructed measurement errors to a hypothesized value permits significance levels to be assigned to the deviations from consistency.

To see how this might work, consider Figure 2, which duplicates the WARP violation that was depicted in Figure 1. In this figure, we can consider what sizes of measurement errors (i.e., adjustments of the observed quantities) would be necessary to eliminate the WARP violation. For example, consider the quantity of beef (good 1) in time 1. In order to eliminate the violation, we could either increase $q_{11}$ to $q_{11}^H$, or reduce $q_{11}$ to $q_{11}^L$, holding the consumption of chicken constant at $q_{21}$. In the first case, increasing $q_{11}$ to $q_{11}^H$ shifts the time 1 budget constraint to the right in parallel to the point where the time 2 consumption bundle is

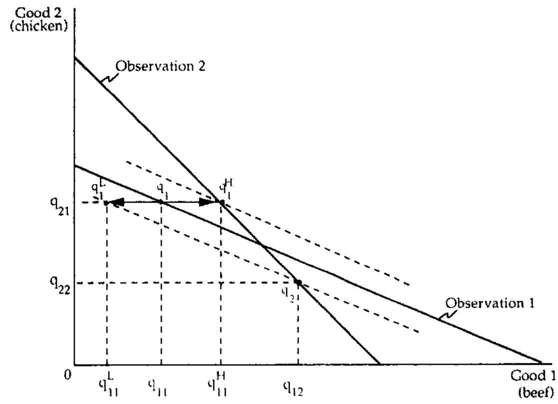


**Figure 2. Measurement Error and WARP Violations—Size of WARP Tests**

no longer revealed preferred to it. In the second case, reducing $q_{11}$ to $q_{11}^L$ shifts the time 1 budget constraint to the left in parallel to the point where the time 1 consumption bundle is no longer revealed preferred to that in time 2. That is, a positive or negative error in measuring the quantity of good 1 in time 1 could have accounted for the WARP violation, and the minimum size of either a positive, or a negative, measurement error necessary to remove the violation can be computed from the data. Another useful insight from Figure 2 is that, everything else equal, a violation of WARP requires that consumption of good 1 in time 1 must lie in the range between $q_{11}^L$ and $q_{11}^H$.

Clearly, since we chose good 1 and time 1 arbitrarily for this illustration, positive or negative measurement errors in either good, at either time period, could account for the violation, so there are eight possible cases to consider (two goods by two time periods by two directions for errors). Thus we can define a set of ranges for each of the goods in each of the time periods that are consistent with the WARP violation. More generally, some combination of such errors could be considered and a programming algorithm may be applied to calculate the minimum sizes of such errors necessary to adjust a data set in order to achieve consistency with GARP (Varian 1985). Such information permits a judgment to be made about whether the measurement-error interpretation of a violation is plausible, or whether the necessary adjustment to the data is too great to be plausible as a measurement error.

Chalfant and Alston found that small adjustments to Martin and Porter's Australian meat consumption data (consistent with two small measurement errors in mutton consumption) were sufficient to achieve consistency with GARP. Burton

[7] The same can be said of prices. Keeping with the practice to date, we take observed prices and expenditures as exact, and focus on measurement errors in the quantities.

and Young found that a similar set of adjustments in U.K. beef consumption could explain the violations of GARP that they found. Another example, in the context of the demand for money, is provided by Belongia and Chrystal. These studies illustrate how one could reduce the size of the test for consistency (i.e., reduce the probability of a false rejection of stable preferences) by allowing small departures from the revealed-preference axioms. However, much more work is required before particular rejection probabilities can be associated with such conditions.[8]

### *"Power" of the Tests*

There are several notions of power worth considering. All relate to the probability of rejecting the null hypothesis that the data were generated by maximization of a stable, well-behaved utility function. The first concept focuses on quantities and involves measurement error, as did the discussion of size, that is, by how much would a particular quantity have to change before it could no longer be believed to be consistent with the utility function that generated the other observations? By how much would a quantity have to be mismeasured before GARP detected the error? In Figure 2 we saw the range for good 1 in time 1 needed to generate a WARP violation. In order to determine power, we need to see what sizes of measurement errors are necessary to shift the budget constraint into such a range, and thereby generate a WARP violation, when the data are initially consistent with WARP. If that amount were large, power could be said to be low.

Figure 3 corresponds closely to Figure 2, but, in Figure 3, the budget constraint in time 1 is drawn farther to the right so that there is not a WARP violation: $q_1Rq_2$, but the converse is not true. Thus, in order to induce a WARP violation by adjusting $q_{11}$, we can shift the time 1 budget constraint in parallel to the left to a quantity between $q_{11}^L$ and $q_{11}^H$. This is consistent with a positive shock to $q_{11}$ that is at least as great as $e_1 = q_{11}\text{-}q_{11}^H$, but no more than $e_2 = q_{11}\text{-}q_{11}^L$. The power of the test will be large if $e_1$ is small relative to plausible sampling (measurement) errors and $e_2$ is large relative to plausible sampling errors. As with the "size" of the tests, the problem becomes more complicated when we consider more than two goods, more than two observations, measurement errors in prices as well as quantities, joint distributions when we have combinations of multiple measurement errors, and intransitivities (i.e., SARP rather than WARP).

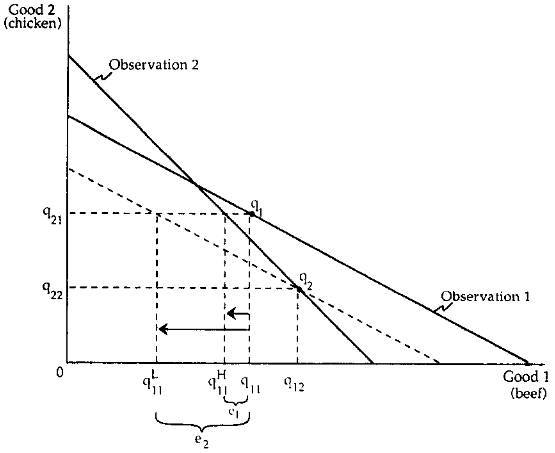


**Figure 3. Measurement Error and WARP Violations—Power of WARP Tests**

This is one concept of power. Another relates to the extent to which the data contain comparable observations (i.e., at least one is revealed preferred to the other) and, where there are comparable observations, the extent to which the data are dominated by income growth. Chalfant and Alston characterized this problem in terms of the properties of the $\Phi$ matrix. The ratios in $\Phi$, $\Phi_{ij} = C_{ij}/C_{ii}$, indicate whether the bundle of time $j$ quantities was affordable at time $i$ prices: a ratio less than one indicates $q_iRq_j$. A WARP violation occurs when two matching off-diagonal elements in the $\Phi$ matrix are both less than one: that is, $\Phi_{ij} < 1$ and $\Phi_{ji} < 1$.

When real expenditures are rising through time, it is likely that the cost at time $i$ of buying bundles purchased earlier in the sample will be less than expenditures at time $i$. Similarly, the cost of any bundle purchased later in the sample, when measured using time $i$ prices, it likely to exceed actual expenditure at time $i$. An extreme case is when every consumption bundle is revealed preferred to every previous consumption bundle and when current expenditure is not sufficient to afford the cost of any future bundle, when measured at current prices. In such a case, all of the elements of $\Phi$ above the diagonal would be greater than one, and all of the elements below the diagonal would be less than one. To have any chance of finding a WARP violation requires that at least some elements of $\Phi$ be less than one on both sides of the diagonal. Thus, intuitively, WARP violations will

[8] Varian (1985, 1988) and Tsur (1989, 1991) provide discussions of tests with measurement errors. See Burton and Young for an application of Varian's test to meat demand.

be unlikely in data sets in which there are few instances of $\Phi_{ij} < 1$ on one side of the diagonal. Any procedure that increases the number of instances of $\Phi_{ij} < 1$ will increase the chances of finding a violation. Chalfant and Alston (p. 404) calculated the numbers of observations above and below the diagonals in their annual U.S. data and their quarterly Australian data. In the Australian data set, they found 1,345 observations of $\Phi_{ij} < 1$ below the diagonal and 1,088 above the diagonal, which led them to suggest that "concerns over power need not be great." This is true for the concern over rising expenditures, at any rate. Their results with U.S. data showed only 73 observations of $\Phi_{ij} < 1$ above the diagonal and 581 below the diagonal, a less satisfactory situation. In our revised Australian data, there are 1,456 cases of $\Phi_{ij} < 1$ above the diagonal and 1,427 cases below. Such evidence is only circumstantial in relation to the power of the nonparametric tests, that is, it indicates only that it is at least feasible to find violations, not whether they are probable nor whether only very large taste changes will be detected.[9]

Finally, power also depends on relative prices, the nature of the consumption data, and the nature of the hypothesized shift in preferences. An example adapted from Gross illustrates this point. Suppose that the underlying utility function is of the Cobb-Douglas form:

$$u = x_1^{\alpha_1} x_2^{\alpha_2}.$$

In time 1, $\alpha_1 = .9$ and $\alpha_2 = .1$, while the values are reversed for time 2. As is well-known, the $\alpha$'s give the utility-maximizing budget shares. Suppose income is \$10,000 in both time periods. Finally, suppose that in time 1, prices are \$99 for good 1 and \$101 for good 2, while in period 2, both prices are \$100. Thus, optimal consumption in time 1 is 10.1 of good 1 and 89.11 of good 2 (ensuring budget shares of .9 and .1, respectively), while in period 2, it is 90 units of good 1 and 10 of good 2.

This is obviously a large change in preferences between two data points; it is much larger than what we think we are testing for in typical meat demand studies. Let us consider checking these data points using WARP. It turns out that $\Phi_{12} = .992$ (Gross, p. 417). Calculating $\Phi_{21}$ yields .9012, and a violation of WARP. That is an encouraging result. Under the assumption that the data were generated without measurement errors, WARP detected the shift in the preferences. However, had consumption in period 2 of the first good been higher by just .81 (due to either measurement error or an increase in expenditures spent only on that good), the ratio $\Phi_{12}$ would equal one and the WARP violation would vanish. This shows that certain patterns of preference shifts and relative prices, although they may represent substantial departures from the null hypothesis, may not be detected. Of course, this example is somewhat artificial, since only two data points were used and there was little variation in relative prices. It does illustrate the importance of determining the region in which WARP or GARP violations would be detected to develop a feel for the power of the test.

A formal approach to this calculation can be taken. A useful thing to know would be, for a given price vector and level of expenditures, what range of observations for quantities would be violations of stable preferences and what range would not. For instance, we should be able to state a decision rule analogous to what can be done with statistical hypothesis tests. If the outcome is in the "critical region," we reject the null hypothesis, while we continue to maintain it for all other outcomes. What quantities of consumption for meats, next period, would lead us to reject stability?

We have outlined already with the figures for the two-good case how one could do such an analysis for any data point. We attempted a simple simulation, for each of our goods, to see by how much we would have to change each data point before we generated a violation of WARP.[10] Simply by adding or subtracting small increments to each observed quantity, one should be able to find out how large an interval around that quantity there is before a violation is encountered. We were surprised, at first, to find that the results were occasionally sensitive to step-size; that is, it was sometimes possible to jump completely over the relatively small region where two data points were comparable. But, as shown in Figure 3, such an outcome is to be expected because there is both an upper and a lower limit of measurement errors that

[9] Chalfant and Alston also discussed the interpretation of the size of the elements of the $\Phi$ matrix as indication of the severity of a WARP violation. They inferred that in a case where both $\Phi_{ij}$ and $\Phi_{ji}$ are both less than one, but very close to one, as they found in the Australian data, the violation might be attributable to small measurement error. Such inferences should be drawn with great care. Gross shows, using Cobb-Douglas utility functions, how two consumers with very different utility functions may be characterized by $\Phi_{ij}$ and $\Phi_{ji}$ close to one. Thus, while small changes in quantities might restore consistency with GARP, this is no guarantee that the underlying preferences were, in fact, stable. He suggested, instead, using a money metric utility function in order to evaluate the importance of violations in terms of the necessary adjustment in expenditures.

[10] For simplicity, we checked only WARP and not GARP, so anything we found about the probability of a violation of consistency is a lower bound on the true probability.

will lead to a WARP violation, complicating the design of experiments to measure power.

Consider two data points, observed at times 1 and 2. Price vectors are $p_1$ and $p_2$, and the corresponding quantity vectors are $q_1$ and $q_2$, so that total observed expenditures are $C_{11}$ and $C_{22}$ (where $C_{ij} = p_i'q_j$, as defined earlier). A violation of WARP occurs if $C_{12} < C_{11}$ ($\Phi_{12} < 1$) and $C_{21} < C_{22}$ ($\Phi_{21} < 1$). This may be stated as: $C_{12} - C_{11} < 0$ and $C_{21} - C_{22} < 0$ (i.e., $p_1'[q_2 - q_1] < 0$ and $p_2'[q_1 - q_2] < 0$). Suppose there currently *is not* a violation of WARP between observations 1 and 2. What could cause one? Below we consider three alternative scenarios and show how to calculate changes in the quantity $q_{1j}$ of a particular good $j$ at time 1 (as adjustments to correct for hypothetical measurement errors) necessary to generate a WARP violation:

*Case 1:* $\Phi_{12} < 1$ ($p_1'[q_2 - q_1] < 0$) and $\Phi_{21} > 1$ ($p_2'[q_1 - q_2] > 0$).

In this case, to cause a WARP violation, we have to increase $C_{22}$ relative to $C_{21}$. To do this we can either increase $q_2$ or reduce $q_1$. Considering the particular observation of good $j$, in time 1, $q_{1j}$, the adjustment necessary to make $C_{21} - C_{22} < 0$ is given by $\Delta q_{1j} < [C_{22} - C_{21}]/p_{2j}$, which is negative. However, reducing $q_{1j}$ also affects $\Phi_{12}$ through $C_{11}$; the adjustment to $q_{1j}$ can cause a WARP violation only so long as it does not also lead to a decrease in $C_{11}$ sufficient to make $C_{12} - C_{11} > 0$. Thus, there is also a maximum reduction in $q_{1j}$ so that $\Delta q_{1j} > [C_{12} - C_{11}]/p_{1j}$, which is also negative. Clearly, as discussed earlier, such a range may or may not exist for each good in each time period. One way to learn about which observations are informative or influential in a particular data set is to construct bounds such as these.

*Case 2:* $\Phi_{12} > 1$ ($p_1'[q_2 - q_1] > 0$) and $\Phi_{21} < 1$ ($p_2'[q_1 - q_2] < 0$).

This case is the converse of case 1. In order to generate a WARP violation, we must increase $q_{1j}$ by an amount that satisfies both inequalities in $[C_{22} - C_{21}]/p_{2j} \geq \Delta q_{1j} \geq [C_{12} - C_{11}]/p_{1j}$, which may or may not be feasible.

*Case 3:* $\Phi_{12} > 1$ ($p_1'[q_2 - q_1] > 0$) and $\Phi_{21} > 1$ ($p_2'[q_1 - q_2] > 0$).

In this case, in order to generate a WARP violation, it is necessary to make at least two adjustments (i.e., increasing $q_{1j}$ would move $\Phi_{12}$ in the right direction, but it would move $\Phi_{21}$ in the wrong direction). For example, one could increase $q_{1j}$ and decrease $q_{2j}$. Generating a WARP violation in this manner should be most likely when the relative prices of the two goods change a lot between the two periods. For instance, increasing $q_{1j}$ to increase $C_{11}$ has the greatest effect when $p_{1j}$ is large and, at the same time, has the least effect on increasing $C_{21}$ when $p_{2j}$ is small. While determining what will cause a violation of GARP involving a number of data points is more complicated, the role of relative price changes seems clear: other things equal, more variation in relative prices will likely increase the power of the nonparametric approach.

## Increasing the Power of the Tests by Imposing Prior Beliefs

We have shown a procedure by which it is possible to describe the events that would lead to a rejection of stable preferences. Alternatively, it permits us to define all combinations of bundles of goods, prices, and expenditures that are compatible with the preferences that are consistent with a given set of observations, which has the potential to improve our understanding of what can be learned from nonparametric tests and what can be done to increase their power. A similar exercise could presumably be undertaken in any parametric study, perhaps using tests for influential data points, although we know of no examples.

The work of Sakong and Hayes represents the most ambitious attempt to date to impose prior beliefs in the nonparametric approach. It is also a major innovation in the nonparametric approach. They noted (as did Chalfant and Alston) that if the preferences that rationalize the data also imply properties that are considered to be implausible, such as that one of the goods is inferior, it might be better to conclude that the data cannot be rationalized. The method of adjusting the data and retesting GARP, as in Chalfant and Alston, is unattractive since results depend on the elasticities chosen for the adjustment. Sakong and Hayes reformulated the problem as one of computing the set of expenditure elasticities that minimizes the taste changes necessary to rationalize the data. This is a straightforward linear programming problem. They then measured the minimum taste changes necessary to rationalize the data, subject to the constraint that expenditure elasticities for all meats are non-negative.

The key insight behind this approach is that it is the compensated demands, adjusted for changes in money income and prices, that should be tested for stability. This way, one avoids altogether the problems associated with rising total expenditure that had been identified in the previous literature. However, it means that expenditure elasticities are used for more than adjusting for changes in expenditure, since they provide information on allowable responses to price changes (in the form of the income effect). One is free to add any other constraints on elasticities, such as a smoothness constraint. Sakong and Hayes added a constraint on how much the expenditure elasticities could change from observation to observation. An alternative is simply to constrain their ranges, since expenditure elasticities might change by a large amount between two data points, if relative prices happened to change greatly.

Sakong and Hayes used meat consumption data sets for a number of countries, including one for U.S. meat consumption that is essentially the same as that used by Chalfant and Alston, except that Sakong and Hayes deleted fish. They used a smaller sample, but it includes the years in which most observers speculate that there was a shift in preferences away from beef and toward chicken.[11] They found that their per capita U.S. data for beef, pork, and chicken were consistent with GARP, but that, when income elasticities were restricted to be non-negative, some violations of WARP were observed. Imposing a smoothness constraint on the fitted expenditure elasticities increased the implied taste changes somewhat, as one would expect.[12] Sakong and Hayes noted that the preference shifts they found were of roughly the same magnitudes as those obtained by Moschini and Meilke, using different data and different techniques, lending some support to both sets of results. As noted earlier, this illustrates one use of the nonparametric approach. Finding that data are consistent with GARP can be used as prior justification for expecting demand theory (i.e., symmetry, homogeneity) to hold in a particular parametric demand system. Sakong and Hayes have taken this one step further, by showing that trends or other "taste change" factors are needed to keep the estimated system consistent with prior beliefs about elasticities.[13]

### *Results from the Australian Data*

We made use of the Sakong and Hayes method to retest the consistency of quarterly meat consumption data from Australia, subject to restrictions on expenditure elasticities. The first set of restrictions we tried was to set every expenditure elasticity equal to 1.0. This was suggested by Chalfant and Alston as a benchmark case for compensating for changes in expenditures. We found, using 48 observations (1977:1 to 1988:4), that a number of data points were inconsistent with the revealed-preference conditions and unitary expenditure elasticities.

Figure 4 shows the implied "taste changes" for beef and chicken: the vertical axis measures the change in demand needed to rationalize the observations, so that a positive value measures a shift toward that good. The sum of all of the values of beef over the 12-year period is 0.57 kg, and the sum of all of the values for chicken is $-0.97$ kg. The mean values of quarterly per capita consumption for these two meats, for the same observations, were 11.28 kg for beef and 5.10 kg for chicken. Thus, the implied taste changes show a cumulative shift *toward* beef and away from chicken. This is certainly contrary to expectations, given the usual hypotheses about taste changes. Pork and lamb each were also subject to some negative shifts, on average, with the sum over the 48 quarters for lamb being $-1.35$ kg (against a mean value of 3.91 for consumption), and the sum for pork equal to $-0.38$ kg (against mean consumption of 3.89 kg). These cumulative sums could be misleading because they mask the variation in the individual observations. For instance, for beef, reducing any one of the three positive spikes—observations 9 (1979:1), 27 (1983:3), and 47 (1988:3)—would be sufficient to eliminate (or reverse) the positive overall effect; similarly, for chicken, reducing either of the two negative

[11] Deleting observations from the nonparametric approach is likely to have little effect on the comparisons between data points that remain. Sakong and Hayes, for instance, obviously cannot say anything about taste changes prior to 1971, since their data include the years 1971 to 1984, but the comparisons within their sample are likely to be a lot less sensitive to the inclusion of the early years than are parametric results. The only way the presence or absence of, say, 1970 could matter in detecting any shifts in preferences between 1971 and 1984 is if there was some intransitivity that would not be detected, of the form $q_{71}Rq_{70}$, $q_{70}Rq_{84}$, and $q_{84}Rq_{71}$, that did not also show up as a WARP violation between 1971 and 1984. This is an important aspect of revealed-preference approaches. There is no need to rely on large-sample properties of estimators, so if only the years 1971 to 1984, say, are of interest, only data for those years need be included.

[12] Similar results were obtained for other countries; imposing non-negative income elasticities led to increased frequencies of violations.

[13] Chalfant and Wallace, in a production study using Varian's (1984) Weak Axiom of Cost Minimization (WACM), found a number of violations of the cost-minimization assumption when analyzing data from trucking firms around the time of deregulation of the trucking industry. They observed that there were characteristics of firms that helped to explain these violations—supported by a logit analysis of the violations—which justified including the characteristics of firms as "shifters" in parametric cost functions. Larson, Klotz, and Chien have experimented with determining how many utility functions are necessary to rationalize a cross-sectional data set.

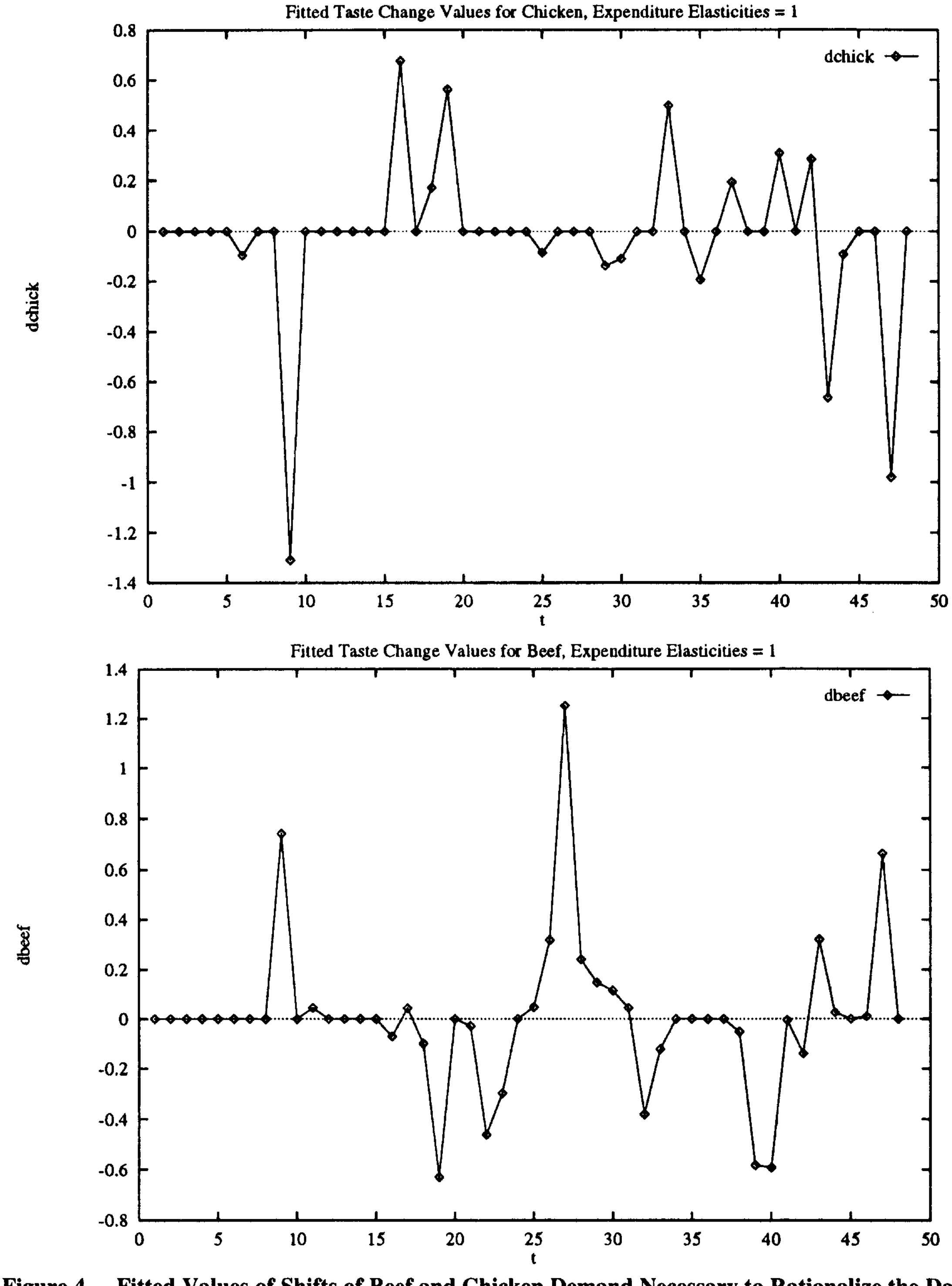


**Figure 4. Fitted Values of Shifts of Beef and Chicken Demand Necessary to Rationalize the Data when all Expenditure Elasticities Are Equal to 1**

spikes—observations 9 and 47—would be sufficient to eliminate the negative overall effect.

One thing that can be learned from these results is that there are interactions among the meats. For instance, as we have already observed, the large positive spikes for beef coincide in two of three cases with large negative spikes for chicken. When the largest deviation from consistency was found for beef, a value of +1.25 kg (in 1983:3), there was a −0.44 kg deviation for lamb and a −1.14 kg deviation for pork. That observation's chicken quantity required no adjustments. Similarly, the largest observed negative value for beef was in observation 19 (1981:3), when a −0.63 kg decline was estimated. At the same time, there were shifts of +0.17 kg in favor of lamb, +0.21 kg in favor of pork, and +0.56 kg in favor of chicken.

Not too much should be made of these results, since the restriction that all expenditure elasticities should equal 1.0 is not plausible. On the other hand, it is clear from the plots that the shift in preferences needed to reconcile the data is by no means a systematic trend away from beef in favor of chicken. The quarter-to-quarter shifts sometimes seem to dwarf the cumulative effect, in magnitude. This suggests that it will be important, in using the Sakong and Hayes procedure, to account for the possible presence of measurement errors and seasonality.

We tried one more experiment with these data. We changed the constraint on the expenditure elasticities, allowing them to vary between 0.5 and 2.0. Overall, this must decrease the extent to which the data are found to be inconsistent with a stable utility function, since a constraint has been relaxed.[14] However, it is also possible that the resulting deviations from consistency will be more systematic. Interestingly, we found very similar results for beef and chicken when this alternative set of constraints was tried. The sum of all fitted taste-change values for beef was +0.58 kg, and for chicken it was −1.07 kg. The most dramatic difference was a large reduction in the cumulative effect on lamb, which dropped to a shift away from lamb of 0.12 kg over the 48 quarters; these results indicated that the expenditure elasticity of 1.0 was too large for lamb, and relaxing that constraint reduced the size of measured taste changes. Figure 5 shows, once more, the implied shifts in preferences. Note that the magnitudes are smaller, as expected, and there appear to be more observations for which no adjustment was necessary. Again, it is interesting to note that the two large negative spikes for chicken (in the same quarters as before) are matched by two large positive spikes for beef.

As to whether to take these as measurement errors or taste changes, we leave that for further research. It is important to note the presence of both positive and negative shifts for both beef and chicken within the 48 observations. Thus, at least some of the shifts have to be considered measurement errors, or else we must treat preferences as shifting back and forth, perhaps due to seasonal shifts in preferences. No doubt a nontrivial component of this shifting back and forth is due to measurement errors, and also some may be due to the fact that the income elasticity has been too tightly bounded for beef. For example, if the income elasticity of meat expenditure was 0.5, the upper bound of 2.0 that we placed on expenditure elasticities would translate to an income elasticity of 1.0, which might be too small for beef. All of our intuition about such elasticities is based on parametric models, so we may be mixing techniques if we bring those to bear on interpreting nonparametric evidence; however, we found for this data set that the fitted expenditure elasticities using typical parametric demand systems are well above 1.0 for beef, although they did not appear to be above 2.0 (Alston and Chalfant 1991b).

In Sakong and Hayes, the fitted taste changes for beef in the United States were mostly non-zero and mostly in the same direction. Thus, Sakong and Hayes observed that there would have to be either a *systematic* measurement error—an undercounting of beef consumption (due, perhaps, to changes in trimming practices)—or a shift in preferences away from beef to cause the sequence they observed. Using the Varian (1985) or Tsur (1989) approach, one could make an assumption about the variance of measurement errors in quantities and construct a statistical test of the significance of the taste changes found by Sakong and Hayes. This ignores one vital piece of information—the *sign* of the individual shifts. If one has a sequence of implied taste changes from this procedure, a runs test or some other test of randomness seems more appropriate than simply applying a test for overall significance.

## Conclusion

Nonparametric demand analysis permits utility theory alone to be "imposed" on a data set. This

[14] This is still more restrictive than the Sakong and Hayes requirement of non-negative expenditure elasticities, although they added a constraint on the size of the shift between elasticities from adjacent observations. While an expenditure elasticity of 2.0 may seem large for meat products, note that this is with respect to meat expenditure, not total expenditure, so to the extent that the income elasticity of overall meat expenditure is less than 1.0, this number should be scaled down somewhat to impute an overall income elasticity.

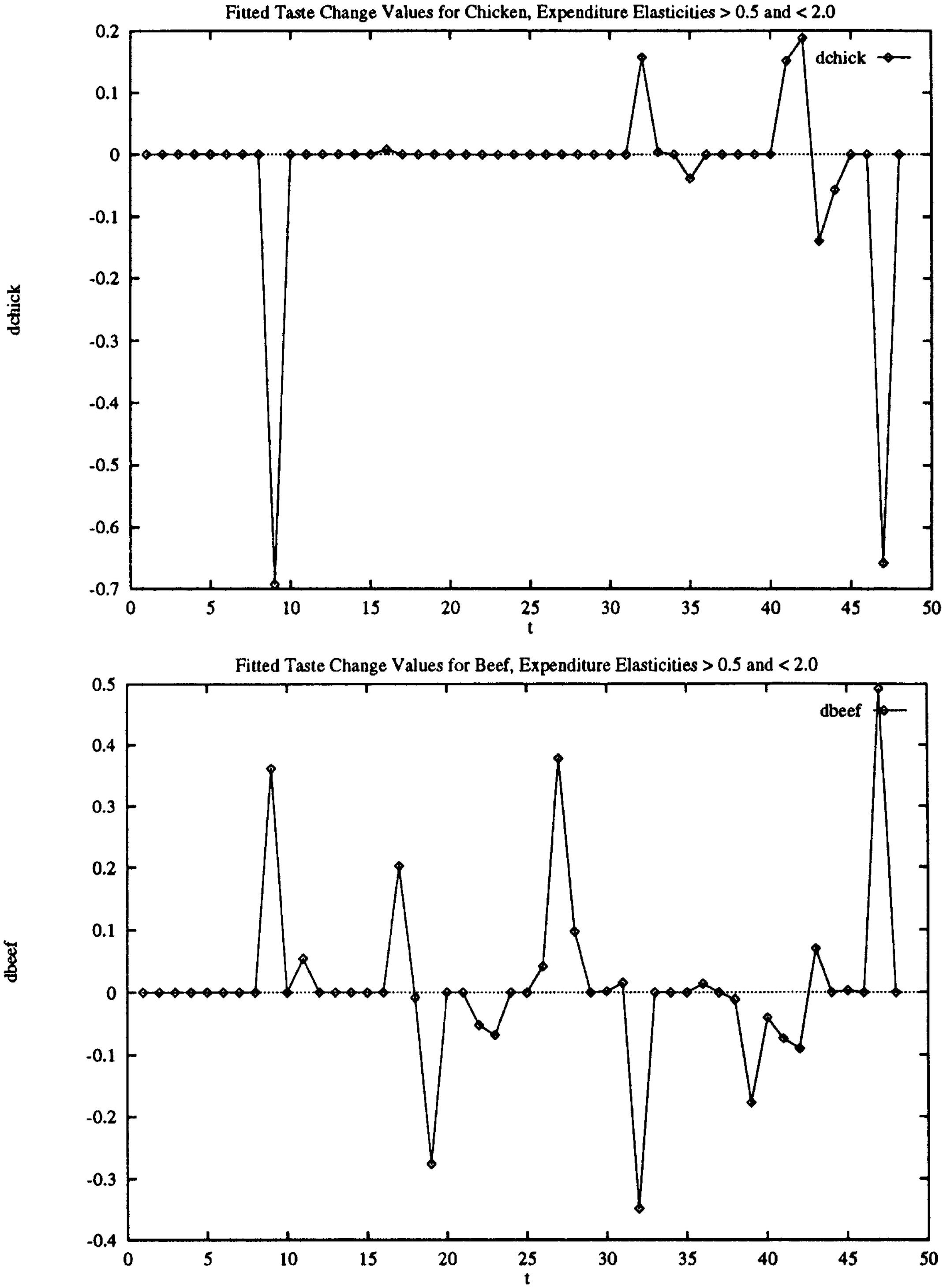


**Figure 5. Fitted Values of Shifts of Beef and Chicken Demand Necessary to Rationalize the Data when Expenditure Elasticities Are between 0.5 and 2.0**

has the potential to be more revealing than the typical parametric study, if one is concerned over possible specification errors and the difficulty of interpreting a multitude of diagnostic tests for structural change and/or model misspecification. On the other hand, concerns have been raised about the power of nonparametric tests. It is possible to find examples where two data points are known to be inconsistent with stable preferences, yet WARP or GARP are not violated. Little is known about the relative chances of GARP and parametric tests detecting structural change in typical data sets, although, if the frequencies of such findings in the literature are any guide, the nonparametric approach seems less likely to imply that data are inconsistent with a stable, well-behaved utility function. Of course, this may not mean that the parametric approach has greater power. There is a growing body of evidence about the problem of false inferences from misspecified parametric models. Much progress has been made in adding power to the nonparametric approach and in reducing the tendency toward false rejections using parametric methods, and they are becoming increasingly good complements to one another.

It is not clear that the nonparametric approach will reject less often than the parametric approach when prior information is imposed. Sakong and Hayes reached essentially the same conclusions with the nonparametric approach, for instance, as did Moschini and Meilke with an almost ideal demand system. To the extent that prior beliefs about elasticities are less arguable than those about functional forms, the Sakong and Hayes procedure represents a promising innovation in the nonparametric approach. It should not be difficult to extend their approach to restrict substitution responses as well as income effects. For instance, could we constrain the partial effects in both beef and chicken consumption, after a decrease in the pork price, to be negative?

We hope to have some results available soon that show the effects of imposing prior beliefs about substitution effects and also that make use of exogenous variables to detect any patterns in the taste changes. One could do this with some measure of health concerns, for example, to see if the implied taste changes yielded by the Sakong and Hayes approach can be explained by news about cholesterol. Also, the effects of advertising can be tested in this manner. For instance, the measured shifts in demand could be regressed against advertising variables to provide a new measure of the effectiveness of advertising. Finally, there may be seasonal patterns in the inconsistencies that may provide a basis for testing for seasonality and for adjusting for it.

It seems essential to continue to work toward finding a measure of standard error for the fitted taste-change values. We doubt whether any of the accumulated sums of taste-change effects that we found would be statistically significantly different from zero, given the amount of random variation there appears to be. This will no doubt depend, however, on how much can be attributed to systematic components such as any seasonality, and on what additional constraints are imposed. In any event, the results do suggest that the absence of any large taste changes is supported by the Sakong and Hayes procedure. Even with a strong restriction on expenditure elasticities, there does not appear to be a large change that is systematic away from red meats toward chicken. Relaxing the constraint to allow income elasticities to lie within a plausible range based on prior beliefs leads to essentially the same conclusion.

In conclusion, there is much potential and great appeal in the nonparametric approach to demand analysis. There is also much reason for caution. No one would suggest that it will replace familiar parametric methods based on regression models. On the other hand, it appears that the method can complement parametric studies in many ways. It focuses attention on the data, rather than the properties of particular tests for structural change; permits utility theory alone to be imposed on the data; and makes it convenient for prior beliefs about economic behavior, as opposed to functional forms, to be included in the analysis. Especially as progress is made in incorporating measurement errors and providing a firm statistical foundation, the method should prove increasingly useful.

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