Nonrobustness of Dynamic Dual Models of the Northeastern and U.S. Dairy Industries

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The robustness of dynamic dual models examined by Howard and Shumway is reevaluated with the proper specification for the modified generalized Leontief (GL) and normalized quadratic (NQ) functional forms. In an application to the Northeastern and U.S. dairy industries, the theoretical properties, adjustment rates, and predictive ability were similar under both functional forms. However, elasticity measures differed significantly.

In a recent *NJARE* article, Howard and Shumway examine the robustness of dynamic dual models to the choice of functional form. The use of these models to examine industry structure has grown from the initial conceptualization by Epstein due to the models' ability to determine an adjustment path for each quasi-fixed input which is theoretically consistent with the optimizing behavior of the firm. However, in order for the micro-level theory of dynamic duality to be applied at the macro level, the value function must be specified such that it depends only on the aggregate stock of quasi-fixed assets in the region and not on their distribution across firms. In the Howard and Shumway paper, this restriction is incorporated into the modified generalized Leontief (GL) functional form of the value function but not into the other functional form tested which is the normalized quadratic (NQ). The inconsistent specification between the alternative functional forms could consequently lead to erroneous conclusions regarding the robustness of the models.

The purpose of this paper is to discuss aggregation of the NQ model since the use of dynamic dual models is likely to continue to increase and the results should aid future researchers in defining their models. The robustness issue is then reevaluated with the proper specification for both functions using the Howard and Shumway example of the U.S. dairy sector. The dynamic dual methodology is also applied to the Northeastern dairy sector. Although general conclusions can only be made under an experimental approach or for a large number of examples, employing two empirical examples aids in the evaluation of the robustness issue. In addition, the national and Northeast results can be compared to determine if adjustment responses differ.

Methodology

Using the same variables and notation as Howard and Shumway, it is assumed that an industry production function exists which includes gross investment as an argument, $F(X, Z, I, T)$, where X is the variable input feed concentrate, Z is the vector of the quasi-fixed inputs milk cows and labor which are fixed in the short run, and I is gross investment in Z. A time trend, T, is included to represent disembodied technical change.[1] Assuming the production function is well behaved, which implies F is twice continuously differentiable, concave, with F_X, $F_z > 0$ and $F_I < 0$, then Epstein has shown there exists a duality between F and the value function. The optimal value function, V, represents the discounted future stream of rents accruing to the quasi-fixed inputs in the initial time period. The value function is determined by solving the following infinite-horizon nonautonomous

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[1] The consistent incorporation of technical change in dynamic optimization models is discussed by Larson.

problem under the assumption of static price expectations:[2]

$$(1)\quad J(P, W, R, Z_0, t_0) = e^{-rt_0} V(P, W, R, Z_0, t_0),$$

where $V(P, W, R, Z_0, t_0)$

$$= \max_{X(t),\, I(t)} \int_{t_0}^{\infty} e^{-r(t-t_0)} (P\, F(X, Z, I, T) - WX - RZ)dt$$

subject to $\dot{Z} = I - \delta Z$,
$X(t), I(t) > 0, Z(t_0) = Z_0$,

where r is the required rate of return, P is the milk price, W is the price of feed concentrate, R is the (1×2) vector of rental prices for the quasi-fixed factor Z, δ is the depreciation rate, and $\dot{Z}$ is net investment in Z. The value function attains a maximum in any period provided it satisfies the Hamilton-Jacobi equation.

The first step in empirically estimating the structural equations through dynamic duality is to specify the functional form of the value function. As Howard and Shumway note, both the GL and NQ functional forms are linear homogeneous in prices, concave with respect to the quasi-fixed inputs, and can express net investment in the form of a flexible accelerator. However, in order for the micro-level theory to be applied at the aggregate level, both forms of the value function must be specified such that they are affine in Z which involves setting $V_{zz} = 0$ (Blackorby and Schworn). This restriction implies the firms are aggregated linearly so that the value function depends only on the aggregate stock of quasi-fixed inputs and not on their distribution across firms (Chambers and Lopez). Formally, the linear aggregation can be written as

$$V(P, W, R, Z, t_0) = \sum_i V(P, W, R, Z_i, t_0) \text{ and } Z = \sum_i Z_i,$$

where i represents the number of firms in a region. This theoretical restriction does not preclude the existence of empirical aggregation problems.

With this condition, the dual value function for the GL is

$$(2)\quad V(P, W, R, Z_0, t_0) = [PW]AZ + R'\, B^{-1}R + [P^{0.5}\, W^{0.5}]ER^{0.5} + R^{0.5\prime}KR^{0.5} + [P^{0.5}W^{\,0.5}]G[P^{0.5}W^{0.5}]' + t_0\, H\, [PWR]',$$

where the parameters to be estimated, A, B^{-1}, E, K, and G, are (2×2) matrices and H is a (4×1) vector.

The NQ functional form of the value function is

$$(3)\quad V(w, r, Z_0, t_0) = a[1\; w\; r'Z']' + r'\; b^{-1}Z + 0.5\; gw^2 + wcZ + wer + 0.5r'kr + t_0h[1wr'],$$

where $w = W/P$ and $r = R/P$ are the normalized prices. The parameter a is a (1×6) vector, b and k are (2×2) matrices, c and e are (1×2) vectors, g is a scalar, and h is a (1×4) vector. Note the parameter n in the Howard and Shumway formulation is not included in order to satisfy the restriction that the value function be affine in Z.

The equations for milk supply, feed demand, and net investment in the quasi-fixed inputs are determined by applying the envelope theorem to the Hamilton-Jacobi equation. The GL behavioral equations, which are based on relative prices, will be[3]

$$(4)\quad F(P, W, R, Z_0, t_0) = rV_P - V_{PZ}\dot{Z} - V_{Pt_0},$$

$$(5)\quad X(P, W, R, Z_0, t_0) = -rV_W - V_{WZ}\dot{Z} - V_{Wt_0},$$

$$(6)\quad \dot{Z}(P, W, R, Z_0, t_0) = V_{RZ}^{-1}\,(rV + Z - V_{Rt_0}).$$

For the NQ, feed and quasi-fixed input demands are the same as those for the GL above except all prices are normalized by milk price. However, the milk supply equation is formed by adding normalized expenditures to the Hamilton-Jacobi equation.

$$(7)\quad f(w, r, Z_0, t_0) = rV + wX + r'Z - V_Z\dot{Z} - V_{t_0}.$$

Thus the systems of equations to be estimated are (4), (5), and (6) for the GL, and (5), (6), and (7) for the NQ. Several modifications used by Howard and Shumway are first incorporated before the two systems are estimated. Error terms are appended to each of the equations, a discrete approximation of net investment is used, and lagged milk price is used as a proxy for expected milk price.

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[2] The assumption of static price expectations is necessary to establish the duality between the production function and value function (Taylor).

[3] A typographical error is made in the feed demand equation (4) by Howard and Shumway. It should read $X(P, W, V, Z) = -rJ_W + J_{zW} Z$.

Data

Data sources and variable definition at the regional and national level are similar to that employed by Howard and Shumway for the price and quantity of milk, feed, and cows collected on a statewide basis for the eleven states comprising the Northeastern production region as defined by the U.S. Department of Agriculture (USDA). However stock of labor used is proxied in this paper by the number of hours required for milk cows, which is provided on a regional level by *Economic Indicators of the Farm Sector: Production and Efficiency Statistics* rather than through construction from aggregate agricultural labor data. In addition, the wage rate is estimated by wages paid to all hired farm labor obtained from *Agricultural Statistics* rather than by dividing the total expenditure on hired labor by the number of hired workers since this information is not available at the regional level for recent years. The resulting variables are aggregated to the regional level by summing state totals for the quantity variables or averaging for the price variables with the weights based on state milk production shares for the eleven states comprising the Northeastern production region as defined by the USDA. Additional differences from the Howard and Shumway data are that both quasi-fixed inputs are not adjusted for quality and that annual data are collected for an extra three years from 1950 to 1985.

Empirical Comparison

Using the Gallant and Jorgenson T^0 test statistic, the theoretical properties of the value function were examined for both the GL and NQ functional forms. The results are reported in Table 1. Symmetry was imposed on the models by Howard and Shumway, but the findings here indicate that symmetry is rejected in the GL at both the national and regional level. Global convexity in prices is also rejected in the GL which is in contrast to the results obtained by Howard and Shumway. Symmetry was accepted for the NQ as was global convexity under the ad hoc test procedure of calculating the sign of the determinant for the matrix of price parameters. In both functional forms, net investment for each of the two quasi-fixed inputs was found to depend on the stock level of the other period within the period.

The Northeast and national adjustment rates for the accepted models of both functional forms are presented in Table 2. In contrast to Howard and Shumway, the estimated adjustment rates are very similar for both the GL and NQ which is expected since both imply a multivariate flexible accelerator in the quasi-fixed inputs. With a real discount rate of 3%, cow numbers in the Northeast complete approximately 30% of their movement toward long-run equilibrium within one year while labor response is only one-tenth as fast. At the national level, the adjustment rates are again close for both functional forms. However, U.S. cow numbers fully adjust to their equilibrium levels within five years, which is approximately one-third slower than the estimated rate for the Northeast. This rate is faster than the 0.11 rate obtained by Howard and Shumway using the GL functional form. However, the major difference, besides the consistency of the GL and NQ adjustment rates, is that the estimated adjustment response for labor is much slower (3%

Table 1. Hypothesis Tests for GL and NQ Functional Forms

Hypothesis	Test Statistic		Critical Value
	Northeast	U.S.	
Symmetry			
GL: $K_{12}=K_{21}$, $G_{12}=G_{21}$	6.788	13.480	$\chi^2_{2,\ .05}=5.991$
NQ: $k_{12}=k_{21}$	3.390	2.035	$\chi^2_{2,\ .05}=3.841$
Convexity			
GL: $E_{ij}<0$, $i,j=1.2$; K_{ij}, $G_{ij}<0$, $i=j$	15.924	22.916	$\chi^2_{6,\ .05}=12.592$
NQ: $\begin{bmatrix} g & e_1 & e_2 \\ e_1 & k_{11} & k_{12} \\ e_2 & k_{21} & k_{22} \end{bmatrix}$ positive definite	Ad hoc, parameters within one standard deviation		
Independent Adjustment			
GL: $M_{12} = M_{21} = 0$	6.028	24.197	$\chi^2_{2,\ .05}=5.991$
NQ: $m_{12} = m_{21} = 0$	18.973	18.312	$\chi^2_{2,\ .05}=5.991$

Table 2. Northeast and U.S. Adjustment Rates for the GL and NQ Functional Forms

Functional	Northeast		U.S.	
Form	Cows	Labor	Cows	Labor
GL	−.277	−.036	−.203	−.025
NQ	−.302	−.031	−.182	−.003

versus 10%). The difference can be attributed to the definition of the labor stock.

The other parameter estimates are not reported here but are available upon request. Approximately 50% of these parameter estimates were significant at the 5% level for the GL and slightly less for the NQ at both the regional and national level. Similar results were found by Howard and Shumway for the GL, but they found only 12% of the parameter estimates to be significant for their misspecified NQ functional form. The relative explanatory power of each model was compared through the use of an historical simulation. Both functional forms demonstrated an ability to track past data reasonably well. In addition, the relative predictive ability of each variable is the same between both models with cow numbers being the best explained with a percentage root mean square error (RMSE) of 2% and labor demand showing the poorest statistical fit with an RMSE of approximately 7%.

Own-price milk supply elasticities are calculated under each functional form in Table 3 for both the Northeastern and U.S. dairy sectors. Unlike the adjustment rates, the elasticity measures show more of a divergence between functional form than between the regional and national level. In general, the NQ form generates elasticities that are more inelastic in the short run and exhibit more change between the initial and long-run response. The GL specification is expected to lead to less-elastic measures since it is better suited to data sets that exhibit a high degree of aggregation of inputs leading to limited substitution between the aggregated input variables.[4] The absolute response for the other elasticity measures also differed significantly between the GL and NQ models, but there were few occurrences where the direction of response differed between the two functional forms unlike the Howard and Shumway estimates.

[4] The author wishes to acknowledge the comment of a reviewer for this point.

Conclusions

Dynamic dual models are likely to be used with increasing frequency by researchers given their ability to incorporate adjustment response in a theoretically consistent manner. This paper has reexamined the choice of functional form for the value function in the dynamic dual model which was originally discussed by Howard and Shumway. In an application to the Northeastern and U.S. dairy industries and under proper specification for both the modified generalized Leontief (GL) and normalized quadratic (NQ) functional forms, it was found that the theoretical properties, adjustment rates, and predictive ability were similar under both functional forms for both levels of aggregation. However, the absolute value of the elasticity measures did differ significantly. The lack of robustness across functional forms in terms of elasticity measures is not unique to dynamic dual models: similar findings have been reached for static primal (Chang and Shumway) and dual models (Swamy and Binswanger).

Although the results, with the exception of the elasticity measures, do not differ significantly for the alternative functional forms, the analysis pointed out two additional considerations. The first is that the adjustment rate for milk cows is somewhat faster for the Northeast region in comparison to the national rate but that its milk supply response to own-price is slightly slower. The second is that the adjustment rates are sensitive to data specifications. The major difference between the data used here

Table 3. Northeast and U.S. Milk Own-Price Supply Elasticities Derived from the GL and NQ Functional Forms, 1985

Functional	Northeast		U.S.	
Form	Short Run	Long Run	Short Run	Long Run
GL	.249	.324	.287	.363
NQ	.029	.499	.092	.290

and that employed by Howard and Shumway is the definition for the stock of labor. The significant divergence between labor adjustment rates under the alternative definition implies variable specification has a much more important role than choice of functional form.

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