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Environmental technical efficiency and phosphorus pollution abatement cost in dairy farms: A parametric hyperbolic distance function approach

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Abstract

The dairy sector is an important sector in Northern Ireland being the single largest contributor to its agricultural economy. However, the sector contributes more to soil phosphorus (P) surplus compared to other agricultural sectors. Consequently, the goal of this research is to analyse the environmental technical efficiency of dairy farms making use of a novel parametric hyperbolic distance function approach. The model is able to internalise P surplus as undesirable output in the dairy production process by treating desirable and undesirable outputs asymmetrically, thereby allowing for the maximum expansion of the desirable output and an equi-proportionate contraction of the undesirable output. The stochastic production frontier model is analysed simultaneously with an inefficiency model to explain variability in efficiency scores assuming the existence of heteroskedasticity in the idiosyncratic error term. Additionally, we estimated the shadow price and pollution cost ratio of P surplus in dairy farms. Our results showed that the average environmental technical efficiency estimates for dairy farms in Northern Ireland is 0.93. Intensification resulting in increased use of concentrates feed was found to be negatively related to environmental technical efficiency. We also found that age of the farmer and share of milk output have a positive relationship with environmental technical efficiency.

Key words: Dairy farms; Environmental efficiency; Pollution abatement cost; Phosphorus surplus; Shadow price

Introduction

The dairy sector is an important sector in Northern Ireland being the single largest contributor to its agricultural economy, contributing about 32% of the total agricultural output (DAERA, 2017). However, the sector contributes more to soil phosphorus (P) surplus compared to other agricultural sectors, putting pressure on the environment (Adenuga, *et al.*, 2018a). Already about 50% of farmed grassland have plant-available P (Olsen-P) values greater than the critical value of 25mg/kg (Bailey, 2015; Kleinman *et al.*, 2015, Cave and McKibbin, 2016). Also, more than 50% of all rivers in the country are classified as “moderate/poor status” and about 70% of lakes are still classed as eutrophic under the “water framework directive” (Kleinman *et al.*, 2015; Cave and McKibbin, 2016). Soil P level unlike other nutrients is fixed in supply. The implication of this is that, excess P in soils is detrimental not only from an economic point of view but also from an environmental sustainability point of view. Negative externality from agricultural production in the form of P surplus has detrimental effects on water quality and biodiversity. There is increasing pressure from the policy angle on the need to ensure that agricultural production is sustainable especially given the effect of agricultural pollution on the environment.

Given the foregoing, the objective of this study is to estimate the environmental technical efficiency of dairy farms in Northern Ireland, incorporating P surplus as undesirable output in a parametric hyperbolic distance function modelling framework. The stochastic production frontier model is analysed simultaneously with an inefficiency model to explain the variability in efficiency scores assuming the existence of heteroskedasticity in the idiosyncratic error term. Additionally, by employing the duality between the distance function and the maximisation of the profitability function, we estimated the shadow price (marginal abatement cost) and consequently pollution cost ratio of P surplus in dairy farms. The shadow price is defined as the opportunity cost of reducing one more unit of P in terms of reduction of the revenue from dairy production activities (Hailu and Veeman, 2000; Färe *et al.*, 2006; Zhou *et al.*, 2014). The estimates of the marginal abatement cost can serve as an important parameter for the design of agri-environmental policy instruments required to optimize agricultural pollution abatement policies. For example, setting a nutrient surplus taxation policy or the derivation of the optimal subsidy to farmers per unit of the pollutant reduced. This will be relevant as empirical evidence in the implementation of government’s policies to reward farmers for sustainable and environmentally friendly practices. It will also be useful in the development of a manure market which may be necessary in transferring nutrient surplus from areas of higher concentration to areas of lower concentration.

The contribution of this study to the existing literature is threefold. Firstly, the study provides the first attempt to simultaneously analyse the environmental technical efficiency and marginal

abatement cost (shadow price) of P surplus on dairy farms using farm level panel data in Northern Ireland. The measure reflects not only the level of environmental pressure but also the level of competitiveness and economic efficiency of the dairy production systems in the country. Secondly and from a methodological perspective, the study employed the hyperbolic environmental technology distance function which is less restrictive compared to the output or input distance functions. Previous studies have employed mainly the radial output/input distance functions, which assumes proportional expansion of all outputs (both desirable and undesirable outputs), or contraction of all inputs, in the same direction (Färe *et al.*, 1993; Chung *et al.*, 1997; Hadley 1998; Hailu and Veeman, 2000). In contrast, the hyperbolic distance function treats outputs asymmetrically by seeking to simultaneously expand the desirable outputs and contract the undesirable outputs given a fixed level of inputs (Cuesta and Zofio, 2005; Hou *et al.*, 2015; Mamardashvili *et al.*, 2016; Wang *et al.*, 2017; Pena *et al.*, 2018; Cuesta *et al.*, 2009). This analogy is in line with the government's policy that seek to reduce negative agricultural externalities and increase production of desirable outputs simultaneously (Hailu and Veeman, 2000; Färe *et al.*, 2006). The methodology allows for the estimation of a more robust environmental efficiency measure and shadow price of P surplus. Lastly, unlike previous studies that employed the farm gate approach, our estimation of the P surplus was based on the soil surface balance approach which involved the development of a novel nutrient requirement model employed to estimate P output from grazed grass. The findings from this study will assist policy makers in understanding the factors that influence environmental technical efficiency and the trade-offs between revenue from dairy production and the negative impact of P surplus. This will be important in devising balanced policies to improve current operations and enhance sustainability of dairy production systems.

The remaining part of this paper is divided into five sections. In section two, we present the theoretical framework for the hyperbolic distance function methodology. The data and empirical specification of the model are presented in section three while the results are reported in section four. The paper ends with a summary of the main conclusions in section five.

Theoretical framework for the hyperbolic environmental technology distance function

The parametric hyperbolic environmental technology distance function is based on the multiplicative homogeneity property of the Shephard's distance function. The term hyperbolic is derived from the hyperbolic path implied by the distance function toward the efficient frontier (Färe *et al.* 1989; Cuesta and Zofio, 2005; Cuesta *et al.*, 2009; Duman and Kasman, 2018). In line with the pareto optimality condition, the methodology seeks to provide a composite index that simultaneously captures economic and environmental balance in dairy production systems in line with the best production practices in the region. It provides a lot of flexibility in the evaluation of environmental

technical efficiency in the presence of environmentally sensitive undesirable outputs (Färe *et al.*, 2005; Hailu and Chambers, 2012; Du *et al.*, 2015). Unlike the non-parametric DEA approaches, it accounts for statistical noise, it is differentiable and is able to conduct statistical inference without bootstrapping (Färe *et al.*, 2005; Wei *et al.*, 2013). The results obtained using the methodology are also less sensitive to outliers which may negatively affect the accuracy of results (Boyd *et al.*, 2002; Simar and Wilson, 2007; Wei *et al.*, 2013; Pérez-Urdiales *et al.*, 2016; Skevas, 2018; Adenuga *et al.*, 2018b). An alternative to the hyperbolic distance function approach is the directional output distance function which also simultaneously accounts for the expansion of desirable outputs and reduction of undesirable outputs by modifying the direction in which to search for an efficient counterpart of each farm towards the production frontier (Färe *et al.*, 2005; Manello, 2012; Adenuga *et al.*, 2018b). However, the directional distance function does not satisfy the property of commensurability (scale invariance) such that the efficiency measure depends on the units of measurement of the input and output variables that enter the model (Peyrache and Coelli, 2009; Skevas, 2018). Also, while the hyperbolic distance function is based on the multiplicative homogeneity property of the Shephard's (1970) distance function, the directional distance function makes use of the translation property which is an additive analogue of the multiplicative homogeneity property of the hyperbolic distance function (Chambers *et al.*, 1998; Fare *et al.*, 2005; Cuesta and Zofio, 2005; Cuesta *et al.*, 2009).

Few studies that have employed the hyperbolic distance function in the economics literature. Examples of such studies are outlined below. Cuesta and Zofio (2005) employed the translog hyperbolic distance function to estimate the efficiency of Spanish savings banks, while Cuesta *et al.* (2009) used the hyperbolic and the enhanced hyperbolic distance function approach to estimate the efficiency scores for a set of U.S. electric industries and consequently estimated the shadow price of SO₂ emissions which was considered as undesirable output. Suta, *et al.*, (2010) using the hyperbolic distance function approach, calculated the environmental technical efficiency scores of selected EU farms. Glass *et al.*, (2014) employed the enhanced hyperbolic distance function to measure the relative performance of Japanese cooperative banks, modelling non-performing loans as an undesirable output. Mamardashvili *et al.* (2016) applied the hyperbolic distance function to analyse the environmental performance and estimated the shadow price of nitrogen surplus for conventional and organic Swiss dairy farms using cross sectional data. Despite the popularity of this approach in other production sectors, only a few studies Suta *et al.* (2010) and Mamardashvili *et al.* (2016) have employed it in the context of agriculture. The theoretical framework for the hyperbolic distance function is as follows:

Suppose there are n ($n=1,2,\dots,N$) decision making units, i.e. dairy farms in this case, employing multiple inputs denoted by vector $x_n = (x_{1n}, x_{2n}, \dots, x_{jn}) \in R_+^J$ to produce a vector of

desirable outputs $y_n = (y_{1n}, y_{2n}, \dots, y_{Mn}) \in R_+^M$ and a vector of undesirable outputs $s_n = (s_{1n}, s_{2n}, \dots, s_{1Kn}) \in R_+^K$. Then, the environmental production technology can be represented by the output possibility set $T(x)$ given in equation (1) (Chung *et al.*, 1997; Cuesta, *et al.*, 2009; Zhou *et al.*, 2016; Duman and Kasman, 2018).

$$T(x) = \{(x, y, s): x \in R_+^J \text{ can produce } (y, s); y \in R_+^M, s \in R_+^K\} \quad (1)$$

The hyperbolic environmental technology distance function (D_H) represents the maximum expansion of the desirable output vector (y) and the equi-proportionate contraction of the undesirable output vector (s) that places a producer on the boundary of the technology (Färe *et al.*, 1989; Cuesta, *et al.*, 2009). It is formally defined in equation (2).

$$D_H(x, y, s) = \inf \left\{ \eta > 0: (x, \frac{y}{\eta}, s\eta) \in T \right\} \quad (2)$$

As indicated by η in equation (2), the desirable and undesirable output changes in the same proportion but in opposite direction. The range of the hyperbolic environmental technology distance function is $0 < D_H(x, y, s) \leq 1$. Farms are said to be fully efficient if $D_H = 1$ implying that the estimated observation is on the boundary of the production frontier and it will not be possible to reduce P surplus or increase dairy output at the same time. If the value of the distance functions is less than 1 ($D_H < 1$), then the farm is inefficient which leaves room for enhancing efficiency of the dairy farms by increasing dairy output and reducing P surplus.

Following Cuesta *et al.*, (2009) and Mamardashvili *et al.*, (2016), the almost homogeneity property can be employed to derive the hyperbolic environmental technology distance function. Given a set of inputs data, desirable output and undesirable output, the function can be expressed as given in equation (3)

$$D_H(x, \phi y, \phi^{-1} s) = \phi D_H(x, y, s), \phi > 0 \quad (3)$$

Given that ϕ in equation (3) is greater than 0, then imposing the almost homogeneity condition by setting $\phi = \frac{1}{y_m}$ (where y_m is, without loss of generality, the Mth output), the hyperbolic environmental technology distance function can be expressed as presented in equation (4):

$$D_H \left(x_i, \frac{y_i}{y_m}, s_i y_m \right) = \frac{1}{y_m} D_H(x_i, y_i, s_i) \quad (4)$$

The model can be specified based on the translog functional form given that it provides a flexible approximation to the production technology. It is differentiable and quite amenable to the

imposition of almost homogeneity conditions (Cuesta and Zofio, 2005; Cuesta *et al.*, 2009). It is also linear in parameters, easy to compute and has been extensively used in the empirical literature (Färe *et al.*, 1989; Coelli *et al.*, 2005; Cuesta *et al.*, 2009; Penal, *et al.*, 2018; Mamardashvili, *et al.* 2016). The hyperbolic distance function is specified in a stochastic frontier analysis (SFA) framework. The SFA provides room for the estimation of the frontier of best production practices that envelop the data while assuming the existence of an idiosyncratic error term. The function $D_H(x_i, y_i, s_i)$ is almost homogenous of degrees r_1, r_2, r_3 and r_4 if,

$$D_H(\phi^{r_1}x, \phi^{r_2}y, \phi^{r_3}s) = \phi^{r_4}D_H(x, y, s), \forall \phi > 0 \quad (5)$$

From this definition the environmental hyperbolic distance function D_H is homogenous of degrees 0, 1, -1, 1. It is non-decreasing in desirable outputs, $D_H(x, \eta y, s) \leq D_H(x, y, s)$, $\eta \in [0,1]$; non-increasing in undesirable output $D_H(x, y, \eta s) \leq D_H(x, y, s)$, $\eta \geq 1$ and non-increasing in inputs $D_H(\eta x, y, s) \leq D_H(x, y, s)$, $\eta \geq 1$ (Cuesta and Zofio, 2005; Cuesta *et al.*, 2009). Assuming that our distance function $D_H(x, y, s)$ is continuously differentiable, to be almost homogeneous it must satisfy the following expression in equation (6).

$$r_1 \sum_{j=1}^J \frac{\partial D_H}{\partial x_j} x_j + r_2 \sum_{m=1}^M \frac{\partial D_H}{\partial y_m} y_m + r_3 \sum_{k=1}^K \frac{\partial D_H}{\partial s_k} s_k = r_4 D_H \quad (6)$$

Departing from equation (6) and given that the hyperbolic distance function satisfy the homogeneity degrees of 0, 1, -1, 1 corresponding to r_1, r_2, r_3 and r_4 respectively we have

$$\sum_{m=1}^M \frac{\partial \ln D_H}{\partial \ln y_m} y_m - \sum_{k=1}^K \frac{\partial \ln D_H}{\partial \ln s_k} s_k = 1 \quad (7)$$

The Translog specification of the hyperbolic environmental technology distance function $D_H(x, y, s)$ following (4) takes the form;

$$\begin{aligned}
\ln D_H(x, y, s) = & \alpha_0 + \sum_{j=1}^J \alpha_j \ln x_{ji} + \frac{1}{2} \sum_{j=1}^J \sum_{j'=1}^J \alpha_{jj'} \ln x_{ji} \ln x_{j'i} + \sum_{m=1}^M \beta_m \ln y_{mi} \\
& + \frac{1}{2} \sum_{m=1}^M \sum_{m'=1}^M \beta_{mm'} \ln y_{mi} \ln y_{m'i} + \sum_{k=1}^K \gamma_k \ln s_{ki} + \frac{1}{2} \sum_{k=1}^K \sum_{k'=1}^K \gamma_{kk'} \ln s_{ki} \ln s_{k'i} \\
& + \sum_{j=1}^J \sum_{m=1}^M \delta_{jm} \ln x_{ji} \ln y_{mi} + \sum_{j=1}^J \sum_{k=1}^K \psi_{jk} \ln x_{ji} \ln s_{ki} \\
& + \sum_{m=1}^M \sum_{k=1}^K \mu_{mk} \ln y_{mi} \ln s_{ki} \tag{8}
\end{aligned}$$

The relevant partial derivatives for the translog hyperbolic distance function presented in equation (8) yields the elasticities presented in equations (9) (10) and (11)

$$\frac{\partial \ln D_H}{\partial \ln x_j} = \alpha_j + \sum_{j'=1}^J \alpha_{jj'} \ln x_{j'i} + \sum_{m=1}^M \delta_{jm} \ln y_{mi} + \sum_{k=1}^K \psi_{jk} \ln s_{ki} \quad (j = 1, 2, \dots, J) \tag{9}$$

$$\frac{\partial \ln D_H}{\partial \ln y_m} = \beta_m + \sum_{m'=1}^M \beta_{mm'} \ln y_{m'i} + \sum_{j=1}^J \delta_{jm} \ln x_{ji} + \sum_{k=1}^K \mu_{mk} \ln s_{ki} \quad (m = 1, 2, \dots, M) \tag{10}$$

$$\frac{\partial \ln D_H}{\partial \ln s_k} = \gamma_k + \sum_{k'=1}^K \gamma_{kk'} \ln s_{k'i} + \sum_{j=1}^J \psi_{jk} \ln x_{ji} + \sum_{m=1}^M \mu_{mk} \ln y_{mi} \quad (k = 1, 2, \dots, K) \tag{11}$$

Taking y_{0-th} output as the normalising variable to satisfy the almost homogeneity condition, and appending a random error term, $v_{it} \sim N(0, \sigma_v^2)$ to equation (8), the stochastic translog hyperbolic environmental technology distance function can be specified as presented in equation (9). The model is enhanced by allowing for a multi-period framework making use of panel data, hence, all variables are indexed with a year subscript t

$$\begin{aligned}
\frac{\ln D_H(x_i, y_i, s_i)}{\ln y_{m_0, it}} = & \alpha_0 + \sum_{j=1}^J \alpha_j \ln x_{j, it} + \frac{1}{2} \sum_{j=1}^J \sum_{j'=1}^J \alpha_{jj'} \ln x_{j, it} \ln x_{j', it} + \sum_{m=1}^{M-1} \beta_m \ln y_{m, it}^* \\
& + \frac{1}{2} \sum_{m=1}^{M-1} \sum_{m'=1}^{M-1} \beta_{mm'} \ln y_{m, it}^* \ln y_{m', it}^* + \sum_{k=1}^K \gamma_k \ln s_{k, it}^* + \frac{1}{2} \sum_{k=1}^K \sum_{k'=1}^K \gamma_{kk'} \ln s_{k, it}^* \ln s_{k', it}^* \\
& + \sum_{j=1}^J \sum_{m=1}^{M-1} \delta_{jm} \ln x_{j, it} \ln y_{m, it}^* + \sum_{j=1}^J \sum_{k=1}^K \psi_{jk} \ln x_{j, it} \ln s_{k, it}^* + \sum_{m=1}^{M-1} \sum_{k=1}^K \mu_{mk} \ln y_{m, it}^* \ln s_{k, it}^* \\
& + v_{it} \tag{12}
\end{aligned}$$

Where $y_{m, it}^* = \frac{y_{m, it}}{y_{m_0, it}}$; $s_{k, it}^* = s_{k, it} \times y_{m_0, it}$. $\alpha, \beta, \gamma, \delta, \psi$ and μ are the parameters to be estimated. Equation (12) cannot be directly estimated given that $\ln D_H(x_i, y_i, s_i)$ is not directly observed. This problem can be solved by making use of the logarithmic properties and denoting $\ln D_H(x_i, y_i, s_i) = u_{it}$ (this can be interpreted as a one-sided error term which is assumed to account for farm-specific effects following Aigner *et al.*, 1977). Moving it to the right-hand side of the equation, an estimable form of the model can be obtained as presented in equation (13). Terms involving the normalising output $y_{m_0, it}$ are null. This is because the ratio $y_{m, it}^*$ is equal to one. The distance function elasticity with respect to the desirable output can however be recovered by making use of the almost homogeneity condition (Cuesta *et al.*, 2009).

$$\begin{aligned}
-\ln y_{m_0, it} = & \alpha_0 + \sum_{j=1}^J \alpha_j \ln x_{j, it} + \frac{1}{2} \sum_{j=1}^J \sum_{j'=1}^J \alpha_{jj'} \ln x_{j, it} \ln x_{j', it} + \sum_{m=1}^{M-1} \beta_m \ln y_{m, it}^* \\
& + \frac{1}{2} \sum_{m=1}^{M-1} \sum_{m'=1}^{M-1} \beta_{mm'} \ln y_{m, it}^* \ln y_{m', it}^* + \sum_{k=1}^K \gamma_k \ln s_{k, it}^* + \frac{1}{2} \sum_{k=1}^K \sum_{k'=1}^K \gamma_{kk'} \ln s_{k, it}^* \ln s_{k', it}^* \\
& + \sum_{j=1}^J \sum_{m=1}^{M-1} \delta_{jm} \ln x_{j, it} \ln y_{m, it}^* + \sum_{j=1}^J \sum_{k=1}^K \psi_{jk} \ln x_{j, it} \ln s_{k, it}^* \\
& + \sum_{m=1}^{M-1} \sum_{k=1}^K \mu_{mk} \ln y_{m, it}^* \ln s_{k, it}^* + (v_{it} - u_{it}) \tag{13}
\end{aligned}$$

The composed error term $\varepsilon_{it} = (v_{it} - u_{it})$ includes u_i , the one-sided error term that captures environmental technical inefficiency, that is, the distance that separates a farm from the production frontier. It is assumed to be independently and identically distributed across observations. v_{it} is the standard random term which captures the statistical noise and is assumed to be symmetrically distributed around zero, $v_{it} \sim N(0, \sigma_v^2)$. To obtain unbiased estimates of the frontier function's parameters as well as the estimates of inefficiency, we assumed the existence of heteroskedasticity in the inefficiency u_{it} and in the parameter of the idiosyncratic error term v_{it} as presented in equations (14) and (15) (Caudill, *et al.*, 1995; Hadri 1999; Kumbhakar and Lovell, 2000; Wang and Schmidt, 2002; Belotti *et al.*, 2013).

$$\sigma_{u_i}^2 = \exp(z_i' \varphi) \quad (14)$$

$$\sigma_{v_i}^2 = \exp(h_i' \phi) \quad (15)$$

Where z_i' and h_i' are vectors of variables that affect the variance of the two error terms and φ and ϕ are vectors of parameters to be estimated.

Shadow price estimation

To obtain the shadow price of P surplus, we employed the Shephard duality lemma between the stochastic hyperbolic distance function and the maximisation of the profitability function (Shephard, 1970; Färe *et al.*, 2002).

Given y_m as the vector of desirable outputs and p_m as its corresponding prices, the shadow price of P surplus can be derived from the profitability maximising function presented in equation (16) (van Ha *et al.*, 2008; Cuesta *et al.*, 2009; Mamardashvili *et al.*, 2016).

$$\rho(x, y, s) = \max_{y, s} \{p_m y_m / p_s s : D_H(x, y, s) \leq 1\} \quad (16)$$

Differentiating with respect to y_m and s respectively by taking the first order condition, where p_s the (unknown) price of the undesirable output and λ is the Lagrange multiplier we have:

$$\frac{p_m}{p_s s} = \lambda \frac{\partial D_H(x, y, s)}{\partial y_m}, m = 1, 2, \dots, M \quad (17)$$

$$-\frac{\sum_{m=1}^M p_m y_m}{p_s s^2} = \lambda \frac{\partial D_H(x, y, s)}{\partial s} \quad (18)$$

Taking the ratio of the last condition to any first-order condition in the first set, we have

$$\frac{\sum_{m=1}^M p_m y_m}{s} = -p_m \frac{\frac{\partial D_H(x, y, s)}{\partial s}}{\frac{\partial D_H(x, y, s)}{\partial y_1}} \quad (19)$$

Given that the frontier of the production possibility set is a representation of the locus of points for which the distance function is equal to unity, the ratio of partial derivatives on the right-hand side of equation (19) can be expressed as the slope of the relationship between y_m and s at the frontier. That is, by applying the implicit function theorem on the distance function, the equation becomes;

$$p_m \frac{\frac{\partial D_H(x, y, s)}{\partial s}}{\frac{\partial D_H(x, y, s)}{\partial y_m}} = p_m \left. \frac{dy_m}{ds} \right|_{D_H(x, y, s)=1} \quad (20)$$

This can be interpreted as the shadow price of s in terms of y_m . That is, the extent to which the revenue from desirable outputs y_m is reduced if the undesirable output s is reduced by one unit when the point (x, y, s) is on the production frontier. Given that the shadow price is estimated on the frontier, the values refers to the marginal abatement cost for the most efficient farms since farms operating below the frontier are able to reduce undesirable output without reducing the desirable output.

Nutrient budget methodology

In estimating P surplus, we used the OECD/Eurostat soil surface budget methodology. A detailed description of the methodology is given in Adenuga *et al.*, (2018b) and is therefore described only briefly in this paper. It is estimated as the difference between total P input into the soil from chemical fertiliser, livestock manure and seeds and total P output from the soil in the form of grazed

grass, crop production, hays and silages. A complex part of this methodology is in the estimation of nutrient output from grazed grass. Previous studies have employed expert judgement, and assuming a fixed amount of nutrient output per hectare (Humphreys *et al.*, 2008; Loro *et al.*, 2013). However, such an assumption does not take into consideration the differences in the production management systems across the dairy farms. To overcome this deficiency, we developed a nutrient requirement model based on the difference between the net energy (NE) provided by feed purchased from off the farm (dry matter of concentrates and forages) and the total NE requirements of livestock on the farm for, maintenance, milk production, pregnancy and other activities. This was used to estimate nutrient output from grazed grass. It can be described as a back-calculation approach based on the number of grazing animals on the farm, the area under consideration and milk production data (McCarthy *et al.*, 2011). The NE supplied by grass can be obtained by subtracting the total NE from feed (concentrates and supplements), from the total NE requirement for all grazing livestock on the farm (McCarthy *et al.*, 2011). The total NE requirements, converted to units of feed for lactation (UFL) and adapted to local farm conditions, are computed based on relevant equations published in the National Research Council (NRC) publication on “nutrient requirement for dairy cattle” (NRC, 2001). It was assumed that 1 kg dry matter of grass equals 1 unit of feed for lactation (UFL) (McCarthy *et al.*, 2011). Stocking rate was expressed in terms of livestock units (LU) per hectare. The amount of nutrient output from grass was subsequently obtained by multiplying the quantity of grazed grass by the P coefficients in grass (Eurostat, 2013).

Data and empirical specification of the model

Study Area

The study was carried out in Northern Ireland (Latitude: 54°38'N. Longitude: 6°13'W). Northern Ireland is located in the north-east of the island of Ireland with six administrative counties. It is also the smallest region of the United Kingdom with a population of 1.87 million people which make up about 28% of the island of Ireland's total population (NISRA, 2018). The Northern Ireland landscape is dominated by two large lakes (Lough Neagh and Erne), which together drain approximately 6000 square kilometres or 40 per cent of the land area of Northern Ireland plus a further

2500 square kilometre of the Republic of Ireland (Dairyman, 2011). Figure 1 shows the map of the study area.

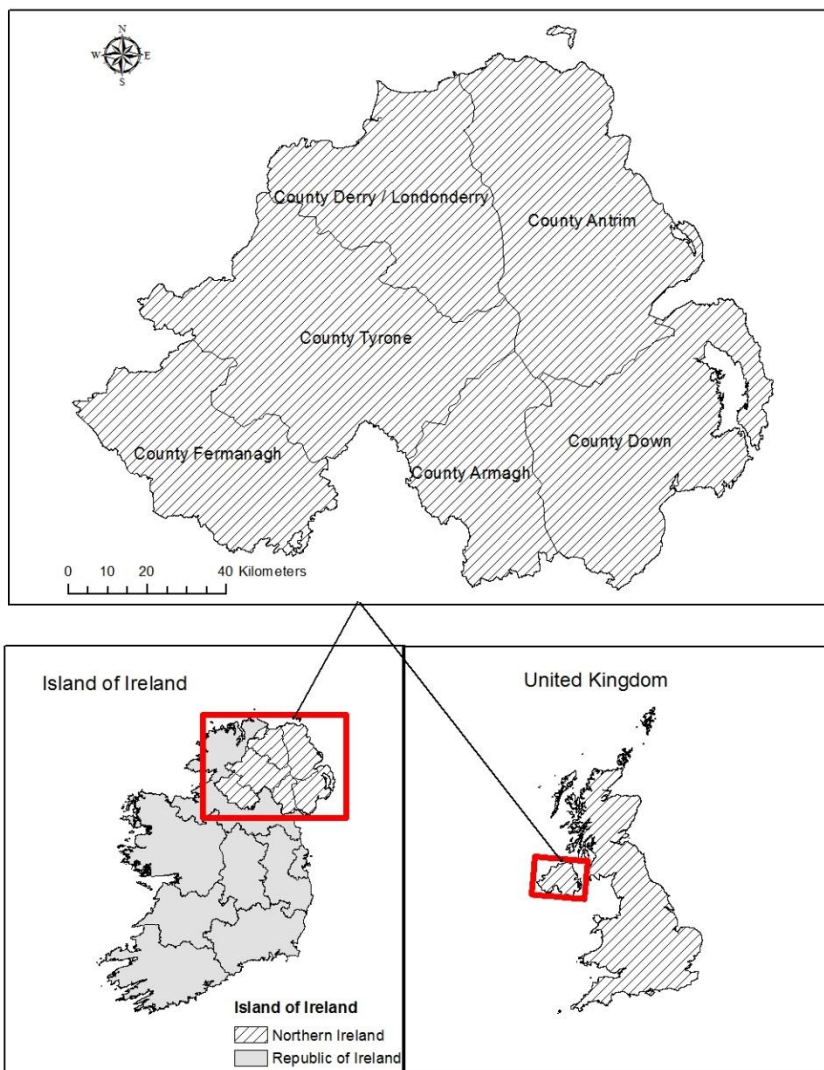


Figure 1: Map of the study area: inset is the map of United Kingdom and the Island of Ireland
Source: Authors' compilation

Analytical Techniques

The empirical specification of the stochastic hyperbolic environmental technology distance function following Aigner *et al*, (1977) is presented in equation (26). Where $i = 1, 2, \dots, N$ represents the observed dairy farms in time $t = 1, 2, \dots, T$ time periods. The study employed unbalanced panel dataset (2009-2014) obtained from the Northern Ireland Farm Business Survey (FBS, Northern Ireland) as part of the EU Farm Accountancy Data Network (FADN) requirements. The data consist of 498 observations from 83 specialist dairy farms. To impose the almost homogeneity condition, the milk output (y_1) was chosen for normalising. A time variable to capture the presence of neutral

technical change as well as other temporal effects is also incorporated. The equation was analysed assuming heteroskedasticity of the inefficiency as well as in the parameter of the idiosyncratic error term (Battese and Coelli 1995; Wang and Schmidt 2002; Cuesta et al. 2009).

$$\begin{aligned}
-\ln y_{1,it} = & \alpha_0 + \sum_{j=1}^5 \alpha_j \ln x_{j,it} + \frac{1}{2} \sum_{j=1}^5 \sum_{j'=1}^5 \alpha_{jj'} \ln x_{j,it} \ln x_{j',it} + \beta_2 \ln \frac{y_{2,it}}{y_{1,it}} + \frac{1}{2} \beta_{22} \left(\ln \frac{y_{2,it}}{y_{1,it}} \right)^2 \\
& + \gamma_o \ln(s_{it} y_{1,it}) + \frac{1}{2} \gamma_{oo} (\ln(s_{it} y_{1,it}))^2 + \sum_{j=1}^5 \delta_{j2} \ln x_{j,it} \ln \frac{y_{2,it}}{y_{1,it}} \\
& + \sum_{j=1}^5 \psi_{jo} \ln x_{j,it} \ln(s_{it} y_{1,it}) + \mu_{2o} \ln \frac{y_{2,it}}{y_{1,it}} \ln(s_{it} y_{1,it}) + \sum_{\tau=1}^T \rho_{\tau} d_{\tau}^t \\
& + (v_{it} - u_i)
\end{aligned} \tag{21}$$

The equation is estimated based on the Green (2005) time varying fixed effect model which allows one to disentangle time varying inefficiency from unit-specific time-invariant unobserved heterogeneity (Green, 2005; Bellotti *et al.*, 2013). The conditional distribution of the inefficiency component was obtained following a truncated normal distribution. The inefficiency component, was specified as a function of explanatory variables, including farm-specific characteristics and policy variables. The parameters of the stochastic frontier and the determinants of inefficiency were simultaneously analysed in a single step procedure. This has been noted to be a better approach than the two-step procedure which has been shown to result in biased results (Wang and Schmidt, 2002; Greene, 2012; Belotti *et al.*, 2013).

Following standard practice in the literature, all variables apart from the time variable were scaled by their geometric mean to avoid convergence issues in the maximum likelihood algorithm and allow for the interpretation of the estimated first order parameters as elasticities at the sample mean of the data (Färe *et al.*, 2005; Cuesta, *et al.*, 2009). The variables measured in monetary units were corrected for inflation using the appropriate annual producer price indices published by Department of Agriculture, Environment and Rural Affairs (DAERA, 2018). The time invariant efficiency estimates EE was calculated for each farm by using the point estimator proposed by Battese and Coelli (1988) given in equation (27):

$$EE = E(e^{-ui}|\varepsilon_i) \quad (22)$$

Where E is the mathematical expectation operator. The analysis was done using STATA statistical software (Belotti *et al.*, 2013). The variables considered in the analysis are based upon the production process of specialised dairy farms. The five inputs included in the specification of hyperbolic environmental technology distance function include:

- i. total utilized agricultural area measured in hectares;
- ii. the number of livestock units on the farm measured in standardized livestock units (LU);
- iii. capital measured in terms of depreciation values for building and machinery;
- iv. variable inputs which consist of costs of livestock feed, fertilizers, seed and others measured in monetary units and
- v. labour measured in standardized labour units.

The desirable outputs are:

- i. revenue from the sales of milk and
- ii. revenue from the sales of other outputs (sales of crops and other livestock).

The undesirable output

- i. nitrogen surplus estimated based on the soil surface balance approach and measured in Kg.

To analyse the determinants of environmental technical efficiency, a number of farm characteristics obtained from the empirical literature were included in the model. The following variables were hypothesized to influence environmental technical efficiency. They include: age of the farmer (Z_1), stocking density (Z_2), milk sales as a share of total revenue (Z_3), the cost of concentrates (Z_4), farm size (Z_5), land type (Z_6), and access to environmental subsidy. (Z_7). It was hypothesized that the age of the farmer should have a negative relationship with environmental technical efficiency. This is on the basis that younger farmers are usually more amenable to adoption of sustainable dairy production practices (Mbehoma and Mutasa, 2013). We expect stocking density to have a negative effect on environmental technical efficiency as a high stocking density is likely to result in increased manure input to the soil per hectare of utilised agricultural area. The share of revenues from milk production was included in the model to capture the effect of degree of specialisation in milk production on environmental technical efficiency. We hypothesized a positive effect of the share of revenue from

milk on environmental technical efficiency as the more specialised farms are expected to be more amenable to adoption of improved environmental technology adoption. Increasing the amount of concentrates fed per dairy cow could result in increased amount of P going to the soil from manure. As a result, we hypothesized that it will have a negative relationship with environmental technical efficiency. We expect higher efficiency for farmers with larger farm size (z_5), which may indicate having more land to spread excess manure. Farm land is categorised into two types in the study, namely farms in less favoured areas (severely disadvantaged (SDA) and disadvantaged (DA) areas) and farms on lowland. The land type variable was measured as a dummy variable in the model. It is hypothesized that farms on lowland should have a positive relationship with environmental technical efficiency. Finally, we hypothesize a positive effect of access to environmental subsidy on environmental technical efficiency as the subsidy should provide farmers with more fund to comply with environmental regulations aimed at maintaining the environment. We were unable to include more variables in the model due to unavailability of data. Nevertheless, their effects will be controlled for in the fixed effect specification adopted for this study. The following variables were included in the model to account for heteroscedasticity of v_{it} . They include: age of the farmer (h_1); milk yield (h_2); amount of concentrates (h_3); and land type (h_4). A summary statistic of the variables included in the model is given in Table 1

Table 1: Structural and socioeconomic variables (averages across 6 years period)

Variables	Mean	SD
Desirable outputs		
Dairy gross output (£)	218762.00	238145.1
Other outputs (£)	57859.80	40749.27
Inputs		
Utilised agricultural area (ha)	81.18	51.93
Variable Input (£)	147667.5	183408.6
Capital Overhead costs (£)	81889.89	73508.75
livestock Units(LU/ha)	163.46	125.59
Herd size (Cow)	106.60	84.02
Labour units	1.73	0.80
Undesirable output		
Gross phosphorus balance (Kg/ha)	13.56	10.95
Socioeconomic and Structural variables		
Milk yield (litres/cow)	6513.19	1175.34
Age(years)	59.61	12.97
Stocking density(LU/ha)	2.11	0.54
Concentrates fed (£)	81994.3	103217

Results and discussions

The parameter estimates and associated standard errors of the parametric hyperbolic distance function model are presented in Table 2. Most of the parameters are statistically significant at the 1% level. Considering the negative sign of the left side variable “ $\ln y_{1,it}$ ” in equation (21), the parameter estimates (distance elasticities) of inputs, other outputs and undesirable outputs variables in the model all possess the expected sign at the mean of the data. For example, the positive sign of the other output parameter implies that for a given input vector x and undesirable output vector s , an increase in outputs will bring the observation closer to the production frontier. On the other hand, the negative sign of the undesirable output parameter implies that for a given output vector y and input vector x , an increase in undesirable output will move the observation to the right, and further away from the production frontier (farms become less efficient). Likewise and all things being equal, the negative sign of the inputs parameters implies that increase in inputs moves the boundary of production possibilities set upward such that the observation become further away from the production frontier. On the basis of these results, the monotonicity conditions are fully satisfied at the sample mean for the estimated hyperbolic environmental technology distance function (Skevas *et al.*, 2018; Cuesta and Zofío, 2005).

Table 2: Parameter estimates of the hyperbolic environmental technology distance function

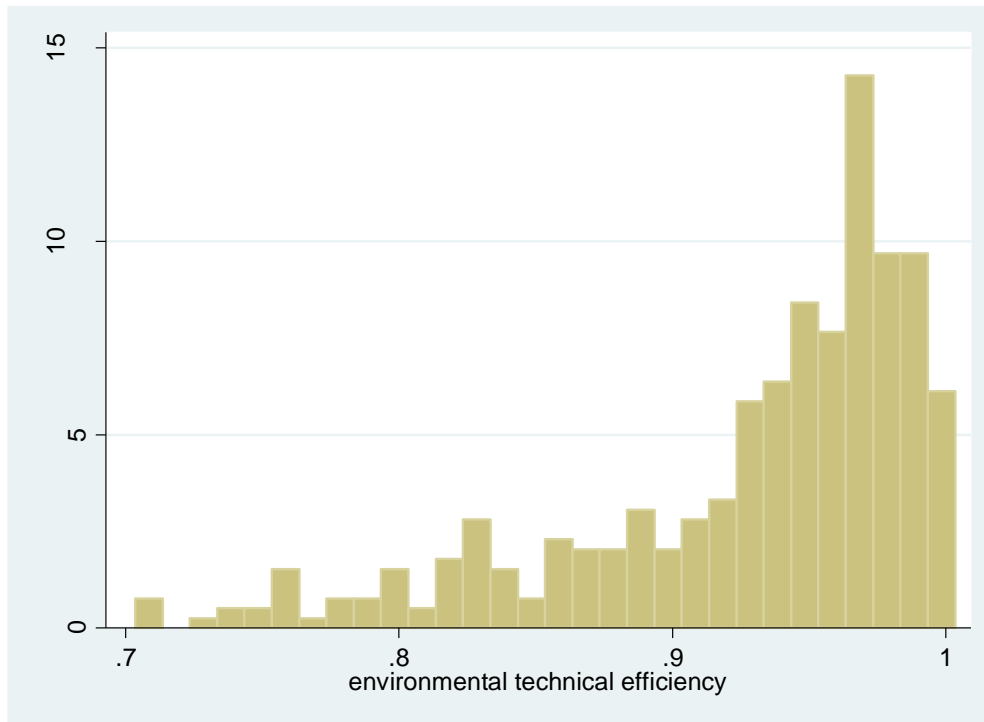
Parameter	Estimate	Std. Err
α_1 (labour)	-0.051	.047
α_2 (land)	-0.031	0.071
α_3 (capital)	-0.817***	0.044
α_4 (livestock units)	-0.354***	0.089
α_5 (variable inputs)	-0.381***	0.048
α_{11}	-0.119	0.111
α_{22}	0.057	0.176
α_{33}	-0.252	0.502
α_{44}	0.077	0.216
α_{55}	0.114	0.349
α_{12}	0.042	0.112
α_{13}	0.232	0.207
α_{14}	0.031	0.115
α_{15}	-0.411***	0.157
α_{23}	.066	.194
α_{24}	-0.069	0.141
α_{25}	-0.169	0.159
α_{34}	0.078	0.246
α_{35}	0.209	0.345
α_{45}	-0.105	0.174
β_2 (other output)	0.349***	0.022
β_{22}	0.103***	0.014
γ_s (phosphorus surplus)	-0.045***	0.009
γ_{ss}	-0.013*	0.007
δ_{12}	0.059	0.065
δ_{22}	-0.077	0.069
δ_{32}	0.279***	0.099
δ_{42}	-0.198***	0.074
δ_{52}	-0.145*	0.082
ψ_{1s}	0.040*	0.021
ψ_{2s}	0.002	0.029
ψ_{3s}	-0.017	0.052
ψ_{4s}	-0.039	0.035
ψ_{5s}	0.061**	0.028
μ_{2s}	0.017*	0.010
ρ_τ	-0.028***	0.005
α_0 (constant)	-0.010	0.015
Heteroskedasticity in σ_u		
φ_1 (age)	-0.031**	0.015
φ_2 (stocking density)	0.315	0.392
φ_3 (milk share)	-5.677***	0.810
φ_4 (concentrates value)	0.00001***	4.19e-06
φ_5 (farm size)	-0.0014	0.0059
φ_6 (land type)	-0.2562	0.4495
φ_7 (environmental subsidy)	0.5877	0.5289
φ_0 (constant)	3.3406**	1.3596
Heteroskedasticity in σ_v		
ϕ_1 (age)	0.0381****	0.0126
ϕ_2 (milk yield)	-0.00006	0.00015
ϕ_3 (concentrates value)	-9.55e-07	1.78e-06
ϕ_4 (land type)	0.0659	0.3541
ϕ_0 (constant)	-7.4323***	1.1469
Log-Likelihood	468.88	
Mean EE	0.93	
SD	0.06	
Min.	0.70	
Max	1	

Notes: Single, double, and triple asterisks (*, **, ***) indicate significance at the 10%, 5%, and 1%

The results of the analysis showed that the average environmental technical efficiency estimates of dairy farms in Northern Ireland is 0.93. The standard deviation of 0.06 reflects a low level of heterogeneity in environmental technical efficiency across the dairy farms. Similar results was obtained by Adenuga *et al.*, (2018b) in which they employed the directional distance function approach to analyse the environmental technical efficiency for dairy farms on the island of Ireland. This estimate gives an indication of how far the inefficient dairy farms are from the production frontier. It implies that on the average, dairy farmers in the country can improve their productive performance by increasing desirable output from dairy production by 7.53% ($1/0.93= 1.075$) and simultaneously contract P surplus by 7% ($1-0.93=0.07$) using current technology. The histogram of the hyperbolic efficiency estimates exhibits a left-skewed pattern with a minimum efficiency of 70.3% indicating that the least efficient farm can increase revenue from milk production by as much as 42% while simultaneously reducing P surplus by 29.7% (Figure 2).

The results imply that a reasonable percentage of the dairy farms are highly efficient. The relatively high level of efficiency may have resulted from the fact that milk quota wasn't binding in Northern Ireland during the milk quota years such that they were able to increase revenue from milk production by increasing cow numbers and milk yield per dairy cow. The parameter estimates of the year variable (ρ_τ), which is intended to capture the neutral technical change has the expected negative sign and was statistically significant providing evidence of technical progress in Northern Ireland's dairy farms over the years considered. The result implies that the efficient farms are able to increase dairy production output while making use of more environmentally friendly technologies.

Figure 2: Distribution of environmental technical efficiency in dairy farms



Determinants of environmental technical efficiency

The set of parameters (ϕ_1 - ϕ_7) presented in Table 2 explains the determinants of environmental technical efficiency. A negative parameter sign implies that the variable has a negative relationship with environmental technical inefficiency and consequently increases efficiency when the variable is increased by one more unit and vice versa. Our results showed that the age of the farmer has a significant negative influence on environmental technical efficiency. This implies that the older farmers are more efficient compared to the younger farmers. This does not conform to our a priori expectation. The reason may be that the older farmers are more experienced and conservative such that they ensure better management of nutrients. Besides, with an average age of 59 years, a good number of the farmers are still active, hence could make good use of their experiences to improve environmental technical efficiency. Reinhard *et al.*, (1999) and Weersink *et al.*, (1990) obtained a contrasting results in measuring the relationship between age and technical efficiency where they found younger farmers to be more efficient than the older farmers.

We find a positive and significant relationship between amount of concentrates fed per dairy cow and environmental technical inefficiency. This implies that an increase in the use of concentrates has a negative impact on environmental technical efficiency. This result is in contrast to that obtained by studies which estimated technical efficiency of dairy farms without incorporating undesirable output such as P surplus in the modelling framework. For example Bajrami *et al.*, (2017); Ma *et al.*, (2018), Cabrera *et al.*, (2010) found significant, positive relationship between feed use intensification and technical efficiency. The result of this study has shown that the reverse occurs in the case of

environmental technical efficiency. Our result is in line with that obtained by Reinhard *et al.*, (2002) in which they found feed use intensification to be negatively related to environmental efficiency. The implication of this result is that excess use of concentrates in dairy farms especially considering the P content of the concentrates can have detrimental effect on the environment. This result also confirms the suggestion by Ma *et al.*, (2018), that intensification of dairy farms might be associated with negative environmental effects. The negative relationship may have resulted from the fact that during the milk quota years, quota was not a constraint in Northern Ireland such that dairy farms increased milk output by increasing the use of concentrates and reducing feed intake from grazed grass. This results in more nutrient going into the soil from manure than it is taken out from it in form grazed grass.

The share of revenue from milk production was also found to have a statistically significant effect on environmental technical efficiency. This is in line with our *a priori* expectation implying that farms with greater share of revenue from milk are more efficient. This may have resulted from the notion that the more specialised dairy farms generates more revenue from the desirable outputs such that the ratio of ratio of desirable to undesirable output is higher resulting in higher level of environmental technical efficiency. Similar results were obtained by Skevas *et al.*, (2018).

Although the sign of the land type variable implies that land quality has a positive relationship with environmental technical efficiency, it is not statistically significant. This implies that there is no statistically significant difference in environmental technical efficient between lowland and less favoured area dairy farms. This may have resulted from the fact that majority of dairy farms in Northern Ireland are located on good land. Stocking density and farm size although with the expected sign were also not statistically significant.

Shadow price and pollution cost of P surplus in dairy farms.

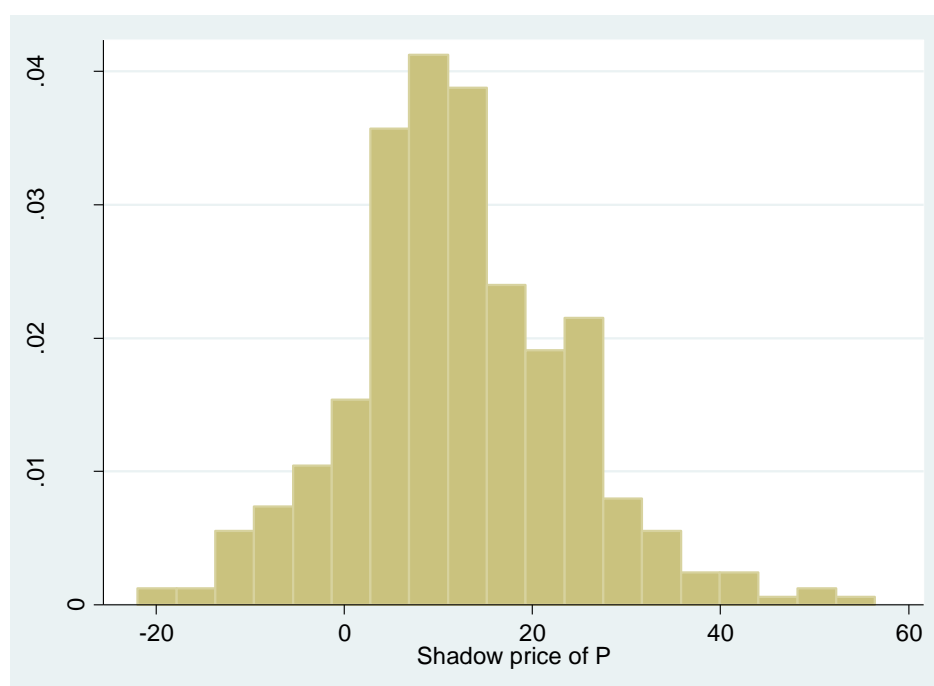
The results of the shadow price analysis based on equation (19) is presented in Table 3. The values represent a measure of the amount that has to be given up by a farm in order to reduce P surplus by one additional unit. The frontier shadow price was inflated by multiplying the ratio of the average value of output by the average value of P surplus because all input and output variables have been normalized to estimate the unknown parameters (Färe *et al.*, 2005; Tang *et al.* 2016). The price of the desirable outputs in the model are also implicitly normalised to 1 given that they are measured in monetary units (Mamardashvili *et al.*, 2016; Tang *et al.*, 2016).

Table 3: Shadow price and pollution cost ratio of P surplus in dairy farms

Year	Shadow price(£/kg)	Pollution cost ratio
2009	5.26 (10.12)	0.02 (0.10)
2010	9.47 (11.41)	0.05(0.07)
2011	10.85 (10.91)	0.04(0.05)
2012	13.60 (10.42)	0.05(0.06)
2013	17.28(12.67)	0.09(0.12)
2014	15.93(10.69)	0.07(0.07)
Average	12.29(11.74)	0.05(0.08)

Standard deviations are in brackets

The marginal abatement cost (shadow price) evaluated at the mean of the data was £12.29/Kg. This implies that on the average, £12.29 of revenue from milk production has to be given up to reduce P surplus by 1Kg. It must be emphasized that for farms bellow the frontier, their shadow price will be zero because this group of farms can still reduce P surplus without reducing revenue. The distribution of the shadow price estimate shows that a good number of the dairy farms have their shadow price around the average. However, a good number of the farms also have values above the average shadow price with reasonable degree of variation in the spectrum of shadow prices across the dairy farms. For example, the 25th percentile is 5.08 £/kg, while the 75th percentile is 19.79 £/kg and the maximum value is 56.41 £/kg (Figure 3). About 10% of the observations had negative shadow values. Mamardashvili *et al*, (2016) and Bokusheva and Kumbhakar (2014) also obtained negative values in their estimation of shadow value of N surplus in dairy farms. It is argued that a negative shadow price is feasible if pollution abatement is implemented in compliance with the regulations because there is a resource-use for abatement (Van Ha *et al.*, 2008; Mamardashvili *et al.*, 2016)

Figure 3: Distribution of shadow price of P in dairy farms

We estimated the P pollution abatement cost per farm by multiplying the derived average shadow price of P surplus by the average estimated volume of P surplus per farm for each year. With a threshold of 5Kg/ha, the value beyond which P surplus becomes a problem (Bailey, 2016), the results of our estimation showed that for an efficient farm (i.e. farms at the hyperbolic production frontier) it will cost about £20,624 per farm to abate P surplus to the 5Kg/ha threshold. These values constitute about 7.5% of revenue from dairy production output over the stipulated time period.

Also, to be able to relate pollution costs to dairy production output, we computed the pollution costs ratio for each farm. An average pollution cost ratio is obtained by dividing the aggregated pollution cost by the aggregated value of output from dairy production. The results are also presented in Table 3. It can be observed from the results that there has been a relative increase in the shadow price of P surplus and pollution cost ratio over the years considered. This implies that it has become increasingly costly to reduce P surplus in the dairy farms. This upward trend in shadow price of P surplus is consistent with those of previous studies for example, Hailu and Veeman (2000), Shaik *et al.*, (2002), Bokusheva and Kumbhakar (2014), and Adenuga *et al.*, (2018c) in which they found increasing shadow prices for nutrient surplus over time. In the interpretation of our results, it should be noted that the estimated shadow price is a measure of opportunity costs based on the assumption of full efficiency of the dairy farms. That is, farms located on the production possibility frontier. The implication of this is that the average shadow price of farms located within the production frontier may not be as high as what we have estimated. (Murty *et al.* 2007).

Conclusion

This study analysed the environmental technical efficiency and shadow price of P surplus in dairy farms employing the hyperbolic environmental technology distance function in a stochastic frontier analysis framework. The methodology allows for the internalisation of negative externalities (P surplus) in the specification of the production function. The model simultaneously account for the expansion of desirable outputs (milk and other outputs) and reduction in undesirable outputs (P surplus) thereby providing a standardized environmental technical efficiency index for each decision making unit. The empirical analysis was based on unbalanced panel data set of 83 dairy farms observed over the period of 2009-2014.

Our results revealed that dairy farms operating below the production frontier in Northern Ireland have the potential to improve their productive performance by simultaneously increasing desirable output and reducing P surplus in the production process. Intensification resulting in increased use of concentrates feed was found to be negatively related to environmental technical efficiency. We also found that age of the farmer and share of milk output in total production have a positive relationship with efficiency. This study has shown that intensification resulting from increasing use of feed concentrates in the dairy diet might results in higher animal productivity and higher desirable output

production at the farm level, it nevertheless also have a detrimental effect on the environment if not properly managed. It therefore means that appropriate strategies must be put in place by the government to ensure improved concentrates use efficiency in dairy farms for example by reducing the P content of dairy concentrates and improving the genetic potentials of the dairy cows. The results of the study also provide a possibility for the internalisation of negative externalities in dairy production in Northern Ireland as it gives an indication of how much has to be given up in order to abate one more unit of P surplus in each farm.

Finally, it is worth noting that the estimation of the shadow price is based on the assumption that farms are operating on the frontier such that, they cannot reduce P surplus without reducing desirable outputs. However, as shown by the environmental technical efficiency estimates, farms operating below the frontier can reduce P surplus without reducing desirable output. The measurement of the level of environmental technical efficiency of the dairy sector would improve the knowledge of sustainable dairy production systems and aid in the understanding of livestock sector impacts. The derived results will also help policy makers in understanding the trade-offs between the desirable and undesirable outputs when farm-level inefficiencies are eradicated, assisting them in devising balanced policies to improve current operations and enhance sustainability.

References

- Adenuga A. H., Davis J., Hutchinson G, Donnellan T., Patton M. (2018a). Estimation and determinants of phosphorus balance and use efficiency of dairy farms in Northern Ireland: A within and between farm random effects analysis, *Agricultural Systems*, 164:11-19
- Adenuga A. H., Davis J., Hutchinson G, Donnellan T., Patton M. (2018b). Modelling regional environmental efficiency differentials of dairy farms on the island of Ireland, *Ecological Indicators*, 95(1):851-861
- Aigner, D.J., C.A.K. Lovell, and P. Schmidt (1977). Formulation and estimation of stochastic frontier production function models. *Journal of Econometrics* 6:21-37.
- Bailey, J.S. (2015). *Assessing the environmental risks associated with newly revised P application limits for farmland* in NI. In: Proceedings of the Agricultural Research Forum 2015 (ed. M. McGee and Agricultural Research Forum Committee), Co. Meath, Ireland, Teagasc.
- Bailey J. S. (2016). Phosphorus management for sustainable dairy production. Paper Presented at the Step to Sustainable Livestock International Conference, Bristol, United Kingdom (2016), p. 51. Available at: <http://www.globalfarmplatform.org/wp-content/uploads/2016/04/GFP-Steps-to-Sustainable-Livestock-2016-Conference-Abstracts.pdf>, Accessed 20th Sep 2017
- Battese, G. E., and T. J. Coelli. (1988). Prediction of firm-level technical efficiencies with a generalized frontier production function and panel data. *Journal of Econometrics* 38: 387–399.
- Battese, G. E., and T. J. Coelli (1995). A model for technical inefficiency effects in a stochastic frontier production function for panel data. *Empirical Economics* 20: 325–332.

- Belotti, F., Daidone, S., Ilardi, G., Atella, V. (2013) "Stochastic Frontier Analysis using Stata". *Stata Journal*, 10(3), 458-481.
- Bajrami E., Wailes E. J., Dixon B. L., Arben Musliu A. And Alvaro Durand-Morat A. (2017). Factors affecting the technical efficiency of dairy farms in Kosovo. *Journal of Central European Agriculture*, 18(4):823-840
- Bokusheva R and Kumbhakar S. C. (2014). *A Distance Function Model with Good and Bad Outputs*. Paper presented at the EAAE 2014 Congress 'Agri-Food and Rural Innovations for Healthier Societies' August 26th to 29th, Ljubljana, Slovenia. Available at: https://ageconsearch.umn.edu/bitstream/182765/2/Bokusheva-Distance_function_model_with_good_and_bad_outputs-258_a.pdf
- Boyd, G.A., Tolley, G., Pang, J., (2002). Plant level productivity, efficiency, and environmental performance of the container glass industry. *Environ. Resource. Econ.* 23 (1), 29–43
- Cabrera V. E., Solís D., and del Corral J (2010). Determinants of technical efficiency among dairy farms in Wisconsin, *Journal of Dairy Science* (93)(1):387-393.
- Caudill, S. B., J. M. Ford, and D. M. Gropper. 1995. Frontier estimation and firm specific inefficiency measures in the presence of heteroscedasticity. *Journal of Business and Economic Statistics* 13: 105–111.
- Cave, S. and McKibbin, D., 2016. *River Pollution in Northern Ireland: An Overview of Causes and Monitoring Systems, with Examples of Preventative Measures*. Belfast, NIAR 691-15. Research and Information Service, Northern Ireland Assembly, Retrieved from. <http://www.niassembly.gov.uk/globalassets/documents/raise/publications/2016/environment/2016.pdf>
- Chambers R. G., Chung Y. H., and Färe R. (1998). Profit, directional distance functions, and Nerlovian efficiency. *J Optimiz Theory Appl* 198:351–64.
- Chung Y. H, Färe R. and Grosskopf S. (1997). Productivity and Undesirable Outputs: A Directional Distance Function Approach. *Journal of Environmental Management*, 51:229–240
- Coelli T. J., Rao D.S.P., O'Donnell C.J., Battese G.E. (2005). *An Introduction to Efficiency and Productivity Analysis* (second ed.), Springer
- Cuesta A. R., Lovell C. A. K., and Zofio J. L. (2009). Environmental efficiency measurement with translog distance functions: A parametric approach. *Ecological Economics* 68 (2009) 2232–2242
- Cuesta, R. A., and J. L. Zofio (2005). "Hyperbolic Efficiency and Parametric Distance Functions: With Application to Spanish Savings Banks." *Journal of Productivity Analysis* 24:31–48. doi: 10.1007/s11123-005-3039-3.
- Dairyman (2011). *An Assessment of Regional Sustainability of Dairy Farming in Northern Ireland*. Work package 1 Action 1. Available at, http://www.interregdairyman.eu/upload_mm/2/6/4/bba8dca7-6df5-41ae-ab68-2ae245e47a25_NI-Regional_Report_-_WP1A1.pdf. 56pp.
- Department of Agriculture, Environment and Rural Affairs (DAERA) (2016a). *Northern Ireland Agri-Food Sector, Key Statistics*, Northern Ireland, United Kingdom. Author 16pp.

Retrieved from <https://www.daera-ni.gov.uk/sites/default/files/publications/daera/Northern%20Ireland%20Agri-food%20Sector%20Key%20Statistics%202016%20Final.pdf>.

- Department of Agriculture, Environment and Rural Affairs (DAERA), (2016b). *Nitrates action programme 2015–2018 and phosphorus regulations* Guidance Booklet. 125pp. Available at: <https://www.daera-ni.gov.uk/sites/default/files/publications/dard/nap-2015-2018-and-phosphorus-regulations-guidance-booklet-final-may-2016.pdf>, Accessed date: 15 June 2016
- Department of Agriculture, Environment and Rural Affairs (DAERA), (2017). Northern Ireland Agri Food Sector Key Statistics CAP, Policy, Economics and Statistics Division, Belfast. Retrieved from: <https://www.daera-ni.gov.uk/sites/default/files/publications/daera/Northern%20Ireland%20Agri-food%20Sector%20Key%20Statistics%202017.pdf>. Accessed 6th December 2017
- Department of Agriculture, Environment and Rural Affairs (DAERA) (2018). Indices of producer prices 1981 onwards. Available at: <https://www.daera-ni.gov.uk/publications/indices-producer-prices-1981-2012>
- Du, L., Hanley A., and Wei C. (2015). Marginal abatement costs of carbon dioxide emissions in China: a parametric analysis. *Environ Resour. Econ.* 61: 191-216
- Duman Y. S. and Kasman A (2018). Environmental technical efficiency in EU member and candidate countries: A parametric hyperbolic distance function approach. *Energy*, 147:297-307
- Eurostat (2013). *Nutrient Budgets – Methodology and Handbook*. Version 1.02, Luxembourg, Eurostat and OECD.
- Ewing, W.N., (2002). *The Feeds Directory: Commodity Products Guide*. Context
- Färe R., Grosskopf S., Lovell C. A. K., C. Pasurka C. (1989). Multilateral productivity comparisons when some outputs are undesirable: a nonparametric approach. *Rev Econ Stat*, 71:90-98
- Färe R, Grosskopf S, Lovell C. A., and Yaisawarng S. (1993). Derivation of shadow prices for undesirable outputs: a distance function approach. *Rev Econ Stat*; 75: 374–80
- Färe, R., Grosskopf, S., and Zaim, O. (2002). Hyperbolic efficiency and return to the dollar. *European Journal of Operational Research*, 136(3), 671-679. doi:[https://doi.org/10.1016/S0377-2217\(01\)00022-4](https://doi.org/10.1016/S0377-2217(01)00022-4)
- Färe, R., Grosskopf S., Noh, D. W. and Weber, W. L. (2005). Characteristics of a polluting technology: Theory and practice. *Journal of Econometrics*, 126: 469-492
- Färe, R., Grosskopf S., and Weber, W. L. (2006). Shadow prices and pollution costs in U.S. agriculture. *Ecological Economics*, 56:89-103
- Glass C. J., McKillop D. G., Quinn B. and Wilson J. (2014) Cooperative bank efficiency in Japan: a parametric distance function analysis, *The European Journal of Finance*, 20 :(3), 291-317, DOI: 10.1080/1351847X.2012.698993

- Greene, W. (2005). Reconsidering heterogeneity in panel data estimators of the stochastic frontier model. *Journal of Econometrics* 126: 269–303
- Greene, W. (2012). *Econometric Analysis*. 7th ed. Upper Saddle River, NJ: Prentice Hall.
- Hadley D. (1998). *Estimation of Shadow Prices for Undesirable Outputs: An Application to UK Dairy Farms*. Paper presented at the American Agricultural Economics Association Annual Meeting, Utah. Available at: <http://ageconsearch.umn.edu/record/20977>. Accessed 15th May 2017.
- Hadri, K. 1999. Estimation of a doubly heteroscedastic stochastic frontier cost function. *Journal of Business and Economic Statistics* 17: 359–363
- Hailu A., and Veeman, T. S. (2000). Environmentally sensitive productivity analysis of the Canadian pulp and paper industry, 1959-1994: An input distance function approach. *Journal of Environmental Economics and Management*, 40(3), 251-274
- Hailu, A., Chambers, R.G., (2012). A Luenberger soil-quality indicator. *J. Prod. Anal.* 38 (2), 145–154.
- Hou L., Hoag D. L. K. and Keske C. M. H. (2015). Abatement costs of soil conservation in China's loess plateau: balancing income with conservation in an agricultural system. *Journal of environmental management*, 149: 1-8
- Huhtanen, P., Nousiainen, J. and Turtola, E. (2011). Dairy farm nutrient management model: Evaluation of different strategies to mitigate phosphorus balance. *Agricultural Systems*, 104, 383–391.
- Humphreys J., Connell K. O. and Casey I. A. (2008). Nitrogen flows and balances in four grassland-based systems of dairy production on a clay-loam soil in a moist temperate climate. *Grass and Forage Science*, 63, 467–480
- Kleinman, J. A. P., Sharpley A. N., Withers J. A. P., Bergstrom L., Johnson T. L., and Doody D. G. (2015). Implementing agricultural phosphorus science and management to combat eutrophication. *AMBIO* 44(Suppl. 2): 297-310. doi: 10.1007/s13280-015-0631-2
- Kumbhakar, S. C., and C. A. K. Lovell. 2000. *Stochastic Frontier Analysis*. Cambridge: Cambridge University Press
- Loro P., Arzandeh M., Brewin D., Akinremi W., and Ige D. (2013). *Estimating soil phosphorus budgets for rural municipalities in Manitoba*. Technical Report DOI: 10.13140/2.1.5063.3767
- Ma W., K. Bicknell and Renwick A (2018). Feed use intensification and technical efficiency of dairy farms in New Zealand. *Australian Journal of Agriculture and Resource Economics*, 00:1–19 <https://doi.org/10.1111/1467-8489.12283>
- Mamardashvili P., Emvalomatis G., and Jan P. (2016). Environmental Performance and Shadow Value of Polluting on Swiss Dairy Farms. *Journal of Agricultural and Resource Economics* 41(2):225–246. Available at: <http://www.waeonline.org/UserFiles/file/JAREMay2016Mamardashvili225-246.pdf>
- Manello A. (2012). *Efficiency and productivity in presence of undesirable outputs*. Published PhD. Thesis, University of Bergamo, Italy

- Mbehoma P. M. and Mutasa F. (2013). Determinants of Technical Efficiency of Smallholders Dairy Farmers in Njombe District, Tanzania. *African Journal of Economic Review*, 1(2):15-29
- McDonald, P., Edwards, R., Grenhalgh, J., Morgan, C.,(2002). *Animal Nutrition*. Prentice
- Macpherson, A.J., Principe, P.P., and Smith, E.R. (2010). A directional distance function approach to regional environmental-economic assessments. *Ecological Economics*, 69(10), 1918-1925.
- Northern Ireland Statistics and Research Agency (NISRA)(2018). 2017 *Mid-year Population Estimates for Northern Ireland*. Retrieved from <https://www.nisra.gov.uk/news/2017-mid-year-population-estimates-northern-ireland>.
- NRC – National Research Council. (2001). *Nutrient requirements of dairy cattle*. 7th revised edition. Washington, National Academy Press. 408pp
- Peña C. R., Serrano A. L. M., de Britto P. A. P., Franco V. R., Guarnieri P., Thomé K. M. (2018). Environmental preservation costs and eco-efficiency in Amazonian agriculture: Application of hyperbolic distance functions, *Journal of Cleaner Production*, 197(1):699-707. <https://doi.org/10.1016/j.jclepro.2018.06.227>
- Pérez-Urdiales, M., Lansink, A.O. and Wall, A. (2016). Eco-efficiency among dairy farmers: The importance of socio-economic characteristics and farmer attitudes. *Environmental and Resource Economics*, 64(4): 559-574. DOI <https://doi.org/10.1007/s1064>
- Peyrache, A., and T. Coelli. “A Multiplicative Directional Distance Function.” Working Paper WP02/2009, University of Queensland Centre for Efficiency and Productivity Analysis (CEPA), Queensland, Australia, 2009. Available online at <http://www.uq.edu.au/economics/cepa/docs/WP/WP022009.pdf>.
- Reinhard, S., C.A.K. Lovell and G. Thijssen (1999). “Econometric Estimation of Technical and Environmental Efficiency: An Application to Dutch Dairy Farms.” *Amer. J. Agr. Econ.* 81:44–60.
- Reinhard S., Lovell C. A. K., and Thijssen G. (2002). Analysis of environmental efficiency variation, *Amer. J. Agr. Econ.* 84(4):1054–1065
- Shaik, S., Helmers, G.A. and Langemeier, M.R. (2002). Direct and Indirect Shadow Price and Cost Estimates of Nitrogen Pollution Abatement, *Journal of Agricultural and Resource Economics* 27, 420-432.
- Shephard R. W. (1970). *Theory of cost and production functions*. Princeton: Princeton University Press.
- Skevas I., Zhu X., Shestalova V., and Emvalomatis G. (2018). The Impact of Agri-Environmental Policies and Production Intensification on the Environmental Performance of Dutch Dairy Farms. *Journal of Agricultural and Resource Economics* 43(2):423–440
- Simar, L. and Wilson, P. W. (2007). Estimation and inference in two-stage semi-parametric models of production processes, *Journal of Econometrics* 136: 31–64
- Suta, C. M., A. Bailey, and S. Davidova (2010). *Environmental efficiency of small farms in selected EU NMS.* Paper presented at the 118th Seminar of the European Association of Agricultural Economics, August 25-27, Ljubljana, Slovenia.

- Tang K., Gong C. and Wang D. (2016). Reduction Potential, Shadow Prices and Pollution Costs of Agricultural Pollutants in China. *Science of Total Environment* 541: 42-50
- Van Ha, N., S. Kant, and V. Maclaren (2008). “Shadow Prices of Environmental Outputs and Production Efficiency of Household-Level Paper Recycling Units in Vietnam.” *Ecological Economics* 65:98–110. doi: 10.1016/j.ecolecon.2007.06.003.
- Wang, H.-J., and P. Schmidt. (2002). One-step and two-step estimation of the effects of exogenous variables on technical efficiency levels. *Journal of Productivity Analysis* 18: 129–144
- Wang K., Che L., Ma C. and Wei Y. (2017). The shadow price of CO₂ emissions in China’s iron and steel industry. *Science of total environment* 598: 273-281.
- Weaver, D. M. and Wong, M. T. F. (2011). Scope to improve phosphorus (P) management and balance efficiency of crop and pasture soils with contrasting P status and buffering indices. *Plant and Soil*, 349, 37–54.
- Weersink, A., C.G. Turvey and A. Godah.(1990) “Decomposition Measures of Technical Efficiency for Ontario Dairy Farms.” *Can. J. Agr. Econ.* 38:439–56
- Wei C., Loschel A. and Liu B., (2013). An empirical analysis of CO₂ shadow price in Chinese thermal enterprises. *Energy Econs* 40: 22-31
- Zhou P., Zhou X., and Fan, L. W. (2014). On estimating shadow prices of undesirable outputs with efficiency models: A literature review. *Applied Energy* 130: 799-806