Optimal Agricultural Pest Management with Multiple Species

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Increased concern for the environmental effects of pesticides has led to considerable interest in optimal management strategies for controlling pest populations affecting agricultural production. This issue has been considered by a number of researchers (Feder and Regev; Hall and Norgaard; Hueth and Regev; Lazarus and Swanson; Marsolan and Rudd; and Talpaz and Frisbie). With the exception of Feder and Regev, these studies considered only one species. This approach involves serious limitations since a grower is generally confronted with multiple species during the production period. For example, insect prey-predator relationships may exist in the field (Feder and Regev). Alternatively, as an insect pest develops through a number of growth stages, multiple pests in effect exist (Reichelderfer and Bender).

The biological interaction of multiple species influencing a management strategy has been examined by Larkin (1966), and Shoemaker (1973a, 1973b). However, investigation of the economic implications of species interaction influencing a producer's management strategy has received limited attention. Feder and Regev were concerned only with interactions occurring between a prey-predator relationship and did not consider the case of multiple pests.

The objective of this paper is to extend the theoretical models of earlier research to accommodate multiple species. A general dynamic model of two pests with interaction is specified in an optimal control framework. The maximum principle is utilized to derive theoretical solutions for the problem which illustrates the impact of multiple species on optimal pest management strategies.

Basic Theoretical Model

A common concept of pest management is to utilize economic thresholds based on pest population levels and economic returns from pest control. Management programs of this type involve multiperiod analyses and are necessarily dynamic. Our approach assumes producers operate within a multi-period optimization framework. Producers control decisions influence two insect species, one possibly a beneficial, and the other a pest, or alternatively, one species with two distinct growth stages, in a single crop environment. The planning horizon is one growing season with the season partitioned into periods beginning in period t_0 and ending in period T.

Given a two-species, one-crop management model, the state variables of the system at the beginning of period t are:

q_t = measure of plant yield,
x_{1t} — the density of species 1, and
x_{2t} — the density of species 2.

The producer in each time period has control of u_t, a species control input, which is a composite of various control inputs directed at the species. For example, u_t may be composed of various insecticides for controlling an insect pest.

The objective of the model is to maximize the net benefits or net returns over a growing season with other control inputs besides species control inputs at their optimal levels. Given a concave benefit function $B(q_T)$ and a convex cost function $c(u_t)$ the objective is to:

(1)   Maximize $B(q_T) - \sum_{T=t_0}^{T-1} c(u_t)$.

The basic problem includes three sets of constraints in difference equation form. Two constraint sets represent the species control equations. These constraints model the effect of the control inputs on the population level of a species from period t to t + 1 and are given by:

(2)   xn-t-i = x_{lt} + f Kxu, x_{2t}, u_t, t),
                    (i = 1,2) (t = to, . . . , T - 1),

where f' is the change in the species population. Equation (2) is a very simplified difference equation for resource level determination. Similar models have been employed by Hall and Norgaard, and Hueth and Regev. The main difference in these equations, compared with previous models, is the inclusion of possible species interaction. For example, X_j could be a predator to x_t resulting in $fxjt^1 < 0$. Other properties of f * are $f_{xit}* > 0$ and $f_{ut}^! \wedge 0$. The argument t enters explicitly in (2) since the species population level may change through time.

   The third constraint set corresponds to the plant growth equations. Plant growth at t + 1 is,

(3) q_{t+i} = q_t + w(q_t, x_u, X_{2t}, t),
                              (t = to, . . . , T - 1),

where w is the increase in plant size from t to t + 1 given any damage that may have occurred during this interval. If x_{lt} is a pest then $w_{xit} < 0$, if it is a beneficial then $w_{xit} > 0$. Equation (3) is, once again, a common plant growth equation with the main exception being the influence of multiple species. In most applications, qx is actual final output of the crop and q_t is potential final output assessed in period t.

Optimal Control Problem

The optimal control problem is one of maximizing (1) subject to constraint sets (2) and (3). To find the necessary conditions to this maximization problem a procedure analogous to static optimization with Lagrange multipliers is used. The constraining relations must hold at each t over the entire interval t_0 to T, so a multiplier function rather than a single Lagrange multiplier for each constraint is employed. The Lagrangean function is then,

(4) $$L = \overset{\wedge}{\underset{t=t_0}{}} H + X_{t+},(q_t - q_{t+1})$$

$$+ 2 \overset{2}{\underset{i=I}{}} rit+i(Xit - x_{it+1}) \overset{1}{}$$

where X and y_t are known as costate, auxiliary, adjoint or influence variables and are the dynamic equivalents of the Lagrange multipliers of static problems. H, the Hamiltonian function, is,

(5) $$H = -c(u_t) + X_{t+1}w(q_t, X\ll, X_{2t}, t)$$

$$+ \overset{2}{\underset{i=I}{}} X Tfc+if *(X_{lt}, X_{2t}, u_t, t).$$

The Hamiltonian function is defined as the intermediate function, $c(u_t)$, of the objective functional plus the product of the costate variables and functions defining the rate of change of the state variables. Imagine H to be multiplied by A, a change, then H is the sum of the total cost incurred in the interval A plus the accrual of the state variables during the interval valued at their marginal values. Thus, AH is the total contribution of the activities that go on during the interval A, including both the direct effect to the objective functional and the change in the value of the state variables during the interval. Differentiating (4) with respect to u_t and equating the derivative to zero results in the following necessary condition.

(6) $aL/3u_t$ $^=$ — flc/dut + $yit+ifut^1$

$$+ T2_{t+1} f_{ut}^2 = 0,$$

This states that along the optimal path of the decision variable at any time the marginal short-run effect of a change in decision must just counter-balance the effect of that decision on future benefits. Therefore, u_t should be chosen so that the marginal immediate cost just equals the marginal long-run benefit, which is measured by the increment in the total value of the objective functional, (-yit+i, 721+1), multiplied by the effect of the decision on the change in the state variables. The remaining necessary conditions are obtained by differentiating (4) with respect to q_t and Xn-

(7) $dL/3q_t$ = $Xt+iwqt$ + X_{t+1} — X_t = 0,

(8) dL/aXu = $Xi+iw_{xit}$ + $-yit+ifxit^1$
         + $yat+ifxn^2$ + $yit+1 - y^\wedge = 0,$
                                              (i = 1,2).

   The rates at which the value of a state variable are changing are given by — (X_{t+1} — X_t)

and $-(y_{it+1} - 7u)$. Equations (7) and (8) assert that when the optimal time path of changes in the state variable is followed, the change in value of a unit of the state variable in a short interval of time is its contribution to enhancing the value of the state variable at the end of the interval. Or expressed in a different manner, systemic changes in crop and insect states optimally controlled, will lead to conditions in each subperiod within the production horizon, which will generate the necessary conditions for optimality at the end stage. These rates can also be interpreted as the loss that would be incurred if the change in a state variable were postponed for a short time, or alternatively the net contribution of change in the level of the state variables to future benefits. Finally, the transversality conditions for this model are,

(9) $x_T = dB/aq_T = 0$, and y_{iT}
                $= dB/dX_{iT} = 0$, $(i = 1,2)$.

The economic interpretation of these conditions is clarified by obtaining the following expressions for $A_{\cdot t}$, y_{lt} and $y_2 t$ from equations (7) through (9), (see Appendix):

(10) $A_{\cdot t} = (aB/aq_T)OqT/3q_t)$, and

(11) $y_{it} = E_t^{l} + At^1$

where:

$E t^1 = \underset{r=t}{JT} \ (aB/aq_T)(aq_T/aq_{r+1})$

                              $(aq_{r+j}/dX_{ir}) \ (\ aX_{ir}/aX_{it})$,

$A,' = \underset{r=t}{\overset{T-l}{2 \ N}}$

        $\underset{l}{JE_{r+1}^{k}} + \underset{r=t+l}{£ \ D^\text{fin-i}^1} \quad \backslash$

     $+ \underset{r=t+2}{]T \ D_r}(E_{r+1}^{*} + \underset{r=t+3}{£ \ Dp^E^1} \quad V$

     $+ \underset{r=t+4}{\overset{T-l}{£ \ D_r'}} \ldots + \underset{r=t-l}{\overset{T-l}{X \ IVE,''}} \quad ^{Y1}_{///}$

$(i \ ^ \ k; i, \ k = 1, 2)$, and $D_r^* = (ax_{kt}W \ ax_{ir}) \ (ax_{ir}/ax_{it})$.

Equation (10) can be clearly interpreted. λ_t is the additional level of benefits received from a given amount of plant growth in period t. Equation (11) is, however, a bit more complex. This equation measures the effect that species population levels exert on plant growth and thus on net benefits. Interaction

between the species is measured by the expression, At^1. Given independence between the species, i.e., no interaction, At^1 equals zero and (11) simply becomes,

(12) $y_{it} = Et^1$.

Thus, y_{it} measures the change in the level of benefits from a given population level of the species in period t. Considering interaction A_t^l measures the rippling effect through time caused by the interaction among species. Or more explicitly, the effects of multi-pest or pest-predator interaction in the current period are distributed over subsequent periods due to abberations in population dynamics imposed by the intruding species. Specifically, D^l measures the population level of species k in the next period given a change in the population level of species i. Combining D_r' and E_{r+1}^{k} results in a measure of the change in benefits given a change in species k caused by a change in species i. A similar process occurs with expressions D_r^{k} and E_{r+j}^{1} except species k is now influencing benefits through species i. This interaction may continue forward in time until the terminal state T is obtained. However, as with most rippling effects, the process will probably dampen out after two or three periods, as the initial effect is incorporated into the growth process.

The effect of species interaction on benefits depends on whether the interaction is a prey-predator or multiple pest relationship. For example, assuming species one is a predator that reduces the population of species two and also that species two has no effect on species one then (11) would reduce to:

              $TH = E t^1$,
              $Tfet = E_t^2 + A_t^2$.

In this case the interaction is negative, $D_r^2 < 0$, resulting in an increase in net benefits.

Economic Thresholds

The optimal timing and amount of control applied to field crops can be determined from investigating (6). Substituting (10) and (11) into (6) gives:

(13) $-ac/5u_t$
       $+ (Et_{-,/} + A_{t+1}^1)(ax_{lt+1}/3u_t)$
          $+ (E_{t+l}^2 + A_{2t+1}^2)(ax_2t_{+1}/aut) = 0$.

The terms in (13) are the marginal values associated with the application of a control vari-

able in period t. The first term is the marginal cost of the control in period t. The following two terms are the marginal values of the accumulative effects of controlling species population levels with u_t. Equating these marginal values to zero maximizes net benefits. If these species are detrimental pests, then these terms are marginal benefits. If, however, a species is beneficial and an application u_t reduces its population level, then the marginal benefit transfers to a marginal cost.

It is interesting to note that the economic threshold is still influenced by the presence of multiple species even if no interaction occurs within the system. In this case (13) reduces to:

$$(14) \quad -ac/du_t + E_{t+1}i^1(dx_{1t+1}/du_t)$$
$$+ E_{t+1}^{2}(dx_{2t+1}/3u_t) = 0,$$

and, as is evident, u_t remains a function of both species population levels.

Equation (13) can also be interpreted to indicate the impact of abstracting from multiple species. If species two is ignored:

$$(15) \quad -ac/au_t + Et+xKdXu+i/aut) * 0.$$

If x_2 is a beneficial species then the full level of marginal resource cost is not considered in (15). In this case the producer will administer the control prematurely when marginal costs are greater than marginal benefits. Similarly if x_2 is a detrimental species, marginal costs will be less than marginal benefits and the producer will have applied the control either too late or in insufficient quantities. These conclusions hold regardless of the interaction among species. That is, a comparison between (14) and (15) reveals that a bias still exists in the economic threshold if one of the species is ignored.

The implications of these results for economic threshold determination are illustrated in Figure 1 for two pest populations in period v within a production horizon. An additional pest control option with cost MC is compared with the marginal benefits of the control, MBi and MB_2, associated with pest 1 and 2, respectively. The marginal benefit curves are concave following the assumption that the positive effects of the control action decline through time. Considering pest 1, point A in Figure 1 is the level of direct benefits in period v associated with a pest control. In subsequent periods the population of pest 1 will be reduced as a result of the pest control in period v. The level of these additional benefits are measured in Figure 1 by B-A. However, only

considering pest 1 results in the cost of the pest control, C, exceeding the total benefits, B, and no control should be taken. With the inclusion of pest 2, but ignoring any possible pest interaction, as described by (14), control cost may still exceed benefits. This is indicated in Figure 1 where point D, the total benefits associated with this control action, is less than its cost, C. If pest interaction occurs in a dynamic population context, as described by (13), marginal benefits of control, point E, will exceed costs, point C, and a control action should be enacted. Thus, failure to consider multiple species interaction in a dynamic context may result in a suboptimal cost action.

Approaches to Application

The dynamic and stochastic nature of natural processes contribute to the difficulties of finding closed form solutions to specific empirical problems. Dimensionality constraints place further limits on acceptable approaches to feasible solutions. Various empirical methodologies have been applied to pest control problems. Reichelderfer and Bender approached their study through simulation methods. Regev, Gutierrez and Feder; Marsolan and Rudd; and Talpaz et al. have used nonlinear optimization techniques to solve their problems numerically. These numerical search procedures have computational problems as the number of variables increase. Nonseparability of the movements of state variables from one period to another rules out the possibility of solving this type of control system analytically. Specifically, pest densities and control decisions are interdependent among periods. Methods capable of incorporating these linkages are needed to effectively model the evolution of the system states.

Dynamic programming and polyperiod programming are two approaches sufficiently comprehensive in their methods to accommodate the specific problems of agro-ecosystem modeling. Optimization of a multiperiod model within a consistent dynamic theoretic framework, requires incorporation of all significant interrelationships between periods. Modeling multiple pest populations, and several growth stages for each pest, requires a large number of state variables to effectively approximate field conditions. An advantage of polyperiod programming over dynamic pro-
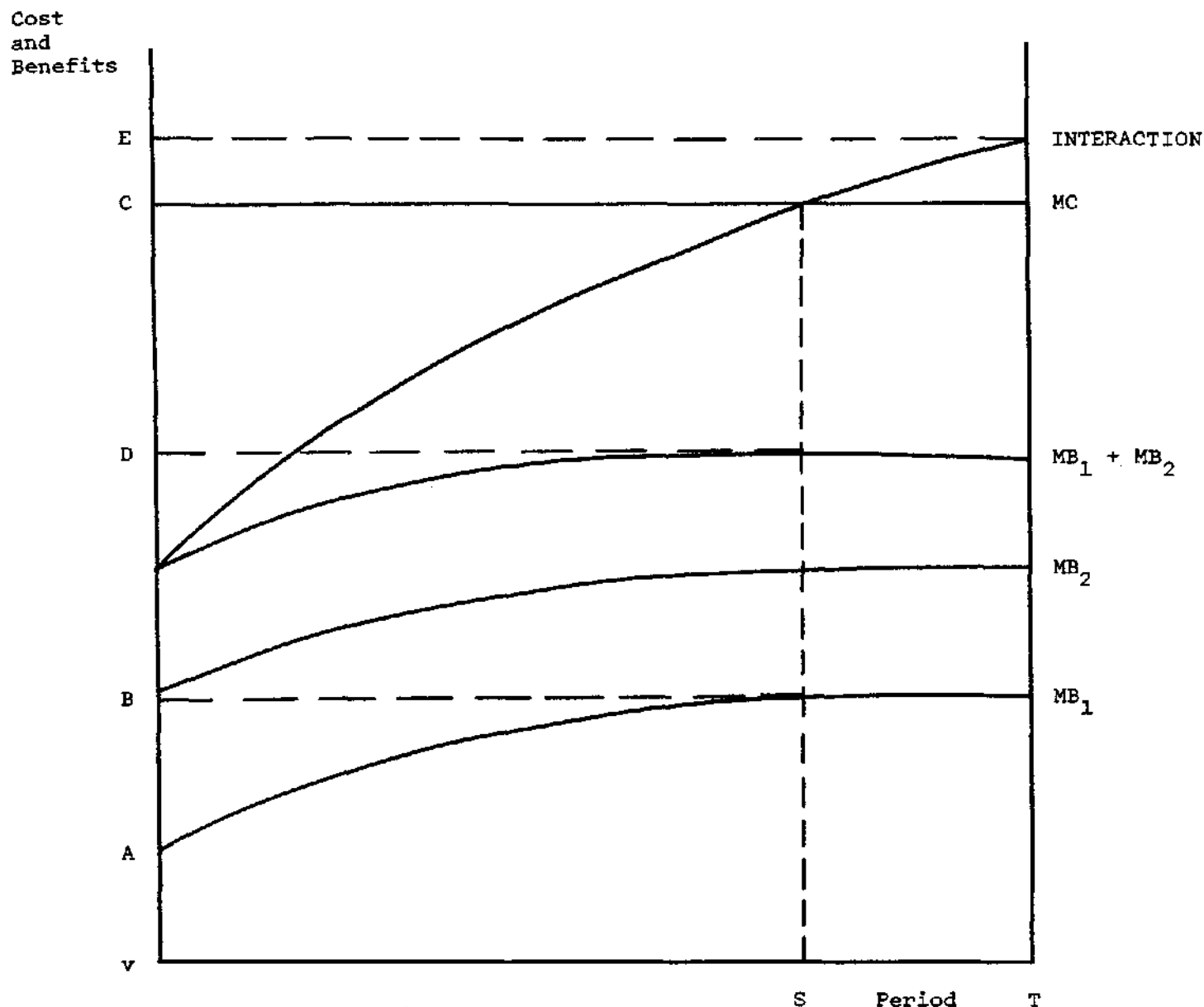
Cost
and
Benefits

Figure 1.   The Effect of Pest Interaction on Economic Threshold Determination in Period v.

gramming is the ability of polyperiod models to accommodate this requirement. In addition, this methodological approach provides improvements over simulation which only gives the best outcome from alternative exogenously determined control methods.

Another attractive method in modeling dynamic systems is mixed integer programming, which is appropriate when one or more of the control variables can take on only integer values. This is generally the case for pesticide applications where the decision to apply a pesticide is discrete. The objective function of such a model is to maximize returns above

neously select the optimal combinations of control variables and pest densities at all time periods. Activities can be divided into three sets: species activities, treatment activities and damage activities. Intertemporal relationships among the state variables and their relationship with the control variables are explained by row restrictions. For instance, growth of a pest may be sensitive to variations in weather and other environmental conditions. While the full stochastic environmental influences cannot be represented by this type of model, the influences in different time periods for various growth stages of pests can be

and rows for each growth stage at each time period. Data required for estimating pest population densities are the densities of pests at various growth stages for each time period.

Treatment and damage activities can be modeled in a similar fashion. Equations for the treatment activities would be based on the kill efficiency rates of various control methods. Estimation of pest damage equations are a more difficult methodological problem due to the continuous nature of crop injury through the growth period. The usual approach is estimating damages in terms of final yield reduction. Damage data may be obtained from experiments where pests were allowed to attack the crop at varying time durations.

Two major weaknesses of polyperiod models are their nonstochastic nature and requirements of linear activities. The latter problem can be accommodated by developing a linear spline type technology through estimation of linear pest treatment and damage activities for each time period. This provides for possible nonlinear relationships of the activities. The former problem is a more difficult one to reconcile. The stochastic nature of the crop growth system requires the incorporation of a modeling process reflecting these random traits. However, stochastic programming models become unwieldy as the number of activities and controls increase to reflect the stochastic nature of the underlying processes. Even very simple models can reach unmanageable proportions (Anderson, Dillon, and Hardaker). The alternative is to resort to simulation where the optimality algorithm has been removed and thus both the nonlinear and stochastic nature of the problem can be investigated.

Conclusions

The results of this paper indicate that the development of optimal management programs for the control of crop pests should consider the existence and possible interaction of multiple species. Research investigating these interactions will lead to improvements in economic threshold determination and aid decision-making relative to control of the production environment. Two areas of further research are logical follow-ups to this study. First, theoretically investigating the possible interactions of multiple crops on pest popula-

tions. For example, one crop may be a host for an insect which affects another crop. This could be accomplished by modifying equation (2) to incorporate plant growth. Second, investigate empirically the influence of multiple species and crops on economic thresholds. Unfortunately, current data limitations in many cases constrain research of this type. Further developments in biological research designed to uncover significant systematic relationships will facilitate future empirical economic investigations.

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Appendix

Solving the difference equations (5) and (6) for A_t, y_u and y_2t respectively results in:

(AI) $$X_t = X_T \, Y\!/ \, (1 + {}^w \!\ll r)$$
$$\overset{T-I}{\underset{i-t}{}}$$

(A2) $$Y_U = X_{t+}, w_{Xu} + -X_{kt+}if x_{lt}{}^h$$

$$+ X \, x_{v+1} w_{Xtv} \, f[\, a + w>$$
$$\underset{v=t+I}{} \qquad \underset{s=t}{}$$

$$+ \chi^T \! 7k_{V+1} f_{\chi iv}{}^h \, n_{}^{v-1}$$
$$\underset{v-t+I}{} \qquad \underset{s=t}{}$$

$$(i + w> + V\!IT \, n \, (i + V),$$
$$\underset{r-t}{}$$

$$(i * k; \; i, k = 1, 2).$$

Note that:

(A3) $$w_{x,t}{}^{j} = aq_{jt+i}/S_{xu},$$

(A4) $$f^*{}_{lt}{}^{k} = ax_{kt+l}/3x_{lt}.$$

From equation (3):

$$q_T = q_T - i + W_T - i$$
$$= qT - 2 + {}^w T - 2 \; "*" \; WT - I$$

$$= q_t + w_t + w_t + i \ldots + WT^\wedge I$$

hence (A5) $$3q_T/{}^\wedge q_t = 1 + dwt/flqt$$
$$+ \, dwt + i/aqt \ldots + aw_T - !/aq_t.$$

since

$$dWt + i/Sqt = (dWt + i/aq_{t+}t)(aq_{t+l}/aq_t) = w_{Qt+1}(I + w_{qt}),$$

it can be shown that (A5) becomes

(A6) $$3q_T/aq_t = \underset{r=t}{\overset{r-i}{]\!\sim]}} (1 + w_{,r}).$$

Similarly, from equation (2)

(A7) $$ax_{iv}/ax_{tt} = \underset{s=t}{\overset{\sqcup\text{-}\!\urcorner}{Y\!/}} (i + f_{x \cdot s}'),$$

(A8) $$ax_{)T}/ax_{it} = {}^T_L \, V\!i + f_{x)r}').$$
$$\underset{r=t}{}$$

Substituting equations (9), (A3), (A4) and (A6) through (A8) into (AI) and (A2) and solving for y_u and y_{2t} results in equations (10) and (11).