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# The Production Structure of Pennsylvania Dairy Farms

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The production structure of a selected cross-section sample of family owner-operated dairy farms is investigated using a translog variable cost function. Elasticities of scale, input substitution, and input own- and cross-price elasticities are estimated. At the sample mean herd size of 67 cows producing at 15,173 pounds of milk per cow, the elasticity of scale parameter was 1.00, implying constant returns to scale. The elasticities of substitution between feeds and hired labor and the own- and cross-price elasticities were inelastic.

Dairy producers' responses to changing input and output prices have long been of interest to agricultural economists. Traditional approaches to measurement of both short- and long-run elasticities of output supply and input demand have often relied upon direct least squares estimates of supply or production functions. Studies by Conneman, Halvorson, Wipf and Houck, Stammer, and Matulich reported results using supply or cost functions and Hoch, Dahlgram, Paris, Malossini, Pilla and Romita, and Heady, Jacobson, Madden and Freeman reported results using a production function. However, direct least squares estimates of production relationships may not result in good estimates of the underlying technological parameters. A better approach would be to estimate input demand and supply and infer the underlying technological parameters (Varian, p. 126). In recent years flexible functional forms, the most popular of which is the transcendental logarithmic (translog) function, have been used to obtain estimates of input demand and supply. In this paper, we use a translog variable cost function to estimate input demand and then infer the production structure from a selected cross-section sample of Pennsylvania dairy producers for the year 1981. Specifically, partial static equilibrium elasticities of input substitution, own- and cross-price elasticities of input demand, and scale elasticity are estimated. The production structure is also tested for

homotheticity, homogeneity, and unitary elasticity of substitution.

Although the translog form of the cost function has been widely used in agricultural studies at the aggregate level (Binswanger, Kako, Ray, Ball and Chambers, and Brown and Christensen), it has not been widely applied in published studies using cross-section farm level data. A recent study by Weaver and McSweeney used a translog profit function to estimate a set of short-run elasticities from a cross-section sample of Pennsylvania dairy producers using 1974 USDA cost of production survey data. They considered the multiple output situation for feed, livestock, and milk production. We elect to use the cost rather than the profit function. An important limitation of the cost approach is that it assumes output is exogenous and, thus, not affected by factor price changes. Use of the profit function implies a stronger behavioral assumption, profit maximization. Whether family owner-operated dairy farms minimize cost given a targeted level of output or are profit maximizers is an empirical question. For an annual period, the assumption of cost minimization is not unreasonable given that the technical merit of the dairy livestock is fixed.

The paper is organized into four main sections and a summary. In the first section the model used in estimation and methods used to calculate the elasticities are developed. The area of study and a description of the data used are presented in section two. Section three presents the results and an evaluation of the model is discussed in section four.

## Modeling the Structure of Production

Duality theory allows the derivation of input demand without specifying the form of the production technology. Using the duality principal a production structure can be represented by a cost function from which the optimal derived demand for each input can be investigated. Importantly, the translog form can be used to model production situations without imposing stringent restrictions on the elasticity of substitution (Christensen, Jorgensen, and Lau). It allows scale economies to vary with the level of output, a feature essential to enable the unit cost curve to attain the classical U-shape (Christensen and Greene).

In applications of cost functions, an important assumption is that all inputs are in full static equilibrium. However, the firm can be assumed to be in static equilibrium with respect to a subset of inputs conditional on the observed levels of the remaining inputs. This framework is referred to as partial static equilibrium by Brown and Christensen. In addition to their study, another application of the variable cost function has been provided by Caves, Christensen, and Swanson. Lau and Yotopoulos provided an earlier application using the variable profit function. As shown by Lau, estimates of the structure of production can be obtained from either the total or variable cost function under a set of general regularity conditions. In our application, we estimate the variable cost function. The total cost function could not be estimated because of data limitations. The elasticities computed are thus partial, rather than full, static equilibrium elasticities.

Five variable inputs and five fixed factors are used. The variable inputs are concentrates (m), silage (w), hay (h), pasture (g), and hired labor (l). The fixed factors are family and operator labor (f), all capital less dairy live-stock (t), herd size (d), average cow age (a), and other operating costs (o). The other operating cost variable is included as a fixed factor since it is reported in dollar amounts only. The translog variable cost function for the five variable inputs and the five fixed factors can be written as

$$(1) \ln C = \alpha_0 + \alpha_q \ln Q + \sum_i \beta_i \ln P_i + \sum_k \gamma_k \ln Z_k + \frac{1}{2} \sum_i \sum_j \beta_{ij} \ln P_i \ln P_j + \frac{1}{2} \sum_q \alpha_{qq} (\ln Q)^2 + \frac{1}{2} \sum_r \sum_k \gamma_{kr} \ln Z_k \ln Z_r + \sum_i \phi_{iq} \ln P_i \ln Q + \sum_i \sum_k \theta_{ik} \ln P_i \ln Z_k + \sum_k \rho_{kk} \ln Z_k \ln Q,$$

where  $i, j = m, h, w, g$ , and  $l$ ;  $k, r = f, t, d, a$ , and  $o$ ; and  $C$  is the total cost of the five variable inputs.  $Q$  is milk produced,  $P_i$  is the unit price of input  $i$ , and  $Z_k$  is the level of the fixed factor  $k$ . Each observation of the variable inputs was divided by its sample arithmetic mean and normalized by the unit price of hay (h) before taking its logarithm. In addition, each observation of the fixed factors and the quantity of milk produced was divided by its mean before taking the logarithm. The parameters to be estimated are  $\alpha_0, \alpha_q, \beta_i, \gamma_k, \beta_{ij}, \alpha_{qq}, \gamma_{kr}, \phi_{iq}, \theta_{ik}$ , and  $\rho_{kk}$ .

From Young's theorem on the equality of the second cross partial derivatives we impose symmetry on the model. This implies  $\beta_{ij} = \beta_{ji}$  and  $\gamma_{kr} = \gamma_{rk}$ . Since a cost function must be homogeneous of degree 1 in input prices, we impose in (1) the restrictions  $\sum_i \beta_i = 1$  and  $\sum_j \beta_{ij} = 0$ .

Differentiating (1) with respect to each input price and invoking Shephard's lemma, the share equations ( $S_i$ ), which form the basis for estimation, are

$$(2) S_i = \partial \ln C / \partial \ln P_i = \beta_i + \sum_j \beta_{ij} \ln P_j + \phi_{iq} \ln Q + \sum_k \theta_{ik} \ln Z_k.$$

The dependent variable of the share equation is  $S_i = X_i P_i / C$ , where  $X_i$  is the quantity of the  $i^{\text{th}}$  input. Note that there are only four linearly independent input share equations since hay (h) was used as the normalizing input. The total shares of all five inputs must sum to unity.

The constant output own-price elasticity of input demand can be computed directly from the translog cost function in the following manner (Binswanger):

$$E_{ii} = \frac{\partial \ln X_i}{\partial \ln P_i} = \frac{\beta_{ii} + s_i^2 - S_i}{S_i} = S_i \sigma_{ii},$$

where

$$\sigma_{ii} = \frac{\beta_{ii} + s_i^2 - S_i}{S_i^2}.$$

Similarly, the constant output cross-price elasticity between inputs is

$$E_{ij} = \frac{\partial \ln X_i}{\partial \ln P_j} = \frac{\beta_{ij} + S_i S_j}{S_i} = s_j \sigma_{ij},$$

where

$$\sigma_{ij} = \frac{\beta_{ij} + S_i S_j}{S_i S_j},$$

and  $\beta_{ii}$  and  $\beta_{ij}$  are parameter estimates and  $S_i$  and  $S_j$  are fitted cost shares computed at the

sample means of the respective variables. The term  $\sigma_{ij}$  is the Allen-Uzawa partial elasticity of substitution between the  $i^{\text{th}}$  and  $j^{\text{th}}$  operating inputs and  $\sigma_{ii}$  is the own partial elasticity of substitution.

Following Hanoch, elasticity of scale ( $\epsilon_s$ ) is the reciprocal of the elasticity of cost ( $\epsilon_c$ ) with respect to output along the expansion path. Under the variable cost function framework the elasticity of scale is conditional on the observed levels of fixed factors. Following Caves *et al.*, the elasticity of scale for the variable cost function can be calculated from (1) as

$$(3) \quad \epsilon_s = (1 - \sum_k (\partial \ln C / \partial \ln Z_k)) / (\partial \ln C / \partial \ln Q).$$

For  $\epsilon_s > 1$ ,  $\epsilon_s < 1$ , and  $\epsilon_s = 1$ , the production technology exhibits increasing, decreasing, and constant returns to scale, respectively (Ball and Chambers).

A cost function is homothetic if it can be written as a separable function of input prices and output (Shephard, and Denny and Fuss). Homotheticity implies the optimal input combination is independent of the scale of output. The expansion path is thus linear. From the share equation in (2), homotheticity requires

$$\phi_{iq} = \rho_{kq} = 0.$$

Homogeneity in output for the cost function requires the elasticity of cost with respect to output to be constant. From (2), homogeneity imposes the restrictions

$$\alpha_{qq} = \phi_{iq} = \rho_{kq} = 0.$$

Therefore, homogeneity is tested with two additional restrictions as a hypothesis conditional upon the acceptance of homotheticity.

The cost function exhibits Cobb-Douglas technology if the partial elasticity of substitution between all inputs is unity. This requires all squared and cross product term to be set equal to zero.

Four models are estimated after imposing the restrictions for linear homogeneity in input prices and the translog symmetry conditions. The estimated models are the unrestricted translog cost function, homothetic cost structure, homogeneous cost structure, and the Cobb-Douglas form that imposes both unitary elasticity of substitution and homogeneity. For all models, the cost function and four share equations for concentrates, silage, pasture, and hired labor are estimated as a system. The system is estimated by Zellner's seemingly unrelated regression technique. Iterating the Zellner procedure is a computa-

tionally efficient method for obtaining maximum-likelihood estimates.

The various parameter restrictions of the three restricted models are tested with the likelihood ratio in the following manner (Nadiri and Schankerman):

$$(4) \quad \lambda = \Omega(R) - \Omega(U)$$

where  $\Omega(R)$  and  $\Omega(U)$  are the log likelihood values under the restricted and unconstrained versions, respectively. Then,  $-2\lambda$  is distributed asymptotically as a chi-squared variate with degrees of freedom equaling the number of independent restrictions tested.

### Area of Study and Variable Specification

The sample consists of 106 Pennsylvania family owner-operated dairy farms for the year 1981. Individual farm data were obtained by merging information from the Pennsylvania Farmers Association (PFA) financial management records and the Pennsylvania Dairy Herd Improvement Association (DHIA) records. The farms had herd sizes ranging from 28 to 188 cows, with an average size of 67. On average, 96 percent of total farm receipts were from milk sales. Farms that did not hire labor were excluded because of the requirement that all input quantities and prices be greater than zero when taking logs. The farms studied are thus a selected sample and may not be representative of farms participating with PFA and DHIA or farms in general in the state.

Pennsylvania dairy farms typically produce multiple outputs (milk, heifers, calves, grain, and forage) from a set of multiple inputs (labor, purchased feed, produced feed, etc.). By using DHIA data on the quantities of feed fed and feed prices the complex issue of multiple outputs can be reduced to the single output case. The DHIA data are reported for feed fed to milk producing cows. Thus, the production of feeds and other livestock need not be considered in the investigation of milk production.

The use of a translog cost function requires firm level information on the unit prices and quantities of each operating input and the quantity of output produced. Characteristics of the data used are detailed in Table 1. Quantities of concentrates, hay, silage, and pasture (grazing days per cow) fed are estimated jointly by the farm operator and DHIA representative. Farm produced feeds are valued at farm-gate market prices estimated by the farm operator and DHIA representative and

**Table 1. Selected Characteristics of the Sample of Pennsylvania Dairy Producers, 1981**

Variables	Arithmetic Mean	Standard Deviation
Price of hay/cwt. (\$)	3.63	0.86
Cwt. of hay/cow	24.95	12.15
Price of concentrates/cwt. (\$)	7.96	1.33
Cwt. of concentrates/cow	63.61	13.71
Price of silage/cwt. (\$)	1.42	0.35
Cwt. of silage/cow	134.80	27.69
Price of pasture/grazing day (\$)	0.35	0.20
Grazing pasture days/cow	86.79	55.48
Price of hired labor/day (\$)	22.74	9.38
Days of hired labor	425.49	318.92
Capital per farm (\$)	324,063	191,905
Days of family labor	548.02	195.89
Average age of cows (mos.)	53.41	6.68
Other variable costs (\$)	38,436	26,671
Herd size (cows)	67	32
Milk/cows (lbs.)	15,173	2,207

purchased feeds are valued at the purchase price. Differences in feed prices across farms emerge because some farms produce larger percentages of their feed requirements and because there is regional variation in market prices. An unknown factor in the measurement of both purchased and produced feeds is their quality. Qualitative differences can have a significant impact on the estimated prices of produced feeds and the total quantity of feed used. Errors in data measurement can occur as a result of these differences. Qualitative measures on the feeds used were not available. Data on prices and quantities of hired labor were taken from PFA records.

The five fixed factors were included to account for structural differences across farms. Capital is the total dollar value of land, buildings, machinery and equipment employed. Herd size is defined as the average number of cows in the herd over the annual period. Cow age, recorded in months, was included as a livestock quality variable. Up to a certain age, older cows consume larger quantities of feed and produce larger quantities of milk than younger cows. Family labor includes both the operator and other family labor. The other operating cost variable includes the annual expenses for utilities, building repair, livestock supplies, veterinary, breeding, and testing services, insurance, interest, and miscellaneous items. While normally considered variable factors, they were included as fixed factors because the data were recorded as expenses only.

## Empirical Results

Estimated results for the unrestricted model are reported in Table 2. These estimates are used in the calculation of the elasticities reported below. Results for the homothetic, homogeneous, and Cobb-Douglas models are not reported in order to conserve space. The model fits the data quite well as the system R-square ( $\bar{R}^2$ ) for the unrestricted cost function was 0.88. This statistic is calculated as  $1 - R_1/R_2$ , where  $R_1$  is the determinant of the full information maximum likelihood estimated residual moment matrix and  $R_2$  is determinant of the moment matrix of the actual dependent variables. The hypothesis for homothetic, homogeneous, and Cobb-Douglas production structure was tested using the likelihood ratio in equation (4). All were rejected at the 0.01 probability level (Table 3). For the homotheticity test this result implies the underlying production technology can be written as a separable function of input prices and output. Because homogeneity is a special case of homotheticity, it was rejected by the strong rejection of homotheticity. The Cobb-Douglas test, which is a joint hypothesis test of homogeneity and unitary elasticities of substitution, was also rejected. Thus, the data suggest the farms studied exhibit a nonunitary elasticity of substitution production technology.

The elasticity of scale was calculated from (3) using the parameters from (1). At the sample mean the calculated elasticity was 1.0009. This implies that constant returns to scale occurred at the sample mean level of production of 1,016,591 pounds of milk per farm, or an average of 15,173 pounds per cow for the sample mean herd size of 67 cows. Elasticities at other levels of production were not calculated because the estimated parameter for the square of the output variable ( $\alpha_{qq}$  in Table 2) was not significant.

The estimated Allen-Uzawa own-price elasticities and elasticities of substitution between operating inputs are reported in Table 4. The own-price elasticities are shown on the diagonal and the elasticities of substitution are shown on the off-diagonal. Own-price elasticities are expected to be negative. An increase in the price of an input results in a decrease in the quantity demanded. The signs for concentrates, hay, pasture, and hired labor were negative as expected and all but hay were significant. The sign for silage was found

**Table 2. Cost Function Parameter Estimates, Unrestricted Model**

Coefficient	Estimate	Coefficient	Estimate	Coefficient	Estimate
$\alpha_o$	11.069 (0.028) <sup>b</sup>	$\phi_{ql}$	-0.122*** (0.048)	$\theta_{ha}$	0.015 (0.039)
$\alpha_q^a$	0.485*** (0.136)	$\rho_{qt}$	0.213 (0.249)	$\theta_{ho}$	-0.022** (0.012)
$\beta_m$	0.530*** (0.008)	$\rho_{qd}$	0.684 (1.018)	$\theta_{hf}$	-0.004 (0.015)
$\beta_h$	0.089*** (0.005)	$\rho_{qa}$	0.226 (0.684)	$\beta_{gl}$	-0.007** (0.003)
$\beta_w$	0.203*** (0.005)	$\rho_{qo}$	-0.344 (0.214)	$\theta_{gt}$	0.011* (0.007)
$\beta_a$	0.032*** (0.003)	$\rho_{qt}$	-0.033 (0.308)	$\theta_{gd}$	0.017 (0.020)
$\beta_l$	0.146*** (0.008)	$\beta_{mh}$	-0.037* (0.015)	$\theta_{ga}$	-0.018 (0.020)
$\gamma_t$	-0.004 (0.056)	$\beta_{mw}$	-0.104*** (0.012)	$\theta_{go}$	-0.014** (0.006)
$\gamma_d$	0.427*** (0.172)	$\beta_{mg}$	0.006 (0.005)	$\theta_{gf}$	0.005 (0.008)
$\gamma_a$	0.007 (0.170)	$\beta_{ml}$	-0.053*** (0.013)	$\theta_{lt}$	0.004 (0.022)
$\gamma_o$	0.084 (0.058)	$\theta_{mt}$	0.034 (0.022)	$\theta_{ld}$	0.158*** (0.066)
$\gamma_f$	-0.022 (0.062)	$\theta_{md}$	-0.208*** (0.064)	$\theta_{la}$	0.033 (0.067)
$\alpha_{qq}$	-0.342 (0.881)	$\theta_{ma}$	-0.099 (0.065)	$\theta_{lo}$	0.002 (0.020)
$\beta_{mm}$	0.188*** (0.023)	$\theta_{mo}$	0.020 (0.020)	$\theta_{lf}$	-0.064*** (0.025)
$\beta_{hh}$	0.071*** (0.033)	$\theta_{mf}$	0.070 (0.024)	$\gamma_{td}$	-0.152 (0.302)
$\beta_{ww}$	0.167*** (0.013)	$\beta_{wh}$	-0.033** (0.012)	$\gamma_{ta}$	-0.323 (0.344)
$\beta_{aa}$	0.014*** (0.002)	$\beta_{wg}$	-0.012*** (0.004)	$\gamma_{to}$	-0.042 (0.108)
$\beta_{ll}$	0.079*** (0.011)	$\beta_{wl}$	-0.018*** (0.007)	$\gamma_{tf}$	-0.012 (0.104)
$\gamma_{tt}$	0.111 (0.152)	$\theta_{wt}$	-0.032*** (0.015)	$\gamma_{da}$	0.041 (0.870)
$\gamma_{dd}$	-0.663*** (1.272)	$\theta_{wd}$	0.066 (0.044)	$\gamma_{do}$	0.004 (0.266)
$\gamma_{aa}$	-0.245 (1.395)	$\theta_{wa}$	0.069 (0.045)	$\gamma_{df}$	-0.130 (0.389)
$\gamma_{oo}$	0.215 (0.145)	$\theta_{wo}$	0.015 (0.014)	$\gamma_{ao}$	0.043 (0.340)
$\gamma_{ff}$	-0.297 (0.224)	$\theta_{wf}$	0.012*** (0.004)	$\gamma_{af}$	0.290 (0.349)
$\phi_{qm}$	0.154*** (0.047)	$\beta_{hg}$	0.027*** (0.004)	$\gamma_{of}$	0.205* (0.118)
$\phi_{qh}$	0.027 (0.019)	$\beta_{hl}$	-0.0004 (0.009)		
$\phi_{qw}$	-0.039 (0.032)	$\theta_{ht}$	-0.016 (0.014)	$\bar{R}^2$	0.88 <sup>c</sup>
$\phi_{qo}$	-0.020 (0.015)	$\theta_{hd}$	-0.033 (0.039)		

<sup>a</sup> Variable notation: q = milk produced, m = concentrates, h = hay, w = silage, g = pasture, l = hired labor, f = family and operator labor, t = capital less livestock, d = herd size, a = cow age, o = other operating cost.

<sup>b</sup> Estimated standard errors in parentheses: Single asterisk denotes significant at 10% level, double asterisk denotes significant at 5 % level, and triple asterisk denotes significant at 1% level.

<sup>c</sup> System R-square.

**Table 3. Chi-square Statistics for Hypotheses Tests**

Model	Calculated Value	Number of Restrictions	Critical Value 1%
Homotheticity	104.81	10	23.209
Homogeneity	99.51	11	24.725
Cobb-Douglas	74.31	66	50.892

to be positive, but was not significant. The assumption of concavity in input prices was thus not satisfied for silage. We return to this important violation of the maintained hypotheses of the model below. For the four inputs with negative signs, the own-price elasticities were less than one and, thus, inelastic. The elasticity for hay was the most inelastic at  $-0.090$ , followed by concentrates at  $-0.113$ , hired labor at  $-0.185$ , and pasture at  $-0.530$ .

The result for concentrates is of particular interest. Concentrates are an important source of protein and energy, both of which are necessary to achieve high levels of production. The farms studied here did not make large reductions in the quantity fed for increases in its price. This suggests high levels of milk production per cow are an important objective of operators and they may not reduce the rate of concentrate feeding even if its price increases significantly in the short run. Farmers do, however, have some flexibility to adjust the rate of concentrate feeding over periods as short as one year. Grain, a major component of concentrates, that is produced on the farm need not necessarily be fed since readily available markets for this feed exist. However, because annual, cross-sectional data were used in this study, large changes in the quantity

demanded due to price effects may not be expected.

Inputs are classified as complements if the partial elasticity of substitution has a negative sign and substitutes if it has a positive sign. The elasticities reported on the off-diagonal in Table 4 show silage was complementary with concentrates, hay, and pasture and a substitute with hired labor. Hay with concentrates, pasture, and hired labor, pasture with concentrates, and hired labor with concentrates were all found to be substitutes. However, only three of the elasticities were found to be significant, that between pasture and concentrates, pasture and silage, and pasture and hired labor. With the exception of pasture, these results imply there was little variability in the allocation of feeds and hired labor used. Homogeneity in the rate of feeding is expected to occur over an annual production period even though the farms studied differed by herd size and geographical location within the State. Even though most of the elasticities were not significant, their signs were indicative of the expected relationship. The complementary relationship between silage and concentrates and silage and hay was as expected. Both concentrates and hay are sources of protein, while silage is a roughage feed. Higher rates of protein utilization result in higher rates of roughage feeding. Hay also has a roughage feeding value, but not to the same extent as silage. The substitution relationship found between hay and concentrates was expected since both feeds are good sources of protein. The two elasticities found to be significant are difficult to interpret since pasture was recorded in cow-grazing days and concentrates and silage were recorded in one-

**Table 4. Partial Elasticities of Substitution, Unrestricted Model**

Input	Concentrates	Silage	Hay	Pasture	Hired Labor
Concentrates	$-0.113$ (0.043) <sup>a</sup>				
Silage	$-0.043$ (0.112)	$0.099$ (0.064)			
Hay	$0.179$ (0.378)	$-0.951$ (0.620)	$-0.090$ (0.095)		
Pasture	$1.377$ (0.317)	$-1.120$ (0.548)	$0.762$ (1.422)	$-0.530$ (0.076)	
Hired Labor	$0.155$ (0.172)	$0.161$ (0.254)	$0.965$ (0.630)	$-0.708$ (0.080)	$-0.185$ (0.078)

<sup>a</sup> Approximate standard error in parentheses:  $SE(\sigma_{ij}) = SE(\beta_{ij})/S_i S_j$ .

hundred pound units. Consumption of pasture forage by livestock is to a large extent a function of its quality.

Estimated cross-price elasticities between operating inputs are shown on the off-diagonal in Table 5. The values on the diagonal are own-price elasticities and are identical to those on the diagonal in Table 4. Cross-price elasticities contain much of the same information contained in the elasticities of substitution. However, the cross-price elasticities are not symmetric as in the case of the elasticities of substitution. The reason is that the cross-price elasticity depends heavily on the weight of the individual input shares of the total operating cost. Note, first, that only four of the elasticities were significant. This is not surprising given that only two of the elasticities of substitution were significant. In general the elasticities were small with most having values of less than one-tenth. Changes in the price of one feed did not result in significant adjustments in the quantity demanded of other feeds or hired labor. As in the case of the elasticities of substitution, this result might be expected when using cross-sectional data over a short-run time frame. Large differences in feeding rates were not found for the farms studied.

### Model Evaluation

Because of the unexpected positive sign for the own-price elasticity of the input silage, the estimated results were not consistent with the implied hypothesis of cost minimization. This inconsistency can be due to a number of factors such as the sample selected, inadequacies

in model specification, and measurement error in the data. Errors in the data for silage, hay, and pasture cannot be ruled out. The reason is that the farms studied produced most of their roughage feed requirements. Within farm differences in the quality or feeding value of silage, hay, and pasture can result in less-than accurate estimates of prevailing prices. Feed quality not only depends on the type of crop produced, but also its condition at harvest, the harvesting method used, the environmental conditions at harvest, and the method of storage. Further, the absence of an active market for silage does not allow for observations on prevailing market worth.

There may also be questions regarding the validity of the specification of the model. Over an annual period it is normally assumed that all feeds are variable inputs. This may not be the case on farms that produce most of their roughage requirements. Farms producing their own feed generally feed their production regardless of what they believe the market value to be. Marketing high valued, farm-produced roughage and replacing it with lower valued, purchased roughage is not a common practice on dairy farms.

Two important considerations emerge with respect to modeling milk production. First, including qualitative measures or adjusting feed quantity and prices for qualitative effects may be necessary when farm-produced feeds are used in production. Qualitative measures of farm produced feeds are, however, rarely available. Secondly, treating farm-produced roughage feeds as variable inputs in a short-run feeding program may not be a realistic assumption in the milk production process. Once produced, these feeds may be viewed by

**Table 5. Estimated Own and Cross-Price Elasticities, Unrestricted Model**

Input	Concentrates	Silage	Hay	Pasture	Hired Labor
Concentrates	-0.113 (0.043) <sup>a</sup>	0.005 (0.023)	0.025 (0.036)	0.043 (0.010)	0.040 (0.024)
Silage	-0.030 (0.059)	0.099 (0.064)	-0.080 (0.059)	-0.034 (0.017)	0.044 (0.036)
Hay	0.101 (0.159)	-0.172 (0.127)	-0.090 (0.095)	0.023 (0.052)	0.138 (0.090)
Pasture	0.721 (0.167)	-0.180 (0.112)	0.073 (0.134)	-0.530 (0.076)	-0.084 (0.114)
Hired Labor	0.073 (0.090)	0.049 (0.052)	0.092 (0.060)	-0.029 (0.025)	-0.185 (0.078)

<sup>a</sup> Approximate standard errors are in parentheses:  $SE(E_{ij}) = SE(B_{ij})/S_i$ .



producers as fixed factors in the feeding program. Whatever the cause of the violation of the maintained hypothesis of cost minimization, the results of this study should be interpreted with caution.

## Summary

The production structure of a selected cross-section sample of family owner-operated Pennsylvania dairy farms is investigated using a translog variable cost function. The Allen-Uzawa partial elasticities of substitution between operating inputs, own- and cross-price input elasticities of demand and the elasticity of scale are estimated. The production structure is tested for homotheticity, homogeneity, and unitary elasticity of substitution.

Homothetic, homogeneous, and unitary elasticity of substitution production structures are all rejected at the 0.01 probability level. The results suggest an appropriate model to investigate the production structure would be an unrestricted form that does not impose homotheticity, homogeneity, or unitary elasticities of substitution. The estimated elasticity of scale at the sample mean level of farm milk production is 1.0009.

The own-price elasticities of concentrates, hay, pasture, and hired labor have the expected negative sign and are inelastic. An unexpected positive sign is found for silage, but the size of elasticity is small and not significant. The positive sign, which violates the maintained hypothesis of cost minimization, may be due to differences in feed quality and resulting measurement error in the quantities and prices of the feeds. Silage is found to be a complement with concentrates, hay, and pasture. These latter three feeds are found to be substitutes for each other. Hired labor is a substitute with all feeds except pasture. Similar results are found for the cross-price elasticities which contain the same information on the elasticities of substitution. All of the cross-price elasticities are found to be inelastic.

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