Factor Demand and Returns to Scale in Milk Production: Effects of Price, Substitution and Technology

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A translog cost function was estimated using pooled time series-cross section data from five Northeastern States to study structural changes in the dairy industry. The approach given in the duality theory was found useful in estimating the input demand structure under changing input prices and technology conditions. The estimated Allen partial elasticities of substitution show the existence of substitution between energy and non-energy inputs in dairy farming. Despite input price increases the dairy industry maintained competitiveness as seen by the returns to scale parameters.

During the last two decades, dairy farming in the United States has undergone substantial structural changes. The number of dairy farms and the total cow population of the country continually declined for a number of years (Matulich, Sibold and Nesselroad). The number of farms with small herds fell while farms with large herd sizes increased in number (U.S. Dept. of Commerce). In the Northeast region, the dairy industry followed a similar trend, although cow herd sizes in the region were considerably smaller than those in the West and the Midwest.

The consolidation of farms and herd expansion in the dairy industry have ensued for the purpose of attaining economies of scale and efficiency (Wysong, Matulich). Particularly, the improvements in the technology and the quality of inputs used in cattle breeding, herd management, milking systems, feeding programs, etc., over the years provided enough economic incentives for changes in the patterns of resource allocation and factor demand. On the other hand, changes in prices of direct energy inputs such as fuel oil, natural gas and electricity, and of indirect energy inputs, such as fertilizer, dairy concentrates and machinery, caused farmers to change input ratios. Due to the capacity for substitution of inputs, it is expected that the structure of derived demand for factors also changed in the dairy industry.

The purpose of this paper is to examine the nature of the structural changes that occurred in dairy farming as a result of changes in technology and factor prices. Specifically, the paper estimates the derived demand for inputs, both energy and non-energy, the factor substitutions between categories of inputs and the returns to scale in the northeastern dairy industry. In this study, the characteristics of productive behavior in dairy farming were analyzed by the cost function approach given in the duality theory of production. This approach does not require *a priori* assumptions concerning homotheticity, homogeneity and returns to scale. Furthermore, the approach provides a suitable framework for analyzing productive behavior of the farm when the physical input data are simply not available. These inherent characteristics are considered to be advantages which make the duality theory attractive.

Analytical Model and Estimation Procedure

We assume the existence of a twice differentiable aggregate production function that describes the production technology of dairy farming in the form

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Published with the approval of the Director of the West Virginia University of Vermont which operates through farmers' voluntary tific Article No. 1867. This research was supported with funds appropriated under the Hatch Act.

(1) $Q = f(L, F, U, G, M, C, N, t,),$

where Q is the quantity of milk produced and L, F, U, G, M, C, and N refer respectively to labor, feed, utilities (electricity and natural gas), fuel oil (gasoline and diesel), machinery, capital, and all other inputs, while t is a time variable which serves as a proxy for technology. Under the duality theory of production, if producers purchase specifiable inputs in competitive markets and pursue a cost minimizing behavior, then the production technology of dairy farming could be uniquely represented by a dual cost function (Diewert) of the general form:

(2) $C = g(Q, P_i, t),$
$i = L, F, U, G, M, C, N,$

where C is the total cost and P_i are the prices of inputs. The cost function is a positive and non-decreasing function in Q, linearly homogeneous, concave and continuous in P_i for all positive rates of output and it is twice differentiable with respect to P_i.

The specific functional form of the dual cost function (2) is expressed in terms of the generalized translog cost function (Christensen, Jorgensen and Lau) of the form:

(3) $$\ln C = \alpha_0 + \alpha_Q \ln Q + \Sigma_i \alpha_i \ln P_i + \tfrac{1}{2}\gamma_{QQ}(\ln Q)^2 + \tfrac{1}{2}\Sigma_i \Sigma_i \beta_{ij} \ln P_i \ln P_j + \Sigma_i \gamma_{Qi} \ln Q \ln P_i + \phi_T t + \tfrac{1}{2}\phi_{TT} t^2 + \phi_{TQ} t \ln Q + \Sigma_i \phi_{Ti} t \ln P_i.$$

Linear homogeneity of degree one of the cost, C, in input prices, of course, requires the imposition of the following restrictions on the parameters of (3):

$$\Sigma_i \alpha_i = 1; \quad \Sigma_j \beta_{ij} = \Sigma_j \beta_{ji} = \Sigma_j \gamma_{Qi} = \Sigma_{i\gamma} \phi_{Ti} = 0$$

and $\beta_{ij} = \beta_{ji}$ for all i, j is assumed since the Hessian of the twice differentiable cost function is symmetric. Homogeneity of degree one in prices does not, however, impose homogeneity of degree one on the production function.

By using Shephard's Lemma, which implies that $\partial C/\partial P_i = X_i$, where X_i is the cost minimizing input demand, we find the cost shares of input i, S_i, as

(4) $$\frac{\partial \ln C}{\partial \ln P_i} = \frac{\partial C}{\partial P_i}\frac{P_i}{C} = \frac{X_i P_i}{C} = S_i$$

and the input demand functions expressed in terms of the cost shares are derived from the translog cost function by differentiating (3) as,

(5) $$\frac{\partial \ln C}{\partial \ln P_i} = S_i = \alpha_i + \Sigma_j \beta_{ij} \ln P_j + \gamma_{Qi} \ln Q + \phi_{Ti} t.$$

The Allen partial elasticity of substitution (AES), which measures the effect of a change in the price of the j^{th} input on the quantity demanded of the i^{th} input when output is held constant can be obtained pairwise from the dual cost function (Uzawa).

(6) $$\sigma_{ij} = \frac{(C\partial^2 C/\partial P_i \partial P_j)}{((\partial C/\partial P_i)(\partial C/\partial P_j))}.$$

In the case of the translog cost function, the AES are derived in terms of cost shares and the coefficients of the cost function (Binswanger, Berndt and Wood) as

(7) $$\sigma_{ij} = \frac{(\beta_{ij} + S_i S_j)}{S_i S_j}, \quad \text{for all i and j, } i \neq j;$$

$$\sigma_{ii} = \frac{(\beta_{ii} + S_i^2 - S_i)}{S_i^2}, \text{ for all i.}$$

The AES can also be used to obtain price elasticity of input demand (E_{ij}) by multiplying the AES by the cost shares (Mundlak) as

(8) $E_{ij} = S_j \sigma_{ij}$, for all i and j.

At constant output, positive AES between inputs i and j suggests they are substitutes, while they are complements if AES is negative. Also, even though $\sigma_{ij} = \sigma_{ji}$, in general $E_{ij} \neq E_{ji}$.

The elasticity of scale, which measures relative changes in output resulting from proportional changes in all inputs is described by Hanoch in relation to the total cost and output along the expansion path. It can be obtained from the translog cost function as

(9) $$\epsilon = \frac{1}{\partial \ln C/\partial \ln Q} = (\alpha_Q + \gamma_{QQ} \ln Q + \Sigma_i \gamma_{Qi} \ln P_i + \phi_{TQ} t)^{-1}$$

Thus, if $\epsilon = 1$, then the production function exhibits constant returns to scale. Further, $\epsilon > 1$ and $\epsilon < 1$ imply, respectively, increasing and decreasing returns to scale. If the production function $Q = F(X)$ is homothetic, its dual cost function is multiplicatively separable as $C(Q,P) = h(Q) \cdot C(P)$ where P is the vector of prices. For the translog cost function (3) this requires $\gamma_{Qi} = 0$ and $\phi_{TQ} = 0$ for all i, so that the interaction terms between the output and input prices disappear.

Finally, following Ball and Chambers, and Ohta, factor-augmenting technical change is determined by measuring the cost reducing effect of technical progress as follows:

$$(10)\quad \epsilon_t = -\frac{\partial \ln C}{\partial t}$$

$$= -(\phi_T + \phi_{TT}t + \phi_{TQ}\ln Q + \Sigma_i\phi_{Ti}\ln P_i).$$

If $\phi_{Ti} = 0$ for all i, the technical change is neutral. For input i, the technical change is input-saving or input-using if ϕ_{Ti} is, respectively, less than or greater than zero.

The coefficients of the cost function (3) are generally derived by estimating the cost share equations (5). However, in the present framework, estimating the share equations alone cannot provide all the necessary coefficients since certain parameters needed for determining the elasticity of scale (9) are obtained only from the cost function (3). Thus, it is necessary to estimate both the cost function (3) and the cost share equations (5). To assume randomness in these functions, however, we must add to each equation of (3) and (5) an error term which would represent the errors in cost minimization behavior. It is further assumed that the error terms are independently and normally distributed with mean zero and a nonsingular variance-covariance matrix. Since the share equations must sum to unity, the sum of the error terms across the equations at each observation point is zero and the covariance matrix is singular and non-diagonal. However, according to Barten, nonsingularity in the variance-covariance matrix can be ensured if one equation is dropped from the system of equations in (5) and the rest are estimated by a maximum likelihood technique that would provide independent estimates, irrespective of which equation was dropped.

The Iterative Zellner's Efficient Procedure, IZEF, (Zellner) provides estimates which are identical and computationally equivalent to the maximum likelihood estimates (Kmenta and Gilbert, Ruble). They are also invariant to the equation omitted in (5) and converge asymptotically to maximum likelihood estimates through successive iterations. The IZEF procedure contained in the SAS package is, therefore, used to estimate the system of equations in (3) and (5). To ensure nonsingularity in the variance-covariance matrix, the cost share equation of miscellaneous inputs was dropped during estimation and its price was used as the numeraire to assure the imposition of symmetry and linear homogeneity restrictions.

The Data

Dairy farms are assumed to be involved in milk production with the use of seven categories of inputs. They are labor (L); feed (F) which includes dairy concentrates, nonconcentrate feed and fertilizer; utilities (U), which include electricity and natural gas; fuel oil (G) used in the form of gasoline and diesel oil; machinery (M); capital (C); and all other intermediate material inputs (N). The data required for fitting the translog cost function (3) and the share equations (5) are the cost shares and prices of these inputs.

The cost share data were obtained from the series of Electronic Farm Accounting (ELFAC) Dairy Farm Business Analysis reports published for the years 1967 through 1981. The ELFAC program, which operates in five states of the Northeastern region keeps itemized records of actual income and expenses for participating dairy farms.[1] In the program, the farms are grouped under three general categories according to the sizes of their herds—farms with (i) less than 40 cows, (ii) 40 to 79 cows and (iii) 80 or more cows. Each year, a summary report is published in which the average costs and returns of each herd size group in each state are provided. Each of these group averages was treated as an observation and thus, for every year, cross section data of 15 observations were obtained for the study. When these were pooled over the 15 year time period, they provided enough observations to fulfill the degree of freedom requirements for estimating the large numbers of coefficients contained in the translog cost function.

Although the data set delineated above was adequate in terms of the number of observations and accuracy, it had certain limitations due to the nature of the ELFAC program. First, the participating farms were neither

[1] ELFAC is a farm business record keeping program at the University of Vermont which operates through farmers voluntary participation in a number of Northeastern States of which five are prominent—West Virginia, Connecticut, New Hampshire, Vermont, and Maine. Maryland and Massachusetts are included in the program but have only a few farms. The number of farms from each state included in the program varied yearly, the average ranging from 20 in Connecticut to 126 in Vermont and the total sample ranged between 217 and 303 over the time period.

large in number nor were they randomly selected. Second, though most farms continued in the program over the years, some farms dropped out and others joined in every year.[2] Also most of the participating farms had herd sizes less than 120 cows. All these introduce the possibility of bias and as such caution must be exercised while interpreting the results.

The expense data obtained from the yearly ELFAC Reports from 1967 to 1981 are categorized under the seven input categories mentioned above. To the operating capital expenses of the farm, a fixed cost for investment (at the rate of 9% of total fixed investment) is added. The total cost, therefore, gives the sum of operating and fixed expenses of the farm.

Farm labor wage rates (P_L), prices of gasoline (P_G), and prices of electricity (P_U) were obtained from the *Agricultural Statistics* of the USDA. Prices of the rest of the inputs were obtained in index form from the *Agricultural Price Summaries* of the U.S. Crop Reporting Board. The price indexes with 1977 as the base year are available for the later years. Since the base year for the earlier year's prices was 1914, they were converted to the 1977 base. The price indexes used are: feed (P_F), machinery and implements (P_M), interest on indebtedness of farm real estate (P_C), and farm and other supplies (P_N).

Results

The estimated parameters of the translog cost function are presented in Table 1. Given the ELFAC data base, the estimates are found to be quite satisfactory and the fitted function is well behaved.[3] The R^2 measure shown at the end of Table 1 was quite high.

The parameter estimates of the cost function were used in computing the Allen partial elasticities of substitution, as shown in Table 2. The price elasticities of input demand also were calculated and given in Table 3. In com-

[2] The authors thank one of the reviewers who brought this point to their attention.

[3] To be well behaved a cost function must be monotonic and concave in input prices. Monotonicity is tested by fitting the cost share equations with estimates to check if they are positive at each annual observation. Concavity of the cost function is satisfied if the Hessian matrix based on the parameter estimates is negative semidefinite. From these tests we conclude that the estimated cost function is well behaved within the region given by the data for the time period 1967–81.

Table 1. Estimated Parameters of the Translog Cost Function

	Intercept.	Labor	Feed	Utilities	Fuel Oil	Machinery	Capital	Misc.	Output	Time
		(L)	(F)	(U)	(G)	(M)	(C)	(N)	(Q)	(t)
α	1.5825	−0.2707	.1420	.0046	.0382	.0506	.6381	.3972	.2404	−.0047
	(1.6900)	(.0391)	(.0379)	(.0074)	(.0111)	(.0279)	(.0604)		(.3861)	(.0265)
β_{LJ}		.0162								
		(.0335)								
β_{FJ}		−.0119	.1143							
		(.0142)	(.0172)							
β_{UJ}		.0339	−.0165	.0005						
		(.057)	(.0026)	(.0029)						
β_{GJ}		−.0036	−.0049	−.0044	.0174					
		(.0084)	(.0039)	(.0026)	(.0047)					
β_{MJ}		.0330	.0255	−.0280	−.0082	.0393				
		(.0201)	(.0102)	(.0067)	(.0091)	(.0303)				
β_{CJ}		.0485	−.0766	−.0238	−.0289	−.1228	.0818			
		(.0361)	(.0195)	(.0088)	(.0136)	(.0352)	(.0660)			
β_{NJ}		−.1729	−.1311	.0019	−.0125	.0146	.4335	−.5679		
		(.0818)	(.0370)	(.0304)	(.0510)	(.1053)	(.1116)			
γ_{QJ}		.0486	.0220	−.0037	−.0047	−.0114	−.0430	−.0078	.0882	
		(.0029)	(.0039)	(.0005)	(.0007)	(.0019)	(.0042)		(.0441)	
ϕ_{TJ}		−.0061	.0059	.0042	.0027	.0118	−.0006	−.192	−.0030	−.0034
		(.0027)	(.0015)	(.0005)	(.0008)	(.0020)	(.0044)		(.0026)	(.0019)
	R^2 = .9872									

* Standard errors in parentheses.

Table 2. Estimated Allen Partial Elasticities of Substitution (σ_{ij})

i \ j	Labor	Feed	Utilities	Fuel Oil	Machinery	Capital	Other Inputs
Labor	−7.7989 (3.7838)						
Feed	0.0793 (0.3839)	−0.8031 (0.1110)					
Utilities	23.5046 (3.7591)	−1.6207 (0.4207)	−59.5209 (11.4979)				
Fuel Oil	−0.3465 (3.1809)	0.5523 (0.3538)	−8.7207 (5.7386)	−12.5812 (5.8917)			
Machinery	6.3855 (3.2837)	1.9990 (0.3994)	−25.9120 (6.4802)	−3.5033 (5.0093)	−5.0789 (7.1651)		
Capital	2.9423 (1.4463)	0.2657 (0.1843)	−4.6489 (2.0868)	−2.8859 (1.8225)	−6.1290 (2.0425)	−1.6087 (0.9396)	
Other Inputs	−12.2795 (6.2866)	−1.4105 (0.6807)	1.8456 (13.7793)	−2.2197 (13.1308)	2.6257 (11.7179)	12.8276 (3.0457)	35.9386

Standard error in parentheses.

puting Allen partial elasticities of substitution and price elasticities of demand, the average expenditure shares for the time period 1967–81 are used.

The Allen partial elasticities of substitution given in Table 2 show the existence of substitutability among the various inputs as well as between the pairs of a number of energy and non-energy inputs. In dairy farming, the substitution between utilities (electricity and natural gas) and labor is high. Between fuel oil and labor, however, complementarity is observed. Similarly, substitution is high between utilities and miscellaneous inputs but complementarity is observed between fuel oil and miscellaneous inputs. On the other hand, utilities show a strong complementarity with feed, machinery and capital. However, fuel oil is a substitute for feed, but is complementary to machinery and capital. Overall, increases in the price of utilities tend to lead to an increase in demand for labor and miscellaneous inputs but a decrease in demand for feed, machinery and capital. Increases in the price of fuel oil, however, lead to declines in the demand for labor, machinery and capital but increases in the demand for feed. Therefore, while energy price increases have a mixed effect on labor and feed use, they tend to decrease the intensity of machinery and capital use in dairy production. These findings are similar to the findings of other studies as regards production in other sectors of the economy such as meat (Ball and Chambers), manufacturing (Berndt and Wood), and dairy (Gempesaw).

The own price and cross-price elasticities of demand for inputs shown in Table 3 confirm these expectations. The demand for utilities is price responsive and more elastic (−0.3535). It is also confirmed by the high cross-price elasticity between utilities and labor (2.2131) that increases in utility prices are associated with elastic responses in the demand for labor. The demand responses of other inputs to price increases in utilities or fuel are mostly negative.

Among the non-energy inputs, capital and machinery are both found to be substitutes for

Table 3. Estimated Own-Price and Cross-Price Elasticities of Factor Demand (E_{ij})

i \ j	Labor (L)	Feed (F)	Utilities (U)	Fuel Oil (G)	Machinery (M)	Capital (C)	Other Inputs (N)
Labor (L)	−0.7343	0.0640	2.2131	−0.0326	0.6012	0.2770	−1.1562
Feed (F)	0.2672	−0.3160	−0.6376	0.2173	0.7864	0.1045	−0.5549
Utilities (U)	0.4081	−0.0259	−0.9511	−0.1393	−0.4140	−0.0738	0.0295
Fuel Oil (G)	−0.0097	0.0155	−0.2451	−0.3535	−0.0984	−0.0811	−0.0624
Machinery (M)	0.4151	0.1299	−1.6843	−0.2277	−0.3301	−0.3984	0.1707
Capital (C)	0.7800	0.0704	−1.2245	−0.7651	−1.6248	−0.4264	3.4001
Other Inputs (N)	−1.6983	−0.1950	0.2552	−0.3070	0.3631	1.7741	−4.9693

labor, feed and miscellaneous inputs. Capital and machinery maintain complementarity. The own-price elasticities of capital and machinery are low, ranging from −.4264 to −.3301. The demand responses of these inputs to increases in their own prices are slightly inelastic. The effects of wage increases on labor demand is high, but wage effects on the demand for other inputs are low.

The factor demand responses of dairy farms to the price increases in the direct and indirect energy inputs, which characterized the last decade, are clearly discerned from the analysis of substitution effects. For example, the demand for fuel oil is inelastic—primarily due to the absence of substitution of other types of inputs. Therefore, dairy farmers do not significantly reduce fuel oil use when fuel prices increase. Utilities, on the other hand, are better substitutes for labor or miscellaneous inputs. Thus, an increase in utility prices leads to a reduction in utility use, but a rise in labor demand. The demand for labor, on the other hand, can also rise due to an increase in the prices of capital and machinery. Conversely, in the event of wage increases, the demand for utilities, capital and machinery tends to rise. On the other hand, a rise in the interest rate for borrowed capital would cause a decrease in the demand for machinery.

Technological changes during the time period studied led dairy farmers toward attaining increasing returns to scale. This is apparent from the yearly elasticity of scale shown in Table 4. It is observed that the returns to scale in dairy farming increased slowly but steadily over the years and attained constancy from around 1979. Thus signifies decreasing returns prior to 1979.

Scale economies in the dairy industry are further explained by the nature of technical progress in the industry. First, technical progress, as shown in Table 4, was steadily infused into the industry through the period of our study. The rate of technical progress increased from about 4 percent in 1967 to about 8 percent in 1981. Second, the technical change coefficients shown in Table 1 suggest that the new technology going into the dairy industry has been labor saving, machine oriented and energy using. Even during the energy crisis period, the technical progression continued undaunted. The effects of energy price increases were felt due to the inelastic demand conditions of fuel oil and utilities. But, since the energy share of costs in dairy farming was quite low, only about 5 percent, it might have led farmers to ignore the effects of energy price increases in favor of changes in technology.

Table 4. Estimated Elasticities of Scale (ϵ), and Rates of Technical Progress (ϵ_t)

Year	Elasticity of Scale (ϵ)	Rate of Technical Progress (ϵ_t)
1967	0.9520	0.0406
1968	0.9597	0.0445
1969	0.9625	0.0479
1970	0.9634	0.0517
1971	0.9608	0.0546
1972	0.9632	0.0577
1973	0.9632	0.0590
1974	0.9703	0.0623
1975	0.9838	0.0656
1976	0.9848	0.0669
1977	0.9897	0.0698
1978	0.9919	0.0729
1979	1.0017	0.0767
1980	1.0019	0.0788
1981	1.0093	0.0820

Summary and Conclusions

In the absence of production input use data, the dual cost function approach can be effectively utilized to evaluate farm production behavior. Empirical results from this study of the dairy industry in the Northeast suggest that effects of input price changes can be better understood by a study of factor substitution and technical changes in the industry. Increases in the prices of energy inputs have caused dairy farmers to change input ratios via factor substitution. The dairy industry has also undergone significant changes in technology. But, despite the rapid increases in input prices, the industry showed a surprising ability to remain competitive during the post-energy crisis years.

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