Issues and Methods in Estimating Adjustment Costs

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Introduction

The practice of econometrics entails choosing which equations to include in a system, and which equations to inclue in a system, and what restrictions to impose on the parameters. There is a widespread, but not universally held, preference for making these choices on the basis of theory. Until recently, this theory has rested on the hypothesis that agents solve static optimization problems. For agents' behavior to be consistent with this story, certain restrictions must hold. These restrictions are used in econometric models as maintained or testable hypotheses.

Over the past ten or fifteen years, the techniques of dynamic optimization have become widespread in economics. For both theoretical and prescriptive purposes, it has become popular to replace the static optimization problem with a dynamic problem. There has been less progress in using dynamic theory to construct econometric models. A possible reason is that econometricians consider the story that consumers and firms solve dynamic problems less plausible than the story that they solve static problems. A second explanation has to do with the technical difficulty of imposing the restrictions implied by dynamic theory. A third explanation considers the development of a competing approach to modeling dynamics. This approach questions whether theory provides a useful basis for the estimation of dynamic relations. It tries to discover empirical regularities by means of time series analysis. Its major concern is with forecasting, not with the interpretation of observed regularities. These three explanations for the relatively slow adoption of dynamic theory by applied econometricians are neither exhaustive nor mutually exclusive.

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This paper discusses two questions related to these explanations. The first is, "Is it reasonable to impose the restrictions implied by dynamic theory?" A more positive restatement of the question is, "Why should econometricians base their models on dynamic theory?" The second question is, "What are the restrictions and how should the econometrician go about imposing them, assuming that he wishes to do so?" The first question has elements of the philosophical; the second is largely technical. In addressing the second question, a distinction is made between restrictions that are implied by a linear-quadratic model and restrictions implied by a more general model. Three methods of dealing with the linear-quadratic and two of dealing with the general model are discussed.

Rational Expectations and Dynamic Optimization

Before turning to these issues, it is useful to clarify the relationship between rational expectations models and dynamic optimization models. The former category consists of those models which include as explanatory variables the expectation of some variable(s), either exogenous or endogenous. These may be the expectation of the current, but still unknown, values of the variables, conditioned upon previous information, or the expectation of future values. The expectations are "rational," as opposed to *ad hoc*, because they are the mathematical expectation conditioned on relevant information. The hypothesis that agents have rational expectations means that, given their information, it is impossible systematically to improve on their predictions. (Hayashi (1980) provides an excellent summary of ways to estimate these models.)

By "dynamic optimization model," or more briefly, "dynamic model," we mean an econometric model which *explicitly* incorporates restrictions from some dynamic optimization problem. This definition excludes unrestricted vector autoregressive models and most models based on *ad hoc* specifications. Rational expectations models imply an underlying dynamic optimization problem. This is so because if agents solved static optimization problems they could base their decisions entirely on the current environment; their expectations of future events would not affect their decision rules, and hence would not affect the economic environment. The dynamic optimization underlying a rational expectations model may, however, be let implicit. For example, the modeler may claim that supply in period t depends on the expectation, at $t - 1$, of the period t price, but not attempt to formally justify that claim, or the exclusion of other expectations. On the other hand, a rational expectations model may be derived from the first order conditions of a stochastic dynamic optimization problem. Such models form the intersection of rational expectations and dynamic optimization models. They are the chief topic of this discussion.

Instead of imposing restrictions implied by the stochastic dynamic problem, the corresponding certainty-equivalent problem may be used. These models generally assume that expectations are static or adaptive. In the following discussion they are referred to as deterministic dynamic models. There are two ways to determine the restrictions implied by these models. The direct, or primal method, specifies the objective function in a control problem. The optimal control rules for this problem are the equations to be estimated. This requires solving the control problem. The indirect, or dual method, specifies the functional form of the value of the maximized control problem (the value function). The form of the control rules is inferred from this function.

For and Against Dynamic Models

If the hypothesis that economic agents solve optimization problems is seriously entertained, it is natural to conjecture that many of those problems are dynamic. The econometrician's response to such a conjecture may range between two extremes, represented by the "empiricist" and the "structuralist." These designations indicate relative positions, and should be interpreted loosely. A more neutral description would label partisans of respective positions as "one who holds weak priors" and "one who holds strong priors" or, "one who estimates the reduced form" and "one who estimates the structural form." The first nomenclature is adopted for reasons of convenience.

The empiricist may tentatively accept the hypothesis that agents solve dynamic problems. This suggests there should be some relation between current and past variables. He undertakes to discover what that relation is, imposing as few prior restrictions as possible. The typical method of doing this is to fit the data to a vector autoregressive-integrated-moving-average (ARIMA) model. The result is useful for in-sample forecasting and, if the structure remains constant, for out-of-sample forecasting.

The structuralist is not only willing to hypothesize that agents solve dynamic problems, he is willing to specify details of the problem. These include the form of the objective function (the primal approach) or the value function (the dual approach), the constraints, and the manner in which agents form expectations of future variables. The control problem implies a specific form of the behavioral equations, or control rules, which are estimated jointly with the expectation-generating equations.

The empiricist and structuralist estimate, respectively, the reduced and structural forms of economic relations. We will assume that the purpose of this estimation is to provide, however indirectly, a guide to policy makers. Throughout the first part of the 70's economists estimated reduced form dynamic equations and used these to simulate the effect of government policies, or to determine optimal policies by solving control problems. This procedure was criticized on the grounds that the reduced form equations are not invariant to government policies. The reduced form equations incorporate the public's behavior, and this depends on government policies and the public's expectation of future government policies. This is the essence of the "Lucas critique" of early uses of reduced form dynamic models (Lucas). The criticism is applicable whether the public has perfect foresight or forms its expectations of future events in a rational manner. In either case, the remedy calls for determining structural relations, i.e., those relations which are invariant to policy changes. The Lucas critique spurred interest in, or at least provided an additional justification for, rational expectations models. Deterministic dynamic models also involve estimation of structural equations and thus address Lucas's central criticism.

The empiricists have at least two responses: a) reduced form equations may not be as unreliable as the Lucas critique suggests, and b) restrictions imposed by structuralists may introduce more noise than information. The first response (Sims) is based on the observation that seldom are the policy options under consideration radical departures from previous policies. If such a policy proposal does emerge, say as the solution to a control problem, it should be discarded out of hand. Since reduced form models provide reliable in-sample predictions, and since reasonable policy alternatives are at least close to that sample, predictions from the reduced form may be used with some confidence. However, without attempting to identify the structure of the economy, how does one know what is meant by a policy alternative that is "close" to the historical sample? The argument is a prescription for conservative behavior, which may be sage advice, or a recommendation to repeat the mistakes of the past.

Even if the proposed policy is not close to the sample, short run predictions from reduced form models are adequate if the underlying structure changes slowly. Agents probably do not radically change their expectations or behavior in the short term in response to changes in their environment. Sims concludes that although structural models might provide a better indication of long run effects of changes in the environment, reduced form models are better for short run predictions. The hypothesis that structure changes slowly is plausible, but in order to test it, it appears necessary to estimate structural models.

The empiricist's second defense is that the restrictions used to identify structural models are as apt to be wrong as right. Possession of an incorrect structural model may give the econometrician unjustified confidence in his out-of-sample predictions, creating a danger that does not exist for the more modest empiricist. The reasons for doubting the restrictions implied by dynamic theory include all the reasons for doubting the restrictions implied by static theory. The greater complexity of dynamic problems strengthens those reasons. It is difficult to imagine a typical agent solving an optimal control problem, or at least the particular problem chosen by the econometrician. It is harder still to imagine aggregate behavior resembling the solution to such a problem.

This argument against the estimation of structural parameters works equally well as a defense of any modest proposal against one that is more ambitious: the less one tries to say, the less danger of falling into error. There is certainly a line which one does not wish to cross in making inferences from data; it is not clear where the line is drawn. To the extent that a researcher regards economics as a science, he is more inclined to err on the side of caution. To the extent that he regards it as a speculative undertaking, he is more willing to impose structure.

Whatever the misgivings about the assumptions used to identify particular models, attempts to estimate the structural form will persist. The alternative restricts the field of enquiry too greatly. The next two sections review several methods of estimating dynamic structural models. We then discuss the stationarity assumption which is involved to one extent or another in all of these methods.

The Linear-Quadratic Model

The linear-quadratic control problem provides the most tractable, and hence the most popular, dynamic model. Given a quadratic objective function, a constraint in the form of a linear difference equation, and the initial condition on the state, the optimal control rule, which is linear in the state, may be easily derived. Estimation of the model entails solving the inverse problem. One observes (or estimates) the system of linear difference equations which governs the evolution of the state vector, and the linear control rule, and infers the quadratic objective function.

Without *a priori* restrictions, the objective function is always non-unique. That is, for any linear state equation and control rule, there exist infinitely many quadratic objective functions such that the control problem implied by that objective function and state equation, has as its solution the observed control rule. (See Jameson and Kreindler, and below.) The structuralist imposes prior restrictions on the objective function to insure identification of its parameters. In the case of exact identifications this results in no restrictions on the control rule and state equation. Typically, however, the model is overidentified, and this implies non-linear cross equation restrictions on the control rule and state equation.

A cause of overidentification is that elements in the state vector may follow autoregressive processes of order higher than one. These lagged variables are included in the state vector since they help predict the future. They are also arguments in the control rule, since the current optimal control depends on expectations of future variables, which are linear functions of past variables. It is often reasonable to suppose that the objective function in the current period does not depend directly on these lagged variables; this implies zero-restrictions on the objective function parameters that involve those variables. In general, a higher autoregressive order implies a higher degree of overidentification.

There are two approaches for deriving the restrictions implied by the linear-quadratic model. These are based on the classical and dynamic programming (DP) solutions to the control problem. In addition to these, we discuss a simpler method, also based on the DP approach.

An important assumption in each approach is that the problem results in a stationary control rule of the form

$$x_t = Gy_{t-1} \tag{1}$$

where x_t and y_t are, respectively, the control and state vector at time t. The matrix G is time invariant, but time can be introduced linearly by defining one element of y as time; non-linear time trends cannot be accommodated. The state evolves according to the difference equation

$$y_t = Ay_{t-1} + Cx_t + e_t \tag{2}$$

where e_t is a random term. The vector y may include the control vector, and will probably include certain variables, such as prices, which agents take as given. The stochastic process governing these may be explosive, but must satisfy a condition (see below) to insure that G in (1) exists.

An error term, u_t, is added to (1) for estimation; the justification is that agents base their decisions on certain variables not observed by the econometrician. The properties of the error terms u_t and e_t involve subtleties not considered here (see Hansen and Sargent). We consider only the cross-equation restrictions involving A, C, and G. The empiricist estimates equations similar to (1) and (2) without imposing these restrictions.

a) The Classical Method

The most common dynamic model is based on a cost of adjustment problem. The vector y is partitioned into vectors y_1, which includes all variables that agents take as given (e.g., prices) y_2, all variables that they completely control (e.g., input stocks) and x, the controls. Thus, (2) can be written as

$$y_t = \begin{bmatrix} y_1 \\ y_2 \\ x \end{bmatrix}_t = \begin{bmatrix} A_1 & O & O \\ O & I & O \\ O & O & O \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ x \end{bmatrix}_{t-1} + \begin{bmatrix} O \\ I \\ I \end{bmatrix} x_t + \begin{bmatrix} e_1 \\ O \\ O \end{bmatrix}_t . \tag{2'}$$

The objective function is

$$E_0 \sum_{t=0}^{\infty} \beta_t y_t K y_t \tag{3}$$

where β is the discount rate, K is a constant matrix and E_0 is the expectation at time O. The upper left block K, which contains the coefficients of terms which are quadratic in y_1, is arbitrarily set to 0, since these parameters are not estimable. The algebraic demonstration is given below, but the intuition is obvious: agents are unable to influence the part of the payoff that is quadratic in y_1, so their behavior is invariant to its value; changing the coefficients that multiply the quadratic part of y_1 is equivalent to adding a (constant) term to the utility function.

Details of the classical solution to the problem are quite involved. A description of the method is given in Hansen and Sargent, and Sargent. Applications are found in Sargent and Blanchard. The control x is eliminated using the identity $y_{2,t} - y_{2,t-1} = x_t$. The first order conditions at t indicate that the optimal $y_{2,t}$ is a function of $y_{2,t-1}$ and $E_t y_{2,t+1}$. The latter can be replaced by a geometrically declining series of future expected values of $y_{1,t}$. These are replaced by their linear predictors, using $y_{1,t} = A_1 y_{1,t-1} + e_t$. The decision rule (the equation for $y_{2,t}$) involves parameters in A_1 and K. Since the system for y_1 also involves A_1, joint estimation of the system of y_1 and y_2 involves cross-equation restrictions.

b) The Dynamic Programming Approach

This method determines the cross equation restrictions directly from the dynamic programming solution to the problem given by (2) and (3). This solution is (see, e.g., Chow)

$$(4) \qquad G = -(C'HC)^{-1} C'HA,$$

where H satisfies

$$(5) \quad H = K + \beta(A + CG)'H(A + CG).$$

The last equation is the algebraic Ricatti matrix equation; it is the steady state solution of the difference equation formed by subscripting H on the right side of (5) with t + 1, and on the left side with t. To determine the restrictions on the stochastic process for the uncontrollable state vector y_1, write (2) as

$$(2'') \quad Y_t = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}_t = \begin{bmatrix} A_2 & O \\ A_2 & A_3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}_{t-1} + \begin{bmatrix} O \\ C_1 \end{bmatrix} x_t + \begin{bmatrix} e_1 \\ O \end{bmatrix}_t$$

which is slightly more general than (2′). Define $R = A + CG$, and partition G to conform to the partition of C, so that

$$R = \begin{bmatrix} A_1 & O \\ A_2 + C_1G_1 & A_3 + C_1G_2 \end{bmatrix}.$$

The Ricatti difference equation can be written as $H_t = K - \tilde{R}H_{t+1}\tilde{R}$, $\tilde{R} = \sqrt{\beta}R$. For the backward solution of this to converge, it is necessary that the roots of $\tilde{R}$ be less than 1 in absolute value, which requires the root of R be less than $1/\sqrt{\beta}$ in absolute value. Since R is block triangular, its roots are the roots of A_1 and $A_3 + C_1G_2$. Thus, a necessary condition for H to converge, and for the stationary G to exist, is that the roots of A_1 be less than $1/\sqrt{\beta}$ in absolute value. This places a bound on the rate at which the independent stochastic process can explode.

Chow suggested using constrained maximum likelihood to estimate the model. Equations (1) and (2) are estimated subject to the restrictions given by (4) and (5). We are unaware of any applications using this approach. Its attractions are that it is very easy to set up, and the derivatives of the likelihood function may be obtained analytically. Chow provides an iterative algorithm, but it may be easier to use a packaged program such as MINOS.

A possible problem involves the constraint (5). If H is obtained by solving the Ricatti difference equation, then under fairly weak conditions the second order conditions will be satisfied. However, methods such as Chow's iterative approach may converge to an estimate of H which does not satisfy the second order conditions.

The solutions implied by the classical and dynamic programming approach are equivalent, but are very different in appearance. Chow shows that where (2) is given by (2′) it is possible to obtain the classical solution from the dynamic programming solution. Hansen and Sargent recommend the classical approach; their reasons are most compelling when only a small number of state variables enter the objective function, and the order of autoregression is high. In this case the classical approach results in a small problem with an easily interpreted solution; the DP approach results in a large problem. If the objective function depends on a "large" number (say, more than 2) of states, the DP approach may be preferable, since the constraints are much easier to derive.

c) Chow's Consistent Estimators
(A Simpler Method)

Chow suggested a very easy way of obtaining consistent estimates of the structural parameters. The idea is to obtain consistent estimates of (1) and (2) and then use (4) and (5) to infer H, and thus K. We restate his suggestions in different notation. This notation makes it easier to implement his proposal, and also leads to an algebraic explanation for a point alluded to above.

Let p and q be, respectively, the number of states and controls. Define S as a p^2xp^2 permutation matrix, such that the $(i-1)p+j$ row of S contains a 1 in the $(j-1)p+i$ column, $i,j = 1,2 \ldots p$. Use the fact that H and K are symmetric, and define H^* and K^* as upper triangular matrices such that $H = H^* + H^{*\prime}$, $K = K^* + K^{*\prime}$. Define $h = \text{vec } H^*$, so $\text{vec } H = (I + S)h$, and $k = \text{vec } K^*$, so $\text{vec } K = (I + S)k$. Write (4) and (5) as

$$(4') \qquad C'HR = 0$$

$$(5') \qquad K = H - \beta R'HR,$$

where $R = \hat{A} + \hat{C}\hat{G}$, the consistent estimates of A, C, and G.

Apply the vec operator to (4′) and (5′), and use the above definitions to write (see Dhrymes, Chapter 4)

$$(6) \qquad (R' \otimes C')(I + S)h = 0$$

$$(7) \quad (I + S)k = [I - \beta(R' \otimes R')](I + S)h.$$

These equations contain the *a posteriori* information. Write the prior restrictions on K^* as $L(I + S)k = 0$; let m be the number of rows of L (the number of prior restrictions). Nonhomogenous restrictions result in only minor changes of the following. Premultiply (7) by L and write

$$L[I - \beta(R'\otimes R')](I + S)h = 0. \quad (8)$$

By construction, $(p^2 - p)/2$ elements of h are equal to zero. Those elements can be deleted from h, forming h^*; the columns of (6) and (8) corresponding to the deleted elements can also be deleted. The resulting systems can be stacked to form the system

$$Qh^* = 0, \quad (9)$$

where Q is a $(pq + m)x(p^2 + p)/2$ matrix.

A sufficient condition for identification is that the rank of Q be equal to $(p^2 + p)/2 - 1$. The necessary condition for identification is $m \geq (p^2 + p)/2 - pq - 1$. In addition to providing a systematic way of constructing Q, the vec notation is useful because it makes it easy to determine cases where the necessary condition may hold, but the system is not identified. We mentioned above the impossibility of estimating unrestricted coefficients of terms which are quadratic in states which cannot be influenced by agents. The algebra behind this is apparent from inspection of systems (6) and (8), or system (9). Suppose that y_1 is a scalar. Since the first row of C is a 0 vector (see (2″)) the first column in $(R'\otimes C')(I + S)$ is a 0 vector The first column in $[I - \beta(R'\otimes R')](I + S)$ consists entirely of 0's except for the first element. Therefore, the first column of the stacked system, Q, is a 0 vector unless there is a nonzero element in the first column of L; that requires a restriction on the (1,1) element of K. If y_1 is a vector, the same argument can be used to show that restrictions are required on all coefficients of terms which are quadratic in y_1.

Chow (pg. 252) mentions several ways of solving (9) for the overidentified case. One of these involves normalizing a particular element of h^*, say the last, and partitioning the system as

$$Q_1h_1^* + q_2 = 0, \quad (10)$$

and solving this using least squares: $h^*_1 = -(Q'_1Q_1)^{-1}Q'_1q_2$. A slight variation of this, which corresponds to generalized least squares, is to choose $h^*_1 = -(Q'_1PQ_1)^{-1}Q'_1Pq_2$ where P is a "precision matrix" which weights the different constraints. This is useful if the researcher believes that certain constraints are more likely to be correct than others. For example, certain of the prior constraints, those represented by L, may be essentially definitions, which should hold almost exactly.

One problem with this method of estimating structural parameters is that there is no guarantee that the resulting H will satisfy second order conditions. However, the method is so inexpensive, and makes such modest demands on data, that in many cases experimentation is worthwhile.

More General Models

The linear-quadratic framework involves very strong assumptions. Depending on the model, these may include the assumption of linear technology and symmetric adjustment costs. Efforts have been made to allow econometricians to use other functional forms. These efforts have proceeded along two lines, based on stochastic and deterministic dynamic problems.

a) The Stochastic Problem

Hansen and Singleton provide a method of estimating a general stochastic model. Pindyck and Rotenberg apply the technique. The first step derives the first order conditions to the stochastic dynamic problem. These are the stochastic Euler equations, mentioned above in connection with the classical approach to the linear-quadratic problem. The condition in period t typically involves functions of the state which the agent controls, evaluated at and before period t, and the expectation, at period t, of future values. For simplicity, suppose that only the expectations at t + 1 are relevant. In the linear-quadratic case, the expectation of the t + 1 value of the controlled state is eliminated, and the expectation of the sum of all future values of the uncontrolled states appear in the first order condition. For more general functions it is not possible to make this substitution. Hansen and Singleton suggest using the "method of moments" which is an instrumental variables technique. The first order conditions at t (without the expectations operator) are multiplied by instruments, which consist of variables in the agent's information set at t, and which are observed by the econometrician. The resulting function is summed over t. The quadratic form of this sum is minimized with respect to the unknown parameters. The authors discuss the determination of the optimal quadratic weights.

b) The Deterministic Problem

The deterministic cost of adjustment model can be estimated in two ways. The direct method specifies a deterministic, stationary objective function, and solves for the control, or investment rule. Craine uses this method with a production function that is homogenous of degree 1 in quasi-fixed and variable inputs, and a separable, quadratic adjustment cost. Another approach is to specify a profit function that is linearly homogenous in quasi-fixed inputs and investment. The investment rule is then a function of Tobin's "q," the ratio of the capitalized value of the firm to the quasi-fixed stock (see Hayashi (1982)). This can be generalized by allowing the profit function to be homogenous of degree γ. The investment rule is then a function of γq.

The indirect, or dual approach, specifies the functional form of the value of the maximized problem. Standard duality arguments are then used to derive the investment rule and the demand system for variable inputs. McLaren and Cooper, and Epstein did the basic theoretical work. Applications include Epstein and Denny, Bernstein and Nadiri and Vasavada and Chambers. These are all based on a continuous time version of the problem. A discrete formulation may be more natural; the investment rule from the continuous time model is a first order Taylor approximation of the investment rule from the discrete framework (Karp). If the investment rule is linear, the two are equivalent. A linear decision rule may be preferred, for reasons of consistent aggregation (see Blackorby and Schworm, and Epstein and Denny). This depends on the level of aggregation of the data and on the researcher's preference for consistent aggregation relative to a more general specification. Even if the investment rules are the same with the discrete and continuous time formulation, the demand system for variable inputs will be different due to discounting.

The Stationarity Assumption

The approaches discussed above involve assumptions which ensure the existence of stationary control rules. These assumptions differ for the various approaches, but they all place strong restrictions on the type of dynamic problem agents are presumed to solve. In some cases the data may be at odds with this presumption. One response is to adjust the data, which typically involves some form of detrending.

The stationarity assumptions are strongest in the dual approach to estimating the deterministic cost of adjustment models, and weakest in Hansen and Singleton's instrumental variables approach to estimating the stochastic model. The dual approach assumes, among other things, that technology is fixed. Most time series data is not consistent with this assumption. To make it consistent, the series on capital stock, for example, may be quality adjusted. Unfortunately, the theory does not explain how agents' investment decisions are related to quality adjusted capital stock. Hansen and Singleton's method assumes that the stochastic process is ergodic, so that a limiting probability distribution exists. This is analogous to the "constant in repeated samples" assumption used in asymptotic theory of the standard linear model. The ergodicity assumption is sufficient, but it is not clear that it is necessary (Hansen and Singleton, p. 1275, note 7). Pindyck and Rotenberg ignore the assumption in their application.

Conclusion

Dynamic theory has proven its value in theoretical economics and operations research applications. Its usefulness in constructing econometric models is problematical. This is partly because the risk of misspecification is proportional to the detail of specification, which is very high in dynamic models; it is partly because of technical problems associated with estimation of these models. The technical problems may diminish, but the risk of misspecification is in the nature of the endeavor. Continued empirical work may allow us to feel more comfortable in taking that risk.

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