On Technological Change and Yield Resiliency in Canadian Crop Yields

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Abstract

Feeding nine billion people by 2050, yield resiliency, climate change, and remaining economically competitive have received significant attention in the literature. Technological change in agriculture will largely dictate our ability to meet these challenges. Although there is significant literature on technological change in U.S. crop yields, very little has been done with Canadian yields. Moreover, the adoption and effect of various technologies and their interaction with climate tend to be crop-region specific. To this end, we model technological change in county-level yields for barley, canola, corn, oats, soybean and wheat in Canada. We use mixtures to allow and test for heterogeneous rates of technological change within the yield data generating process. While we tend to find increasing but heterogeneous rates of technological change, increasing and asymmetric yield volatility, and increasing *absolute* but decreasing *relative* yield resiliency, our results do differ across crops and exhibit spatial bifurcations within a crop. Using a standard attribution model, we find changing climate has differing effects across crops. We also consider the public funding implications of technological change for Canadian Business Risk Management programs.

Some key words: technological change, yield volatility, yield resiliency, changing climate, crop insurance

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1. **Introduction**

Meeting expected future growth in food demand is considered a major challenge for global agriculture (Pretty et al., 2010; Conforti et al., 2011; Searchinger et al., 2014; McKenzie and Williams, 2015). Some have argued for food waste reduction as a possible solution (Parfitt, Barthel, and Macnaughton, 2010; Kummu et al., 2012; Gustafsson et al., 2013; Lipinski et al., 2013; Grafton, Daugbjerg, and Qureshi, 2015) while others have argued for a shift towards greater reliance on plant-based diets (White, 2000; Pimentel and Pimentel, 2003; De Boer and Aiking, 2011; Ranganathan et al., 2016). Neither are consistent with economic theory or past empirical evidence: historical increases in global wealth led to increases in both global meat consumption and global food waste (Godfray et al., 2010; Machovina, Feeley, and Ripple, 2015). However, technological advances have led to significant and sustained gains in average yields per acre since the mid-twentieth century. In the United States -- the world's largest producer of corn and soybean -- average yields have more than quintupled and doubled, respectively (USDA NASS, 2018). In Canada, average yields have roughly quadrupled for corn, tripled for canola, and doubled for soybean and wheat (Statistics Canada, 2018). There is no doubt that technological change will continue to play a significant role in meeting food demand at affordable prices, a challenge exacerbated by a changing and possibly increasingly volatile climate.

The resiliency of crop yields to weather variations and pest infestations has driven vast amounts of private and public funds into seed genomics and has received significant attention in the plant science/breeding literature (Lin, 2011; Bennett et al., 2014; Kole et al., 2015; Mondal et al., 2016; Altieri, Nicholls, and Montalba, 2017). Yield resiliency, which can be defined by multiple measures of mass in the lower tail of the yield distribution, is paramount to the entire economy of developing countries and the agricultural economy of developed countries.[2] Developed countries funnel significant public monies into assisting farmers in managing the financial consequences of low yield realizations via publicly subsidized crop insurance programs (Mahul and Stutley, 2010). Karlan et al. (2014) found that the lack of effective yield risk management in developing countries was the largest impediment to social and economic progress. The innovation and adoption of certain technologies, such as triple-stacked seeds, will no doubt play a significant role in enhancing yield resiliency and, by default, alter the public monies spent on agricultural risk management programs in developed countries and the economic progress in developing countries.

Finally, understanding technological change in Canadian agriculture is critically important given our dependence on export markets and the need to remain economically competitive. Canola seed, canola oil, wheat, and soybean currently represent the top four agri-food exports. In 2017, 50% of canola production,

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[2]In the crop science literature, yield resiliency is typically defined as the ability of a crop to retain its productivity following environmental stresses (Holling, 1973). Methods to measure yield resiliency include, but are not limited to, determining the plant biomass after recovery and resurrection from stress (Lukac et al., 2011; Gaudin et al., 2013; Griffiths et al., 2016) and estimating the ratio of crop productivity to severeness of stress (Simelton et al., 2009).

49% of wheat production, and 48% of soybean production were exported (Canadian Grain Commission, 2019). Conversely, the majority of barley, corn, oats, and soybean are domestically consumed in the production of beef and pork products. In 2017, beef and pork exports totalled \$6.4 billion (Agriculture and Agri-Food Canada, 2019). Technological change is and will continue to be paramount to the competitiveness of Canadian agriculture.

The objective of this manuscript is to estimate technological change in Canadian crop yields. As discussed, the study of technological change is critical for meeting future food demand, enhancing yield resiliency, mitigating any negative externalities from a changing climate, and remaining internationally competitive; all of these are issues that have received increasing and plentiful attention across various strands of literature. With respect to the agricultural economics literature, the modelling of technological change has almost exclusively been in regards to U.S. yields (e.g. Skees and Reed (1986); Kaylen and Koroma (1991); Goodwin and Ker (1998); Just and Weninger (1999); Sherrick et al. (2004); Ramirez and McDonald (2006); Woodard and Sherrick (2011); Tack, Harri, and Coble (2012); Wu and Zhang (2012); Tack (2013); Tolhurst and Ker (2015); Park, Brorsen, and Harri (2019)). In contrast, there is no corresponding literature in regards to Canadian crop yields despite technological change in yields tending to be crop-region specific. To fill this gap, we estimate the structure of technological change in barley, canola, corn, oats, soybean, and wheat using Canadian county-level yield data. To that end, we generalize and use a mixture model approach forwarded by Tolhurst and Ker (2015). The advantage of using mixtures in our analysis is that it allows us to test if rates of technological change are homogeneous across sub-populations (mixtures) of the yield distribution. In addition, the mixture model allows us to test if the probability of sub-populations is changing with technological change. Specifically, we consider the following economically interesting questions: (i) what are the rates of technological change in Canada by crop-region combination; (ii) do the rates of technological change differ within the yield distribution; (iii) do rates of technological change differ between regions within a crop; (iv) has technological change affected overall yield volatility; (v) has technological change affected conditional (within a mixture or sub-population) yield volatility; and (vi) has technological change increased or decreased yield resiliency.

For illustrative purposes, in Figure 1(A) we plot the 1949-2016 Middlesex, Ontario, county-level corn yields and the estimated conditional quantiles (Koenker and Bassett Jr, 1978). Figure 1(B) illustrates the estimated (using a mixture model with time-varying parameters) conditional yield densities at various years and the predicted conditional yield density in year 2050. A number of points are worth noting in regards to these Figures: (i) mean yields have roughly tripled; (ii) the rate of technological change differs by quantile; (iii) technological change has increased yield volatility; and (iv) technological change has led to asymmetric changes (between the upper and lower tails) in yield volatility. Interestingly, overall mean yields
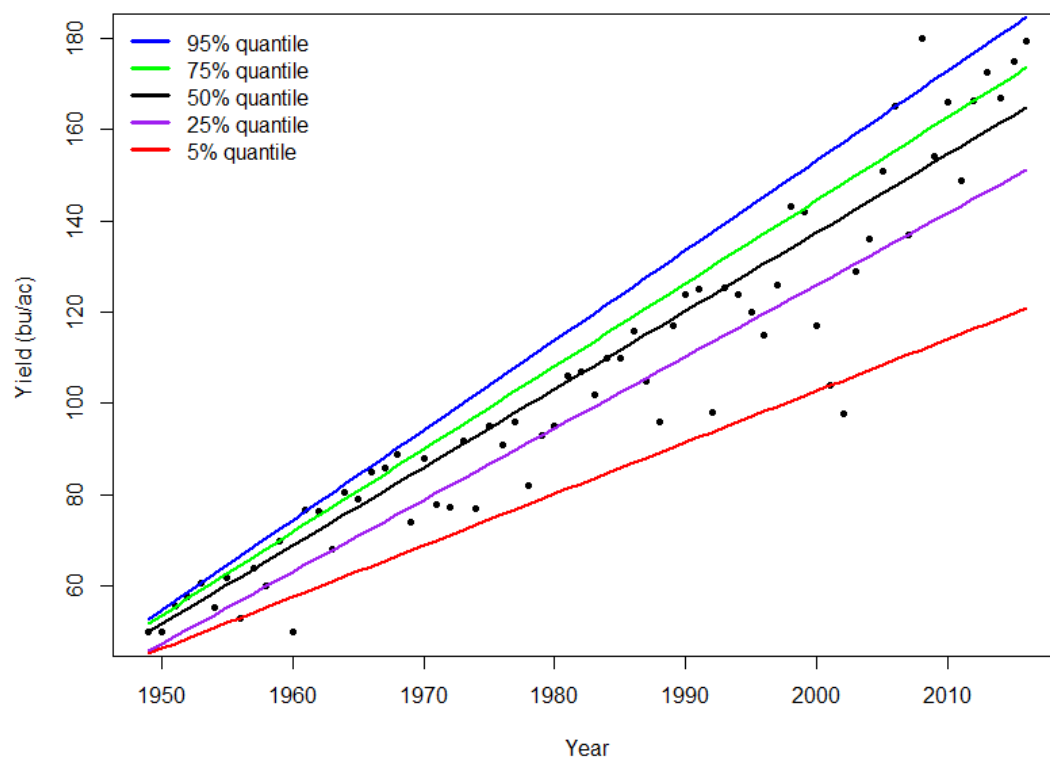
(A) Conditional Quantile Regression
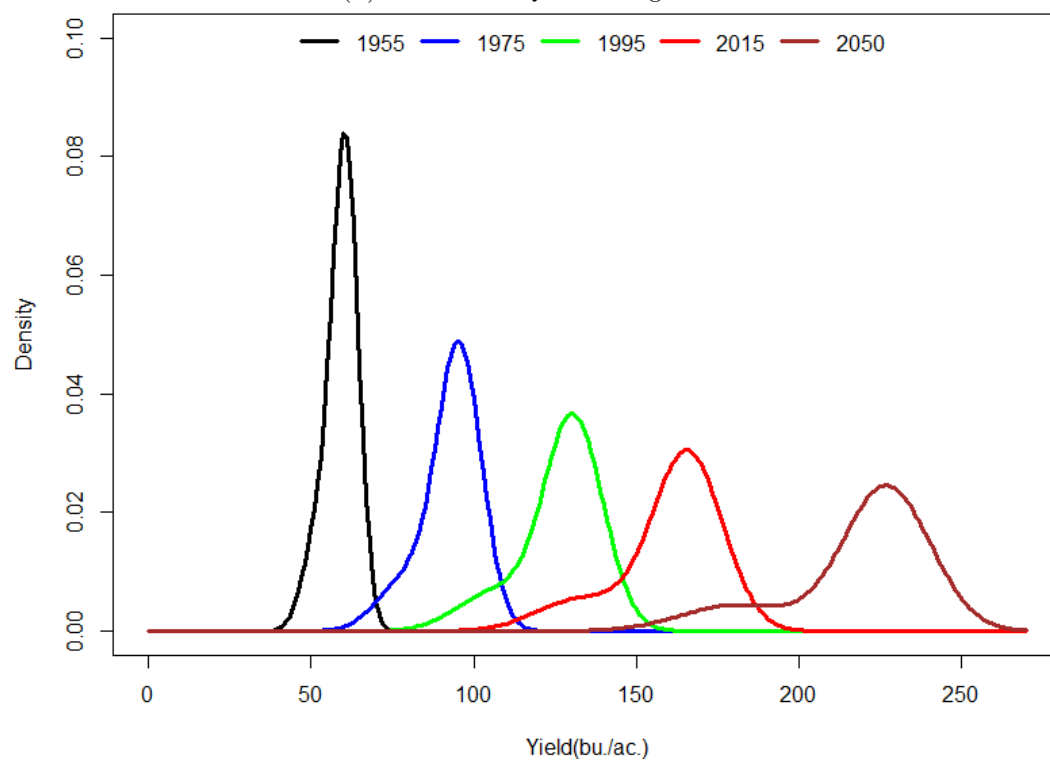
(B) Estimated Conditional Yield Distribution

FIGURE 1. Middlesex County, Ontario, Corn Yields, 1949 - 2016.

for Middlesex corn are expected to increase by 33% by 2050, which is notably greater than the predicted population increase of 20%. However, the probability of 25 and 50 bushels per acre shortfalls is expected to increase by 17% and 47%, respectively, between 2018 and 2023, which is the time frame for the Canadian Agricultural Policy (CAP) framework and accompanying Business Risk Management (BRM) programs. To what extent these results can be generalized to other crop-region combinations in Canadian agriculture is unknown and the gap the manuscript intends to fill.

In addition, we consider the following two extensions. First, we use the spatial structure of our results to identify the effect of changing climate on various estimated technological change parameters. To this end, we use an attribution model approach proposed by Lobell and Asner (2003) and commonly employed in the climate change literature. This extension fills a second gap by providing a sense of how climate change has altered the yield data generating process. Second, we forecast technological change and the conditional yield densities over the next five and ten years and compare crop insurance premium rates. This extension fills a third gap by providing a sense of how public BRM spending may change over the next decade. Note that BRM programs are the main avenue in Canada -- as well as in most developed countries -- of funnelling public monies into the agricultural production sector. Federal-provincial monies budgeted for BRM under CAP are $3 billion.

The manuscript proceeds as follows. The second section outlines the yield and climate data used for our empirical analyses. The third section discusses estimation of conditional yield densities. The fourth section details our estimation results while our hypothesis test results are presented in section five. The sixth section outlines the attribution model and presents the results. The seventh section forecasts premium rates and discusses the policy implications for Canadian BRM programs. Our conclusions are outlined in the final section.

2. Data

Unfortunately, county-level yield data of any historical length only exists for Alberta, Saskatchewan, Manitoba, and Ontario.[3] Yield data are available from 1978-2017 for Alberta, from 1938-2016 for Saskatchewan (only 1970-2016 for canola), from 1993-2017 for Manitoba, and from 1949-2016 for Ontario. We decided not to include Alberta and Manitoba given the insufficient length of yield data for the mixture models. In Ontario, we have corn, soybean, and winter wheat yield data. Ontario accounted for 62%, 49%, and 77% of

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[3]Ideally, farm-level yield data would be used to empirically investigate the effects of technological change given that the adoption decision is at the farm-level. Unfortunately, such data does not exist across time or space in sufficient quantities to be of use and thus we necessarily -- as does almost all the literature -- use county-level data. While this level of aggregation masks farm-level heterogeneity, to the extent that technological effects are dominant across farms within a county, we will be able to identify them with the county-level data. Coble, Dismukes, and Thomas (2007), Cooper et al. (2009), and, Claassen and Just (2011) illustrate that year-to-year farm-level variation is generally double or more the variation in county-level yield data and thus our results regarding volatility and tail effects are likely to be muted relative to farm-level results.

national corn, soybean, and winter wheat production in 2017, respectively (Statistics Canada, 2018).[4] We have 32 counties for corn, six for soybean, and 26 for winter wheat. Our counties represent 97%, 40%, and 95% of Ontario corn, soybean, and winter wheat production, respectively.[5] In Saskatchewan, we have yield data for barley, canola, oats, and spring wheat. Saskatchewan accounted for 40%, 52%, 53%, and 39% of national barley, canola, oats, and spring wheat production in 2017, respectively (Statistics Canada, 2018).[6] We have 204 counties for barley, 144 for canola, 131 for oats, and 267 for spring wheat. For all four crops, our counties represent over 95% of Saskatchewan production. In total, we have seven crops and 810 county-crop combinations. Table 1 presents a summary of the yield data for Ontario and Saskatchewan.

Recall that we will use a standard attribution model from the climate literature to consider the effect of a changing climate on various estimated technological change parameters (Lobell and Asner (2003)). We exclude soybean as we only have six counties. We use daily minimum and maximum temperature and precipitation data from 1950-2017 for each of our 34 Ontario counties and 290 Saskatchewan counties. The temperature and precipitation data are at the longitude and latitude geographical centroids of each county. The daily estimates are based on smoothing from nearby Environment Canada weather stations and were provided by Natural Resource Canada (McKenney et al., 2011). We construct the usual climate variables employed in the literature: growing degree days (GDD); harmful degree days (HDD), vapor pressure deficit (VPD), and, precipitation (PCP). We also include PCP and VPD for only July and August following Tolhurst and Ker (2015). We follow the approach of Roberts, Schlenker, and Eyer (2012) to estimate the distribution of temperature using a sine curve approximation bounded by the minimum and maximum temperature for each day. GDD is simply the sum of daily heat exposure between the lower and upper temperature threshold

TABLE 1. Summary of Crop Yield Data

Province	Observations		Summary Statistics of Crop Yield (bu/ac)				
Crop	Counties	Period (Years)	Min.	Mean	Median	Max.	Std. Dev.
Ontario							
Corn	32	$1949 - 2016 \ (68)$	20.2	92.9	85.0	192.9	33.3
Soybean	6	$1949 - 2016 \ (68)$	16.0	33.0	32.0	56.4	9.0
Winter Wheat	26	$1949 - 2016 \ (68)$	22.0	50.8	46.3	109.5	17.1
Saskatchewan							
Barley	204	$1938 - 2016 \ (79)$	1.0	37.8	38.0	102.7	15.5
Canola	144	$1970 - 2016 \ (47)$	1.5	23.0	22.2	59.5	8.2
Oats	131	$1938 - 2016 \ (79)$	1.0	48.7	48	155.8	20.6
Spring Wheat	267	$1938 - 2016 \ (79)$	1.0	24.9	25.0	70.3	9.9

[4]We collected the yield data from the annual Agricultural Statistics Reports published by the Ontario Ministry of Agriculture, Food and Rural Affairs (OMAFRA).

[5]Soybean production in Ontario increased dramatically by the early 1980s.

[6]We collected the yield data from Saskatchewan's Ministry of Agriculture.

(expected to be beneficial to plant growth) over the growing season. HDD is the summation of daily exposure above the upper temperature threshold (expected to be harmful to plant growth). VPD is the difference between the amount of water the air can hold and the amount of water it currently holds; a high VPD indicates hot and dry conditions. PCP is the summation of precipitation over the growing season.

We use different temperature thresholds and growing seasons for each province-crop combination. Table 2 summarizes the temperature thresholds and growing season dates used for construction of our variables. For Ontario corn, we follow Tolhurst and Ker (2016). The upper temperature threshold for corn is 29°C, whereas the lower threshold is 10°C. The corn growing season starts April 1 for each county and ends by the first day after September 1 with a minimum temperature of −2°C. For winter wheat, Tolhurst and Ker (2016) proposed a series of thresholds: above 12°C with no lower bound (September to November), between 3°C and 15°C (December to February), and, between 9°C and 18°C (March to end of growing season). The start of growing season varies widely across the province from September 5 in the northern and central regions to October 10 in the south. The end of growing season varies from June 15 in the subsequent year to July 20. For Saskatchewan crops, we follow Robertson et al. (2013): 5°C to 28°C for barley, 3°C to 29°C for canola, 5°C to 29°C for oats, and 5°C to 29°C for spring wheat. The growing season for all Saskatchewan crops starts April 1 and ends September 1.

TABLE 2. Summary of Growing Seasons and Critical Temperature Thresholds

Province	Length of Growing Season		Critical Temperature Threshold	
Crop	Start of Season	End of Season	GDD Temp Low	GDD Temp High
Ontario				
Corn	April 1	Sept 1	10°C	29°C
Winter Wheat	Sept 5-25;	June 15-25;	n/a	12°C [a]
	Oct 10	July 5-20	3°C	15°C [b]
			9°C	18°C [c]
Saskatchewan				
Barley	April 1	Sept 1	5°C	28°C
Canola	April 1	Sept 1	3°C	29°C
Oats	April 1	Sept 1	5°C	29°C
Spring Wheat	April 1	Sept 1	5°C	29°C

[a] From September to November.
[b] From December to February.
[c] From March to end of growing season.

3. Estimating Technological Change and Conditional Yield Distributions

The effects of technological change on yields are almost exclusively measured by estimating change in productivity with respect to time. Given the vast number of technological advances in seed, machinery,

inputs, and farm management technologies with varying and unknown rates of adoption, pinpointing the effect of a given technology is empirically impossible unless experimental plot data is used. As a result, technological change is measured by time and reflects not only changing technology but also its interaction with changing policy, changing climate, changing farm management strategies, etc. These effects can be conditioned out; however, for our hypotheses we want the unconditional effects of technological change. That is, the effect of technological change interacting within its production environment or "realized" technological change as measured through the yearly changes in the yield distribution. Therefore, consistent with the literature, we use time to model technological change and explicitly recognize that what we are capturing is technological change interacting within its production and policy environments.

For the most part, the literature employs a two-step process for estimating the conditional yield distribution over time (Zhu, Goodwin, and Ghosh, 2011). In the first step, technological change or the temporal process of yields is estimated by some function of time. Examples of deterministic approaches include a linear spline (Skees and Reed, 1986), a polynomial trend (Just and Weninger, 1999), and a robust linear spline (Ker and Tolhurst, 2019). Examples of stochastic approaches include a Kalman filter (Kaylen and Koroma, 1991) and an ARIMA(p, d, q) model (Goodwin and Ker, 1998; Ker and McGowan, 2000). The estimated residuals are often corrected for possible heteroscedasticity as per Harri et al. (2011). In the second step, the time-conditional yields are assumed to be independent and identically distributed, and parametric or nonparametric methods are used to estimate the distribution. There is a great amount of literature on both approaches (Botts and Boles, 1958; Day, 1965; Gallagher, 1987; Nelson and Preckel, 1989; Moss and Shonkwiler, 1993; Ramirez, 1997; Goodwin and Ker, 1998; Goodwin, Roberts, and Coble, 2000; Ker and Goodwin, 2000; Ramirez, Misra, and Field, 2003; Atwood, Shaik, and Watts, 2003; Ker and Coble, 2003; Sherrick et al., 2004; Stochs and LaFrance, 2004; Ramirez and McDonald, 2006; Woodard and Sherrick, 2011; Tack, Harri, and Coble, 2012; Wu and Zhang, 2012; Tack, 2013; Annan et al., 2014; Tolhurst and Ker, 2015). Note that none of the above cited literature is with respect to Canadian crop yields.

Technological change is often only measured at mean yields, or, if heteroscedasticity is considered, at the mean of the year-to-year volatility. However, technological change moves mass all around the yield distribution; thus measuring the effects of technological change beyond the means of the first two moments can be economically informative. Figure 1(A) illustrated that technological change in Canadian crop yields appears to be having differential effects across the distribution. First, the rate of technological change in the lower quantiles is increasing at a lower rate than in the upper quantiles. Second, technological change is conditionally asymmetric between the upper and lower tails, suggesting that common estimates of heteroscedasticity which assume symmetric changes may hide information.[7] An alternative approach using

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[7]For asymmetric modelling of heteroscedasticity, see Ker and Tolhurst (2019).

maximum likelihood methods while incorporating time directly into the parameters of the likelihood have been proposed by Zhu, Goodwin, and Ghosh (2011) and Tolhurst and Ker (2015). Zhu, Goodwin, and Ghosh (2011) proposed a beta distribution with time-varying parameters, whereas Tolhurst and Ker (2015) used a mixture model, also with time-varying parameters. Because the parameters are time-varying beyond the conditional mean, these methods are able to capture the effects of technological change throughout the distribution. Economically, using mixtures is appealing as the mixtures themselves identify heterogeneity which is most often the result of differing underlying economic drivers. Moreover, mixture models are exceedingly flexible and can approximate any density to a desired level of bounded error (Everitt and Hand, 1981). Somewhat surprisingly, mixtures have been sparingly used in the agricultural economics literature. Notable exceptions include Hall, Brorsen, and Irwin (1989) to model commodity futures, Goodwin, Roberts, and Coble (2000) to model price uncertainty, and Li et al. (2017) to model differing GARCH processes in commodity markets.

4. **Estimation Results**

A normal mixture model is defined as:

$$(1) \qquad y_t \sim \sum_{k=1}^{K} \lambda_k N(\mu_k(t),\, \sigma_k^2(t))$$

where y_t is the yield, K is the number of mixture components, λ_k is the probability of a particular component k (subject to $\lambda_k > 0$ and $\sum_{k=1}^{K} \lambda_k = 1$), $N(\mu_k(t),\, \sigma_k^2(t))$ are time-varying normal distributions with mean $\mu_k(t)$ and variance $\sigma_k^2(t)$. Empirically, we considered mixtures of 1, 2, and 3 components as well as constant versus time-varying variances for each of the 810 county-crop combinations. Given we allow for time-varying variances, our mixture model is a generalization of Tolhurst and Ker (2015). As is commonly done in the literature, the mixture models were estimated using the EM-algorithm, and AIC was used to choose among the six different model forms.[8]

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[8]The EM algorithm involves two steps: an E-step and M-step. To begin with, we arbitrarily assigned an initial probability (ρ_t) to each point (y_t) of belonging to the lower component. This allowed us to recover an initial set of parameters by maximizing weighted likelihoods. The E-step subsequently computes ρ_t for each y_t using the current value of the parameters, whereas the M-step subsequently computes a new set of parameters based on the new weights (ρ_t). The algorithm iterates between the E-step and M-step until convergence. Two points are worth noting. First, the EM algorithm may converge at a local maximum rather than the global maximum (Karlis and Xekalaki, 2003; McLachlan, 2018). Therefore, we used multiple starting values (Finch, Mendell, and Thode Jr, 1989; Atwood et al., 1992; Karlis and Xekalaki, 2003) and chose the parameters that maximized the likelihood across the multiple starting values. Often, the different starting values led to the same parameter estimates. We used conditional quantile (equally spaced over 20%-80%) regression for our starting probabilities. Second, the EM algorithm is biased in small samples towards equal component probabilities. Therefore, using our parameter estimates from the EM algorithm as starting values we subsequently maximized a penalized (in the direction of the bias) likelihood. A standard squared term in the direction of the bias was used as the penalty and 50 levels of tuning parameters were considered (the tuning parameter puts more or less weight on the penalty relative to the likelihood). This led to 50 additional sets of parameter estimates. Our chosen final estimate was the set of parameters that maximized the (un-penalized) likelihood from amongst the 51 sets of parameters.

For Ontario corn, a mixture of two normals with time-varying conditional variances minimized AIC in the most counties:

$$(2) \qquad y_t \sim \lambda \mathcal{N}(\alpha_l + \beta_l t, \gamma_l + \delta_l t) + (1 - \lambda)\mathcal{N}(\alpha_u + \beta_u t, \gamma_u + \delta_u t).$$

Note, α_l and β_l are the intercept and slope coefficient in the lower component mean function, γ_l and δ_l are the intercept and slope coefficient in the lower component variance function, α_u and β_u are the intercept and slope coefficient in the upper component mean function, γ_u and δ_u are the intercept and slope coefficient in the upper component variance function, and λ is the mixing parameter representing the probability of the lower component.

Conversely, for the other six province-crop combinations, a mixture of two normals with constant conditional variances minimized AIC in the most counties within the crop:

$$(3) \qquad y_t \sim \lambda \mathcal{N}(\alpha_l + \beta_l t, \sigma_l^2) + (1 - \lambda)\mathcal{N}(\alpha_u + \beta_u t, \sigma_u^2).$$

Again, we denote α_l and β_l as the intercept and slope coefficient in the lower component mean function, σ_l^2 as the lower component variance, α_u and β_u as the intercept and slope coefficient in the upper component mean function, σ_u^2 as the variance in the upper component, and λ as the mixing parameter representing the probability of the lower component.

Equation (2) was estimated for each of the 32 Ontario corn counties while equation (3) was estimated for the remaining 778 county-crop combinations for a total of 810 distinct estimates of conditional yield densities. We imposed two restrictions on our mixture models. First, the component variances were restricted to be above zero within the sample and 10 years forward (until 2028). This was only an issue with the slope parameters (δ_l, δ_u) being sufficiently negative relative to the constants (γ_l, γ_u) in the time-varying component variance equations. We tested this restriction for all counties and both components (64 tests) using a likelihood ratio test and only one restriction was rejected which is well below the size of the test. Second, the lower and upper component means were restricted from crossing. This happened exclusively in the beginning of the yield series where there is strong clustering. We tested this restriction for all counties and both components (1620 tests) using a likelihood ratio test and 104 restrictions were rejected (slightly above size of the test). A notable benefit of using mixture models is that one can recover the *estimated* probability that an observation belongs to a given component or mixture. We define the estimated probability of observation y_t belonging to the lower component as $\hat{\rho}_t$.[9] These probabilities can be subsequently modelled

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[9]The estimated probability of observation y_t belonging to the upper component is necessarily $1 - \hat{\rho}_t$.

as functions of explanatory variables. For example, Tolhurst and Ker (2015) modelled these probabilities as a function of climate variables.

One of our objectives is to estimate the effect of technological change on yield resiliency. The mixture model allows us to consider three unique measures of yield resiliency not previously considered in the literature. First, we define increased *absolute* yield resiliency as mean yields in the lower component increasing through time ($\beta_l > 0$). Second, we define increased *relative* yield resiliency as mean yields in the lower component moving closer to mean yields in the upper component ($\beta_l > \beta_u$). Third, we define increased *frequency-based* yield resiliency as the decreasing (through time) probability of a yield from the lower component. For this latter measure, we estimate the following for each county-crop combination:

$$(4) \qquad\qquad \hat{\rho}_t = a + bt + \varepsilon$$

where $\hat{b}$ represents the increase or decrease in the probability of the lower component through time. That is, $b > 0$ corresponds to a decrease in *frequency-based* yield resiliency whereas $b < 0$ corresponds to an increase.

Table 3 summarizes the median parameter estimates for all 810 county-crop combinations. The minimum, maximum, mean, median, and standard deviation of all parameter estimates are available in the online appendix. Also in the online appendix are maps of each estimated parameter by crop-province combination (to identify any spatial concentrations). Figure 2 illustrates representative estimates of the conditional mean equations and densities at the county-crop level. We also illustrate the expected yield densities in year 2050 (corresponding to the issue of population growth and food security). There are a number of interesting results that arise.

First, the yield distributions have changed markedly through time. This is not surprising given the past 70 years reflect very significant innovations in both seed and farm management technologies. Also of interest is how stable the expected 2050 yield distributions appear given we are predicting 30+ years forward in function space. Note, the yield distributions for all crops tend to exhibit negative and increasing skewness. Ker et al. (2017) suggested that increasing asymmetric volatility is consistent with producers substituting subsidized crop insurance for other risk-reducing technologies. The finding of increasing negative skewness is also consistent with the findings of Ker and Tolhurst (2019) for U.S. yields. We do see some bi-modal distributions. This results from constant component variances (chosen by AIC) and predicting 30+ years forward. Note, a time-varying component variance structure (like in the corn model) spreads the component mass relative to the spread in the component means adn thus significantly reduces any bi-modality. However, for our small sample sizes, AIC, not surprisingly, tended to suggest fixed component variances.

Table 3. Median of Estimated Parameters, by Crop and Province

Province	Parameters									
Crop	λ	α_l	β_l	γ_l	δ_l	α_u	β_u	γ_u	δ_u	b
Ontario										
Corn	0.42	46.54	1.06	4.13	2.01	47.24	1.54	6.46	1.67	-0.00175
Soybean	0.19	19.00	0.19	17.74	n/a	19.70	0.40	8.03	n/a	-0.00351
Winter Wheat	0.19	26.14	0.38	15.83	n/a	26.14	0.77	30.10	n/a	-0.00424
Saskatchewan										
Barley	0.17	14.52	0.31	54.24	n/a	20.52	0.53	52.59	n/a	-0.00176
Canola	0.47	15.25	0.17	19.38	n/a	17.27	0.42	13.64	n/a	-0.00060
Oats	0.33	19.18	0.44	86.58	n/a	28.14	0.65	101.29	n/a	-0.00110
Spring Wheat	0.18	8.85	0.19	16.28	n/a	16.36	0.28	28.24	n/a	-0.00194

Second, the estimated probability (or size) of the lower component, $\hat{\lambda}$, tends to be significantly smaller than the upper component in all but Ontario corn and Saskatchewan canola. Corn has been relatively more sensitive to climate variations, and canola has been relatively more prone to pest infestations. Of particular interest is the spatial bifurcation of $\hat{\lambda}$ for each crop (see online appendix). In particular, Ontario winter wheat in the west, tends to have a smaller probability of a low component realization than the eastern part. Similarly, Saskatchewan spring wheat in the north tends to have a smaller probability of a low component realization than spring wheat in the southern counties.

Third, the yearly change in the probability of the lower component ($\hat{b}$) tends to be negative for all county-crop combinations (plotted in Figure 4 in the appendix). This suggests that technological change is increasing *frequency-based* yield resiliency. Fourth, $\hat{\beta}_l$, which represents the slope of the lower component mean, tends to be positive for all county-crop combinations. This suggests that technological change is increasing *absolute* yield resiliency. Fifth, the slope in the upper component mean, $\hat{\beta}_u$, tends to be higher than in the lower component mean, $\hat{\beta}_l$. This result suggests differing rates of technological change within the yield distribution and, given $\beta_u > \beta_l$, that technological change is causing increasing volatility and decreasing *relative* yield resiliency. The differing rates of technological change are not surprising given that technologies do not simply move the mean upwards and leave the rest of the distribution unaffected. That "realized" technological change is increasing yield volatility asymmetrically is also not surprising under subsidized crop insurance.

Sixth, if we consider the ratio β_u/β_l by county-crop combination, we see that the rates of technological change in the upper component are increasing at roughly double those in the lower component for soybean, winter wheat, barley, canola, and spring wheat. For corn and oats, the rates of technological change in the upper component are increasing at 150% those in the lower components. As a result, yield volatility is
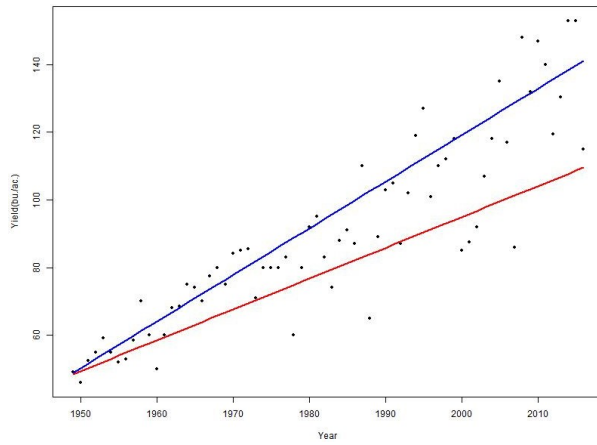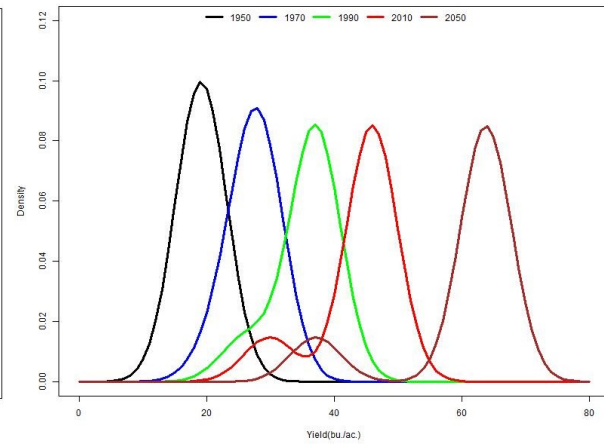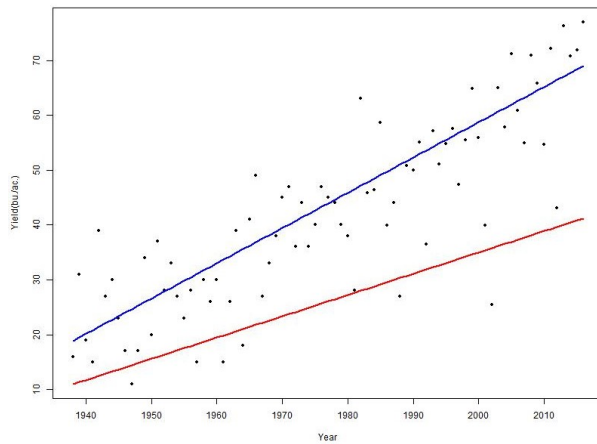
(A) Peel, Ontario, Corn
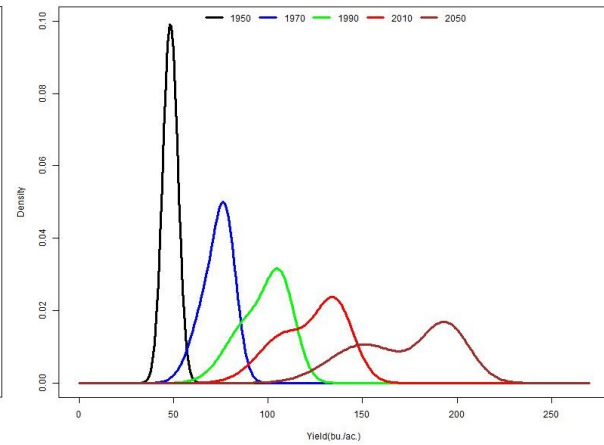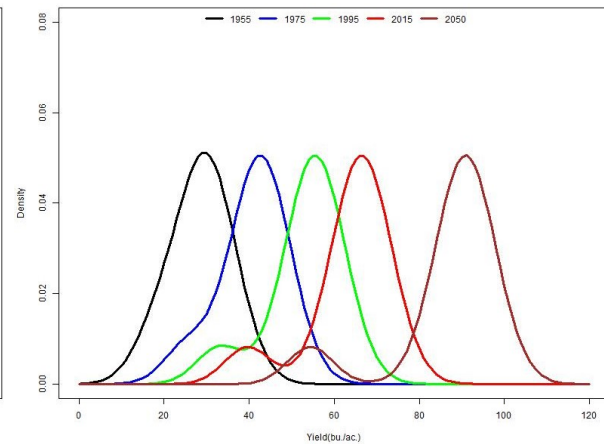

(B) Peel, Ontario, Corn
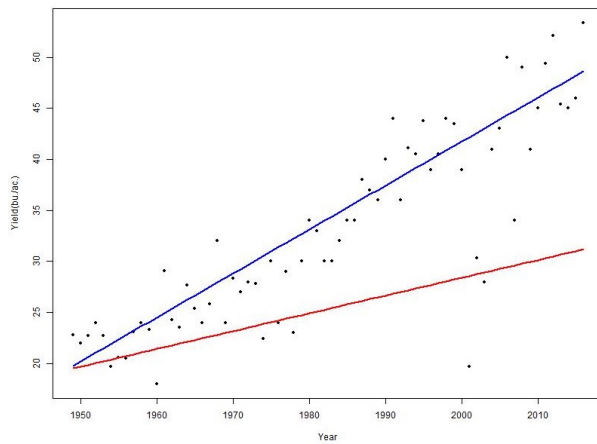
(C) Middlesex, Ontario, Soybean
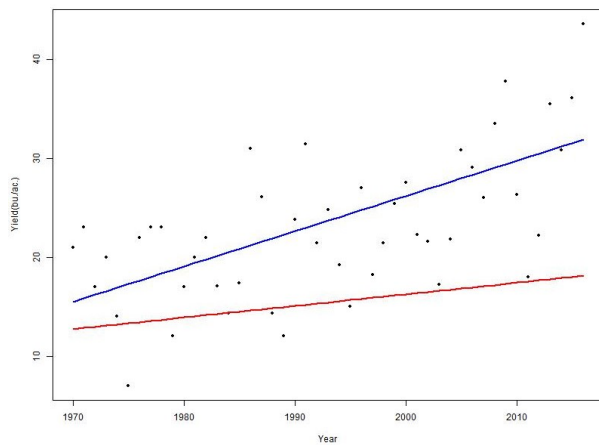

(D) Middlesex, Ontario, Soybean
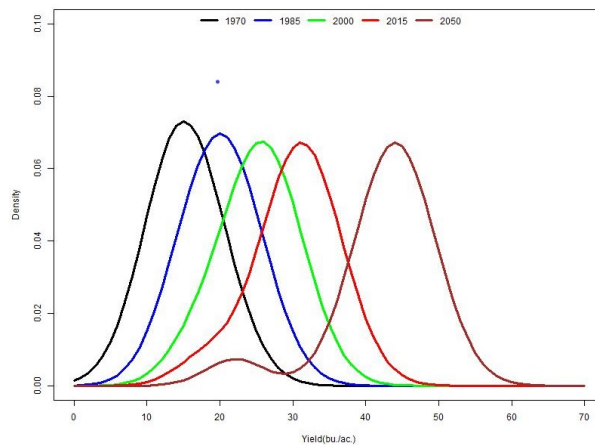

(E) Hoodoo, Saskatchewan, Barley
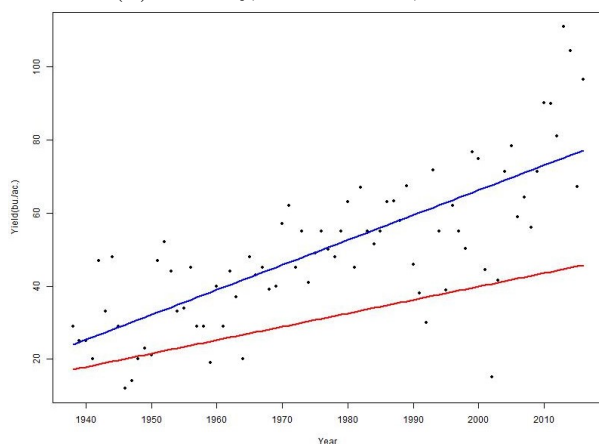

(F) Hoodoo, Saskatchewan, Barley

FIGURE 2. Representative Technological Trends and Estimated Yield Distribution, by Crop
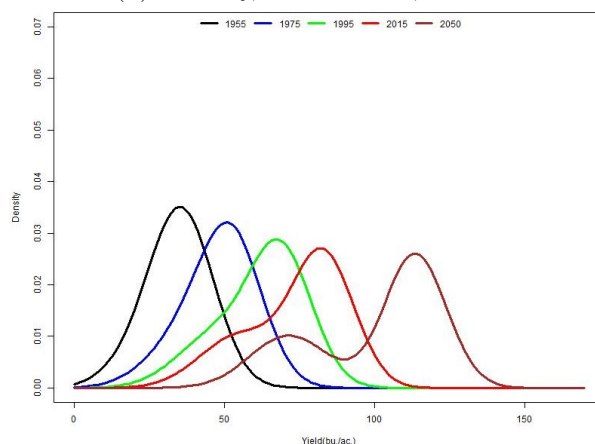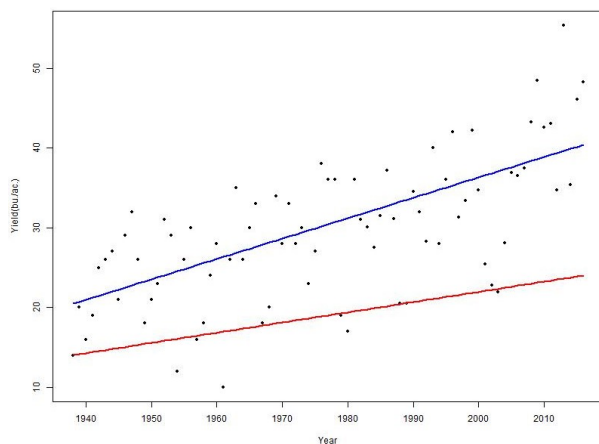
(G) Wolseley, Saskatchewan, Canola

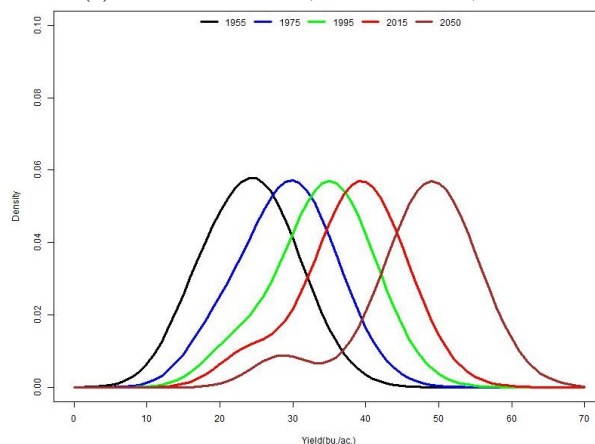(H) Wolseley, Saskatchewan, Canola

(I) Frenchman Butte, Saskatchewan, Oats

(J) Frenchman Butte, Saskatchewan, Oats

(K) Abernethy, Saskatchewan, Wheat

(L) Abernethy, Saskatchewan, Wheat

FIGURE 2. Representative Technological Trends and Estimated Yield Distribution, by Crop

expected to increase more in soybean, winter wheat, barley, and canola relative to corn, oats, and spring wheat. Figure 4 (appendix) plots the difference $\hat{\beta}_u - \hat{\beta}_l$. Interestingly, we see spatial bifurcations for each crop. For both Ontario corn and winter wheat, the difference is much larger in the western part of the province, suggesting an area of greater yield volatility. For all four Saskatchewan crops, the difference is much larger in the northern part of the province, again suggesting an area of greater yield volatility.

Finally, if we consider the rate of technological change of the overall mean, yield is increasing at a greater rate than population growth for all crop-province combinations. In fact, the median increases in yields between 2018 and 2050 are 30%, 25%, and 29% for Ontario corn, soybean, and winter wheat, respectively. Similarly, the median increases in yields are 27%, 31%, 26%, and 24% for Saskatchewan barley, canola, oats, and spring wheat, respectively. Recall that population is supposed to increase by roughly 17-20% between 2018 and 2050. If we consider the overall standard deviation, we see that yield volatilities are expected to increase by 42%, 41%, and 43% for Ontario corn, soybean and winter wheat, respectively, between 2018 and 2050. With respect to Saskatchewan barley, canola, oats, and spring wheat, yield volatilities (as measured by overall standard deviation) are expected to increase by 27%, 59%, 29%, and 23%, respectively. Interestingly, the coefficient of variation for Ontario corn, soybean, and winter wheat as well as Saskatchewan canola is increasing, while the other crop-province combinations appear to be roughly constant.

In summary, our estimation results produce a number interesting findings: (i) technological change is having pronounced effects on aspects of the yield distributions beyond the overall mean; (ii) technological change is having noticeably different effects throughout the yield distribution; (iii) technological change is increasing yield volatilities; (iv) technological change is increasing yield volatilities asymmetrically between the two tails, with greater volatility in the lower tail; (v) technological change is increasing *absolute* and *frequency-based* yield resiliency but decreasing *relative* yield resiliency; and (vi) technological change, if it continues and to the extent that Ontario and Saskatchewan are representative, will increase mean yields at a greater rate than expected population increases.

5. Hypothesis Test Results

The mixture model allows us to consider hypothesis tests that would otherwise not be testable. We test if the rates of technological change are statistically different across the two components of the yield distributions ($H_0 : \beta_l = \beta_u$). For this we use a standard likelihood ratio test. In the appendix, we plot $\hat{\beta}_u - \hat{\beta}_l$ by county-crop combination on maps to identify any spatial clustering. We also test if the conditional variances – variances within the components – are increasing or decreasing with technological change. Specifically, we test $H_0 : \delta_l = \delta_u = 0$ for Ontario corn, as the other crop-province combinations assumed a model (from AIC model selection) with constant conditional variances. Again, we use a standard likelihood ratio test. Finally,

we consider and test our three different measures of yield resiliency. First, we defined increased *absolute* yield resiliency as mean yields in the lower component increasing through time. This tests $H_0 : \beta_l = 0$ using a standard likelihood ratio test. Second, we defined increased *relative* yield resiliency as mean yields in the lower component getting closer through time to mean yields in the upper component. This is simply our previous test ($H_0 : \beta_l = \beta_u$). Third, we defined increased *frequency-based* yield resiliency as the probability of yields from the lower component decreasing through time ($H_0 : b = 0$). To undertake this test we use the estimated probabilities of realizations belonging to the lower component, regress them against time, and conduct a simple t-test. However, the usual standard errors do not account for the estimated regressor, and we therefore report rejections using jackknife standard errors as well as traditional standard errors.[10] In the appendix, we also plot the slope parameter ($\hat{b}$) by county-crop combinations on maps, again to identify any spatial clustering. The four test results are located in Table 4. Note, the size of all tests is 5%. Given we are doing multiple tests per crop (one per county) we also consider the Holm-Bonferroni multiple testing procedure although this is not commonly done in the literature. This procedure rejects only the null hypotheses with ordered p-values less than the sequence ($\alpha/m, \alpha/(m-1), ..., \alpha$) to ensure the risk of rejecting one or more true nulls is at most α, where α is the size of the test and m is the number of tests (Holm, 1979).[11] This approach assumes statistical independence between tests in constructing the null of the order statistics and is therefore overly conservative given we suspect strong spatial correlation. That said, a single rejection under Holm-Bonferroni is sufficient to reject the "joint" null across all tests under the family-wise rate α.

For Ontario corn, we reject homogeneous rates of technological change across the yield distribution in 25 of the 32 counties. In 24 of the 25 instances, we find $\hat{\beta}_u > \hat{\beta}_l$, indicating that almost all counties exhibit increasing overall volatility. This is strong evidence of decreasing *relative* yield resiliency. Note, we reject under Holm-Bonferroni as well. Recall that using AIC (not a statistical test) we found time-varying conditional variances were preferred to constant conditional variances. We tested $H_0 : \delta_l = \delta_u = 0$ and found that 22 of 32 counties rejected the null of constant conditional variance. Again, Holm-Bonferroni rejected as well. We find all 32 counties exhibit statistically significantly increasing *absolute* yield resiliency ($\beta_l > 0$). When we test the probability of the lower component fixed through time, we find only four of

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[10]In general, the bootstrap is preferable to the jackknife. However in our analysis the residual bootstrap is invalid in the presence of conditional heteroscedasticity and the wild bootstrap is invalid given asymmetric heteroscedasticity. Moreover, bootstrapping the original yield data can lead to convergence problems within the components. The jackknife drops one year at a time and recovers T jackknife estimates. The robust jackknife standard error is $(\frac{T-1}{T} \sum_{t=1}^{T} (\hat{\theta}_{-t} - \bar{\hat{\theta}})^2)^{0.5}$. Note, the jackknife is a linear approximation to the bootstrap but is biased upwards relative to the bootstrap (Efron and Tibshirani, 1994). In this sense, our jackknife standard errors are robust but conservative.

[11]Note, as m, the number of tests increases, the threshold p-values get correspondingly closer to zero. For example, for $m = 100$ tests and rejection level of 0.05, the 100 thresholds of the ordered p-values are $(0.000500, 0.000505, 0.000510, ..., .05)$. If we find 25 rejections but the minimum p-value is not below 0.0005, then we would find no rejections using Holm-Bonferroni. That is, once an ordered p-value is not below the corresponding ordered threshold, the test stops.

Table 4. Rejection Counts of Hypothesis Tests

| Province | | | | $H_o : b = 0$ | |
Crop	Counties	$H_0 : \beta_l \leq 0$	$H_0 : \beta_l = \beta_u$	SE_{Conv}	SE_{Jack}
Ontario					
Corn	32	32	25	4	0*
Soybean	6	5	3	3	1
Winter Wheat	26	25	15	12	3
Saskatchewan					
Barley	204	190	89	59	11
Canola	144	131	32	12	4*
Oats	131	119	45	28	6
Spring Wheat	267	238	88	96	15
All	810	740	297	214	40

Notes : SE_{Conv} are conventional standard errors. SE_{Jack} are jackknife standard errors. * denotes a fail to reject of the joint null for a given crop using the Holm–Bonferroni method.

the counties reject the null under conventional standard errors; all four exhibit decreasing probability in the lower component. That is, *frequency-based* yield resiliency is increasing. However, when we consider the estimated regressor issue and use jackknife standard errors, we cannot reject constant probability for any counties. Recall, Figure 4 in the appendix plots $\hat{\beta}_u > \hat{\beta}_l$ and $\hat{b}$ by county-crop combination. We see that there is noticeable spatial bifurcation: $\hat{\beta}_u - \hat{\beta}_l$ tends to be greater and $\hat{b}$ tends to be negative in southwestern Ontario.

For Ontario soybean and winter wheat, we find that 50% and 58% of counties reject equivalent rates of technological change respectively. Note that for both crops all rejections are such that $\beta_u > \beta_l$, suggesting strong evidence of decreasing *relative* yield resiliency. Conversely, for both crops almost all counties reject $\beta_l \leq 0$, indicating very strong evidence of increasing *absolute* yield resiliency. For both tests, we reject under Holm-Bonferroni as well. We find that 50% and 7% of the soybean and winter wheat counties, respectively, reject constant component probabilities using traditional standard errors. In the three soybean cases and 10 of the 12 wheat cases, $\hat{b}$ is less than zero, suggesting decreasing probability of a low component yield realization and thus increasing *frequency-based* yield resiliency. However, when our jackknife standard errors are considered, the number of rejections drop to one and three, respectively (all have $\hat{b} < 0$). These tests do still reject under Holm-Bonferroni. Interestingly, we find spatial concentration for both $\hat{\beta}_u - \hat{\beta}_l$ and $\hat{b}$; as with the corn model, $\hat{\beta}_u - \hat{\beta}_l$ tends to be greater in southwestern Ontario, but, unlike corn, $\hat{b}$ tends to be below zero in eastern Ontario.

For Saskatchewan barley and canola, we find, 44% and 22% of counties reject the null of equal rates of technological change, of which 91% and 100% have $\beta_u > \beta_l$, respectively. These crops also illustrate

strong evidence of decreasing *relative* yield resiliency. Conversely, for both crops almost all counties reject $\beta_l \leq 0$, indicating very strong evidence of increasing *absolute* yield resiliency. For both tests, we reject under Holm-Bonferroni as well. We find that 29% and 8% of the barley and canola counties, respectively, reject constant component probabilities using traditional standard errors. Interestingly, $\hat{b} < 0$ in 92% and 58% of the cases for barley and canola, respectively. For barley, these results suggest increasing *frequency-based* yield resiliency, whereas for canola the results are almost equally divided between decreasing and increasing *frequency-based* yield resiliency. However, when our jackknife standard errors are considered, the number of rejections for barley and canola drop to 11 and four, respectively. Interestingly, of those 11 and four rejections, 9 and 4 have $\hat{b} < 0$. We reject under Holm-Bonferroni for barley but not for canola. For barley we find $\hat{\beta}_u - \hat{\beta}_l$ tends to be higher in the more northern counties, while $\hat{b}$ tends to be more negative in the more northern counties. For canola, we find similar results between the northeast and northwest counties and the southeast and north central counties.

For Saskatchewan oats and spring wheat, we find that roughly 33% of counties reject the null of equal rates of technological change, of which 91% and 73% have $\hat{\beta}_u > \hat{\beta}_l$, respectively. While oats illustrate strong evidence of decreasing *relative* yield resiliency, the results are noticeably less strong for spring wheat. In a number of southwest-southcentral Saskatchewan counties, we find evidence of increasing *relative* yield resiliency for spring wheat. For both crops, the very large majority of counties reject $\beta_l \leq 0$, indicating strong evidence of increasing *absolute* yield resiliency. We reject both tests under Holm-Bonferroni. We find that 21% and 36% of the oats and spring wheat counties, respectively, reject constant component probabilities using traditional standard errors. Interestingly, $\hat{b} < 0$ in all but three cases, suggesting increasing *frequency-based* yield resiliency. However, when our jackknife standard errors are considered, the number of rejections drop to six and 15, respectively (all from $\hat{b} < 0$). Both tests under the jackknife standard errors reject Holm-Bonferroni. We find strong spatial bifurcations of $\hat{b}$ where the tendency in the more northern counties is for $\hat{b} < 0$, indicating increasing *frequency-based* yield resiliency.

Overall, the results show strong evidence of differing rates of technological change for all crops despite the low power of our tests (caused by relatively small samples; relatively high number of parameters; parameters modelling upper moments of the data generating process; and, strong spatial correlation across the counties). Moreover, in almost all county-crop combinations we find $\beta_u > \beta_l$ suggesting increasing volatility and decreasing *relative* yield resiliency with technological change. A notable exception is Saskatchewan wheat; in the northwest we find $\beta_u > \beta_l$ whereas in the southeast we find $\beta_u < \beta_l$. We found very strong evidence of increasing *absolute* yield resiliency ($\beta_l > 0$) across all crops and regions. We did not find significant evidence of $b \neq 0$ or changing component probabilities. However, of the few results we did find, all suggest that the probability of the lower component is decreasing with technology ($b < 0$) and *frequency-based* yield

resiliency is increasing. We find notable spatial concentrations of our test results, which is not surprising given the notable spatial concentrations of our parameter estimates. Finally, we did find that the statistical strength of our tests or the percent of rejections differed significantly across the six crops. Overall, our results are qualitatively similar to what Tolhurst and Ker (2015) found for U.S. corn, soybean, and winter wheat county-level yields.

6. Climate Attribution Model

Changing climate has become a significant area of academic research and central to many national and international policies. In Canada, carbon pricing has been and continues to be a contentious issue, causing divide amongst the provinces and court actions between provincial and federal governments. With respect to agriculture, there has been significant work with respect to changing climate and U.S. crop yields. Very little has been done with respect to Canadian yields. In this section, we consider to what extent changing climate has affected the yield data generating process. To this end, we use the standard attribution model from the climate literature, proposed by Lobell and Asner (2003). This approach makes use of the spatial heterogeneity in rates of change in yields and climate variables to identify changing climate effects. The model is defined as follows:

$$(5) \qquad \Delta\omega_i = \zeta + \theta\Delta climate_i + \eta_i$$

where $\Delta\omega_i$ is the average yearly change in the yield response variable of interest, and $\Delta climate_i$ is the average yearly change in the climate variables where i denotes the county. This model has been heavily used in the literature (see Tao et al. (2006), Lobell and Field (2007), Kucharik and Serbin (2008), Roberts, Schlenker, and Eyer (2012), Tack, Harri, and Coble (2012)).

The climate attribution model is estimated by crop. As discussed in our climate section, the variables of interest are GDD, HDD, VPD, PCP, VPD$_{JA}$, and PCP$_{JA}$. The mixture model approach allows us to consider the effects of a changing climate on a number of different aspects of the yield data generating process. To the best of our knowledge, we have not seen a complete consideration of changing climate on the yield data generating process. In most instances, only mean yields are considered and primarily using U.S. yield data. We consider the effects of a changing climate on a number of technological change parameters from our estimated yield data generating processes. Specifically, we consider the average yearly change in: (i) the lower component mean ($\hat{\beta}_l$); (ii) the upper component mean ($\hat{\beta}_u$); (iii) the overall mean denoted as $\hat{\bar{y}}$; (iv) the volatility as measured by $\hat{\beta}_u - \hat{\beta}_l$; (v) the overall volatility as measured by the variance and denoted $\hat{\sigma}^2$; (vi) the lower component conditional variance ($\hat{\delta}_l$); (vii) the upper component conditional variance ($\hat{\delta}_u$); and (viii) the probability of the lower component ($\hat{b}$). The results for Ontario are located in Table 5 and the

TABLE 5. Climate Attribution Model Results: Ontario

Crop	Location Measures						Volatility Measures	
	$\hat{\beta}_u$	$\hat{\beta}_l$	$\hat{b}$	$\hat{\delta}_u$	$\hat{\delta}_l$	$\hat{\bar{y}}$	$\hat{\beta}_u - \hat{\beta}_l$	$\hat{\sigma}^2$
Corn								
Intercept	1.238***	0.422	−0.003	1.548	0.743	0.999***	0.816**	7.324***
GDD	−0.366	0.348	0.008*	2.315	0.078	−0.162	−0.715	−3.667
HDD	0.316	1.232	0.014	31.997**	−3.125	3.436	−0.916	−21.065
VPD	0.750	−1.639	−0.019	−11.064	−3.367	0.014	2.390	11.051
VPD_{JA}	−0.246	0.596	0.005	10.254	13.993	1.224	−0.842	−0.133
PCP	0.228	0.182	0.000	−0.465	1.331	0.244	−0.155	−0.477
PCP_{JA}	−0.445	−0.729	0.000	5.462**	0.118	−0.256	0.284	6.503*
R^2	0.215	0.163	0.265	0.357*	0.112	0.271	0.166	0.254
Winter Wheat								
Intercept	0.834***†††	0.363***	−0.009***	n/a	n/a	3.905**	0.471***	1.229**
GDD	0.157	−0.025	−0.006***	n/a	n/a	0.406	0.182	0.013
HDD	0.186	0.027	−0.006	n/a	n/a	−0.168	0.159	0.084
VPD	1.264	−0.195	0.037	n/a	n/a	−0.561	−1.070	−0.848
PCP	0.032	−0.007	0.002*	n/a	n/a	0.695	−0.024	−0.194
R^2	0.088	0.153	0.189	n/a	n/a	0.117	0.052	0.115

Note: Statistical significance is indicated by *, ** and *** for the 10%, 5% and 1% levels under conventional standard errors. Statistical significance is indicated by †, †† and ††† for the 10%, 5% and 1% levels under jackknife standard errors. Statistical significance on the R^2 is a joint test of all parameters using a Wald test based on either the conventional or jackknife covariance matrix.

results for Saskatchewan are located in Table 6. Note, as with our earlier tests, we account for our estimated regressor by using jackknife methods.[12]

For Ontario corn, we find that only four of 48 climate estimates are significant at the 10% level (below the size of the test). When we properly account for the estimated regressor using jackknife standard errors (not commonly done in the literature), none are significant at the 10% level. It is worth noting that changing climate does explain more of the changes in the overall mean versus the conditional means. However, this is not true for the conditional variances, where the spatial variation for changes in the variance of the upper component of yields exhibits the second highest R^2 (0.357) of any of the 48 regressions. Interestingly, we only reject the null of no climate effects for the regression of $\hat{\delta}_u$ and under a conventional Wald test. The jackknife Wald test indicates none of the regressions are statistically significant. For Ontario winter wheat, we find that two of 24 climate estimates are significant at the 10% level, again below the size of the test.[13] We cannot reject the null of no climate effects for all six regressions under either a conventional or jackknife Wald test. Recall, the identification strategy in the attribution model is solely dependent on spatial heterogeneity

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[12]To account for the estimated regressor for individual t-tests, we recover the jackknife standard errors. However, for the joint significant test, it was necessary to construct the jackknife covariance matrix using Shao (1992).

[13]We do not include the PCP$_{JA}$ and VPD$_{JA}$ given the winter wheat growing season.

in the trends in yield and climate variables. Our corn and winter wheat results reflect a small geographical area consisting of 32 and 26 counties, respectively. In this sense, our results are not surprising and may be more indicative of the small sample size and relatively homogeneous area than a lack of a climate effect.

For Saskatchewan barley, we find that 15 of 36 climate estimates are significant at the 10% level using conventional standard errors and two using the robust but conservative jackknife standard errors. The greatest significance is in the overall mean regression, which is not surprising because it represents location effects and thus power (in our tests) is much greater. The significant variables are of the expected sign. That is, in the $\hat{\bar{y}}$ regression, HDD and VPD are negative while VPD_{JA} is positive. Interestingly, we reject the null of no climate effects for all location and volatility measures under the conventional Wald test but only the overall mean under the robust but conservative jackknife Wald test. For Saskatchewan canola, we find that 16 of the 36 climate estimates are significant using conventional standard errors but only one using jackknife standard errors. In the regression of $\hat{\beta}_u - \hat{\beta}_l$, we find HDD significant and positive. We would expect that spread in the components or volatility would increase as HDD goes up. Interestingly, we reject the null of no climate effects for all location and volatility measures using a Wald test but none using the robust jackknife covariance matrix. For Saskatchewan oats, we find that 22 of the 36 climate estimates are significant using conventional standard errors and four using jackknife standard errors. Of note is that climate is explaining the largest degree of spatial heterogeneity for the oat yield data generating process versus the other crops. Again, we find the greatest significance in the overall mean equation. As expected, we find VPD negative and significant, and VPD_{JA} and PCP_{JA} significant and positive. Again, using the conventional covariance matrix we reject the null of no climate effects for all location and volatility measures but only for the overall mean when we use the robust jackknife covariance matrix inside the Wald tests. Finally, for spring wheat we find that 26 of the 36 climate variables are significant but only one when we consider the jackknife standard errors. This result is surprising in that GDD has a negative and significant effect on the overall mean. However, given the number of tests considered (216 t-tests), we only find one statistically significant result of the wrong sign. Again, with the conventional Wald test we reject the null of no climate effects for all location and volatility measures but only for the overall mean when we use the robust jackknife Wald test.

Summarizing our climate attribution results, we find: (i) weak overall statistical significance given our small number of counties, relatively homogeneous geographical region, and sampling error in our estimated regressors; (ii) statistical significance is much smaller in Ontario than Saskatchewan; and (iii) climate tends to explain significantly more of the spatial heterogeneity in the technological change in the overall mean as compared to either of the conditional means or volatility parameters. Although lacking in statistical significance from low power, nonetheless these results are interesting and consistent with general expectations.

TABLE 6. Climate Attribution Model Results: Saskatchewan

Crop	Location Measures				Volatility Measures	
	$\hat{\beta}_u$	$\hat{\beta}_l$	$\hat{b}$	$\hat{\hat{y}}$	$\hat{\beta}_u - \hat{\beta}_l$	$\hat{\sigma}^2$
Barley						
(Intercept)	0.134**	0.472***††	−0.003***	0.434***†††	0.338***	1.114***
GDD	0.071**	0.033	0.001	0.019	−0.038	−0.196
HDD	−0.444	−1.357*	0.025**	−1.965***†	−0.913	−2.825
VPD	1.000***	−0.087	0.031***	−0.826***	−1.087**	−1.679
VPD_{JA}	−1.934***	1.021	−0.058***	2.097***††	2.955***	7.435*
PCP	0.048	0.039	−0.002	0.068*	0.087	0.014
PCP_{JA}	−0.031	0.129	−0.001	0.082	0.159	1.694***
R²	0.060*	0.101***	0.215***	0.224***††	0.099***	0.115***
Canola						
(Intercept)	0.072	0.520***†††	−0.007***	0.351***††	0.449***†	1.880***
GDD	0.063*	−0.023	0.002**	−0.008	−0.086**	−0.451***
HDD	−7.110***	0.720	−0.161***	−2.621***	7.826***†	31.803***
VPD	−0.249	−0.231	−0.013	−0.518**	0.017	1.387
VPD_{JA}	1.994**	0.833	0.059*	1.516***	−1.161	−6.824*
PCP	−0.032	−0.051	0.001	−0.032	−0.019	−0.061
PCP_{JA}	0.178	0.196*	0.003	0.131*	0.018	−0.512
R²	0.078*	0.192***	0.124***	0.076*	0.208***	0.252***
Oats						
(Intercept)	−0.169	0.713***††	−0.007***	0.635***†††	0.882***†††	3.899***
GDD	0.219***	−0.064	0.002***	−0.055	−0.283***	−1.494***
HDD	−5.333***	0.004	−0.051**	−1.071	5.338*	44.459***
VPD	0.691	−2.300***	0.024***	−2.077***††	−2.991***	−7.379
VPD_{JA}	−0.774	4.508***	−0.030*	3.663***††	5.282***	11.234
PCP	0.145	0.003	0.004***	−0.005	−0.143	−0.652
PCP_{JA}	−0.118	0.832***†	−0.007***	0.530***††	0.950***	4.931***
R²	0.228***	0.302***	0.295***	0.382 ***†††	0.370***	0.330***
Spring Wheat						
(Intercept)	0.426***	−0.019	−0.008***	0.417***†††	0.444***	0.934***
GDD	−0.084***	0.063***	0.003***	−0.082***†††	−0.147***	−0.334***
HDD	−0.640	−0.088	−0.073***	0.541*	−0.552	−1.472
VPD	−0.290	1.071***	0.022***	−0.403***	−1.362***	−3.773***
VPD_{JA}	0.893*	−2.173***	−0.021*	0.494*	3.066***	8.644***
PCP	0.090 **	0.078*	0.001	0.052**	0.012	0.073
PCP_{JA}	−0.070	−0.272***	0.000	−0.129***	0.202*	0.643**
R²	0.112***	0.156***	0.332***	0.290***†††	0.176***	0.215***

Note: Statistical significance is indicated by *, ** and *** for the 10%, 5% and 1% levels under conventional standard errors. Statistical significance is indicated by †, †† and ††† for the 10%, 5% and 1% levels under jackknife standard errors. Statistical significance on the R^2 is a joint test of all parameters using a Wald test based on either the conventional or jackknife covariance matrix.

Moreover, they provide a more complete analysis of the impacts of changing climate on yields by considering the effects at and beyond the overall mean.

7. Economic Implications for Crop Insurance

Canadian BRM programs are the main avenue by which public monies are funnelled to agricultural producers. The current policy framework -- Canadian Agricultural Partnership (CAP) -- is a five-year commitment (2018 to 2023) jointly funded ($3 billion total) by federal and provincial governments. Agricultural support programs were first offered in the 1958 Agricultural Stabilization Act. A thorough history of the Canadian agricultural support programs can be found in Barichello (1999). The current BRM program consists of a suite of products: AgriInvest, AgriStability, AgriRecovery, and AgriInsurance. AgriInvest allows producers to deposit funds and receive matching dollars up to a maximum amount. AgriStability is a net profit farm-level insurance program that, despite requiring only payment for sign-up, has been much maligned because of the transaction costs and low coverage rates and has seen continuous and significant decline in participation levels over the last seven years. AgriRecovery is a catastrophic loss program at the region level and requires the province to make an application to trigger payments. AgriInsurance is producer level, commodity specific, yield insurance. Producers can purchase insurance up to the 90% coverage level in Ontario but only up to the 80% coverage level in Saskatchewan. All programs are designed, to varying degrees, to assist producers with low yield outcomes.

We consider the implications of technological change on producer premium rates for yield insurance contracts in Canada. Of particular note is that the producer paid premium rate is roughly 40% of the actuarially rate, with the federal and provincial governments covering the remaining 60% at a 60-40 split, respectively. Moreover, the federal and provincial governments subsidize the entire administrative and operating costs of the program. Therefore, if producer premiums are expected to increase due to technological change, the public monies that support those programs may (and likely will) correspondingly increase. To that end, we compare the premium rates in 2018 with the premium rates in 2023 and 2028 under our estimated mixture models.

The actuarially fair premium rate for a yield (y) insurance contract at a yield guarantee of y_τ is:

$$(6) \qquad \pi = \int_0^{y_\tau} (y_\tau - y) dF_y$$

where F_y is the distribution of y. Premium rates are defined in bushels per acre. Figure 3 shows box plots of the ratio of the premium rates between 2018 and 2023 at both the 70% and 90% coverage levels and by crop. Note, 2023 represents the end of CAP. The on-line appendix shows the same ratio of premium rates but between 2018 and 2028. The box plots are constructed from the county premium rate ratios. For example,
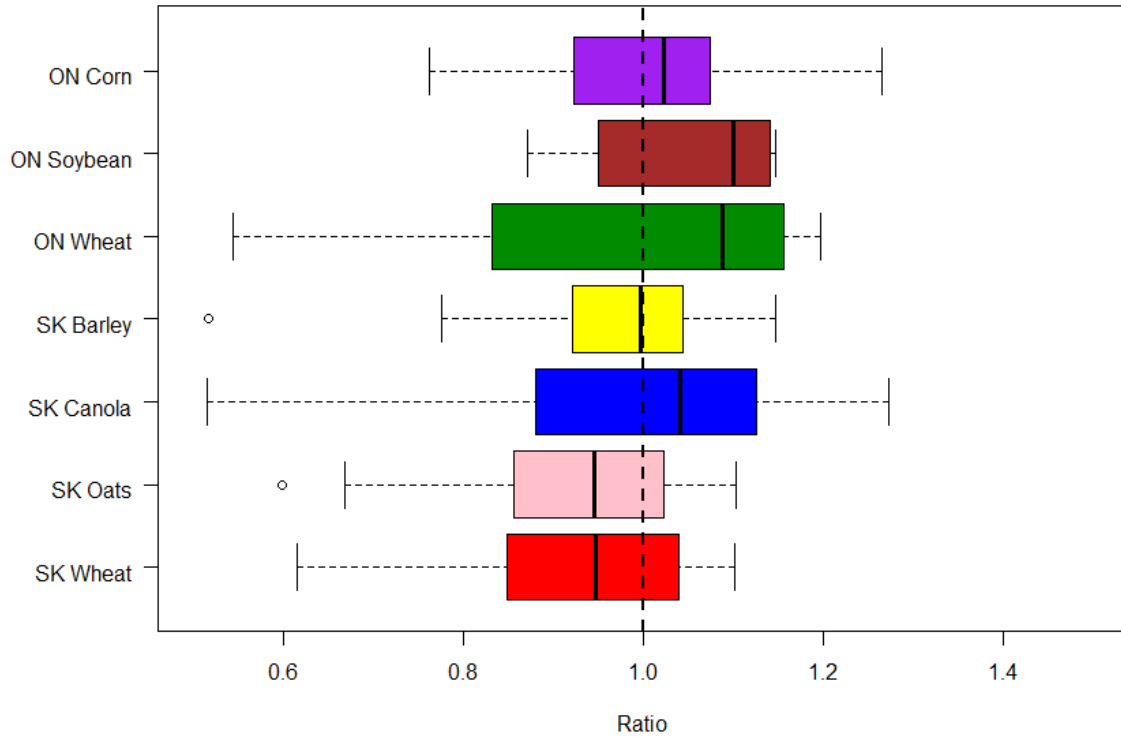
(A) 70% Coverage Level
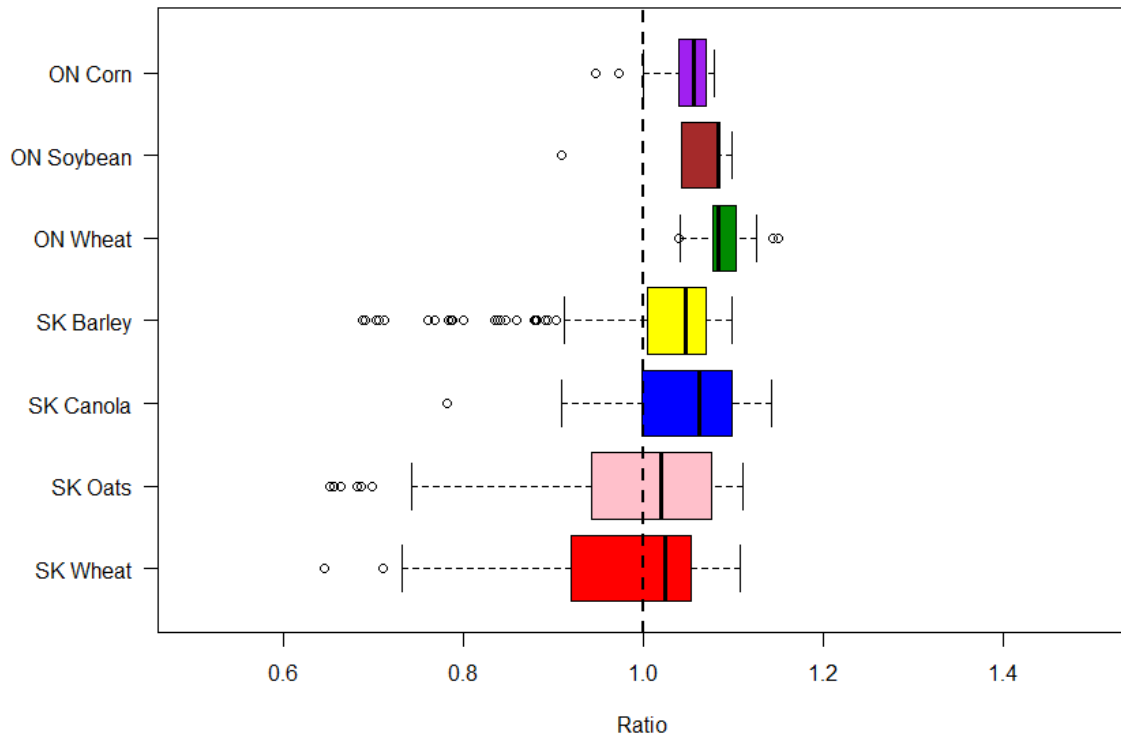
(B) 90% Coverage Level

FIGURE 3. Ratio of Expected Yield Loss: π_{2023}/π_{2018}

the box plot for Ontario corn consists of the 32 ratios of the premium rates corresponding to the 32 counties. It is important to understand the premium rate calculations. First, if mean yield increases and variance is constant, the guarantees would increase and premium rates would decrease. Second, if mean yield is constant and variance increases, guarantees would remain constant but premium rates would increase. Third, if both mean yield and variance increase, guarantees would increase but premium rates may increase or decrease and, moreover, premium rates could increase at one coverage level and decrease at another. That is, mapping changes in the yield distribution to changes in premium rates is both non-linear and intuitively non-trivial. The median of the ratio of premium rate changes between 2018 and 2023, as well as between 2018 and 2028, are located by crop and coverage level in Table 7.

Interestingly, despite widespread volatility increases in the estimated yield distributions, premium rates do not correspondingly increase everywhere. At the 70% coverage level, the majority of premium rates are expected to increase for corn, soybean, winter wheat, and canola, and decrease for oats and spring wheat. Conversely, at the 90% coverage level, the majority of premium rates are expected to increase for all crops, particularly corn, soybean, winter wheat, barley, and canola. Plots of the county-crop premium rate ratios by coverage level are located in the appendix. At the 70% coverage level, we do not see any significant spatial concentrations for Ontario corn. Conversely, at the 90% coverage level we do see spatial bifurcation: the southwestern counties see much greater increases in premium rates than the eastern counties. With respect to Ontario winter wheat, we see an interesting spatial pattern in that the increasing premium rates at the 70% coverage level are spatially correlated with decreases at the 90% coverage level. With respect to Saskatchewan barley, canola, and oats, we see a strong spatial bifurcation in the premium rate changes. The more northern counties experience greater premium rates at both coverage levels relative to the southern counties. Finally, for Saskatchewan spring wheat we see greater premium increases in the northwest counties

TABLE 7. Median Ratio of Premium Rates

Province	π_{2023}/π_{2018}		π_{2028}/π_{2018}	
Crop	70% Coverage	90% Coverage	70% Coverage	90% Coverage
Ontario				
Corn	1.021	1.055	1.037	1.110
Soybean	1.099	1.083	1.205	1.166
Winter Wheat	1.088	1.082	1.176	1.165
Saskatchewan				
Barley	0.996	1.046	0.996	1.092
Canola	1.040	1.062	1.082	1.127
Oats	0.945	1.019	0.893	1.038
Spring Wheat	0.947	1.024	0.899	1.048

versus the southeast counties. Perhaps surprisingly, our results do not suggest widespread and significant increases in BRM spending over the next 5 to 10 years. These results are somewhat contradictory to Tack, Coble, and Barnett (2018) but we would expect different results for different crop-region combinations. Moreover, as noted by Coble, Dismukes, and Thomas (2007); Cooper et al. (2009); Claassen and Just (2011), tail and volatility effects using county-level data are muted relative to on-farm effects.

8. Conclusions

The objective of this manuscript was to estimate technological change in Canadian crop yields. The study of technological change is critical for meeting future food demand, enhancing yield resiliency, mitigating any negative externalities from a changing climate, and remaining internationally competitive. While there exists significant literature modelling technological change in U.S. yields, there is no corresponding literature in regards to Canadian crop yields. We used county-level yield data to estimate the structure of technological change in barley, canola, corn, oats, soybean, and wheat in Canada using mixture models. We contribute to the vast literature in modelling yields by generalizing the mixture model of Tolhurst and Ker (2015) to allow for time-varying component variances. We conducted tests of homogeneous rates of technological change, various yield resiliency measures, and yield volatility. While the results are very interesting in and of themselves, we do contribute to the yield resiliency literature by introducing three new measures of yield resiliency. We considered two interesting extensions that also fill gaps in the literature. First, we estimated the effect of a changing climate on various location and volatility technological change measures recovered from our mixture models (not just the mean). Second, we predicted conditional yield distributions to ascertain the effects of technological change on future premium rates and BRM spending in Canada.

Our empirical analyses and hypothesis tests led to a number of interesting findings: (i) rates of technological change exceed the rate of population growth; (ii) differing rates of technological change; (iii) increasing volatility and decreasing *relative* yield resiliency; (iv) asymmetric volatility changes; and (v) increasing *absolute* yield resiliency. We did not find strong statistical evidence of increasing *frequency-based* yield resiliency. Perhaps one of the more interesting findings was that most of our results showed notable spatial bifurcations within the provinces. With respect to our climate results, we found weak statistical evidence of any climate effects, but those that we did find to be significant were concentrated in our location rather than volatility measures. While our results are not as statistically significant as some, we do properly account for the estimated regressor issue in our statistical tests. With respect to our BRM results, we found that, despite widespread volatility increases in the estimated yield distributions, premium rates do not correspondingly increase everywhere.

Our analyses do suffer from a lack of availability of yield data. Individual farm-level data over the same period covering the same geographical areas (or more) would have been ideal. At a minimum, greater spatial coverage of county-level yield data would have added measurably to our empirical analyses. First, yield data from British Columbia, Alberta, Manitoba, and Quebec would have allowed us to cover more crops and a greater share of crop production in Canada. Second, we would be better able to identify areas of spatial homogeneity and heterogeneity beyond provincial borders. Third, we would have significantly more power in our changing climate regressions.

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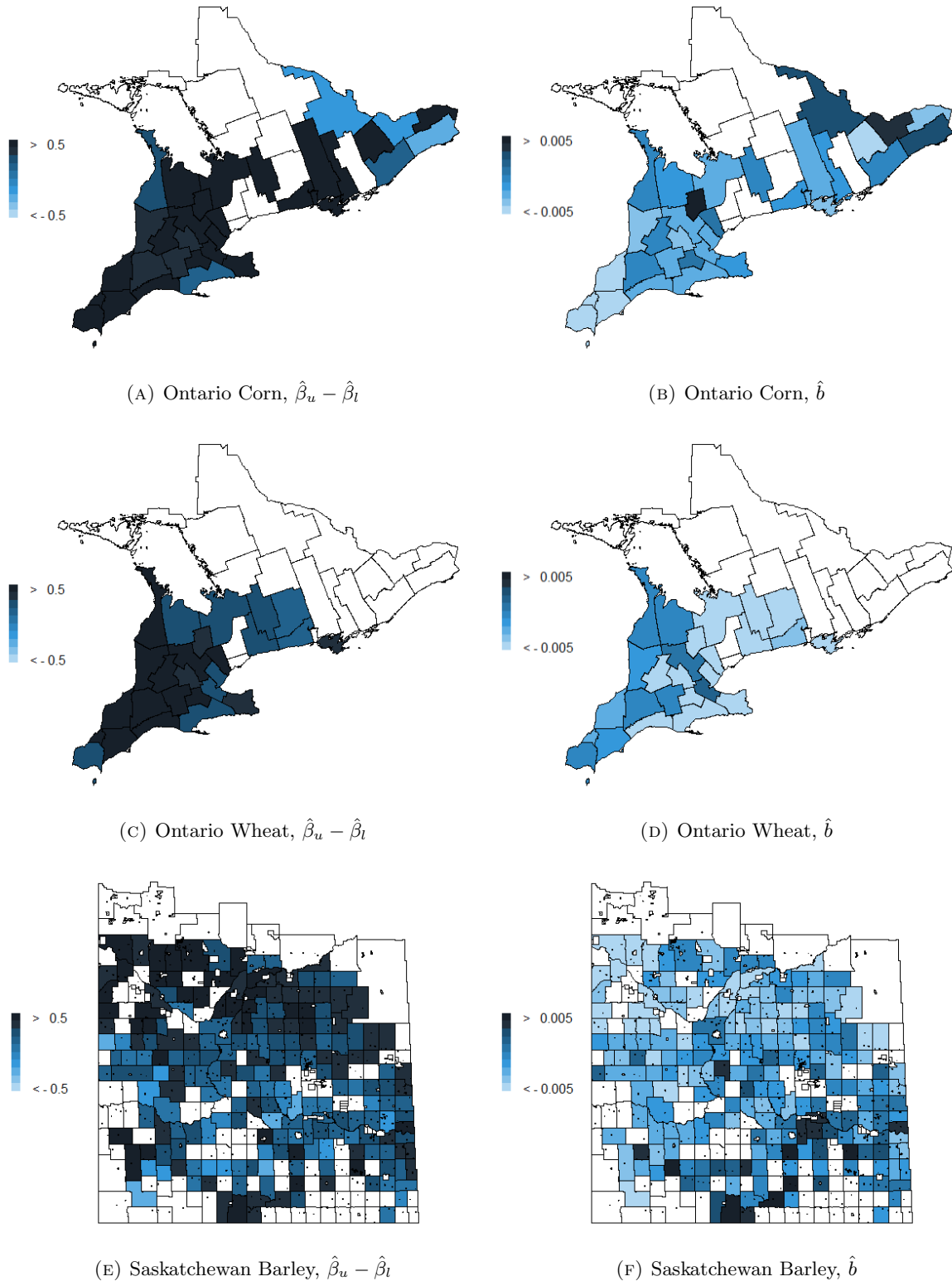
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Appendix

(A) Ontario Corn, $\hat{\beta}_u - \hat{\beta}_l$

(B) Ontario Corn, $\hat{b}$

(C) Ontario Wheat, $\hat{\beta}_u - \hat{\beta}_l$

(D) Ontario Wheat, $\hat{b}$

(E) Saskatchewan Barley, $\hat{\beta}_u - \hat{\beta}_l$

(F) Saskatchewan Barley, $\hat{b}$

FIGURE 4. Spatial Distribution of $\hat{\beta}_u - \hat{\beta}_l$ and $\hat{b}$, Ontario and Saskatchewan
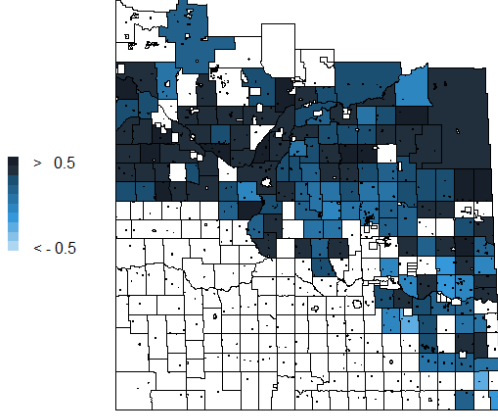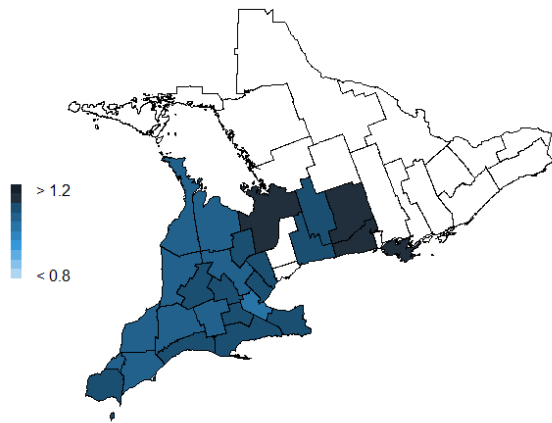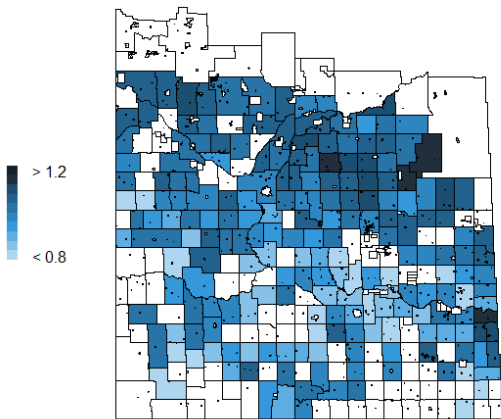
(G) Saskatchewan Canola, $\hat{\beta}_u - \hat{\beta}_l$
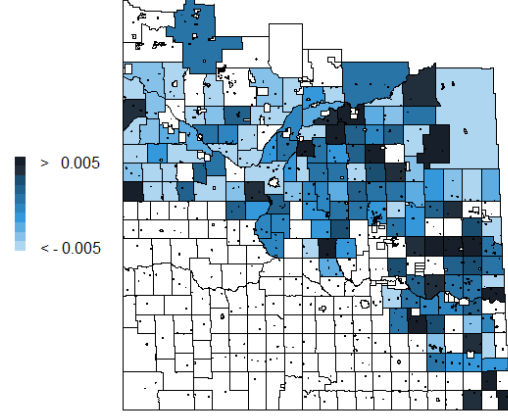
(H) Saskatchewan Canola, $\hat{b}$
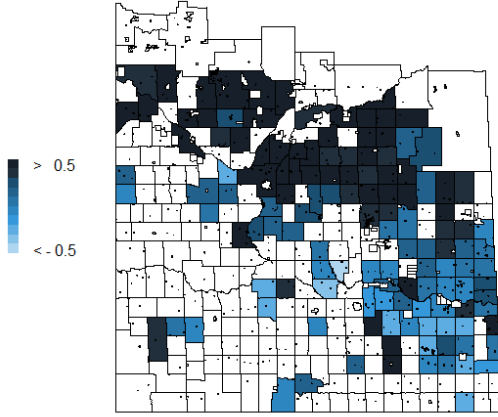
(I) Saskatchewan Oats, $\hat{\beta}_u - \hat{\beta}_l$
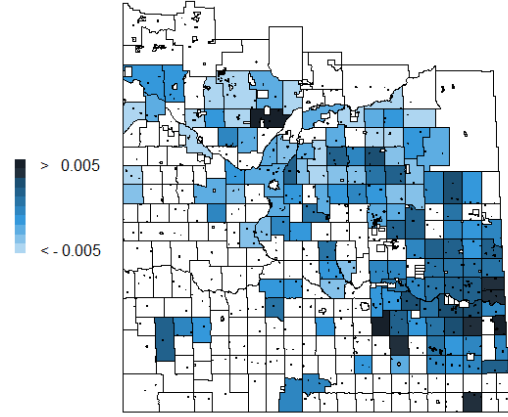
(J) Saskatchewan Oats, $\hat{b}$
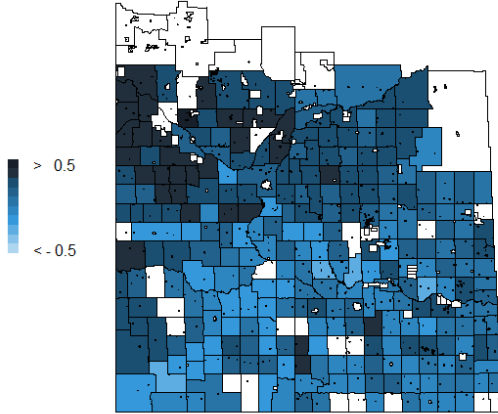
(K) Saskatchewan Wheat, $\hat{\beta}_u - \hat{\beta}_l$
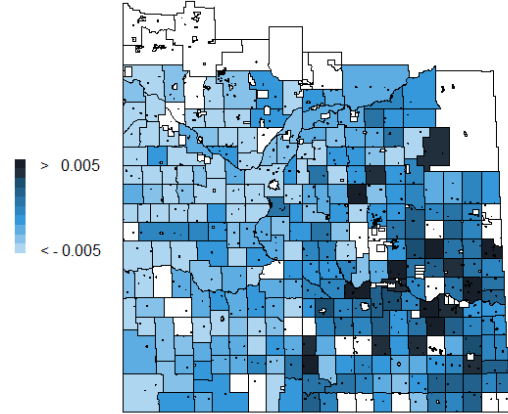
(L) Saskatchewan Wheat, $\hat{b}$

FIGURE 4. Spatial Distribution of $\hat{\beta}_u - \hat{\beta}_l$ and $\hat{b}$, Ontario and Saskatchewan
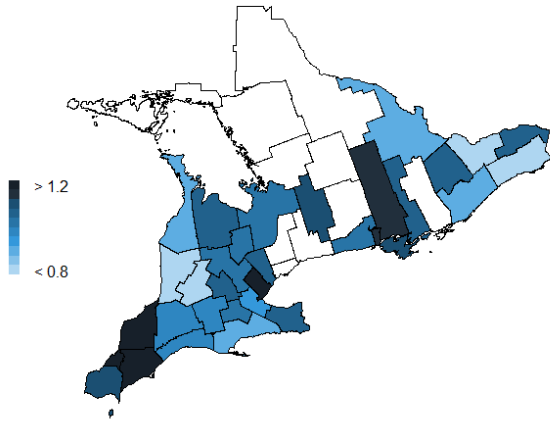
(A) Ontario Corn, 70% Coverage Level
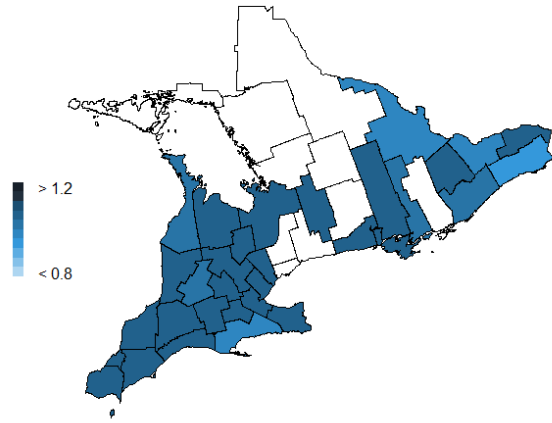
(B) Ontario Corn, 90% Coverage Level
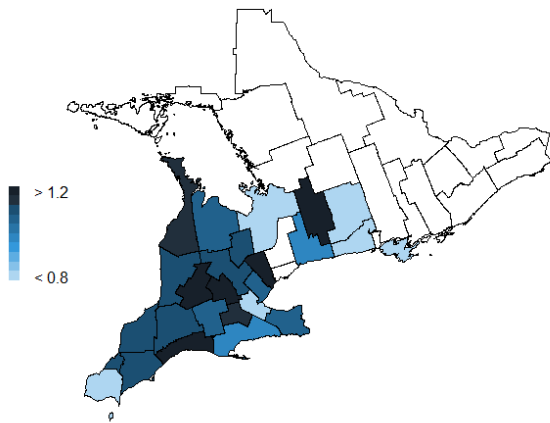
(C) Ontario Wheat, 70% Coverage Level

(D) Ontario Wheat, 90% Coverage Level

(E) Saskatchewan Barley, 70% Coverage Level
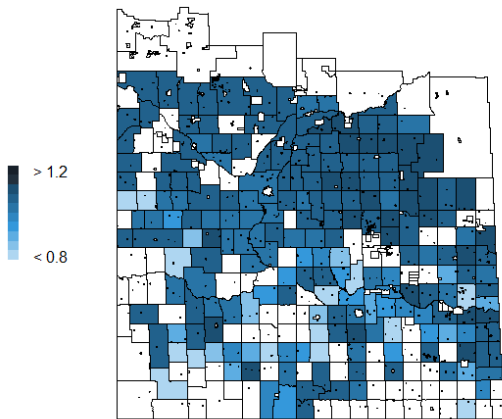
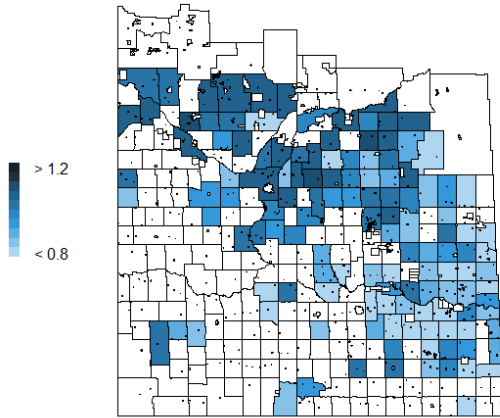(F) Saskatchewan Barley, 90% Coverage Level

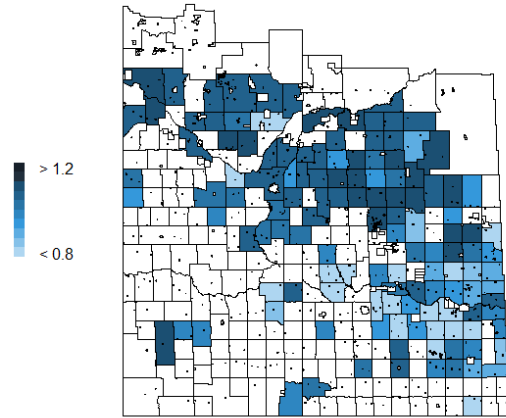FIGURE 5. Spatial Distribution of π_{2023}/π_{2018}, Ontario and Saskatchewan
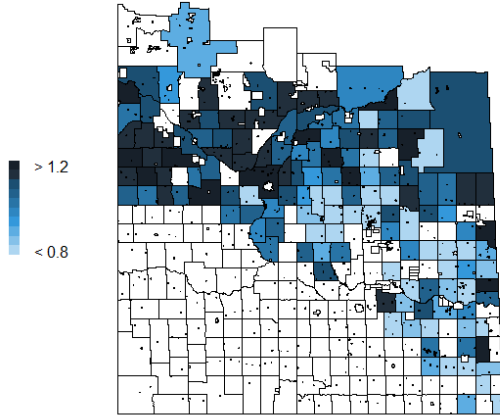
(G) Saskatchewan Canola, 70% Coverage Level
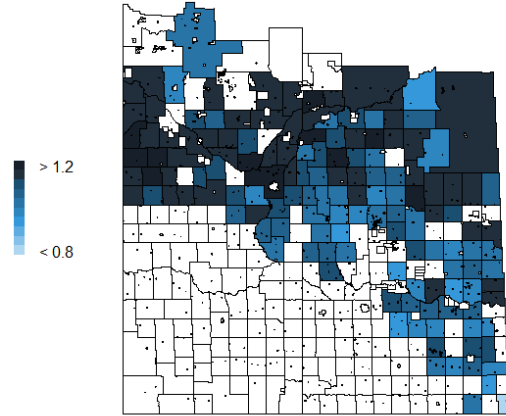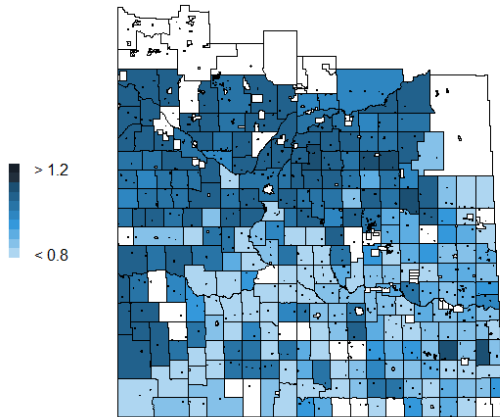

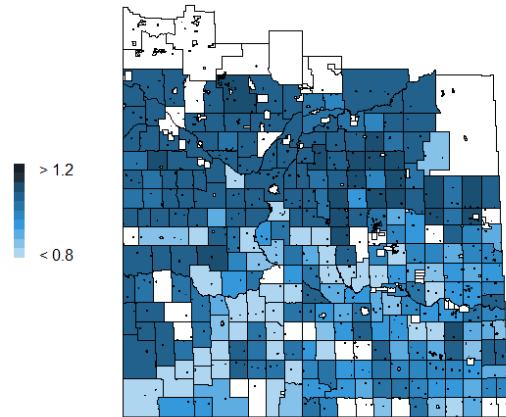(H) Saskatchewan Canola, 90% Coverage Level

(I) Saskatchewan Oats, 70% Coverage Level

(J) Saskatchewan Oats, 90% Coverage Level

(K) Saskatchewan Wheat, 70% Coverage Level

(L) Saskatchewan Wheat, 90% Coverage Level

FIGURE 5. Spatial Distribution of π_{2023}/π_{2018}, Ontario and Saskatchewan