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IMPLICATIONS OF VOLATILITY MODELING ON QUANTIFYING MARKET RISK

Abstract

Due to fluctuations in financial assets, market risk represents the most prevalent risk in the category of financial risks. The process of market risk management includes its quantification and control. Measure that quantifies the maximum potential loss in a given period of time with a certain statistical confidence level is the value at risk, VaR. Treating financial assets prices as a time series that could be described as a random walk with drift or returns of financial assets as a white noise typically underestimates the value at risk. Back testing shows that the estimation of the risk with variance modeled as a generalized conditional autogressive heteroscedastic (GARCH) model is a reliable method for a quantification of risk.

Key words: *value at risk, constant volatility, variable volatility*

JEL Classifications: G11, G17, G32

ИМПЛИКАЦИЈЕ НАЧИНА МОДЕЛОВАЊА ВОЛАТИЛИТЕТА НА КВАНТИФИКОВАЊЕ ТРЖИШНОГ РИЗИКА

Апстракт

Тржишни ризик који настаје као последица колебања цена финансијске активе представља најраспростањенији ризик у категорији финансијских ризика. Процес управљања тржишним ризиком подразумева његову квантификацију и контролу. Мера којом се квантификује највећи потенцијални губитак у датом временском периоду са одређеним статистичком нивоом поверења представља вредност под ризиком, ВаР («value at risk»). Третирање цена финансијске активе као временске серије која се може описати као случајни ход са прирастом („random walk with drift“) односно приноса финансијске активе као бели шум по правилу потцењује вредност под ризиком. «Back testing»-ом се показује да процена вредности под ризиком са варијабилитетом моделованим као генерализовани ауторегресиони условни хетероскедастични (GARCH) модел представља поуздану методу за квантификацију ризика.

Кључне речи: *вредност под ризиком, стални волатилитет, променљиви волатилитет*

Introduction

Expected return and its variability are the key issues in the process of risk control and it’s valuation. The assumption of stationarity of time series of financial assets’ return, which in practice often is not fulfilled leads to underestimation of the level of market risk.

Its calculation perform a significant number of professional investors (banks calculate interest rate VaR etc)[1]

Financial assets variability

Starting from the common assumption that the prices of financial assets have stochastic character which can be described as a random walk with drift:

$$x_t = \mu + x_{t-1} + \varepsilon_t \text{ , } E(\varepsilon_t) = 0 \text{ , } E(\varepsilon_t^2) = \sigma^2 \text{ , } E(\varepsilon_t \varepsilon_s) = 0 \quad \text{...........................(1)}$$

return can be calculatet as:

$$y_t = x_t - x_{t-1} = \mu + \varepsilon_t \text{ , } E(\varepsilon_t) = 0 \text{ ; } E(\varepsilon_t^2) = \sigma^2 \text{ , } E(\varepsilon_t \varepsilon_s) = 0 \text{ za } t \neq s \quad \text{............ (2)}$$

If we assume that the return on financial assets is calculated continuously, which is equation (2) has the form:

$$y_t = \ln(S_{t+1}) - \ln(S_t) = \mu + \varepsilon_t \text{ , } E(\varepsilon_t) = 0 \text{ , } E(\varepsilon_t \varepsilon_s) = 0 \text{ , } E(\varepsilon_t \varepsilon_s) = 0 \text{ , } t \neq s \quad \text{.............(3)}$$

where $r_{t,t-1}$ denotes return and S_{t+1}, S_t are prices.

Then, if the assumptions of model are met: $E(\varepsilon_t) = 0$, $E(\varepsilon_t^2) = \sigma^2$ and $E(\varepsilon_t \varepsilon_s) = 0$ for $t \neq s$ (which means that returns are independently and identically distributed with mean μ and variancy σ^2) then applies:

$$E(\ln(S_{t+1}) = t * \mu \quad VAR(\ln(S_{t+1}) = t * \sigma^2 .$$

This relationship stems from the fact that the linear combination of variables with normal distribution (which is a common assumption about the distribution of return) is also normally distributed. If the variable does not come from a normal distribution, based on the central limit theorem their mean has an approximate normal distribution.

Assumptions of random walk model are in practice often not met. Time series are characterized by the existence of correlation in residuals, with the variance that is variable in time (heteroscedasticity) and symmetrical distribution with thick tails. Periods of high volatility are followed by periods of lower volatility and vice versa. Autoregressive model can be evaluated using least squares if the time series is covariance stationary, and residuals are not correlated (Weiseberg, 2005). If the autocorrelation of the residuals is significantly different from zero, model is not well specified.

As it has been previously mentioned, heteroscedasticity indicates residuals variance dependence on independent variables (Nelson, 1991, Bollerslev 1996). In contrast, homoscedacity presents random error variance independence of the independent variables. In order to produce reliable models it is important to detect heteroskedasticity in time series. Robert Engle gave a solution to test the interdependence of the variance in one period of the variance in the previous period (Christoffersen, 2000). This type is called autoregressive conditional heteroskedastity (ARCH).

ARCH (1) model has the form:

$$\varepsilon_t \sim N(0, a_0 + a_1 \varepsilon_{t-1}^2) \quad \text{...,,,,,..................(4)}$$

[1] About bank interest rate risk see.Radević, Lekpek (2010)

with distribution ε_t, conditional on its value in the past ε_{t-1}, has a normal distribution with a mean 0 and variance a_0. If a = 0, the variance of the random errors in each period is a_0. If $a_1 > 0$ the variance in the observed period depends on how much was the variance in the previous period. If the variance is significant in one period, then over the next period, will be even more significant.

For example consider the correlation structure of daily returns of the company & Goodyear Tire (GT) in the period from August 2008 to July 2009 (Fig. no.1).

Fig.1 Autocorrealation of residuals, Goodyear & Tire

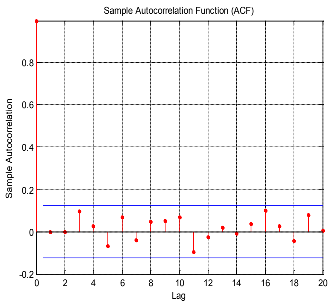

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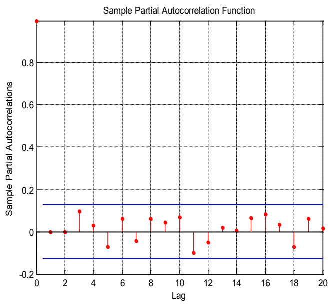
Fig.2 Partitial autocorrelation of residuals, Goodyear & Tire



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The graph shows the correlation structure of 1,2, ... 20 lags, with a confidence interval of 95%, which is shown by horizontal lines. Correlations within this interval are not considered statistically significant. The autocorrelation graph did not show any characteristic pattern. As in Fig. no. 2 it can be seen that even partial autocorrelation graph is also not typical. Assumptions of the model described by equation (2) are satisfied from the existence of serial correlation of residuals. Correlogram of squared returns given in Fig. no. 3 indicates that, although the yields are not necessarily correlated, their variance is characterized by the existence of serial autocorrelation.

For quantification of serial correlation of residuals, it can be used formal tests that are based on hypothesis testing, such as Ljung-Box-Pierce Q-test and Engle's ARCH test.

Ljung-Box-Pierce analysis of independence of random variables is based on Q-statistics $Q = N(N+2)\sum_{k=1}^{L}\frac{r_k^2}{(N-k)}$ ....(5)

where N is the sample size, L number of lags for which autocorrelation is calculated, while r_k^2 is the serial correlation at lags k. Q-statistic has a χ^2 distribution.

The null hypothesis of no serial correlation at lags k is rejected if the test statistics with confidence interval $(1-\alpha)$, for the L-lag is greater than the critical value, ie:

$$Q > \chi^2_{1-\alpha,L}$$

In this example the obtained result is statistically significant, which confirms the presence of serial correlation in residuals.

Fig. 3 Autocorrealation, Goodyear & Tire-squared returns

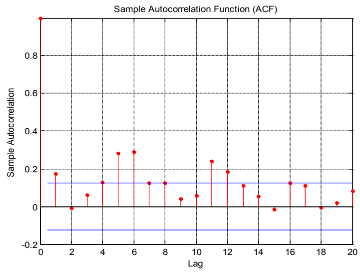

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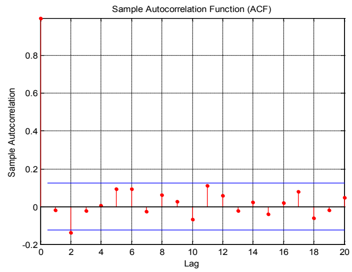
Fig.4 Autocorrealation, Goodyear & Tire-squared standardized residuals



author's calculation

A formal test for autoregressive conditional heteroskedasticity was also statistically significant and indicates that the observed series is characterized by existence of ARCH effects.

The basic model for a random walk with drift must therefore be adjusted in order to fulfill basic assumptions of the model. In practice for modeling the time variable, conditional variance is most commonly used in simple GARCH (1,1) model described by the equations:

$$y_t = \mu + \varepsilon_t \quad \text{(6a)}$$

$$\sigma_t^2 = k + G_1\sigma_{t-1}^2 + A_1\varepsilon_{t-1}^2 \quad \text{(6b)}$$

The general form of the GARCH (P, Q) model is:

$$\sigma_t^2 = k + \sum_{i=1}^{P} G_i\sigma_{t-i}^2 + \sum_{j=1}^{Q} A_j\varepsilon_{t-j}^2, \quad \text{(6c)}$$

where σ_{t-i}^2 is the estimated value of the variance in the next period expressed as a linear combination of the estimated number of variances from previous periods σ_{t-i}^2 and realized errors ε_{t-j}^2.

Residuals with stochastic character can be presented in the form of:

$$\varepsilon_t = \sigma_t z_t, \quad \text{(7)}$$

where σ_t^2 is the estimated value of the variance in the next period expressed as a linear combination of the estimated number of variances from previous periods σ_{t-i}^2 and realized errors ε_{t-j}^2. Thus standardized residuals are independent and equally distributed (Barone-Ades, 1997).

Estimation of parameters in regression equations (6a) (6b) gives the model that best describes the data sample:

$$y_t = 0.00032133 + \varepsilon_t \text{......} (8)$$

$$\sigma_t^2 = 0.00014172 + 0.8788\sigma_{t-1}^2 + 0.089556\varepsilon_{t-1}^2 \text{......} (11)$$

Correlogram of squares of standardized residuals indicates that the variances are not correlated with each other.

Evaluation of value at risk by Monte Carlo Simulation with constant and variable volatility

In order to determine the maximum loss over a period of specific period at given statistical level of confidence $(1-\alpha)$ by Monte Carlo simulations with assumed constant volatility, from assumptive distribution (usually standardized multivariate normal distribution) are generating a large number of equally probable outcomes with probability in the interval from 0 to 1 that represents realization of random variable X.

For each realization of random variable X, marked as X_m.

Vector of portfolio returns are calculated as:

- $R_m = f_m(X) = KX_m + \mu$, where:

-m indicates the m-th simulation;

-k is the Cholesky decomposition of the matrix Σ ;

- X_m is the return vector of the m-th simulation;

- X_m is the m-th realization of the random variable X from the standardized multivariate normal distribution;

- μ is a vector of the expected value of random variable X.

The values for mean μ and variance σ are determined based on a sample of historical data for a specific period..

Daily VaR for the required level of confidence (1 -) is determined as the percentile of the set which consists of a large number of simulations. Value at risk for an investment horizon T is estimated from equation:

$$VaR_{\alpha,T} = VaR_\alpha * \sqrt{T} \cdot$$

Note again that this method of calculating the value at risk is made with the assumption that the variance and covariance remain constant value over time (Hamzagić, 2010).

In contrast, GARCH models give more realistic predictions of variance (Baillie, 1992). In order to determine the variance in the number of time periods in this paper, will be used GARCH (1,1) model.

After determining total standard deviation for all securities in the portfolio value of the variance covariance matrix, with the approximation, which involves a constant correlation (not observed for the overall period, but at a level of observation), among the securities in the portfolio is obtained as the product matrix:

$$COV = \begin{bmatrix} \sigma_1 & . & . & 0 \\ . & \sigma_2 & . & . \\ . & . & . & . \\ 0 & . & . & \sigma_n \end{bmatrix} kor \begin{bmatrix} \sigma_1 & . & . & 0 \\ . & \sigma_2 & . & . \\ . & . & . & . \\ 0 & . & . & \sigma_n \end{bmatrix}.$$

In the diagonal matrix, are present total standard deviations (i.e. the total regression error), while kor marks correlation matrix of returns.

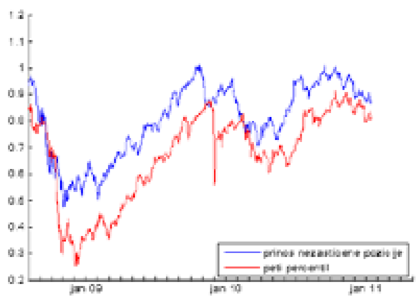
The only difference in estimating value at risk by Monte Carlo simulation with assumed constant and variable volatility is in tha way of calculating variance-covariance matrix. It should be noted that this method predicts the value at risk over the next T periods, because in the variance covariance matrix are given overall standard deviation, as opposed to the Monte Carlo simulation with assumed constant volatility when the value at risk for one is determined first and then multiplied with $\sqrt{T}$.

Back testing of evaluation of value at risk by Monte Carlo simulation (consatnt and variable volatility)

Reliability assessment of a certain level of risk is done by comparing the estimated VaR using Monte Carlo simulations (constant volatility) and Monte Carlo simulations (variable volatility) for a given period and return achieved in the same period.

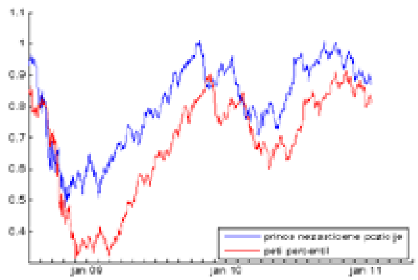
Taking into consideration freely selected portfolio weighted by market capitalization from different industries and with the following structure: Industrial Metals & Minerals: Alpha Natural Resources (ANR), Independent Oil & Gas: Petrohawk Energy Corporation (HK), Biotechnology: Celgene Corporation (CELG), Gilead Sciences (Gilda), Dairy Products: Dean Foods (DF), Plastics & Rubber: Goodyear Tire & Rubber (GT), Networking & Communication Devices: Juniper Networks (JNPR), Internet Information Providers: Google (GOOG), Semiconductor: Trina Solar (TSL) Financial: Nasdaq OMX Group (NDAQ).

Fig. 5 Monte Carlo simulation (variable volatility)



author's calculation

Fig. 6 Monte Carlo simulation (constant volatility)



author's calculation

Monte Carlo simulations (variable volatility) was performed for the selected portfolio in the period from 8/8 /2008 to 6/10/2011. There where 716 observations and 31 times realized returns were lower than those introduced by the 95% VaR. That is 4.3% of all observations and it can be considered as a satisfactory result (Fig. no. 5).

By contrast, method Monte Carlo simulation (constant volatility) gave unreliable results (Fig. no. 6). For the same set of data (previously designated portfolio and in the same period) realized returns were lower than those introduced by the 95% VaR in 56 cases that is 7.8 % (of 716 observations) and it comes from the corridors of 5%.

Concluding remarks

The issue of variability is essential in many applications related to investment decisions and risk management. Estimation of value at risk represents a legal obligation for a significant number of professional investors. Adopting the assumption that financial asset prices follow a random walk with drift path, and that return is a white noise, with no confirmation of conditions that every autoregresive model must satisfy in order to be well-specified, leads to unreliable results and underestimating the value at risk.

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