Implications of Land Market Imperfections on Policy Design

Dan Tavrov, Oleg Nivievskyi
Kyiv School of Economics, Ukraine

Paper prepared for presentation at the 165. EAAE Seminar
'Agricultural Land Markets –
Recent Developments, Efficiency and Regulation'

Humboldt-Universität zu Berlin, Germany, April 4-5, 2019

Implications of Land Market Imperfections on Policy Design

Dan Tavrov and Oleg Nivievskyi

Abstract
Land markets all over the world are diversely regulated, although a vast stock of empirical literature seems to suggest that unrestricted land market is the best policy design option. Since diversity of regulations proves this unlikely, it is surprising that little attention is paid in academic literature to theory that would allow to choose land market design based on welfare implications of various restrictions.

In this paper, we build upon the framework described in the literature and develop a theoretical model that enables to infer an optimal choice of (maximum land holdings) restrictions in the presence of land market imperfections.

Keywords: land market imperfections, partial equilibrium, rental land market
JEL Code: Q15

1 Introduction

Agricultural land markets all over the world are quite regulated (Deininger and Feder, 2001) and the burden of regulations is quite diverse. For example, there is an enormous heterogeneity of land markets and regulations in the EU, ranging from heavily regulated markets in France and Hungary, to the markets in the UK, Greece and Ireland with very little regulations (Swinnen et al., 2016). Unfortunately, there is no systematic comparative review and quantification of land markets and their regulations for other parts of the world, and, moreover, there is no any kind of a global ranking of land market regulations whatsoever.

Usually, policy makers impose various sets of regulations to address undesirable consequences associated with land markets, and these consequences are driven by market imperfections and historical developments. In Western European countries, a land regulatory environment ended up biased towards a more protection of small tenants' rights in the power struggle with powerful landlords. In Eastern European countries (former Soviet-bloc countries), the power struggle is reversed. Dismantling of the centrally planned economy and privatization of state land (either through restitution or redistribution to rural population) resulted in environments where large farms rent thousands of small land plots from families (Swinnen et al., 2016). For example, in Ukraine, agricultural enterprises rent in total about 20 mln ha of agricultural land from about 6.9 million people (Deininger et al., 2017a).

Typically, land market regulations could be grouped into two broad categories, namely land holdings/ownership restrictions and transferability restrictions (Deininger and Feder, 2001). Land holdings/ownership restrictions usually include maximum ceilings to prevent excessive concentration of land, or minimum ceilings to prevent land fragmentation. Transferability restrictions restrict either transaction or access to land, e.g. pre-emptive rights, restriction for foreigners, lease contracts duration mandates, price regulations etc. (Ciaian et al., 2012a; Ciaian et al., 2012b; Nivievskyi et al., 2016). The extreme case is the moratorium on land sales in Ukraine, where agricultural land is not allowed to be traded and used as a collateral (Deininger et al., 2017b).

A vast stock of empirical literature provides quite unanimous view on the outcomes of the above mentioned restrictions. Deininger (2003) summarizes that the farmland market restrictions have rarely achieved their desired impacts. There were many cases where restrictions on land sales markets seemed justified, but enforcement difficulties generated distortions that only worsened the situation. Governments' measures to improve land markets outcomes all over the globe mainly increased transaction costs for participants or drove land transactions to informal sector, reducing the welfare of all participants (Deininger, 2003; Ciaian et al., 2012a; Ciaian et al., 2012b).

From the policy making point of view, the above mentioned stock of literature seems to be suggesting that a land market with no restrictions would be the best policy design option. However, political economy of the issue at stake renders this outcome unlikely; a diversity of land markets and rigidity of regulations (Swinnen et al., 2016) is a good evidence for that. So policy makers will always be confronted with a need to make an ex-ante choice on relevant land market restrictions and their stringency. Quite surprisingly, however, little attention has been paid in academic literature to the theoretical framework that would allow policy makers to strike an acceptable "land market design" based on welfare implications of various restrictions. This has very important policy implications. For example, there is ongoing debate in Ukraine on the design of its agricultural land sales market (Deininger et al., 2017a; Deininger et al., 2017b). Ukraine imposed a ban on agricultural land sales as a temporary measure back in 2001 and was not able to lift the moratorium since then. One of the restrictions being discussed is the maximum amount of agricultural land that private individuals and legal entities are allowed to own. The cutoff points discussed so far are 200 ha for private individuals persons and 1000 ha for legal entities[1]. There is neither empirical nor theoretical justification of these cutoff points whatsoever.

In this paper, we make an attempt to fill in this gap and build up a partial equilibrium framework that would help analyze welfare implications of various land market restrictions in the presence of land market imperfections in the form of transaction costs and imperfect competition.

The rest of the paper is structured as follows. Section 2 presents a brief discussion of the background approach used as a starting point for our modeling exercise. A modified theoretical model that incorporates land holdings restrictions is given in Sect. 3, where several important cases are treated in a consecutive fashion. Section 4 concludes the paper with a summarizing discussion.

2 Conceptual Framework

In our work, we heavily built on the model of Ciaian and Swinnen (2006). Ciaian and Swinnen (2006) was the first attempt to analyze the impact of imperfections in the land market on the welfare effect of subsidies. It was the first one to explicitly incorporate imperfections, both in the form of transaction costs and local market power exhibited by corporate farms leading to imperfect competition.

Ciaian and Swinnen (2006) divides farms into (large) corporate farms and (small) individual farms. Individual farms can start their business by renting the land currently being cultivated by the corporate forms, incurring transaction costs in the process. Corporate farms, on the other hand, possess market power, and their rental price is an increasing function of land rented.

Both types of farms solve respective profit optimization problems. The authors show that when the corporate farms dominate the market, they in fact use less land than in the case when the market is competitive. Rental prices are also lower for corporate farms in the absence of competition and the presence of transaction costs. Landowners lose in any case, due to decreased rental prices compared to the competitive ones. The total welfare effect is negative.

In the following section, we will extend the models given above by explicitly including additional land holdings restrictions to the profit maximization problems faced by economic agents.

3 Partial Equilibrium Model

Let us denote by A_T the total supply of agricultural land, by A_r^I the amount of land rented by individual farmers, and by A_r^C the amount of land rented by corporate farms, such that $A_r^I + A_r^C \le A_T$. So far we assume that all land transactions take place through lease agreements.

Following the work of Ciaian and Swinnen (2006), we formulate the profit of a representative individual farm as follows:

$$\Pi^I = p_y f^I\left(A_r^I\right) - \left(r + t\right) A_r^I,$$

where p_y is the output price of the product (we assume that agricultural producers are price takers in the output markets); $f^I\left(A_r^I\right)$ is the increasing production function with diminishing returns to scale; *r* is the rental price for land; *t* are transaction costs in the rental market. The profit function of a representative corporate farm is

[1] See Kyiv Post article from 16 June 2017 "Drive stalls to create an agricultural land market"

$$\Pi^C = p_y f^C\left(A_r^C\right) - r\left(A_r^C\right)A_r^C,$$

where $r\left(A_r^C\right)$ is a rental price as an increasing function of land rented; $f^C\left(A_r^C\right)$ is also the increasing production function with diminishing returns to scale. Dependence of r on A_r^C shows that corporate farms may exhibit certain market power in the land rental market.

Combining two problems, we get the following multiobjective profit maximization problem facing individual and corporate farms:

$$\begin{gathered} \max \Pi^I = p_y f^I\left(A_r^I\right) - (r+t)A_r^I \\ \max \Pi^C = p_y f^C\left(A_r^C\right) - r\left(A_r^C\right)A_r^C \quad , \\ A_r^I \le \bar{A}_r^I, \quad A_r^C \le \bar{A}_r^C, \quad A_r^I + A_r^C \le A_T \end{gathered} \tag{1}$$

where $\bar{A}_r^I \le A_T$ and $\bar{A}_r^C \le A_T$ ($\bar{A}_r^I + \bar{A}_r^C \ge A_T$) are policy design variables representing maximum landholding requirements for leased land of individual and corporate farms, respectively.

3.1 Perfect Competition and Zero Transaction Costs

In the simplest setting, when there is perfect competition in the rental market (corporate farms do not enjoy any market power) and individual farmers face zero transaction costs, problem (1) simplifies to

$$\begin{gathered} \max \Pi^I = p_y f^I\left(A_r^I\right) - rA_r^I \\ \max \Pi^C = p_y f^C\left(A_r^C\right) - rA_r^C \quad . \\ A_r^I \le \bar{A}_r^I, \quad A_r^C \le \bar{A}_r^C, \quad A_r^I + A_r^C \le A_T \end{gathered} \tag{2}$$

The Lagrangean function for (2) is

$$L = p_y f^I\left(A_r^I\right) - rA_r^I - p_y f^C\left(A_r^C\right) - rA_r^C - \mu_1\left(A_r^I - \bar{A}_r^I\right) - \mu_2\left(A_r^C - \bar{A}_r^C\right) - \mu_3\left(A_r^I + A_r^C - A_T\right).$$

The first order conditions for this problem are as follows:

$$\begin{gathered} p_y \frac{\partial f^I}{\partial A_r^I} = r + \mu_1 + \mu_3, \quad p_y \frac{\partial f^C}{\partial A_r^C} = r + \mu_2 + \mu_3 \\ \mu_1\left(A_r^I - \bar{A}_r^I\right) = 0, \quad \mu_2\left(A_r^C - \bar{A}_r^C\right) = 0, \quad \mu_3\left(A_r^I + A_r^C - A_T\right) = 0 \\ A_r^I \le \bar{A}_r^I, \quad A_r^C \le \bar{A}_r^C, \quad A_r^I + A_r^C \le A_T, \quad \mu_1, \mu_2, \mu_3 \ge 0 \end{gathered}$$

In the absence of any land holding restrictions, the optimal allocation of land would be determined from equating $p_y \frac{\partial f^I}{\partial A_r^I}\left(A_T - A^*\right) = p_y \frac{\partial f^C}{\partial A_r^C}\left(A^*\right)$. The corresponding optimal rent level in the market is then given by $r^* \equiv p_y \frac{\partial f^C}{\partial A_r^C}\left(A^*\right) = p_y \frac{\partial f^I}{\partial A_r^I}\left(A_T - A^*\right)$.

In order to analyze impact of restrictions $\bar{A}_r^I$ and $\bar{A}_r^C$ on the welfare of corporate farms, individual farms, landowners, and the economy as a whole, it is expedient to consider different cases depending on where the restrictions are located compared to A^*.

The first possible case is depicted in Fig. 1, which corresponds to $A_T - \bar{A}_r^I \le \bar{A}_r^C \le A^*$. Functions of derived demand for land for corporate farms D^C and for individual farms D^I are defined by equations $r = p_y \frac{\partial f^C}{\partial A_r^C}$ and $r = p_y \frac{\partial f^I}{\partial A_r^I}$, respectively.

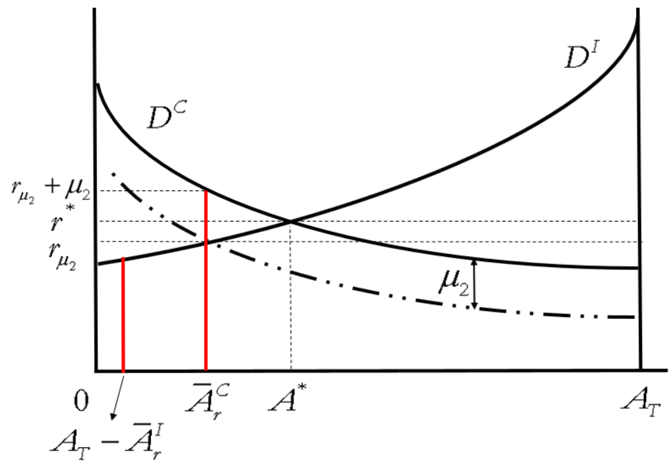

Figure 1 Equilibria in the perfectly competitive rental land market without transaction costs with $A_T - \bar{A}_r^I \le \bar{A}_r^C \le A^*$

In the case in Fig. 1, because of stringent restrictions, corporate farms can obtain only amount $\bar{A}_r^C \le A^*$, which means that $\mu_2 \ge 0$. Individual farms, on the other hand, can afford amount $A_T - \bar{A}_r^C$, in which case $\mu_1 = 0$. This allocation of land can also be found from equating $p_y \frac{\partial f^I}{\partial A_r^I}(A_T - A) = p_y \frac{\partial f^C}{\partial A_r^C}(A) - \mu_2$, which is geometrically equivalent to crossing D^I with D^C shifted by μ_2 units downwards. The corresponding rent in the market received by the landowners is $r_{\mu_2} = p_y \frac{\partial f^I}{\partial A_r^I}\left(A_T - \bar{A}_r^C\right) < r^*$ in this case.

Individual farms enjoy more land at lower rental price. Corporate farms, on the other hand, enjoy less land and incur more economic costs as exemplified by $r_{\mu_2} + \mu_2$. Landowners receive smaller rent for the same amount of land rented out.

Corporate farms' (loss in) change in surplus in this case can be written as follows:

$$\Delta\Pi^C = -\left(\left(r_{\mu_2} + \mu_2 - r^*\right)\bar{A}_r^C + \int_{\bar{A}_r^C}^{A^*} p_y \frac{\partial f^C}{\partial A^C} dA^C - r^*\left(A^* - \bar{A}_r^C\right) \right) =$$
$$= -\left(p_y \frac{\partial f^C}{\partial A^C}\left(\bar{A}_r^C\right)\cdot \bar{A}_r^C + p_y f^C\left(A^*\right) - p_y f^C\left(\bar{A}_r^C\right) - p_y \frac{\partial f^C}{\partial A^C}\left(A^*\right)\cdot A^* \right). \tag{3}$$

Observing that $\frac{\partial f^C}{\partial A^C}(A)\cdot A = \frac{\partial f^C}{\partial A^C}(A)\cdot \frac{A}{f^C(A)}\cdot f^C(A) = E^C(A)\cdot f^C(A)$, where E^C is the output elasticity, (3) can be more succinctly rewritten as

$$\Delta\Pi^C = p_y f^C(A^*)\cdot\left(E^C(A^*)-1\right) - p_y f^C(\bar{A}_r^C)\cdot\left(E^C(\bar{A}_r^C)-1\right). \tag{4}$$

Individual farms' change in surplus can be written as follows:

$$\begin{aligned}\Delta\Pi^I &= \left(r^* - r_{\mu_2}\right)\left(A_T - A^*\right) + \int_{A_T-A^*}^{A_T-\bar{A}_r^C} p_y \frac{\partial f^I}{\partial A^I} dA^I - r_{\mu_2}\left(A^* - \bar{A}_r^C\right) = \\ &= p_y f^I\left(A_T - A^*\right)\cdot\left(E^I\left(A_T - A^*\right)-1\right) - p_y f^I\left(A_T - \bar{A}_r^C\right)\cdot\left(E^I\left(A_T - \bar{A}_r^C\right)-1\right).\end{aligned} \tag{5}$$

Landowners' (loss in) change in surplus can be written as follows:

$$\Delta\Pi^L = -A_T\left(r^* - r_{\mu_2}\right) = -A_T\left(p_y \frac{\partial f^I}{\partial A^I}\left(A_T - A^*\right) - p_y \frac{\partial f^I}{\partial A^I}\left(A_T - \bar{A}_r^C\right)\right). \tag{6}$$

Figure 2 depicts the second case when $A_T - \bar{A}_r^I \le A^* \le \bar{A}_r^C$. In this setting, both corporate and individual farms are able to obtain the optimal amounts of land, as restrictions are sufficiently lax to allow it. This means that $\mu_1 = \mu_2 = 0$, and the actual allocation of land coincides with the optimal one. As an immediate conclusion,

$$\Delta\Pi^C = \Delta\Pi^I = \Delta\Pi^L = 0. \tag{7}$$

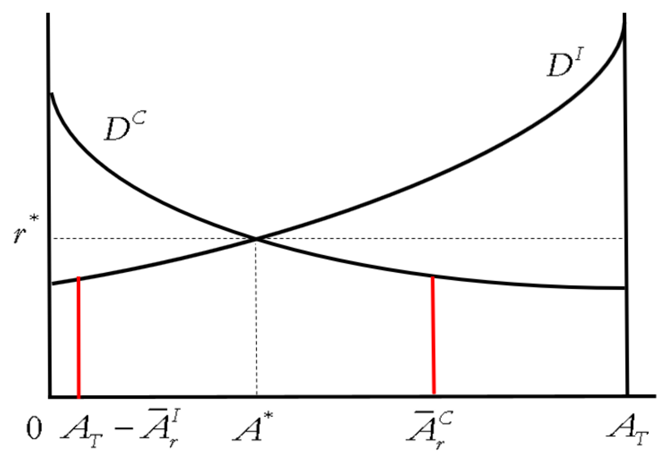

Figure 2 Equilibria in the perfectly competitive rental land market without transaction costs with $A_T - \bar{A}_r^I \le A^* \le \bar{A}_r^C$

Finally, Fig. 3 shows the situation when $A^* \le A_T - \bar{A}_r^I \le \bar{A}_r^C$. In this case, because of stringent restrictions, individual farms can obtain amount $\bar{A}_r^I \le A_T - A^*$, which means that $\mu_1 \ge 0$. On the other hand, corporate farms can afford amount $A_T - \bar{A}_r^I$, in which case $\mu_2 = 0$. The corresponding rent in the market received by the landowners is $r_{\mu_1} = p_y \frac{\partial f^C}{\partial A_r^C}\left(A_T - \bar{A}_r^I\right) < r^*$ in this case.

Corporate farms enjoy more land at lower rental price. Individual farms, on the other hand, enjoy less land and incur more economic costs as exemplified by $r_{\mu_1} + \mu_1$. Landowners receive smaller rent for the same amount of land rented out.

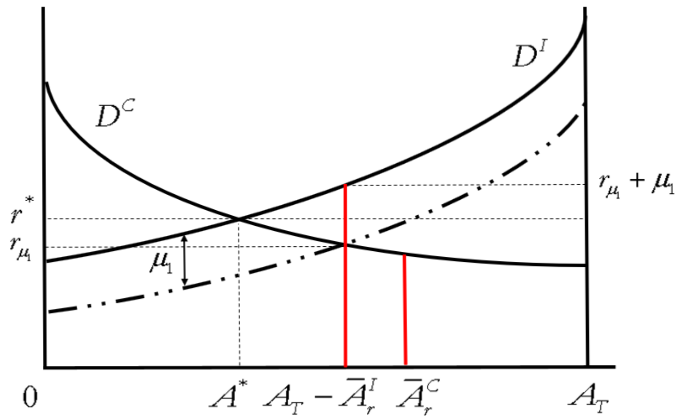

Figure 3 Equilibria in the perfectly competitive rental land market without transaction costs with $A^* \le A_T - \bar{A}_r^I \le \bar{A}_r^C$

Corporate farms' change in surplus in this case can be written as follows:

$$\Delta\Pi^C = \left(r^* - r_{\mu_1}\right)A^* + \int_{A^*}^{A_T - \bar{A}_r^C} p_y \frac{\partial f^C}{\partial A^C} dA^C - r_{\mu_1}\left(A_T - \bar{A}_r^I - A^*\right) = \\ = p_y f^C\left(A^*\right)\cdot\left(E^C\left(A^*\right) - 1\right) - p_y f^C\left(A_T - \bar{A}_r^I\right)\cdot\left(E^C\left(A_T - \bar{A}_r^I\right) - 1\right). \tag{8}$$

Individual farms' change in surplus can be written as follows:

$$\Delta\Pi^I = -\left(\left(r_{\mu_1} + \mu_1 - r^*\right)\bar{A}_r^I + \int_{\bar{A}_r^I}^{A_T - A^*} p_y \frac{\partial f^I}{\partial A^I} dA^I - r^*\left(A_T - \bar{A}_r^I - A^*\right)\right) = \\ = p_y f^I\left(A_T - A^*\right)\cdot\left(E^I\left(A_T - A^*\right) - 1\right) - p_y f^I\left(\bar{A}_r^I\right)\cdot\left(E^I\left(\bar{A}_r^I\right) - 1\right). \tag{9}$$

Landowners' (loss in) change in surplus can be written as follows:

$$\Delta\Pi^L = -A_T\left(r^* - r_{\mu_1}\right) = -A_T\left(p_y \frac{\partial f^C}{\partial A^C}\left(A^*\right) - p_y \frac{\partial f^C}{\partial A^C}\left(A_T - \bar{A}_r^I\right)\right). \tag{10}$$

PROPOSITION 1. In the perfectly competitive land rental market with zero transaction costs, assuming Cobb-Douglas production technology with constant output elasticity with respect to land (<1), maximum surpluses for corporate farms, individual farms, landowners, and the economy as a whole are attained under the following conditions:

– for corporate farms: when individual farms are not allowed to rent land at all ($\bar{A}_r^I = 0$);

– for individual farms: when corporate farms are not allowed to rent land at all ($\bar{A}_r^C = 0$);

– for landowners and the economy as a whole: when corporate and individual farms can afford optimal allocation of land ($\bar{A}_r^C \ge A^*$, $\bar{A}_r^I \ge A_T - A^*$).

Proof of this proposition is given in the Appendix.

3.2 Perfect Competition and Non-Zero Transaction Costs

In the case when there is perfect competition in the rental market, but individual farmers face transaction costs *t*, problem (2) changes to

$$\begin{gathered} \max \Pi^I = p_y f^I\left(A_r^I\right) - \left(r+t\right) A_r^I \\ \max \Pi^C = p_y f^C\left(A_r^C\right) - rA_r^C \quad . \\ A_r^I \le \bar{A}_r^I, \quad A_r^C \le \bar{A}_r^C, \quad A_r^I + A_r^C \le A_T \end{gathered} \tag{11}$$

The first order conditions for (11) can be written as follows:

$$\begin{gathered} p_y \frac{\partial f^I}{\partial A_r^I} = r + t + \mu_1 + \mu_3, \quad p_y \frac{\partial f^C}{\partial A_r^C} = r + \mu_2 + \mu_3 \\ \mu_1\left(A_r^I - \bar{A}_r^I\right) = 0, \quad \mu_2\left(A_r^C - \bar{A}_r^C\right) = 0, \quad \mu_3\left(A_r^I + A_r^C - A_T\right) = 0 . \\ A_r^I \le \bar{A}_r^I, \quad A_r^C \le \bar{A}_r^C, \quad A_r^I + A_r^C \le A_T, \qquad \mu_1, \mu_2, \mu_3 \ge 0 \end{gathered}$$

In the absence of any land holdings restrictions, the optimal allocation of land would be determined from equating $p_y \frac{\partial f^I}{\partial A_r^I}\left(A_T - A_t^*\right) - t = p_y \frac{\partial f^C}{\partial A_r^C}\left(A_t^*\right)$. The corresponding optimal rent level in the market received by the landowners is $r_t^* \equiv p_y \frac{\partial f^C}{\partial A_r^C}\left(A_t^*\right) = p_y \frac{\partial f^I}{\partial A_r^I}\left(A_T - A_t^*\right)$. As noted by (Ciaian and Swinnen, 2006), $A_t^* > A^*$ and $r_t^* < r^*$.

Analysis of welfare implications of the land holdings restrictions in the case with transaction costs closely mimics the one given in Sect. 3.1. Therefore, we will present only major results here.

Figure 4 depicts the case when $A_T - \bar{A}_r^I \le \bar{A}_r^C \le A_t^*$. Derived demand for individual farms D_t^I is defined by equation $r = p_y \frac{\partial f^I}{\partial A_r^I} - t$.

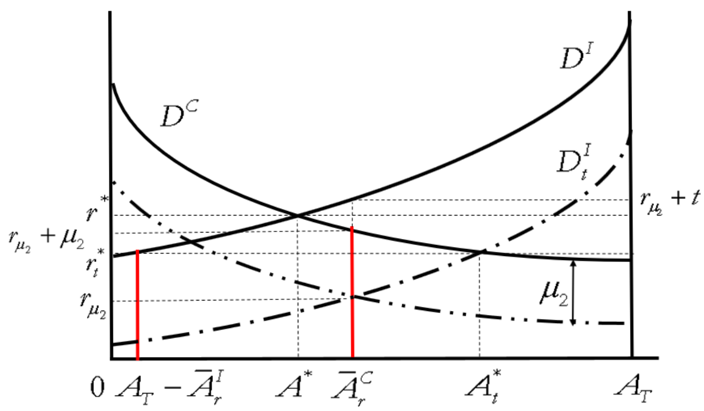

Figure 4 Equilibria in the perfectly competitive rental land market with transaction costs with $A_T - \bar{A}_r^I \le \bar{A}_r^C \le A_t^*$

Because of stringent restrictions, corporate farms in this case can obtain only amount $\bar{A}_r^C \le A_t^*$, which means that $\mu_2 \ge 0$. Individual farms, on the other hand, can afford amount

$A_T - \bar{A}_r^C$, in which case $\mu_1 = 0$. The corresponding rent in the market received by the landowners is $r_{\mu_2} = p_y \frac{\partial f^I}{\partial A_r^I}\left(A_T - \bar{A}_r^C\right) - t < r_t^*$ in this case. Compared to $A_T - A_t^*$, individual farms enjoy more land at lower rental price. Corporate farms, on the other hand, enjoy less land and incur more economic costs as exemplified by $r_{\mu_2} + \mu_2$. Landowners still lose by facing lower rental price to rent out the land in the market.

Changes in surplus for corporate farms, individual farms, and landowners are as follows:

$$\Delta\Pi^C = p_y f^C\left(A^*\right)\cdot\left(E^C\left(A^*\right)-1\right) - p_y f^C\left(\bar{A}_r^C\right)\cdot\left(E^C\left(\bar{A}_r^C\right)-1\right), \tag{12}$$

$$\Delta\Pi^I = p_y f^I\left(A_T - A^*\right)\cdot\left(E^I\left(A_T - A^*\right)-1\right) - p_y f^I\left(A_T - \bar{A}_r^C\right)\cdot\left(E^I\left(A_T - \bar{A}_r^C\right)-1\right), \tag{13}$$

$$\Delta\Pi^L = -A_T\left(p_y \frac{\partial f^I}{\partial A^I}\left(A_T - A^*\right) - \left(p_y \frac{\partial f^I}{\partial A^I}\left(A_T - \bar{A}_r^C\right) - t\right)\right). \tag{14}$$

Figure 5 depicts the second case when $A_T - \bar{A}_r^I \le A_t^* \le \bar{A}_r^C$. In this setting, corporate farms are able to obtain amount A_t^*, whereas individual farms are able to obtain amount $A_T - A_t^*$. This means that $\mu_1 = \mu_2 = 0$ and the actual allocation of land coincides with the optimal one.

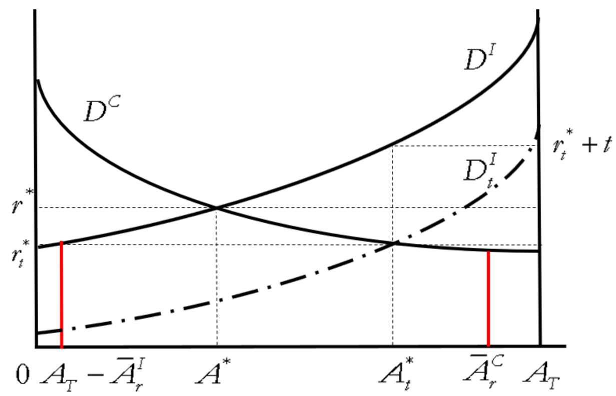

Figure 5 Equilibria in the perfectly competitive rental land market with transaction costs with $A_T - \bar{A}_r^I \le A_t^* \le \bar{A}_r^C$

Using approach outlined above, it is easy to arrive at the following expressions:

$$\Delta\Pi^C = p_y f^C\left(A^*\right)\cdot\left(E^C\left(A^*\right)-1\right) - p_y f^C\left(A_t^*\right)\cdot\left(E^C\left(A_t^*\right)-1\right), \tag{15}$$

$$\Delta\Pi^I = p_y f^I\left(A_T - A^*\right)\cdot\left(E^I\left(A_T - A^*\right)-1\right) - p_y f^I\left(A_T - A_t^*\right)\cdot\left(E^I\left(A_T - A_t^*\right)-1\right), \tag{16}$$

$$\Delta\Pi^L = -A_T\left(p_y \frac{\partial f^I}{\partial A^I}\left(A_T - A^*\right) - \left(p_y \frac{\partial f^I}{\partial A^I}\left(A_T - A_t^*\right) - t\right)\right). \tag{17}$$

Finally, Fig. 6 shows the situation when $A_t^* \le A_T - \bar{A}_r^I \le \bar{A}_r^C$. In this case, because of stringent restrictions, individual farms can obtain amount $\bar{A}_r^I \le A_T - A_t^*$, which means that $\mu_1 \ge 0$. On the other hand, corporate farms can afford amount $A_T - \bar{A}_r^I$, in which case $\mu_2 = 0$. The corresponding rent in the market received by the landowners is $r_{\mu_1} = p_y \frac{\partial f^C}{\partial A_r^C}\left(A_T - \bar{A}_r^I\right) < r_t^*$ in this case. Corporate farms enjoy more land at lower rental price, whereas individual farms enjoy less land and incur more economic costs as exemplified by $r_{\mu_1} + \mu_1$. Landowners still lose by facing lower rental price to rent out the land in the market.

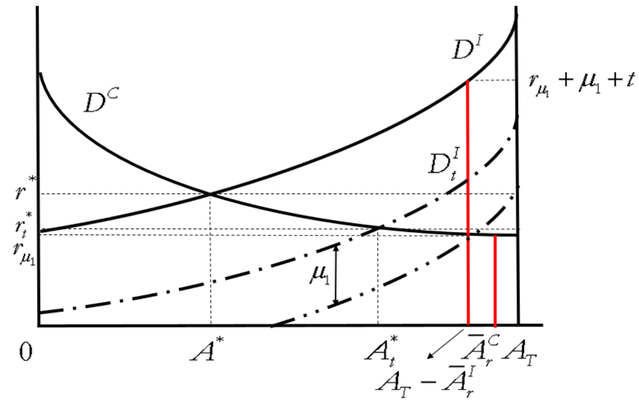

Figure 6 Equilibria in the perfectly competitive rental land market with transaction costs with $A_t^* \le A_T - \bar{A}_r^I \le \bar{A}_r^C$

It is easy to show that the analytic expression for corporate farms' change in surplus $\Delta\Pi^C$ is the same as (8), the analytic expression for individual farms' change in surplus $\Delta\Pi^I$ is the same as (9), and that the analytic expression for landowners' (loss in) change in surplus $\Delta\Pi^L$ is the same as (10).

PROPOSITION 2. In the perfectly competitive land rental market with transaction costs *t* faced by individual farms, assuming Cobb-Douglas production technology with constant output elasticity with respect to land (<1), maximum surpluses for corporate farms, individual farms, landowners, and the economy as a whole are attained under the following conditions:

– for corporate farms: when individual farms are not allowed to rent land at all ($\bar{A}_r^I = 0$);

– for individual farms: when corporate farms are not allowed to rent land at all ($\bar{A}_r^C = 0$);

– for landowners and the economy as a whole: when corporate and individual farms can afford optimal allocation of land under transaction costs ($\bar{A}_r^C \ge A_t^*$, $\bar{A}_r^I \ge A_T - A_t^*$).

Proof of this proposition is given in the Appendix.

3.3 Imperfect Competition and Non-Zero Transaction Costs

In the case when corporate farms possess market power in the rental market, and individual farmers face transaction costs *t*, problem (2) changes to

$$\begin{gathered} \max \Pi^I = p_y f^I\left(A_r^I\right) - (r+t)A_r^I \\ \max \Pi^C = p_y f^C\left(A_r^C\right) - r\left(A_r^C\right)A_r^C \quad . \\ A_r^I \le \bar{A}_r^I, \quad A_r^C \le \bar{A}_r^C, \quad A_r^I + A_r^C \le A_T \end{gathered} \tag{18}$$

The first order conditions for (18) can be written as follows:

$$p_y \frac{\partial f^I}{\partial A_r^I} = r + t + \mu_1 + \mu_3, \quad p_y \frac{\partial f^C}{\partial A_r^C} = r\left(A_r^C\right) + A_r^C \frac{\partial r}{\partial A_r^C} + \mu_2 + \mu_3$$
$$\mu_1\left(A_r^I - \bar{A}_r^I\right) = 0, \quad \mu_2\left(A_r^C - \bar{A}_r^C\right) = 0, \quad \mu_3\left(A_r^I + A_r^C - A_T\right) = 0 \ .$$
$$A_r^I \le \bar{A}_r^I, \quad A_r^C \le \bar{A}_r^C, \quad A_r^I + A_r^C \le A_T, \quad \mu_1, \mu_2, \mu_3 \ge 0$$

In the discussion to follow, we will assume for simplicity that $r\left(A_r^C\right) = a + bA_r^C$.

In the absence of any land holdings restrictions, corporate farms choose amount of land $A_t^M < A_t^*$ that satisfies the equation

$$p_y \frac{\partial f^C}{\partial A_r^C}\left(A_t^M\right) = r\left(A_t^M\right) + A_t^M \frac{\partial r}{\partial A_r^C}\left(A_t^M\right) = a + 2bA_t^M , \tag{19}$$

where the left-hand side represents the marginal benefits (*MB*) and the right-hand side represents the marginal costs (*MC*) of cultivating A_r^C amount of land. Intercept of *MC* line can be found from equating it to the derived demand function for individual farms:

$$a = p_y \frac{\partial f^I}{\partial A_r^I}\left(A_T\right) - t . \tag{20}$$

Individual farms obtain amount of land $A_T - A_t^M \ge A_T - A_t^*$. The rental price $r_t^M = p_y \frac{\partial f^I}{\partial A_r^I}\left(A_t^M\right) - t$ received by the landowners in the market is lower than r_t^*, as "both the transaction costs and the market power of [the corporate farms pushes the] rental price down" (Ciaian and Swinnen, 2006). Compared to the perfectly competitive market with zero transaction costs, corporate farms gain in surplus, whereas for individual farms, the net effect of transaction costs (which raise rental prices and lower land amount) and market power of corporate farms (which lowers rental prices and raises land amount) depends on the relative size of the transaction costs—the lower the costs, the higher is the net change in surplus.

Figure 7 depicts the case when $A_T - \bar{A}_r^I \le \bar{A}_r^C \le A_t^M$. Because of severe restrictions, corporate farms in this case can obtain only amount $\bar{A}_r^C \le A_t^M$, which means that $\mu_2 \ge 0$. Individual farms, on the other hand, can afford amount $A_T - \bar{A}_r^C$, in which case $\mu_1 = 0$. Compared to $A_T - A_t^M$, individual farms enjoy more land at lower rental price $r_{\mu_2} = p_y \frac{\partial f^I}{\partial A_r^I}\left(A_T - \bar{A}_r^C\right) - t$. Corporate farms, on the other hand, enjoy less land and incur more economic costs as exemplified by $r_{\mu_2} + \mu_2$. Landowners lose by facing lower rental price to rent out the land in the market.

Changes in surplus for corporate farms, individual farms, and landowners are as follows:

$$\Delta\Pi^C = p_y f^C\left(A^*\right)\cdot\left(E^C\left(A^*\right) - 1\right) - p_y f^C\left(\bar{A}_r^C\right)\cdot\left(E^C\left(\bar{A}_r^C\right) - 1\right) + \bar{A}_r^C A_t^M \frac{\partial r}{\partial A_r^C}\left(A_t^M\right), \tag{21}$$

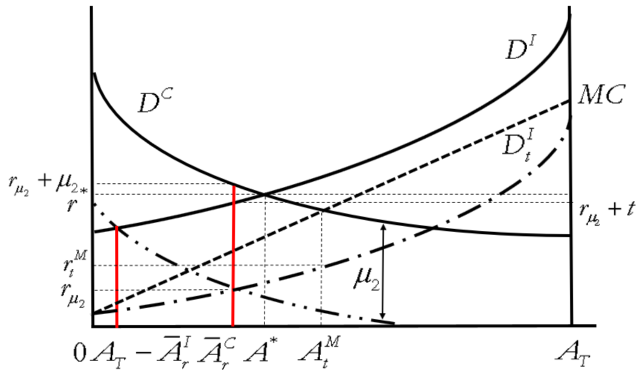

Figure 7 Equilibria in the imperfectly competitive rental land market with transaction costs with $A_T - \bar{A}_r^I \le \bar{A}_r^C \le A_t^M$

$$\Delta\Pi^I = p_y f^I\left(A_T - A^*\right)\cdot\left(E^I\left(A_T - A^*\right)-1\right) - p_y f^I\left(A_T - \bar{A}_r^C\right)\cdot\left(E^I\left(A_T - \bar{A}_r^C\right)-1\right), \quad (22)$$

$$\Delta\Pi^L = -A_T\left(p_y \frac{\partial f^I}{\partial A^I}\left(A_T - A^*\right) - \left(p_y \frac{\partial f^I}{\partial A^I}\left(A_T - \bar{A}_r^C\right) - t\right)\right). \quad (23)$$

Figure 8 depicts the second case when $A_T - \bar{A}_r^I \le A_t^M \le \bar{A}_r^C$. In this setting, both corporate farms are able to obtain amount A_t^M, and, as a consequence, individual farms obtain amount $A_T - A_t^M$. This means that $\mu_1 = \mu_2 = 0$, and the actual allocation of land coincides with the optimal one for this market structure.

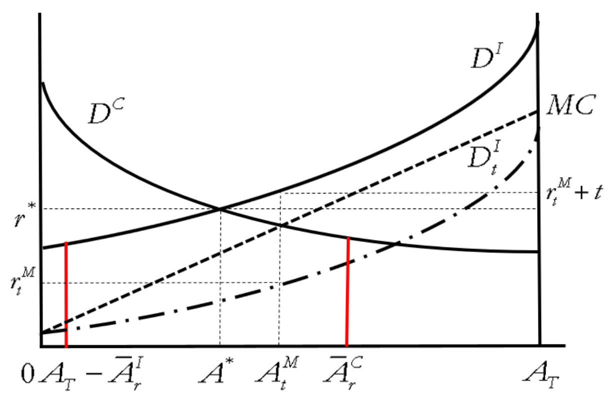

Figure 8 Equilibria in the imperfectly competitive rental land market with transaction costs with $A_T - \bar{A}_r^I \le A_t^M \le \bar{A}_r^C$

Using approach outlined above, it is easy to arrive at the following expressions:

$$\Delta\Pi^C = p_y f^C\left(A^*\right)\cdot\left(E^C\left(A^*\right)-1\right) + p_y f^C\left(A_t^M\right) - p_y E^I f^I\left(A_t^M\right), \quad (24)$$

$$\Delta\Pi^I = p_y f^I\left(A_T - A^*\right)\cdot\left(E^I\left(A_T - A^*\right)-1\right) - p_y f^I\left(A_T - A_t^M\right)\cdot\left(E^I\left(A_T - A_t^M\right)-1\right), \quad (25)$$

$$\Delta\Pi^L = -A_T\left(p_y \frac{\partial f^I}{\partial A^I}\left(A_T - A^*\right) - \left(p_y \frac{\partial f^I}{\partial A^I}\left(A_T - A_t^M\right) - t\right)\right). \quad (26)$$

Finally, Fig. 9 shows the situation when $A_t^M \le A_T - \bar{A}_r^I \le \bar{A}_r^C$. In this case, because of stringent restrictions, individual farms can obtain amount $\bar{A}_r^I \le A_T - A_t^M$ (A_t^M is denoted in the figure by $A_t^{M(0)}$), which means that $\mu_1 \ge 0$. However, for corporate farms, it is still optimal to choose amount A_t^M ($\mu_2 = 0$). As a result, amount of land $A_T - \bar{A}_r^I - A_t^M$ is not demanded and is taken away out of market.

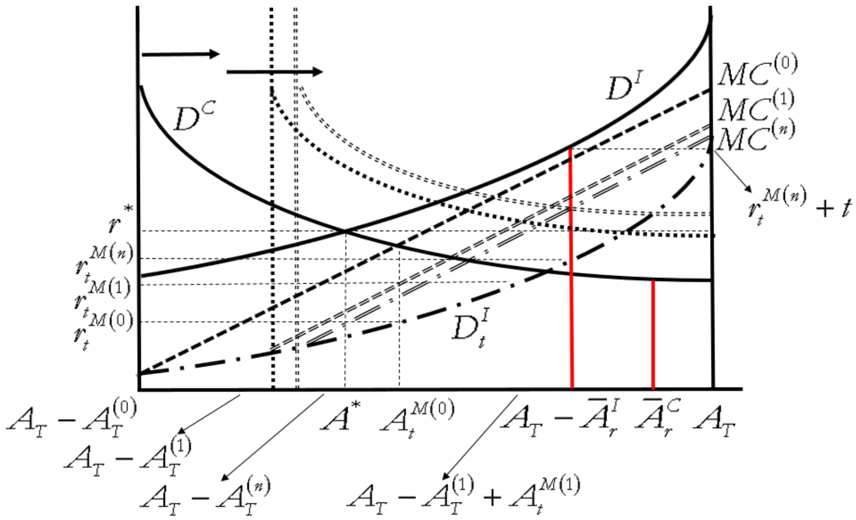

Figure 9 Equilibria in the imperfectly competitive rental land market with transaction costs with $A_t^M \le A_T - \bar{A}_r^I \le \bar{A}_r^C$

Let us denote initial total amount of land supplied in the market by $A_T^{(0)}$, initial amount of land chosen by corporate farms by $A_t^{M(0)}$, initial rental price in the market by $r_t^{M(0)} \equiv p_y \frac{\partial f^I}{\partial A_r^I}\left(A_T^{(0)} - A_t^{M(0)}\right)$, initial intercept of the marginal cost of corporate farms by $a^{(0)}$, and initial amount of land taken away by $gap^{(0)} \equiv A_T^{(0)} - \bar{A}_r^I - A_t^{M(0)}$.

After $gap^{(0)}$ amount of land is taken away from the market, total amount of land available in the market changes to

$$A_T^{(1)} = A_T^{(0)} - gap^{(0)} = A_T^{(0)} - \left(A_T^{(0)} - \bar{A}_r^I - A_t^{M(0)}\right) = \bar{A}_r^I + A_t^{M(0)}. \qquad (27)$$

As a result, intercept of the marginal cost of corporate farms changes to $a^{(1)}$. Direction and magnitude of this change is given in the following lemma.

LEMMA 1. In the imperfectly competitive land rental market with transaction costs faced by individual farms, assuming Cobb-Douglas production technology and linear marginal cost *MC* of cultivating land for corporate farms, change in the total supply of land from $A_T^{(i)}$ to $A_T^{(i+1)} < A_T^{(i)}$ increases the intercept of *MC* from $a^{(i)}$ to

$$a^{(i+1)} = a^{(i)} \cdot \left(1 + \frac{A_T^{(i)} - A_T^{(i+1)}}{A_T^{(i)}}\right). \qquad (28)$$

PROOF. Cobb-Douglas production technology is characterized by unit elastic derived demand for any factor of production, in particular, we can write:

$$-1=\frac{A_T^{(i)}-A_T^{(i+1)}}{\left(p_y\frac{\partial f^I}{\partial A_r^I}\left(A_T^{(i)}\right)-t\right)-\left(p_y\frac{\partial f^I}{\partial A_r^I}\left(A_T^{(i+1)}\right)-t\right)}\cdot\frac{p_y\frac{\partial f^I}{\partial A_r^I}\left(A_T^{(i)}\right)-t}{A_T^{(i)}}. \tag{29}$$

Using (20), (29) can be rewritten as

$$-1=\frac{A_T^{(i)}-A_T^{(i+1)}}{a^{(i)}-a^{(i+1)}}\cdot\frac{a^{(i)}}{A_T^{(i)}}. \tag{30}$$

Expression (28) can be obtained from (30) in a straightforward way. It is easy to see that $a^{(i+1)}>a^{(i)}$ because expression in the parentheses is always greater than unity. Q.E.D.

After *MC* has been shifted by $a^{(1)}-a^{(0)}$ upward, corporate farms choose another amount of land $A_t^{M(1)}$ that satisfies the equation $p_y\frac{\partial f^C}{\partial A_r^C}\left(A_t^{M(1)}\right)=a^{(1)}+2bA_t^{M(1)}$. Direction and magnitude of this change is given in the following lemma.

LEMMA 2. In the imperfectly competitive land rental market with transaction costs faced by individual farms, assuming Cobb-Douglas production technology and linear marginal cost $MC\left(A_r^C\right)^{(i)}=a^{(i)}+2bA_r^C$ of cultivating land for corporate farms, increase in *MC* from $MC\left(A_r^C\right)^{(i)}$ to $MC\left(A_r^C\right)^{(i+1)}=a^{(i+1)}+2bA_r^C$ decreases amount of land chosen by corporate farms from $A_t^{M(i)}$ to

$$A_t^{M(i+1)}=A_t^{M(i)}\cdot\left(\frac{2p_y\frac{\partial f^C}{\partial A_r^C}\left(A_t^{M(i)}\right)-a^{(i+1)}}{2p_y\frac{\partial f^C}{\partial A_r^C}\left(A_t^{M(i)}\right)-a^{(i)}}\right). \tag{31}$$

PROOF. Using (19), we can construct the following system of equations:

$$\begin{cases} a^{(i)}+2bA_t^{M(i)}=p_y\frac{\partial f^C}{\partial A_r^C}\left(A_t^{M(i)}\right) \\ a^{(i+1)}+2bA_t^{M(i+1)}=p_y\frac{\partial f^C}{\partial A_r^C}\left(A_t^{M(i+1)}\right) \end{cases}. \tag{32}$$

Subtracting the second equation from the first one yields

$$a^{(n)}-a^{(n+1)}+2b\left(A_t^{M(n)}-A_t^{M(n+1)}\right)=p_y\frac{\partial f^C}{\partial A_r^C}\left(A_t^{M(n)}\right)-p_y\frac{\partial f^C}{\partial A_r^C}\left(A_t^{M(n+1)}\right).$$

Rearranging the terms, one can arrive at the following expression:

$$\frac{a^{(n)} - a^{(n+1)}}{A_t^{M(n)} - A_t^{M(n+1)}} + 2b = \frac{p_y \frac{\partial f^C}{\partial A_r^C}\left(A_t^{M(n)}\right) - p_y \frac{\partial f^C}{\partial A_r^C}\left(A_t^{M(n+1)}\right)}{A_t^{M(n)} - A_t^{M(n+1)}}. \tag{33}$$

Multiplying both sides of (33) by $A_t^{M(n)}$ and dividing by $p_y \frac{\partial f^C}{\partial A_r^C}\left(A_t^{M(n)}\right)$, and making use of the fact that the Cobb-Douglas production technology is characterized by unit elastic derived demand for any factor of production, one can obtain

$$\frac{a^{(n)} - a^{(n+1)}}{A_t^{M(n)} - A_t^{M(n+1)}} \cdot \frac{A_t^{M(n)}}{p_y \frac{\partial f^C}{\partial A_r^C}\left(A_t^{M(n)}\right)} + \frac{2bA_t^{M(n)}}{p_y \frac{\partial f^C}{\partial A_r^C}\left(A_t^{M(n)}\right)} = -1. \tag{34}$$

Plugging in $2bA_t^{M(n)} = p_y \frac{\partial f^C}{\partial A_r^C}\left(A_t^{M(n)}\right) - a^{(n)}$ from (32) into (34) and rearranging the terms, one can get

$$\frac{a^{(n)} - a^{(n+1)}}{A_t^{M(n)} - A_t^{M(n+1)}} \cdot \frac{A_t^{M(n)}}{p_y \frac{\partial f^C}{\partial A_r^C}\left(A_t^{M(n)}\right)} + 1 - \frac{a^{(n)}}{p_y \frac{\partial f^C}{\partial A_r^C}\left(A_t^{M(n)}\right)} = -1. \tag{35}$$

Expressing out $A_t^{M(n+1)}$ from (35) can be done in a straightforward way. It is easy to see that $A_t^{M(n+1)} < A_t^{M(n)}$, as expression in parentheses in (31) is less than 1 because $a^{(n+1)} > a^{(n)}$ according to Lemma 1. Q.E.D.

After corporate farms choose amount $A_t^{M(1)}$, it is possible that, still, $A_t^{M(1)} < A_T^{(1)} - \bar{A}_r^I$, and amount of land $gap^{(1)} = A_T^{(1)} - \bar{A}_r^I - A_t^{M(1)}$ will be taken away out of market. The total amount of land, similarly to (27), will change to

$$A_T^{(2)} = A_T^{(1)} - gap^{(1)} = A_T^{(1)} - \left(A_T^{(1)} - \bar{A}_r^I - A_t^{M(1)}\right) = \bar{A}_r^I + A_t^{M(1)}.$$

As a result, intercept of *MC* will change to $a^{(2)}$ according to Lemma 1, and the amount of land optimally chosen by corporate farms will change to $A_t^{M(2)}$ according to Lemma 2. This iterative process will continue until some iteration *n* when $gap^{(n)} = A_t^{M(n-1)} - A_t^{M(n)} = 0$. According to the final allocation of land, corporate farms will choose $A_t^{M(n)}$ and individual farms will choose $A_T - \bar{A}_r^I$. Amount $A_t^{M(n)}$ can be determined by iterative application of (28), (31), and the following equation:

$$A_T^{(i)} = A_T^{(i-1)} - gap^{(i-1)} = A_T^{(i-1)} - \left(A_T^{(i-1)} - \bar{A}_r^I - A_t^{M(i-1)}\right) = \bar{A}_r^I + A_t^{M(i-1)}. \tag{36}$$

The equilibrium level of rent $r_t^{M(n)}$ received by landowners is given by the following lemma.

LEMMA 3. In the imperfectly competitive land rental market with transaction costs t faced by individual farms, assuming Cobb-Douglas production technology and linear marginal cost $MC\left(A_r^C\right)=a+2bA_r^C$ of cultivating land for corporate farms, landowners will face rental price

$$r_t^{M(n)}=p_y\frac{\partial f^I}{\partial A_r^I}\left(\bar{A}_r^I\right)-t. \tag{37}$$

PROOF. Rental price at each iteration is obtained as $r_t^{M(i)}=p_y\frac{\partial f^I}{\partial A_r^I}\left(A_T^{(i)}-A_t^{M(i)}\right)-t$. Plugging in (36), one can get $r_t^{M(i)}=p_y\frac{\partial f^I}{\partial A_r^I}\left(\bar{A}_r^I+A_t^{M(i-1)}-A_t^{M(i)}\right)-t$. As $i\to n$, both $A_t^{M(i-1)}\to A_t^{M(n)}$ and $A_t^{M(i)}\to A_t^{M(n)}$, so $r_t^{M(n)}=p_y\frac{\partial f^I}{\partial A_r^I}\left(\bar{A}_r^I\right)-t$. Q.E.D.

It is important to note that $r_t^{M(n)}>r_t^{M(0)}$ because $A_T^{(0)}-A_t^{M(0)}>\bar{A}_r^I$. In other words, rental price in the market increases due to decreased market supply of land as a consequence of stringent land holdings restrictions.

Figure 9 presents one iteration of the above procedure, along with the final allocation of land. Compared to A_t^M, corporate farms enjoy less land at higher rental price $r_t^{M(n)}>r_t^M$, whereas individual farms enjoy less land at the same higher rental price. Landowners lose twice, first due to some land not being rented in the market, and second, because of lower equilibrium rental price.

Changes in surplus for corporate farms, individual farms, and landowners are as follows:

$$\Delta\Pi^C=p_y f^C\left(A^*\right)\cdot\left(E^C\left(A^*\right)-1\right)+p_y f^C\left(A_t^{M(n)}\right)-A_t^{M(n)}\left(p_y\frac{\partial f^I}{\partial A_r^I}\left(\bar{A}_r^I\right)-t\right), \tag{38}$$

$$\Delta\Pi^I=p_y f^I\left(A_T-A^*\right)\cdot\left(E^I\left(A_T-A^*\right)-1\right)-p_y f^I\left(\bar{A}_r^I\right)\cdot\left(E^I\left(\bar{A}_r^I\right)-1\right), \tag{39}$$

$$\Delta\Pi^L=-p_y\frac{\partial f^I}{\partial A_r^I}\left(A_T-A^*\right)\cdot A_T+\left(p_y\frac{\partial f^I}{\partial A_r^I}\left(\bar{A}_r^I\right)-t\right)\cdot\left(\bar{A}_r^I+A_t^{M(n)}\right). \tag{40}$$

PROPOSITION 3. In the imperfectly competitive land rental market with transaction costs t faced by individual farms, assuming Cobb-Douglas production technology with constant output elasticity with respect to land (<1) and linear marginal cost $MC\left(A_r^C\right)=a+2bA_r^C$ of cultivating land for corporate farms, maximum surpluses for corporate farms, individual farms, and the economy as a whole are attained under the following conditions:

– for corporate farms: when restrictions on individual farms preclude any land from being taken away from the market ($\bar{A}_r^I\ge A_T-A_t^M$);

– for individual farms: when corporate farms are not allowed to rent land at all ($\bar{A}_r^C=0$);

– for the economy as a whole: if transaction costs are relatively low, when restrictions on corporate and individual farms preclude land from being taken away from the market ($\bar{A}_r^C\ge A_t^M$, $\bar{A}_r^I\ge A_T-A_t^M$); if transaction costs are relatively high, when individual farms are not allowed to rent land at all ($\bar{A}_r^I=0$).

Proof of this proposition is given in the Appendix.

4 Conclusions and Further Research

In this work, we have developed a theoretical partial equilibrium framework that enabled us to traverse effects of such land market imperfections as transaction costs and imperfect competition on the optimal choice of maximum ownership restrictions on the land being rented by farms.

Perhaps not surprisingly, for all market structures, the optimal welfare in the economy is attained when no restrictions are imposed in the first place. However, in different cases, different agents benefit in a slightly different way.

In perfectly competitive land markets, with or without transaction costs, large corporate farms are better off if small individual farms don't enter the market, and vice versa, whereas for landowners and the economy as a whole, absence of restrictions yields maximum possible economic surplus. In imperfectly competitive land markets with transaction costs, large corporate farms are better off when individual farms are not constrained by stringent restrictions, because otherwise a certain amount of land supplied will not meet adequate demand, driving upward the rental price for the land remaining in the market. Individual farms are still better off when corporate farms don't enter the market. Implications on welfare of the economy as a whole depend on the relative size of transaction costs. If they are sufficiently high, the total welfare is maximized if individual farms don't enter the market; if they are relatively low, again, absence of restrictions yields maximum possible economic surplus.

These conclusions imply, among other things, that the original intention of maximum land holdings restrictions, that is, prevention of excessive land concentration, comes at a cost of reduced overall welfare in the economy. Therefore, other antitrust and regulation measures should be implemented in order to facilitate competition in the land markets, which would not artificially distort market structure.

The model presented in this paper can be augmented by introducing a second market into play, namely, the sales market for land. Effect of sales markets is overlooked in current academic literature, which is a simplification that bears the costs in terms of policy implications, for in many countries rental markets have different weights in the total volume of land market transactions. Studying the effect of land market imperfections in both rental and sales markets treated under one hood will shed important light on the optimal land market policy design.

References

Ciaian, P., K. d'Artis, J. Swinnen, K. Van Herck, and L. Vranken. 2012a. "Rental Market Regulations for Agricultural Land in EU Member States and Candidate Countries." *Factor Markets Working Papers* 15: 1–24.

Ciaian, P., K. d'Artis, J. Swinnen, K. Van Herck, and L. Vranken. 2012b. "Sales Market Regulations for Agricultural Land in EU Member States and Candidate Countries." *Factor Markets Working Papers* 14: 1–32.

Ciaian, P., and J. F. M. Swinnen. 2006. "Land Market Imperfections and Agricultural Policy Impacts in the New EU Member States: A Partial Equilibrium Analysis." *American Journal of Agricultural Economics* 88, no. 4: 799–815.

Deininger, K. 2003. "Land Policies for Growth and Poverty Reduction." A World Bank Policy Research Report. Oxford University Press.

Deininger, K., and G. Feder. 2001. "Land Institutions and Land Markets." In *Handbook of Agricultural Economics, Volume 1, Part A*, edited by Bruce L. Gardner and Gordon C. Rausser, 287–331. Amsterdam: Elsevier.

Deininger, K., D. Nizalov and O. Nivievskyi. 2017a. "Beyond Lifting the Moratorium: Preconditions for Success". VoxUkraine. https://voxukraine.org/en/beyond-lifting-the-moratorium-preconditions-for-success/

Deininger, K., D. Nizalov and O. Nivievskyi. 2017b. "Lifting the Moratorium on Farmland Sales: Why Supply and Demand Must Balance". VoxUkraine. https://voxukraine.org/en/lifting-the-moratorium-on-farmland-sales-why-supply-and-demand-must-balance/

Nivievskyi, O., D. Nizalov, and S. Kubakh. 2016. "Restrictions on Farmland Sales Markets: A Survey of International Experience and Lessons for Ukraine." Project "Capacity Development for Evidence-Based Land & Agricultural Policy Making in Ukraine," Kyiv School of Economics.

Swinnen, J., K. Van Herck, and L. Vranken. 2016. "The Diversity of Land Markets and Regulations in Europe, and (Some of) Its Causes." *The Journal of Development Studies* 52, no. 2: 186–205.

APPENDIX

Proof of Proposition 1

To find the conditions for the maximum level of surplus for corporate farms, it is necessary to solve the problem of maximizing $\Delta\Pi^C\left(\bar{A}_r^C, \bar{A}_r^I\right)$ with respect to its variables. Combining (4), (7), and (8), and using the fact the output elasticities are assumed to be constant, one can arrive at the following analytical expression for $\Delta\Pi^C$:

$$\Delta\Pi^C = \begin{cases} \left(p_y f^C\left(A^*\right) - p_y f^C\left(\bar{A}_r^C\right)\right)\cdot\left(E^C - 1\right), & A_T - \bar{A}_r^I \le \bar{A}_r^C \le A^* \\ 0, & A_T - \bar{A}_r^I \le A^* \le \bar{A}_r^C \\ \left(p_y f^C\left(A^*\right) - p_y f^C\left(A_T - \bar{A}_r^I\right)\right)\cdot\left(E^C - 1\right), & A^* \le A_T - \bar{A}_r^I \le \bar{A}_r^C \end{cases} \quad \text{(A.1)}$$

The first order condition with respect to $\bar{A}_r^C$ is

$$\frac{\partial\Delta\Pi^C}{\partial\bar{A}_r^C} = -p_y \frac{\partial f^C}{\partial A_r^C}\left(\bar{A}_r^C\right)\cdot\left(E^C - 1\right) > 0 \quad \text{(A.2)}$$

when $A_T - \bar{A}_r^I \le \bar{A}_r^C \le A^*$ and $\frac{\partial\Delta\Pi^C}{\partial\bar{A}_r^C} = 0$ otherwise. Expression (A.2) is always positive, since by assumptions $E^C < 1$ and production function is increasing in *A*. Therefore, $\Delta\Pi^C$ increases as $\bar{A}_r^C$ increases from 0 up to the point $\bar{A}_r^C = A^*$, and then stays constant.

The first order condition with respect to $\bar{A}_r^I$ is as follows:

$$\frac{\partial\Delta\Pi^C}{\partial\bar{A}_r^I} = p_y \frac{\partial f^C}{\partial A_r^C}\left(A_T - \bar{A}_r^I\right)\cdot\left(E^C - 1\right) < 0 \quad \text{(A.3)}$$

when $A^* \le A_T - \bar{A}_r^I \le \bar{A}_r^C$ and $\frac{\partial\Delta\Pi^C}{\partial\bar{A}_r^I} = 0$ otherwise. $\Delta\Pi^C$ decreases as $\bar{A}_r^I$ increases from 0 up to the point $\bar{A}_r^I = A_T - A^*$, and then stays constant. In other words, maximum value of $\Delta\Pi^C$ with respect to $\bar{A}_r^I$ is attained when $\bar{A}_r^I = 0$.

Combining the two results, we conclude that maximum $\Delta\Pi^C$ is attained when $\bar{A}_r^I = 0$, $\bar{A}_r^C = A_T$, i.e. when individual farms are not allowed to rent any land, and all available land is rented by corporate farms.

Conditions for the maximum level of surplus for individual farms can be found in a similar fashion. Combining (5), (7), and (9), and using the fact the output elasticities are assumed to be constant, one can arrive at the following analytical expression for $\Delta\Pi^I$:

$$\Delta\Pi^I = \begin{cases} \left(p_y f^I\left(A_T - A^*\right) - p_y f^I\left(A_T - \bar{A}_r^C\right)\right)\cdot\left(E^I - 1\right), & A_T - \bar{A}_r^I \le \bar{A}_r^C \le A^* \\ 0, & A_T - \bar{A}_r^I \le A^* \le \bar{A}_r^C \\ \left(p_y f^I\left(A_T - A^*\right) - p_y f^I\left(\bar{A}_r^I\right)\right)\cdot\left(E^I - 1\right), & A^* \le A_T - \bar{A}_r^I \le \bar{A}_r^C \end{cases} \quad \text{(A.4)}$$

The first order condition with respect to $\bar{A}_r^C$ is

$$\frac{\partial \Delta \Pi^I}{\partial \bar{A}_r^C} = p_y \frac{\partial f^I}{\partial A_r^I}\left(A_T - \bar{A}_r^C\right)\cdot\left(E^I - 1\right) < 0 \qquad \text{(A.5)}$$

when $A_T - \bar{A}_r^I \le \bar{A}_r^C \le A^*$ and $\frac{\partial \Delta \Pi^I}{\partial \bar{A}_r^C} = 0$ otherwise. Expression (A.5) is always negative, since by assumptions $E^I < 1$ and production function is increasing in *A*. Therefore, $\Delta \Pi^I$ decreases as $\bar{A}_r^C$ increases from 0 up to the point $\bar{A}_r^C = A^*$, and then stays constant. In other words, maximum value of $\Delta \Pi^I$ with respect to $\bar{A}_r^C$ is attained when $\bar{A}_r^C = 0$.

The first order condition with respect to $\bar{A}_r^I$ is as follows:

$$\frac{\partial \Delta \Pi^I}{\partial \bar{A}_r^I} = -p_y \frac{\partial f^I}{\partial A_r^I}\left(\bar{A}_r^I\right)\cdot\left(E^I - 1\right) > 0 \qquad \text{(A.6)}$$

when $A^* \le A_T - \bar{A}_r^I \le \bar{A}_r^C$ and $\frac{\partial \Delta \Pi^I}{\partial \bar{A}_r^I} = 0$ otherwise. $\Delta \Pi^I$ increases as $\bar{A}_r^I$ increases from 0 up to the point $\bar{A}_r^I = A_T - A^*$, and then stays constant.

Combining the two results, we conclude that maximum $\Delta \Pi^I$ is attained when $\bar{A}_r^C = 0$, $\bar{A}_r^I = A_T$, i.e. when corporate farms are not allowed to rent any land, and all available land is rented by individual farms.

To find the conditions for the maximum level of surplus for landowners, combine (6), (7), and (10) to arrive at the following expression for $\Delta \Pi^L$:

$$\Delta \Pi^L = \begin{cases} -A_T\left(p_y \frac{\partial f^I}{\partial A_r^I}\left(A_T - A^*\right) - p_y \frac{\partial f^I}{\partial A_r^I}\left(A_T - \bar{A}_r^C\right)\right), & A_T - \bar{A}_r^I \le \bar{A}_r^C \le A^* \\ 0, & A_T - \bar{A}_r^I \le A^* \le \bar{A}_r^C \\ -A_T\left(p_y \frac{\partial f^C}{\partial A_r^C}\left(A^*\right) - p_y \frac{\partial f^C}{\partial A_r^C}\left(A_T - \bar{A}_r^I\right)\right), & A^* \le A_T - \bar{A}_r^I \le \bar{A}_r^C \end{cases} \qquad \text{(A.7)}$$

The first order condition with respect to $\bar{A}_r^C$ is as follows:

$$\frac{\partial \Delta \Pi^L}{\partial \bar{A}_r^C} = -p_y A_T \frac{\partial^2 f^I}{\partial \left(A_r^I\right)^2}\left(A_T - \bar{A}_r^C\right) > 0 \qquad \text{(A.8)}$$

when $A_T - \bar{A}_r^I \le \bar{A}_r^C \le A^*$ and $\frac{\partial \Delta \Pi^L}{\partial \bar{A}_r^C} = 0$ otherwise. Expression (A.8) is always positive, since by assumption production function exhibits diminishing returns to scale. Therefore, $\Delta \Pi^L$ increases as $\bar{A}_r^C$ increases from 0 up to the point $\bar{A}_r^C = A^*$, and then stays constant.

The first order condition with respect to $\bar{A}_r^I$ is as follows:

$$\frac{\partial \Delta \Pi^L}{\partial \bar{A}_r^I} = -p_y A_T \frac{\partial^2 f^C}{\partial \left(A_r^C\right)^2}\left(A_T - \bar{A}_r^I\right) > 0 \tag{A.9}$$

when $A^* \leq A_T - \bar{A}_r^I \leq \bar{A}_r^C$ and $\frac{\partial \Delta \Pi^L}{\partial \bar{A}_r^I} = 0$ otherwise. $\Delta\Pi^L$ increases as $\bar{A}_r^I$ increases from 0 up to the point $\bar{A}_r^I = A_T - A^*$, and then stays constant.

Combining the two results, we conclude that maximum $\Delta\Pi^L$ is attained when $\bar{A}_r^C \geq A^*$, $\bar{A}_r^I \geq A_T - A^*$, i.e. when land holdings restrictions don't preclude the optimal allocation of land to take place in the market.

Finally, the first order conditions for maximizing the total welfare $W = \Pi^C + \Pi^I + \Pi^L$ in the economy can be determined from the problem of maximizing $\Delta W = \Delta\Pi^C + \Delta\Pi^I + \Delta\Pi^L$. In particular, combining results of (A.2), (A.5), and (A.8), one can see that

$$\frac{\partial \Delta W}{\partial \bar{A}_r^C} = -p_y \frac{\partial f^C}{\partial A_r^C}\left(\bar{A}_r^C\right)\cdot\left(E^C - 1\right) + p_y \frac{\partial f^I}{\partial A_r^I}\left(A_T - \bar{A}_r^C\right)\cdot\left(E^I - 1\right) - p_y A_T \frac{\partial^2 f^I}{\partial \left(A_r^I\right)^2}\left(A_T - \bar{A}_r^C\right) \tag{A.10}$$

when $A_T - \bar{A}_r^I \leq \bar{A}_r^C \leq A^*$, and $\frac{\partial \Delta W}{\partial \bar{A}_r^C} = 0$ otherwise. Using the well-known expressions of the first and second derivatives of the Cobb-Douglas production functions $\frac{\partial f}{\partial A}(A) = \frac{E \cdot f(A)}{A}$ and $\frac{\partial^2 f}{\partial A^2}(A) = \frac{E\cdot(E-1)\cdot f(A)}{A^2}$, one can rewrite (A.10) to obtain, after some rearrangements,

$$\frac{\partial \Delta W}{\partial \bar{A}_r^C} = -\frac{p_y f^C\left(\bar{A}_r^C\right)\cdot E^C \cdot\left(E^C - 1\right)}{\bar{A}_r^C} + \frac{p_y f^I\left(A_T - \bar{A}_r^C\right)\cdot E^I \cdot\left(E^I - 1\right)}{A_T - \bar{A}_r^C}\cdot\left(-\frac{\bar{A}_r^C}{A_T - \bar{A}_r^C}\right) > 0. \tag{A.11}$$

Therefore, ΔW increases as $\bar{A}_r^C$ increases from 0 up to the point $\bar{A}_r^C = A^*$, and then stays constant.

Combining results of (A.3), (A.6), and (A.9), one can see that

$$\frac{\partial \Delta W}{\partial \bar{A}_r^I} = p_y \frac{\partial f^C}{\partial A_r^C}\left(A_T - \bar{A}_r^I\right)\cdot\left(E^C - 1\right) - p_y \frac{\partial f^I}{\partial A_r^I}\left(\bar{A}_r^I\right)\cdot\left(E^I - 1\right) - p_y A_T \frac{\partial^2 f^C}{\partial \left(A_r^C\right)^2}\left(A_T - \bar{A}_r^I\right) \tag{A.12}$$

when $A^* \leq A_T - \bar{A}_r^I \leq \bar{A}_r^C$, and $\frac{\partial \Delta W}{\partial \bar{A}_r^I} = 0$ otherwise. One can rewrite (A.12) to obtain, after some rearrangements,

$$\frac{\partial \Delta W}{\partial \bar{A}_r^I} = -\frac{p_y f^I\left(\bar{A}_r^I\right)\cdot E^I \cdot\left(E^I - 1\right)}{\bar{A}_r^I} + \frac{p_y f^C\left(A_T - \bar{A}_r^I\right)\cdot E^C \cdot\left(E^C - 1\right)}{A_T - \bar{A}_r^I}\cdot\left(-\frac{\bar{A}_r^I}{A_T - \bar{A}_r^I}\right) > 0. \tag{A.13}$$

Therefore, ΔW increases as $\bar{A}_r^I$ increases from 0 up to the point $\bar{A}_r^I = A_T - A^*$, and then stays constant.

Combining the two results, we conclude that maximum ΔW is attained when $\bar{A}_r^C \geq A^*$, $\bar{A}_r^I \geq A_T - A^*$, i.e. when land holdings restrictions don't preclude the optimal allocation of land to take place in the market. Q.E.D.

Proof of Proposition 2

To find the conditions for the maximum level of surplus for corporate farms, it is necessary to solve the problem of maximizing $\Delta\Pi^C\left(\bar{A}_r^C, \bar{A}_r^I\right)$ with respect to its variables. Combining (12), (15), and (8), and using the fact the output elasticities are assumed to be constant, one can arrive at the following analytical expression for $\Delta\Pi^C$:

$$\Delta\Pi^C = \begin{cases} \left(p_y f^C\left(A^*\right) - p_y f^C\left(\bar{A}_r^C\right)\right)\cdot\left(E^C - 1\right), & A_T - \bar{A}_r^I \leq \bar{A}_r^C \leq A_t^* \\ \left(p_y f^C\left(A^*\right) - p_y f^C\left(A_t^*\right)\right)\cdot\left(E^C - 1\right), & A_T - \bar{A}_r^I \leq A_t^* \leq \bar{A}_r^C \\ \left(p_y f^C\left(A^*\right) - p_y f^C\left(A_T - \bar{A}_r^I\right)\right)\cdot\left(E^C - 1\right), & A_t^* \leq A_T - \bar{A}_r^I \leq \bar{A}_r^C \end{cases} \quad \text{(A.14)}$$

The first order condition with respect to $\bar{A}_r^C$ is

$$\frac{\partial\Delta\Pi^C}{\partial\bar{A}_r^C} = -p_y \frac{\partial f^C}{\partial A_r^C}\left(\bar{A}_r^C\right)\cdot\left(E^C - 1\right) > 0 \quad \text{(A.15)}$$

when $A_T - \bar{A}_r^I \leq \bar{A}_r^C \leq A_t^*$ and $\frac{\partial\Delta\Pi^C}{\partial\bar{A}_r^C} = 0$ otherwise. The first order condition with respect to $\bar{A}_r^I$ is as follows:

$$\frac{\partial\Delta\Pi^C}{\partial\bar{A}_r^I} = p_y \frac{\partial f^C}{\partial A_r^C}\left(A_T - \bar{A}_r^I\right)\cdot\left(E^C - 1\right) < 0 \quad \text{(A.16)}$$

when $A_t^* \leq A_T - \bar{A}_r^I \leq \bar{A}_r^C$ and $\frac{\partial\Delta\Pi^C}{\partial\bar{A}_r^I} = 0$ otherwise. It is easy to see that (A.15) and (A.16) are exactly the same as (A.2) and (A.3), which allows to immediately arrive at the conclusion that maximum $\Delta\Pi^C$ is attained when $\bar{A}_r^I = 0$, $\bar{A}_r^C = A_T$, i.e. when individual farms are not allowed to rent any land, and all available land is rented by corporate farms.

Conditions for the maximum level of surplus for individual farms can be found in a similar fashion. Combining (13), (16), and (9), and using the fact the output elasticities are assumed to be constant, one can arrive at the following analytical expression for $\Delta\Pi^I$:

$$\Delta\Pi^I = \begin{cases} \left(p_y f^I\left(A_T - A^*\right) - p_y f^I\left(A_T - \bar{A}_r^C\right)\right)\cdot\left(E^I - 1\right), & A_T - \bar{A}_r^I \leq \bar{A}_r^C \leq A_t^* \\ \left(p_y f^I\left(A_T - A^*\right) - p_y f^I\left(A_T - A_t^*\right)\right)\cdot\left(E^I - 1\right), & A_T - \bar{A}_r^I \leq A_t^* \leq \bar{A}_r^C \\ \left(p_y f^I\left(A_T - A^*\right) - p_y f^I\left(\bar{A}_r^I\right)\right)\cdot\left(E^I - 1\right), & A_t^* \leq A_T - \bar{A}_r^I \leq \bar{A}_r^C \end{cases} \quad \text{(A.17)}$$

The first order condition with respect to $\bar{A}_r^C$ is

$$\frac{\partial \Delta \Pi^I}{\partial \bar{A}_r^C} = p_y \frac{\partial f^I}{\partial A_r^I}\left(A_T - \bar{A}_r^C\right)\cdot\left(E^I - 1\right) < 0 \tag{A.18}$$

when $A_T - \bar{A}_r^I \le \bar{A}_r^C \le A_t^*$ and $\frac{\partial \Delta \Pi^I}{\partial \bar{A}_r^C} = 0$ otherwise. The first order condition with respect to $\bar{A}_r^I$ is as follows:

$$\frac{\partial \Delta \Pi^I}{\partial \bar{A}_r^I} = -p_y \frac{\partial f^I}{\partial A_r^I}\left(\bar{A}_r^I\right)\cdot\left(E^I - 1\right) > 0 \tag{A.19}$$

when $A_t^* \le A_T - \bar{A}_r^I \le \bar{A}_r^C$ and $\frac{\partial \Delta \Pi^I}{\partial \bar{A}_r^I} = 0$ otherwise. It is easy to see that (A.18) and (A.19) are exactly the same as (A.5) and (A.6), which allows to immediately arrive at the conclusion that maximum $\Delta\Pi^I$ is attained when $\bar{A}_r^C = 0$, $\bar{A}_r^I = A_T$, i.e. when corporate farms are not allowed to rent any land, and all available land is rented by individual farms.

To find the conditions for the maximum level of surplus for landowners, combine (14), (17), and (10) to arrive at the following expression for $\Delta\Pi^L$:

$$\Delta\Pi^L = \begin{cases} -A_T\left(p_y \frac{\partial f^I}{\partial A_r^I}\left(A_T - A^*\right) - \left(p_y \frac{\partial f^I}{\partial A_r^I}\left(A_T - \bar{A}_r^C\right) - t\right)\right), & A_T - \bar{A}_r^I \le \bar{A}_r^C \le A_t^* \\ -A_T\left(p_y \frac{\partial f^C}{\partial A_r^C}\left(A^*\right) - p_y \frac{\partial f^C}{\partial A_r^C}\left(A_t^*\right)\right), & A_T - \bar{A}_r^I \le A_t^* \le \bar{A}_r^C \\ -A_T\left(p_y \frac{\partial f^C}{\partial A_r^C}\left(A^*\right) - p_y \frac{\partial f^C}{\partial A_r^C}\left(A_T - \bar{A}_r^I\right)\right), & A_t^* \le A_T - \bar{A}_r^I \le \bar{A}_r^C \end{cases} \tag{A.20}$$

The first order condition with respect to $\bar{A}_r^C$ is as follows:

$$\frac{\partial \Delta \Pi^L}{\partial \bar{A}_r^C} = -p_y A_T \frac{\partial^2 f^I}{\partial \left(A_r^I\right)^2}\left(A_T - \bar{A}_r^C\right) > 0 \tag{A.21}$$

when $A_T - \bar{A}_r^I \le \bar{A}_r^C \le A_t^*$ and $\frac{\partial \Delta \Pi^L}{\partial \bar{A}_r^C} = 0$ otherwise. The first order condition with respect to $\bar{A}_r^I$ is as follows:

$$\frac{\partial \Delta \Pi^L}{\partial \bar{A}_r^I} = -p_y A_T \frac{\partial^2 f^C}{\partial \left(A_r^C\right)^2}\left(A_T - \bar{A}_r^I\right) > 0 \tag{A.22}$$

when $A_t^* \le A_T - \bar{A}_r^I \le \bar{A}_r^C$ and $\frac{\partial \Delta \Pi^L}{\partial \bar{A}_r^I} = 0$ otherwise. It is easy to see that (A.21) and (A.22) are exactly the same as (A.8) and (A.9), which allows to immediately arrive at the conclusion that maximum $\Delta\Pi^L$ is attained when $\bar{A}_r^C \ge A_t^*$, $\bar{A}_r^I \ge A_T - A_t^*$, i.e. when land holdings restrictions don't preclude the optimal allocation of land to take place in the market.

The rest of the proof is analogous to the proof of Proposition 1. Q.E.D.

Proof of Proposition 3

To find the conditions for the maximum level of surplus for corporate farms, it is necessary to solve the problem of maximizing $\Delta\Pi^C\left(\bar{A}_r^C,\bar{A}_r^I\right)$ with respect to its variables. Combining (21), (24), and (38), and using the fact the output elasticities are assumed to be constant, one can arrive at the following analytical expression for $\Delta\Pi^C$:

$$\Delta\Pi^C=\begin{cases}\left(p_y f^C\left(A^*\right)-p_y f^C\left(\bar{A}_r^C\right)\right)\cdot\left(E^C-1\right)+\bar{A}_r^C A_t^M\dfrac{\partial r}{\partial A_r^C}\left(A_t^M\right), & A_T-\bar{A}_r^I\le\bar{A}_r^C\le A_t^M\\ p_y f^C\left(A^*\right)\cdot\left(E^C-1\right)+p_y f^C\left(A_t^M\right)-p_y f^I\left(A_t^M\right)\cdot E^I, & A_T-\bar{A}_r^I\le A_t^M\le\bar{A}_r^C\\ p_y f^C\left(A^*\right)\cdot\left(E^C-1\right)+p_y f^C\left(A_t^{M(n)}\right)-A_t^{M(n)}\cdot\left(p_y\dfrac{\partial f^I}{\partial A_r^I}\left(\bar{A}_r^I\right)-t\right), & A_t^M\le A_T-\bar{A}_r^I\le\bar{A}_r^C\end{cases}\quad\text{(A.23)}$$

The first order condition with respect to $\bar{A}_r^C$ is

$$\frac{\partial\Delta\Pi^C}{\partial\bar{A}_r^C}=-p_y\frac{\partial f^C}{\partial A_r^C}\left(\bar{A}_r^C\right)\cdot\left(E^C-1\right)+A_t^M\frac{\partial r}{\partial A_r^C}\left(A_t^M\right)>0\quad\text{(A.24)}$$

when $A_T-\bar{A}_r^I\le\bar{A}_r^C\le A_t^M$ and $\dfrac{\partial\Delta\Pi^C}{\partial\bar{A}_r^C}=0$ otherwise. Expression (A.24) is always positive, since by assumptions $E^C<1$, production function is increasing in *A*, and $\dfrac{\partial r}{\partial A_r^C}\left(A\right)>0\quad\forall A$. Therefore, $\Delta\Pi^C$ increases as $\bar{A}_r^C$ increases from 0 up to the point $\bar{A}_r^C=A_t^M$, and then stays constant.

The first order condition with respect to $\bar{A}_r^I$ is as follows:

$$\begin{gathered}\frac{\partial\Delta\Pi^C}{\partial\bar{A}_r^I}=p_y\frac{\partial f^C}{\partial A_r^C}\left(A_t^{M(n)}\right)\cdot\frac{\partial A_t^{M(n)}}{\partial\bar{A}_r^I}\left(\bar{A}_r^I\right)-\\ -\frac{\partial A_t^{M(n)}}{\partial\bar{A}_r^I}\left(\bar{A}_r^I\right)\cdot\left(p_y\frac{\partial f^I}{\partial A_r^I}\left(\bar{A}_r^I\right)-t\right)-A_t^{M(n)}p_y\frac{\partial^2 f^I}{\partial\left(A_r^I\right)^2}\left(\bar{A}_r^I\right)>0\end{gathered}\quad\text{(A.25)}$$

when $A_t^M\le A_T-\bar{A}_r^I\le\bar{A}_r^C$ and $\dfrac{\partial\Delta\Pi^C}{\partial\bar{A}_r^I}=0$ otherwise. Expression (A.25) is always positive, which can be shown using the following arguments. Expression $-A_t^{M(n)}p_y\dfrac{\partial^2 f^I}{\partial\left(A^I\right)^2}\left(\bar{A}_r^I\right)$ is always positive. Expression $p_y\dfrac{\partial f^C}{\partial A_r^C}\left(A_t^{M(n)}\right)\cdot\dfrac{\partial A_t^{M(n)}}{\partial\bar{A}_r^I}\left(\bar{A}_r^I\right)-\dfrac{\partial A_t^{M(n)}}{\partial\bar{A}_r^I}\left(\bar{A}_r^I\right)\cdot\left(p_y\dfrac{\partial f^I}{\partial A_r^I}\left(\bar{A}_r^I\right)-t\right)$ can be rearranged to look like $\dfrac{\partial A_t^{M(n)}}{\partial\bar{A}_r^I}\left(\bar{A}_r^I\right)\cdot\left(p_y\dfrac{\partial f^C}{\partial A_r^C}\left(A_t^{M(n)}\right)-\left(p_y\dfrac{\partial f^I}{\partial A_r^I}\left(\bar{A}_r^I\right)-t\right)\right)$. Expression in the parentheses is positive because $A_t^{M(n)} < A_T - \bar{A}_t^I$. To show that expression $\frac{\partial A_t^{M(n)}}{\partial \bar{A}_r^I}\left(\bar{A}_r^I\right)$ is also positive, observe that, according to (31),

$$A_t^{M(1)} = A_t^{M(0)} \cdot \left(\frac{2 p_y \frac{\partial f^C}{\partial A_r^C}\left(A_t^{M(0)}\right) - a^{(1)}}{2 p_y \frac{\partial f^C}{\partial A_r^C}\left(A_t^{M(0)}\right) - a^{(0)}} \right).$$

The partial derivative of $A_t^{M(1)}$ with respect to $\bar{A}_r^I$ is

$$\frac{\partial A_t^{M(1)}}{\partial \bar{A}_r^I} = -\frac{A_t^{M(0)} \frac{\partial a^{(1)}}{\partial \bar{A}_r^I}}{2 p_y \frac{\partial f^C}{\partial A_r^C}\left(A_t^{M(0)}\right) - a^{(0)}}.$$

According to (28),

$$a^{(1)} = a^{(0)} \cdot \left(1 + \frac{A_T^{(0)} - A_T^{(1)}}{A_T^{(0)}}\right) = a^{(0)} \cdot \left(1 + \frac{A_T - \bar{A}_r^I - A_t^{M(0)}}{A_T}\right).$$

Therefore, $\frac{\partial a^{(1)}}{\partial \bar{A}_r^I} = -\frac{a^{(0)}}{A_T} < 0$, and thus $\frac{\partial A_t^{M(1)}}{\partial \bar{A}_r^I} > 0$. In other words, as land holdings restrictions on individual farms become less stringent, marginal cost line for corporate farms shifts upward at a lower rate, and amount of land chosen by corporate farms become bigger on each iteration, leading to the overall increase in $A_t^{M(n)}$. Therefore, we conclude that $\frac{\partial A_t^{M(n)}}{\partial \bar{A}_r^I}\left(\bar{A}_r^I\right) > 0$.

In conclusion, $\Delta\Pi^C$ increases as $\bar{A}_r^I$ increases from 0 up to the point $\bar{A}_r^I = A_T - A_t^M$, and then stays constant. In other words, maximum value of $\Delta\Pi^C$ with respect to $\bar{A}_r^I$ is attained when $\bar{A}_r^I \geq A_T - A_t^M$.

Combining the two results, we conclude that maximum $\Delta\Pi^C$ is attained when $\bar{A}_r^I \geq A_T - A_t^M$ and $\bar{A}_r^C \geq A_t^M$, i.e. when restrictions on both types of farms preclude any land from being taken away from the market.

Conditions for the maximum level of surplus for individual farms can be found in a similar fashion. Combining (22), (25), and (39), and using the fact the output elasticities are assumed to be constant, one can arrive at the following analytical expression for $\Delta\Pi^I$:

$$\Delta\Pi^I = \begin{cases} \left(p_y f^I\left(A_T - A^*\right) - p_y f^I\left(A_T - \bar{A}_r^C\right)\right)\cdot\left(E^I - 1\right), & A_T - \bar{A}_r^I \leq \bar{A}_r^C \leq A_t^M \\ \left(p_y f^I\left(A_T - A^*\right) - p_y f^I\left(A_T - A_t^M\right)\right)\cdot\left(E^I - 1\right), & A_T - \bar{A}_r^I \leq A_t^M \leq \bar{A}_r^C \\ \left(p_y f^I\left(A_T - A^*\right) - p_y f^I\left(\bar{A}_r^I\right)\right)\cdot\left(E^I - 1\right), & A_t^M \leq A_T - \bar{A}_r^I \leq \bar{A}_r^C \end{cases} \quad \text{(A.26)}$$

The first order condition with respect to $\bar{A}_r^C$ is

$$\frac{\partial \Delta \Pi^I}{\partial \bar{A}_r^C} = p_y \frac{\partial f^I}{\partial A_r^I}\left(A_T - \bar{A}_r^C\right)\cdot\left(E^I - 1\right) < 0 \qquad \text{(A.27)}$$

when $A_T - \bar{A}_r^I \le \bar{A}_r^C \le A_t^M$ and $\frac{\partial \Delta \Pi^I}{\partial \bar{A}_r^C} = 0$ otherwise. The first order condition with respect to $\bar{A}_r^I$ is as follows:

$$\frac{\partial \Delta \Pi^I}{\partial \bar{A}_r^I} = -p_y \frac{\partial f^I}{\partial A_r^I}\left(\bar{A}_r^I\right)\cdot\left(E^I - 1\right) > 0 \qquad \text{(A.28)}$$

when $A_t^M \le A_T - \bar{A}_r^I \le \bar{A}_r^C$ and $\frac{\partial \Delta \Pi^I}{\partial \bar{A}_r^I} = 0$ otherwise. It is easy to see that (A.27) and (A.28) are exactly the same as (A.5) and (A.6), which allows to immediately arrive at the conclusion that maximum $\Delta \Pi^I$ is attained when $\bar{A}_r^C = 0$, $\bar{A}_r^I = A_T$, i.e. when corporate farms are not allowed to rent any land, and all available land is rented by individual farms.

Combining (23), (26), and (40) to arrive at the following expression for $\Delta \Pi^L$:

$$\Delta \Pi^L = \begin{cases} -A_T\left(p_y \frac{\partial f^I}{\partial A_r^I}\left(A_T - A^*\right) - \left(p_y \frac{\partial f^I}{\partial A_r^I}\left(A_T - \bar{A}_r^C\right) - t\right)\right), & A_T - \bar{A}_r^I \le \bar{A}_r^C \le A_t^M \\ -A_T\left(p_y \frac{\partial f^I}{\partial A_r^I}\left(A_T - A^*\right) - p_y \frac{\partial f^I}{\partial A_r^I}\left(A_T - A_t^M\right)\right), & A_T - \bar{A}_r^I \le A_t^M \le \bar{A}_r^C \\ -p_y \frac{\partial f^I}{\partial A_r^I}\left(A_T - A^*\right)\cdot A_T + \left(p_y \frac{\partial f^I}{\partial A_r^I}\left(\bar{A}_r^I\right) - t\right)\cdot\left(\bar{A}_r^I + A_t^{M(n)}\right), & A_t^M \le A_T - \bar{A}_r^I \le \bar{A}_r^C \end{cases} \qquad \text{(A.29)}$$

The first order conditions are as follows:

$$\frac{\partial \Delta \Pi^L}{\partial \bar{A}_r^C} = -p_y A_T \frac{\partial^2 f^I}{\partial \left(A_r^I\right)^2}\left(A_T - \bar{A}_r^C\right), \qquad \text{(A.30)}$$

$$\frac{\partial \Delta \Pi^L}{\partial \bar{A}_r^I} = \left(1 + \frac{\partial A_t^{M(n)}}{\partial \bar{A}_r^I}\right)\cdot\left(p_y \frac{\partial f^I}{\partial A_r^I}\left(\bar{A}_r^I\right) - t\right) + \left(\bar{A}_r^I + A_t^{M(n)}\right)\cdot p_y \frac{\partial^2 f^I}{\partial \left(A_r^I\right)^2}\left(\bar{A}_r^I\right). \qquad \text{(A.31)}$$

Then, the first order conditions for maximizing the total welfare $W = \Pi^C + \Pi^I + \Pi^L$ in the economy can be determined from the problem of maximizing $\Delta W = \Delta \Pi^C + \Delta \Pi^I + \Delta \Pi^L$. In particular, combining results of (A.24), (A.27), and (A.30), one can see that

$$\begin{aligned} \frac{\partial \Delta W}{\partial \bar{A}_r^C} = & -p_y \frac{\partial f^C}{\partial A_r^C}\left(\bar{A}_r^C\right)\cdot\left(E^C - 1\right) + A_t^M \frac{\partial r}{\partial A_r^C}\left(A_t^M\right) + \\ & + p_y \frac{\partial f^I}{\partial A_r^I}\left(A_T - \bar{A}_r^C\right)\cdot\left(E^I - 1\right) - p_y A_T \frac{\partial^2 f^I}{\partial \left(A_r^I\right)^2}\left(A_T - \bar{A}_r^C\right) \end{aligned} \qquad \text{(A.32)}$$

when $A_T - \bar{A}_r^I \leq \bar{A}_r^C \leq A_t^M$, and $\frac{\partial \Delta W}{\partial \bar{A}_r^C} = 0$ otherwise. Using expressions of the first and second derivatives of the Cobb-Douglas production function, one can rewrite (A.10) to obtain, after some rearrangements,

$$\begin{aligned}\frac{\partial \Delta W}{\partial \bar{A}_r^C} &= -\frac{p_y f^C\left(\bar{A}_r^C\right)\cdot E^C\cdot\left(E^C-1\right)}{\bar{A}_r^C} + A_t^M \frac{\partial r}{\partial A_r^C}\left(A_t^M\right) + \\ &+ \frac{p_y f^I\left(A_T - \bar{A}_r^C\right)\cdot E^I\cdot\left(E^I-1\right)}{A_T - \bar{A}_r^C}\cdot\left(-\frac{\bar{A}_r^C}{A_T - \bar{A}_r^C}\right) > 0.\end{aligned} \tag{A.33}$$

Therefore, ΔW increases as $\bar{A}_r^C$ increases from 0 up to the point $\bar{A}_r^C = A_t^M$, and then stays constant.

Combining results of (A.25), (A.28), and (A.31), one can see that

$$\begin{aligned}\frac{\partial \Delta W}{\partial \bar{A}_r^I} &= p_y \frac{\partial f^C}{\partial A_r^C}\left(A_t^{M(n)}\right)\cdot\frac{\partial A_t^{M(n)}}{\partial \bar{A}_r^I}\left(\bar{A}_r^I\right) - \frac{\partial A_t^{M(n)}}{\partial \bar{A}_r^I}\left(\bar{A}_r^I\right)\cdot\left(p_y \frac{\partial f^I}{\partial A_r^I}\left(\bar{A}_r^I\right) - t\right) - \\ &- A_t^{M(n)} p_y \frac{\partial^2 f^I}{\partial\left(A_r^I\right)^2}\left(\bar{A}_r^I\right) - p_y \frac{\partial f^I}{\partial A_r^I}\left(\bar{A}_r^I\right)\cdot\left(E^I-1\right) + \\ &+ \left(1 + \frac{\partial A_t^{M(n)}}{\partial \bar{A}_r^I}\right)\cdot\left(p_y \frac{\partial f^I}{\partial A_r^I}\left(\bar{A}_r^I\right) - t\right) + \left(\bar{A}_r^I + A_t^{M(n)}\right)\cdot p_y \frac{\partial^2 f^I}{\partial\left(A_r^I\right)^2}\left(\bar{A}_r^I\right)\end{aligned} \tag{A.34}$$

when $A_t^M \leq A_T - \bar{A}_r^I \leq \bar{A}_r^C$, and $\frac{\partial \Delta W}{\partial \bar{A}_r^I} = 0$ otherwise. One can rewrite (A.34) to obtain, after some rearrangements,

$$\frac{\partial \Delta W}{\partial \bar{A}_r^I} = \frac{p_y f^I\left(\bar{A}_r^I\right)\cdot E^I}{\bar{A}_r^I} - t + p_y \frac{\partial A_t^{M(n)}}{\partial \bar{A}_r^I}\left(\bar{A}_r^I\right)\cdot\frac{\partial f^C}{\partial A^C}\left(A_t^{M(n)}\right). \tag{A.35}$$

Sign of (A.35) depends on the value of *t*:

–if $\frac{p_y f^I\left(\bar{A}_r^I\right)\cdot E^I}{\bar{A}_r^I} + p_y \frac{\partial A_t^{M(n)}}{\partial \bar{A}_r^I}\left(\bar{A}_r^I\right)\cdot\frac{\partial f^C}{\partial A^C}\left(A_t^{M(n)}\right) > t$, ΔW increases as $\bar{A}_r^I$ increases from 0 up to the point $\bar{A}_r^I = A_T - A_t^M$, and then stays constant. In other words, when transaction costs are relatively small, maximum ΔW is attained when land holdings restrictions on corporate and individual farms preclude any land from being taken away from the market;

–if $\frac{p_y f^I\left(\bar{A}_r^I\right)\cdot E^I}{\bar{A}_r^I} + p_y \frac{\partial A_t^{M(n)}}{\partial \bar{A}_r^I}\left(\bar{A}_r^I\right)\cdot\frac{\partial f^C}{\partial A^C}\left(A_t^{M(n)}\right) < t$, ΔW decreases as $\bar{A}_r^I$ increases from 0 up to the point $\bar{A}_r^I = A_T - A_t^M$, and then stays constant. In other words, maximum ΔW is attained when $\bar{A}_r^I = 0$, $\bar{A}_r^C = A_T$, i.e. relatively high transaction costs push individual firms away from the market. Q.E.D.