Consumer Welfare Measures: Some Comparative Results

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Consumer's surplus has played an important role in evaluating agricultural price support programs as well as in other applications. Gardner and Wallace provide examples of the use of consumer's surplus in theoretical models, while numerous empirical studies (for example, Anderson, Sedjo and Wiseman, Johnson and Norton) have used the concept to measure welfare costs. Since its introduction by Dupuit and Marshall, the interpretation of consumer's surplus has been subject to change and controversy. In an important paper, Willig demonstrated that consumer's surplus is bounded by compensating variation (C) and equivalent variation (E) and derived rules of thumb showing how closely consumer's surplus approximates these appropriate welfare measures. These rules are compact and readily understandable to researchers doing applied welfare analysis.

The derivation of these rules is based on the assumption of a constant income elasticity. Willig developed another set of bounds for the more realistic case of nonconstant income elasticity. However, these latter bounds are not as readily understandable and are considerably wider than the former set. For example, if over the region under consideration the income elasticity varies from .9 to 1.1 and consumer surplus (A) is 20% of income, then for the wider bounds formula, $\frac{C - A}{A}$ is between .09 and .110. The error bounds with the compact formula are much smaller, .009 to .011, although they presumably might be inaccurate because the income elasticity was not constant.

The purpose of this paper is to empirically investigate how well the more compact formula approximates the actual errors which arise from the use of the consumer's surplus. This investigation is conducted for several alternative demand models including the Linear Expenditure System, the Almost Ideal Demand System, and linear and log-linear single equation models. The parameters of these systems are varied so that a wide range of economic behavior in terms of price and income elasticities is covered. Since it is possible to compute economic welfare measures such as compensating and equivalent variation from these systems, it is also possible to compare how well consumer surplus approximates these true measures. Since Paasche and Laspeyres variations are often considered useful approximations (bounds) to C and E variation, the error in these measures is investigated. The impact of errors in consumer's surplus on the computation of deadweight loss is also investigated.

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I. Welfare Measures from Complete Demand Systems

The cornerstone of consumer welfare measurement is the expenditure function,

$$(1) \qquad m = m(p, \mu),$$

where m is the (minimum) expenditure needed to achieve utility level μ under price vector p. For utility level μ° (utility achieve at p° and m°) consider the welfare measure for a single price change (say p_1° to p_1'),

$$(2) \quad C = \int_{p_1^\circ}^{p_1'} h_1 (p, \mu^\circ)\, dp_1 = m(p', \mu^\circ) - m^\circ,$$

where $h_1(p,\mu^\circ)$ is the Hicksian or compensated demand function for q_1.

Equation (2), is the money metric of the welfare loss from p_1° to p_1', or compensated variation. Similarly,

$$(3) \quad E = \int_{p_1^\circ}^{p_1'} h_1 (p, \mu')dp_1 = m (p', \mu') - m (p^\circ, \mu')$$

is equivalent variation.

The two definitions, (2) and (3), are similar to consumer's surplus in that they integrate a demand function. However, consumer's surplus as it has come to be known over the years, is given by

$$(4) \qquad A = \int_{p_1^\circ}^{p_1'} q_1\,(p, m^\circ)\, dp_1,$$

where $q_1(p, m^\circ)$ is the Marshallian demand function.

Willig has shown that

$$(5) \qquad \frac{\eta_L \mid A \mid}{2m^\circ} \leq \frac{C - A}{\mid A \mid} \leq \frac{\eta_S \mid A \mid}{2m^\circ}$$

and

$$(6) \qquad \frac{\eta_L \mid A \mid}{2m^\circ} \leq \frac{A - E}{\mid A \mid} \leq \frac{\eta_S \mid A \mid}{2m^\circ}$$

where η_L and η_S are, respectively, the largest and smallest values of the expenditure elasticity in the region under consideration. As a precondition these formulae require that

$$(7) \qquad \left| \frac{\eta_S A}{2m^\circ} \right| \leq .05$$

$$(8) \qquad \left| \frac{\eta_L A}{2m_\circ} \right| \leq .05$$

$$(9) \qquad \left| \frac{A}{m_\circ} \right| \leq .9.$$

These bounds were derived under the assumption of a constant income elasticity of demand (Willig, pp. 592–3). However, only complete demand systems arising from homothetic utility functions have constant expenditure elasticities[1]. Furthermore, these constant elasticities must equal unity, so (5) and (6) hold only approximately. The numerical results reported here examine how accurate that approximation is.

To assess the accuracy of the compact bounds, two widely used demand systems, the Linear Expenditure System (LES) and the Almost Ideal Demand System (AIDS) were employed.[2] The methodology employed to assess the approximation is as follows. For each demand system, both composed of two goods, a set of parameters was chosen which satisfies the regularity conditions. These regularity conditions are described by various restrictions on parameters of the utility function that guarantee positive but downward sloping marginal utilities, homogeneity, symmetry, etc. By varying the values of the parameters in each system, it was possible to vary the own-price elasticity from very inelastic ($-.1$) to highly elastic (-5.0). It was possible to vary the cross price effects from substitutes to complements and to make these effects weak or strong. It was also possible to substantially vary the budget share and expenditure elasticity of each good. Hence, a fairly wide range of consumer behavior was represented by these demand systems.

Under constant tastes Laspeyres and Paasche variations are upper and lower limits on compensating and equivalent variation, respectively. The analysis measures how well these sums approximate compensating and equivalent variation. These measures are given by

$$(10) \qquad L = q_1^\circ\,(p_1' - p_1^\circ)$$

and

$$(11) \qquad P = q_1'\,(p_1' - p_1^\circ).$$

Hausman showed that even small errors, introduced by using consumer's surplus to measure compensating variation, could result in large errors in measuring the deadweight loss. For each case considered, the deadweight loss is computed using compensated variation and consumer's surplus. The ratio of these two deadweight losses is presented in the Tables.

The Linear Expenditure System

For the LES, maximization of the utility function

$$(12) \qquad \mu = \prod_{i=1}^{n} (q_i - \gamma_i)_i^{\alpha}, \qquad \Sigma\alpha_i = 1.0, \quad 0 < \alpha_i < 1.$$

yields the demand functions,

$$(13) \qquad q_i = \gamma_i + \alpha_i\, p_i^{-1}\,(m - \sum_{j=1}^{3} p_j\gamma_j), \qquad i = 1, \ldots, 3.$$

Assuming the price of the first good changes, consumer's surplus for the LES is

$$(14) \qquad A_{LES} = [\gamma_1(1-\alpha_1)p_1 + \alpha_1 \ln p_1\,(m - \sum_{j=2}^{3} p_j\gamma_j)]_{p_1^\circ}^{p_1'}.$$

The results for the Linear Expenditure System are given in Table 1. The compact bounds condition held in 14 of the 16 cases. In the two cases where they did not hold, the bounds were very close together (.0002 to .0003) and the discrepancies were extremely small (.0001). Hence, it was found that the compact error bounds held, or came

[1] For a treatment of constant, nonunitary price and income elasticities in an incomplete demand system see LaFrance.

[2] For an example of the former system, see Pollak and Wales, and of the latter see Deaton and Muellbauer.

extremely close to holding, in all cases. Another finding was that the measurement error is proportionate to the percentage price increase. For example, there are various combinations of own-price elasticities, expenditure elasticities, budget shares, and cross price effects. However, when prices increase 10% the consumer's surplus errors in measuring compensating and equivalent variation are almost always 1.4%. With elasticities and budget shares similar to those used for the 10% increase, but with a price increase five times greater, the errors are roughly five times as great, or around 6.5%. Hence, reasonably large errors can arise from the use of consumer's surplus as a measure of either compensating or equivalent variation. This error is in proportion to the price increase and does not appear to be strongly influenced by the own price elasticity, budget share, income elasticity or income level.

Most surprising of all was the extent to which the Laspeyres and Paasche sums failed to approximate compensating and equivalent variation. Based on Table 1 these sums cannot be regarded as reliable measures of consumer welfare. The errors in these sums are roughly four times as great as those found in consumer surplus and average 23% for the 50% price increase case. The errors for both the Paasche and Laspeyres are clearly greatest for goods with highly elastic demand and are greater for the Laspeyres.

This result for the Paasche and Laspeyres might seem to contradict the results of Cory *et al.* Their results showed that for small quantity changes, 10% or less, L and P will be within 5% of consumer's surplus. For many cases with either small price changes or low elasticities such will be the case. However, for others, such as the tax equivalence proposal that would double the price of alcoholic beverages, the percent change in quantity will not be small. It is not that the bounds established by Cory *et al.* are violated, they just increase as price increases. They cite the example that for 10% change in quantity, L and P will be within 5% of A. However, for a 50% change in quantity, L and P will be within 20% of A. Also their bounds require a linear demand curve, which is not used here. Given these problems, the use consumer's surplus is preferable.

The errors in the deadweight loss measurement are very large, confirming Hausman's finding. It is particularly disturbing that the errors are not a function of the amount of the price change. The average deadweight loss error for the 10% price increase case was 38%. For the 50% price increase case the error was 42%. The errors are largest for inelastic demands and/or goods with large budget shares.

The Almost Ideal Demand System

The second set of computations used the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer. The expenditure function for the AIDS model is

$$\text{(15)} \qquad \ln m = \ln P + \beta(p) \cdot \mu,$$

where P and $\beta(p)$ are price indexes given by

$$\text{(16)} \qquad \ln P = \alpha_0 + \sum_{i=1}^{n} \alpha_1 \ln p_i + \frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{n} \gamma_{ij} \ln p_i \ln p_j$$

and

$$\text{(17)} \qquad \beta(p) = \beta_0 \prod_{i=1}^{n} p_i^{\beta_i}.$$

The demand equations for this system are (in budget share form)

$$\text{(18)} \qquad w_i = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln p_j + \beta_i \ln\left(\frac{m}{p}\right), \; i = 1, \ldots n,$$

where $w_i = p_i q_i/m$.

Hence, consumer's surplus is given by

$$\begin{aligned} \text{(19)} \qquad A_{AIDS} = {} & [m^{\circ} \ln p_1(\alpha_1 + \sum_{j=2}^{n} \gamma_{1j} \ln p_j + \beta_1 \ln m^{\circ} + \frac{1}{2}\gamma_{11} \ln p_1) \\ & - \beta_1 m^{\circ} \ln p_1 (\frac{1}{2}\alpha_1 \ln p_1 + \sum_{j=2}^{n} \alpha_j \ln p_j \\ & + \frac{1}{6}\gamma_{11} (\ln p_1)^2 + \frac{1}{2}\sum_{j=2}^{n} \gamma_{jj}(\ln p_j)^2 \\ & + \frac{1}{2}\sum_{j=2}^{n} \gamma_{1j} \ln p_1 \ln p_j \\ & + \sum_{i=2}^{n}\sum_{j=2}^{n} \gamma_{ij} \ln p_i \ln p_j]_{p_1^{\circ}}^{p_1'}. \end{aligned}$$

The results for price increases of 10 and 50% are given Table 2.

The results for the AIDS system were somewhat similar to the LES. For the AIDS system, the income elasticity is more variable as a result of own price change than for the LES. This tends to produce a wider bound. Although, there were more violations of the bounds conditions than in the LES case, the size of these violations was not large. As before, the size of the price increase influenced the amount of error, although the variability is greater than for the LES. Again, the Paasche and Las-

Table 1. Selected Measures for the Linear Expenditure System with Price Increases of 10 and 50 percent

m^0	e_{11}	e_{12}	e_{13}	η^0	η^1	w_1^0	w_1^1	LHS	$\frac{C-A}{A}$	$\frac{E-A}{A}$	RHS	$\frac{L-C}{C}$	$\frac{E-P}{E}$	DWL
-10 Percent Price Increase-														
20.0	−.81	−.02	.02	.81	.79	.37	.38	.0142	.0144	.0142	.0144	.046	.047	1.39
5.0	−1.22	.19	.22	.82	.84	.37	.36	.0141	.0145	.0141	.0145	.046	.044	1.25
10.0†	−1.13	.10	.12	.90	.91	.33	.33	.0142	.0145	.0141	.0144	.041	.039	1.28
5.0	−1.81	−.39	.47	1.73	1.88	.17	.16	.0137	.0146	.0140	.0149	.077	.074	1.17
5.0	−2.27	−1.04	−1.24	4.54	5.21	.066	.058	.0134	.0148	.0138	.0154	.103	.100	1.13
5.0	−.52	−.06	.06	.52	.49	.58	.61	.0136	.0143	.0143	.0146	.010	.010	1.61
5.0†	−.95	−.03	−.02	.99	1.01	.30	.30	.0143	.0144	.0142	.0146	.033	.030	1.33
5.0	−.35	−.04	.04	.35	.33	.86	.92	.0137	.0143	.0143	.0148	.002	.001	1.91
-50 Percent Price Increase-														
20.0	−.81	−.02	.02	.81	.74	.37	.41	.0580	.0624	.0593	.0635	.112	.084	1.45
5.0	−1.22	.19	.22	.82	.93	.37	.32	.0576	.0647	.0573	.0650	.224	.184	1.28
10.0⁺	−1.13	.10	.12	.90	.96	.33	.31	.0590	.0641	.0578	.0630	.194	.158	1.31
5.0*	−1.81	−.39	.47	1.73	2.91	.17	.10	.0494	.0687	.0537	.0828	.422	.362	1.17
5.0*	−2.27	−1.04	−1.23	4.24	12.50	.07	.02	.0428	.0731	.0498	.1176	.634	.553	1.13
5.0*	−.52	−.06	.06	.52	.42	.58	.72	.0545	.0610	.0605	.0676	.045	.024	1.73
5.0⁺	−.95	−.03	−.02	1.00	.97	.30	.31	.0601	.0631	.0587	.0615	.220	.202	1.38
5.0*	−.42	−.04	.05	.42	.32	.72	.93	.0535	.0606	.0609	.0691	.023	.005	1.92

m^0 —Expenditure level m (in \$1,000)

e_{11} —Own price elasticity—good 1.

e_{12} —Cross price elasticity—good 1 with good 2.

e_{13} —Cross price elasticity—good 1 with good 3.

η^0 —Expenditure elasticity evaluated at prices p^0.

η^1 —Expenditure elasticity evaluated at prices p^1.

w_1^0 —Budget share good 1 at prices p^0.

w_1^1 —Budget share good 1 at prices p^1.

LHS —Bound condition, Left Hand Side of (7).

$\frac{C-A}{A}$—Error in measuring compensating variation with consumer surplus.

$\frac{E-A}{A}$ —Error in measuring equivalent variation with consumer surplus.

RHS —Bound condition, Right Hand Side of (8).

$\frac{L-C}{C}$ —Discrepancy between Laspeyres measure and compensating variation.

$\frac{E-P}{E}$ —Discrepancy between Paasche measure and equivalent variation.

DWL—Ratio of deadweight loss using compensating variation to deadweight loss using consumer surplus.
†Does not violate pre-bound conditions (7), (8), and (9). Violates either bound condition (5) or (6).

⁺Violates pre-bound condition (7) or (8). Hence, bounds do not apply. Either $\frac{C-A}{A}$ or $\frac{E-A}{A}$, or both lie outside bound limits.

*Violates pre-bound condition (7) or (8). Hence, bounds do not apply. However, both $\frac{C-A}{A}$ and $\frac{E-A}{A}$ are within the bounds given by (5) and (6).

Table 2. Selected Measures for the Almost Ideal Demand System with Price Increase of 10 and 50 Percent

m^0	e_{11}	e_{12}	e_{13}	η^0	η^1	w_1^0	w_1^1	LHS	$\frac{C-A}{A}$	$\frac{E-A}{A}$	RHS	$\frac{L-C}{C}$	$\frac{E-P}{E}$	DWL
5.0	−2.02	1.09	.62	.31	.28	.07	.07	.0009	.0009	.0009	.0010	.102	.038	1.02
10.0*	−.65	−.09	−.34	1.07	1.07	.14	.14	.0072	.0072	.0071	.0072	.025	.071	1.10
5.0*	−1.39	.36	−.05	1.08	1.23	.13	.13	.0067	.0067	.0067	.0067	.062	.064	1.10
10.0*	−.06	2.05	−3.18	1.19	1.20	.05	.05	.0031	.0032	.0032	.0032	.001	.115	1.03
5.0*	−1.55	.82	1.31	−.58	−.39	.08	.09	−.0020	−.0023	.0021	−.0014	.080	−.105	1.02
5.0*	−.63	−.09	−.36	1.08	1.07	.13	.13	.0068	.0069	.0068	.0068	.024	.030	1.19
-50 Percent Price Increase-														
5.0	−2.02	1.09	.62	.31	−.12	.07	.04	−.0014	.0004	.0024	.0035	.556	.357	1.00
10.0	−.65	−.09	−.34	1.07	1.07	.14	.15	.0313	.0327	.0313	.0327	.114	.133	1.20
5.0*	−1.39	.36	−.05	1.08	1.21	.13	.11	.0267	.0275	.0263	.0271	.302	.231	1.11
10.0	−.06	2.05	−3.18	1.19	1.15	.05	.07	.0147	.0149	.0147	.0152	.021	.118	1.11
5.0*	−1.55	.82	1.31	−.58	−.83	.08	.07	−.0113	−.0119	−.0095	−.0079	.410	.208	.94
5.0*	−.63	−.09	−.36	1.08	1.07	.13	.15	.0305	.0312	.0300	.0308	.112	.097	1.25

*Satisfies pre-bound conditions, but exceeds bound as given by (5) or (6). Definitions are the same as in Table 1.

peyres sums exhibit large errors and are quite variable. The errors in the deadweight loss measures are not as great or as variable as in the LES case, but nonetheless are sufficiently large to cause apprehension concerning their use. Also, the errors appear largest for inelastic demand.

II. Welfare Measures from Single Equation Demand Relations.

The welfare measures in Section I were based on complete systems of demand equations. Knowledge of the complete system was necessary to compute the Hicksian demand functions and the appropriate utility levels required for C and E. However, in many situations it is not feasible to estimate a complete demand system. When demand analysis is required, researchers frequently employ single equation models.

By far the most popular single equation demand models are the linear and log-linear functional forms. Hausman has presented an interesting technique for recovering the indirect utility and expenditure functions for these demand relations.[3] The derivation used in this paper and the actual computations pertain to a two good world. It is possible to work out the approximate measures for a many good case following the method found in LaFrance. The procedure uses the integrability conditions in conjunction with Roy's identity to obtain the indirect utility function for a given utility level. This function is then used to obtain a local expenditure function. Hausman worked out the compensating variation case for linear and log linear demand relations. Compensating variation for the linear case (C_L) is

$$(20) \quad C_L = \frac{1}{\delta} e^{\delta}(p_1' - p_1^{\circ}) [q_1^{\circ}(p_1^{\circ}, m_0) + \frac{\alpha}{\delta}] - \frac{1}{\delta} [q_1'(p_1', m_0) + \frac{\alpha}{\delta}],$$

where

$$(21) \quad q_1 = \alpha p_1 + \delta m + \gamma z.$$

Compensating variation for the log-linear case (C_{LL}) is

$$(22) \quad C_{LL} = \{\frac{(1-\eta)}{(1+\alpha)m_0^{\eta}} [p_1' q_1'(p_1', m_0) - p_1^{\circ} q_1^{\circ} (p_1^{\circ}, m_0)] + m_0^{(1-\eta)}\}^{1-\eta} - m_0,$$

where

$$(23) \quad q_1 = e^{\gamma z} p_1^{\alpha} m^{\eta}.$$

[3] For an alternative method of computing these measures, see Vartia.

Using analogous procedures, equivalent variation for the linear demand curve (E_L) is

$$(24) \quad E_L = \frac{1}{\delta} [q_1(p_1^{\circ}, m_0) + \frac{\alpha}{\delta}] - \frac{1}{\delta} e^{\delta(p_1^{\circ} - p_1')} [q_1(p_1', m_0) + \frac{\alpha}{\delta}],$$

while for log-linear relations E_{LL} is

$$(25) \quad E_{LL} = m_0 - [\frac{(1-\eta)}{(1+\alpha)m^{\eta}} \{p_1^{\circ} q_1'(p_1', m_0) - p_1' q_1^{\circ}(p_1^{\circ}, m_0)\} + m_0^{(1-\eta)}]^{\frac{1}{1-\eta}}.$$

Results of the computations for the log-linear and linear cases are given in Tables 3 and 4 respectively.

Inspection of Tables 3 and 4 reveals results similar to Tables 1 and 2. For the log-linear case, the bounds collapse to a point since the income elasticity does not vary. Nonetheless, as the reader can observe, the $\frac{C-A}{A}$ and $\frac{A-E}{A}$ computations are quite close to this bound and are reasonably small. For identical income levels, price elasticity, and income elasticity the measurement errors are again proportionate to the amount of the price increase. This is obscured by the effect of changing the income elasticity, which for the double-log model is equivalent to changing the income level. Again, the Paasche and Laspeyres measures prove to be poor estimators of equivalent and compensation variation. For the log-linear case, the deadweight loss errors were greater for the 10% price increase case.

For the linear case (which is the example from Hausman) in Table 4, both $\frac{C-A}{A}$ and $\frac{A-E}{A}$ are within the bounds for all cases. Hence, again consumer's surplus is a good approximation. This is most striking for the linear case, as the bounds are extremely close in many cases. The Paasche and Laspeyres sums performed poorly. The deadweight loss measures were in substantial error, but varied little by amount of price increase.

III. Conclusions

The purpose of this paper has been to determine the accuracy of various measures associated with consumer welfare—especially consumer's surplus. In order to accomplish that objective, four demand models, two complete systems and two single equation models, were utilized. The complete sys-

Table 3. Selected Measures for Log Linear Demand Equations with Price Increases of 10 and 50 Percent

m	e_{11}	η	w_1^0	w_1^1	Bound$^+$	$\frac{C-A}{A}$	$\frac{A-E}{A}$	$\frac{L-C}{C}$	$\frac{E-P}{E}$	DWL
					-10 Percent Price Increase-					
10.0	−.1	.9	.743	.809	.03332	.03401	.03262	−.0281	−.0289	8.25
10.0	−.9	.9	.426	.431	.01847	.01868	.01829	.0251	.0239	1.45
10.0	−2.1	.9	.186	.167	.00761	.00764	.00767	.0968	.0884	1.08
5.0	−.1	.9	.796	.868	.03563	.03645	.03494	−.0305	−.0313	8.78
5.0	−2.1	.9	.197	.179	.00810	.00873	.00805	.0963	.0880	1.08
5.0	−.1	.4	.011	.012	.00020	−.00061	−.00062	.0054	.0053	.87
5.0	−.9	.4	.006	.007	.00012	−.00096	−.00118	.0452	.0427	.98
5.0	−2.1	.4	.003	.002	−.00005	−.00552	−.00451	.1113	.0993	.94
10.0	−.1	.4	.007	.008	.00015	.00027	−.00046	.0047	.00514	1.04
10.0	−2.1	.4	.002	.002	.00003	−.00048	.00165	.1057	.0938	.99
					-50 Percent Price Increase-					
10.0	−.5	.9	.563	.689	.11391	.12191	.10652	.0085	.0165	2.32
10.0	−.9	.9	.427	.444	.07960	.08334	.07587	.1154	.0923	1.52
10.0	−2.1	.9	.186	.119	.02735	.02788	.02695	.4872	.3296	1.08
5.0	−.5	.9	.603	.739	.12237	.13133	.11364	.0167	.0247	2.43
5.0	−2.1	.9	.199	.127	.02902	.02989	.02886	.4843	.3283	1.09
5.0	−.1	.4	.011	.016	.00113	.00086	.00088	.0209	.0179	1.05
5.0	−.9	.4	.007	.007	.00054	.00045	.00053	.2078	.1607	1.00
5.0	−2.1	.4	.003	.002	.00018	−.00057	−.00039	.5294	.3479	1.00
10.0	−.1	.4	.007	.011	.00073	.00085	.00074	.0209	.0181	1.04
10.0	−2.1	.4	.012	.001	.00012	.00047	.00049	.5278	.3473	1.00

$^+$Since e_{11} is constant, the bounds collapse to a point (see text for discussion).

Table 4. Selected Measures for Linear Demand Equations with Price Increases of 10 and 50 Percent

m	e_{11}	η^0	η^1	w_1^0	w_1^1	LHS	$\frac{C-A}{A}$	$\frac{A-E}{A}$	RHS	$\frac{L-C}{C}$	$\frac{E-P}{E}$	DWL
8.640	−.20	1.11	1.13	.055	.060	.00300	.00308	.00306	.00310	.0071	.0073	1.30
8.640	−.52	1.40	1.48	.044	.046	.00300	.00300	.00300	.00320	.0236	.0238	1.11
8.640	−1.08	1.92	2.14	.032	.032	.00290	.00310	.00290	.00330	.0536	.0541	1.05
8.640	−2.26	3.02	3.90	.020	.017	.00270	.00390	.00290	.00350	.1236	.1255	1.03
17.28	−.10	1.05	1.06	.059	.064	.00306	.00309	.00306	.00309	.0017	.0017	1.65
17.28	−.56	1.50	1.59	.041	.043	.00299	.00297	.00308	.00317	.0260	.0261	1.10
17.28	−.82	1.75	1.91	.035	.035	.00295	.00285	.00300	.00321	.0401	.0402	1.07
-50 Percent Price Increase-												
8.640	−.2	1.11	1.38	.055	.089	.0277	.0326	.0290	.0346	.0761	.0845	1.29
8.640	−.52	1.40	2.93	.044	.042	.0227	.0351	.0266	.0475	.3063	.3341	1.10
8.640	−1.08	Negative quantity demanded										
17.28	−.10	1.05	1.16	.059	.106	.0293	.0319	.0296	.0324	.0174	−.0208	1.64
17.28	−.56	1.50	3.45	.041	.036	.0221	.0355	.0262	.0507	.346	.377	1.09
17.28	−.82	1.75	10.01	.035	.012	.0181	.0388	.0232	.1039	.640	.697	1.06

tems were the Linear Expenditure System (LES) and the Almost Ideal Demand System (AIDS). The two single equation models were the linear and double log functional forms. Various *a priori* values were assigned to the parameters of these models so that a variety of own-price, cross-price, and income elasticities resulted. Income levels were varied as was the amount of the price change. Hence, the methodology employed is similar to a Monte-Carlo experiment and generality cannot be claimed for the results. However, over a wide range of own-price, of expenditure elasticities and of budget shares several patterns did emerge which were common to all models. For the two complete systems, it was a straightforward matter to compute the appropriate welfare measures of compensating and equivalent variation. For the two single equation models, neither of which possess an explicit utility function, compensating and equivalent variation were computed utilizing the technique of Hausman.

In a previous paper, Willig established limits for the error involved in using consumer's surplus to approximate compensating or equivalent variation. The error bounds formulae are compact and readily understandable. However, they are based on the notion of a constant income elasticity of demand. Since constant income elasticity can result only from a homothetic utility function, the formulae must be considered as approximations for more general demand systems. Although in several cases the bounds were exceeded, the discrepancies were quite small. Hence, on the basis of the demand systems analyzed here, it is possible to conclude that the compact error bounds established by Willig are reliable bounds for the discrepancy between either compensating or equivalent variation and consumer's surplus. This is important because these formulae provide tighter bounds than the more general formulae and are more readily understandable.

A second, and related question is, how well does consumer's surplus approximate compensating and equivalent variation? The answer appears to be, reasonably well. The average absolute percentage error over all cases for $\frac{C-A}{A}$ was 1.22% for price changes of 10%. Similar financial were found for the error between equivalent variation and consumer's surplus.

An interesting result was the large errors found in the Laspeyres and Paasche approximations to compensating and equivalent variation. The average error for the former was 4.8% and 5.0% for the latter for 10% price changes. Equally important was the variation in the errors, with some as high as 11%. The errors for the 50% price change were much higher, averaging 24.1%. This is in contrast to the study by Braithwait which showed that there was little bias in the Laspeyres index (1.5 percent over 15 years) *vis-a-vis* the true cost of living index computed with several alternative demand systems. The Braithwait study used actual price changes over a time period in which relative prices changed very little. This tends to obscure the effect of using a Laspeyres index. The most inaccurate measurement was the ratio of the deadweight loss computed with compensating variation relative to the deadweight loss computed with consumer's surplus.

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