Nonrobustness of Dynamic Dual Models of the U.S. Dairy Industry

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The robustness of dynamic dual model results across functional forms is examined for the U.S. dairy industry. Modified generalized Leontief (GL) and normalized quadratic (NQ) functional forms are compared by examining their consistency with properties of the competitive firm, estimated rates of adjustment for cows and labor, tests of technological change, and elasticities. Homogeneity and symmetry are maintained in both models. Convexity is not rejected by the GL and is not seriously violated by the NQ. Absence of technological change is rejected by both models, but quality indexes on labor and cows fully embody technological change occurring within labor and cows in the NQ but not in the GL. Policy-relevant elasticities differ greatly between the functional forms. Dynamic dual models are found to be non-robust in important ways to choice of functional form.

With excessive stockpiles of dairy products heating up policy debates, considerable recent analytical attention has focused on dynamic adjustment in the U.S. dairy industry (Chavas and Klemme; LaFrance and deGorter). One method for estimating rates of adjustments of herd size and other quasi-fixed inputs, such as labor, is the dynamic dual model (Epstein). Given a flexible functional form, the dynamic dual allows testing and/or maintaining theoretical properties while examining the structure of the industry. However, the robustness of the dynamic dual model to choice of functional form has not been investigated.

This study examines the robustness of dynamic dual model results to the functional form employed for estimation. Epstein suggests four functional forms that meet the required conditions for an intertemporal cost or profit function. Three have been used for estimation in different economic studies: quadratic in prices and quasi-fixed inputs (Epstein and Denny; Vasavada and Chambers, 1986); a modified generalized Leontief (Vasavada and Chambers, 1982); and log quadratic in prices and quadratic in quasi-fixed inputs (Taylor and Monson). However, none of these studies reports results for more than one functional form with the same data.

Research comparing functional forms in static dual models has reported significant differences in (a) test results of theoretical restrictions, (b) estimated price elasticities (Swamy and Binswanger), and (c) elasticities of substitution (Chalfant; Baffes and Vasavada). This study compares two of the functional forms suggested by Epstein in a dynamic dual analysis of the U.S. dairy industry for 1951–82. The purpose is not to determine the "best" functional form, but to determine how robust dynamic dual models are to choice of functional form. Robustness is examined by comparing consistency with theoretical properties, tests of production structure, and elasticities to see if the choice of functional form substantially affects important results.

Method of Analysis

Assume a competitive industry consisting of firms maximizing their net discounted values of production over an infinite planning horizon. Further assume an industry production function, $F(X,Z,\dot{Z})$, where X is a vector of variable inputs, Z a vector of quasi-fixed inputs, and $\dot{Z}$ net investment in Z, such that $\dot{Z} = I - dZ$, where I is gross investment and d is a (constant) depreciation rate. F is twice continuously differentiable, concave, with F_x, $F_Z > 0$ and $F_{\dot{Z}} < 0$, where the subscripts denote derivatives. The first assumptions maintain F as a "well-behaved" production function, and the last assumption means that there are positive costs associated with adjusting the quasi-fixed inputs.

Given the above assumptions on F, a dynamic dual value function, J(P,W,V,Z,r) exists, where P

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is output price, W a vector of variable input prices, V a vector of quasi-fixed input rental rates, and r the discount rate. J is twice continuously differentiable, linearly homogeneous and convex in (P,W,V), and concave in Z. In long-run equilibrium $\dot{Z} = 0$, so the envelope theorem can be applied to establish a duality between J and F (Epstein); i.e., the properties of F are fully manifest in J. Moreover, if $J_{zv} \neq f(P,W,V)$, net investment in quasi-fixed inputs can be expressed in the form of a flexible accelerator,

$$\dot{Z} = M[Z - Z^*],$$

where Z is the original level or endowment of the quasi-fixed input vector, Z* is the desired level, and M is the rate-of-adjustment matrix.

Estimating the rate-of-adjustment matrix M with the dynamic dual approach allows one to test the degree of fixity of any input initially treated as quasi-fixed. Modeling an input such as labor as quasi-fixed and estimating how quickly it adjusts to a new equilibrium level given changes in exogenous variables is preferred to a priori designating it as a variable input. The possibility that it is a variable input (i.e., adjusts within one period to a new equilibrium) can be tested as a nested hypothesis. Additionally, interdependency of adjustment between two or more quasi-fixed inputs can be examined with a dynamic dual model.

Functional forms that maintain linear homogeneity in prices, concavity in quasi-fixed inputs, and flexible accelerator investment in quasi-fixed inputs are employed to estimate the aggregate behavioral equations for the U.S. dairy industry. The modified generalized Leontief (GL) and normalized quadratic (NQ) forms used by Vasavada and Chambers (1982 and 1986, respectively) meet the above requirements.

While the analyst is clearly aware ex-post of changes in prices and technology, it is assumed that producers have static ex-ante price and technology expectations.[1] This assumption of the Markovian property (Hillier and Lieberman, p. 351) is that the economic agents perceive current prices and technology as containing all relevant information about future prices and technology. As the base period changes, new expectations come into being, so time is appended to the vector of regressors in the value function. Decisions made in period t are based on information available in that period. Static price expectations in a dynamic model may trouble some readers; estimation tractability is the usual reason given for using static prices, but there are more valid arguments. Possible reasons why a firm that recognizes the cost of acquiring information may rationally choose to formulate expectations in this manner while continuously updating decisions subject to new information are outlined in Chambers and Lopez. Additionally, Karp, Fawson, and Shumway estimated a rate-of-adjustment matrix for real estate and durable capital that was robust to different assumptions regarding price expectations (p. 18).

The dual value function in the GL form is:

$$\begin{aligned}(1)\quad J(P,W,V,Z) &= [P\ W]AZ + V'B^{-1}Z \\ &+ [P^{.5}\ W^{.5}]EV^{.5} + V^{.5\prime}KV^{.5} \\ &+ [P^{.5}\ W^{.5}]G[P^{.5}\ W^{.5}]' + TH[P\ W\ V']',\end{aligned}$$

where P is the average U.S. blend price of milk, W is the price of feed concentrates, Z is a (2 × 1) vector with Z_1 being the number of dairy cows in the U.S. that have calved and Z_2 the quantity of labor used in the U.S. dairy sector, V is a (2 × 1) vector with V_1 being the annual average rental price of a dairy cow in the U.S. and V_2 the average U.S. agricultural wage rate, and T is year, which is included to capture the effects of disembodied technological change. Parameters A, B^{-1}, E, K and G are each (2 × 2), and H is (1 × 4); K and G are symmetric.

The dual value function in the NQ form is:

$$\begin{aligned}(2)\quad J(w,v,Z) &= a[1\ w\ v'Z']' + v'b^{-1}Z \\ &+ .5\ g\ w^2 + w\ c\ Z + w\ e\ v \\ &+ .5v'k\ v + .5Z'n\ Z + Th[1\ w\ v']',\end{aligned}$$

where w = W/P, and v = V/P. Parameter a is (1 × 6), b, k, and n are (2 × 2), c and e are (1 × 2), g is a scalar, and h is (1 × 4); k and n are symmetric.

The behavioral equations are obtained by applying the envelope theorem to the value function. Under the stated assumptions, output supply, variable input demand, and quasi-fixed input demand for the GL are, respectively:

$$(3)\quad Y(P,W,V,Z) = -rJ_p - J_{zp}\dot{Z},$$

$$(4)\quad X(P,W,V,Z) = -rJ_w - J_{zw}\dot{Z},$$

$$(5)\quad \dot{Z}(P,W,V,Z) = J_{zv}^{-1}(rJ_v + Z).$$

For the NQ, variable and quasi-fixed input demands are equations (4) and (5) with normalized prices. Output supply is obtained by adding normalized expenditures to the normalized value function, which yields:

$$(6)\quad y(w,v,Z) = rJ + wX + v'Z - J_z\dot{Z}.$$

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[1] This assumption is different from the nonstationarity assumption employed by Howard and Shumway.

Equations (3), (4), and (5) are the estimation equations for the GL, and (6), (4), and (5) for the NQ. The elements of the rate-of-adjustment matrices for the GL and NQ are, respectively, $M_{ii} = (B_{ii} + r)$ and $m_{ii} = (b_{ii} + r)$. Because linear homogeneity in prices is maintained by modifying the expansion in the GL and by normalization in the NQ, the number of independent parameters estimated in the NQ exceeds those in the GL. Error terms are added to the estimation equations to account for errors in optimization. $\dot{Z}$ is approximated discretely by $Z_t - Z_{t-1}$. Lagged milk price is used as a proxy for expected milk price. Instruments for the jointly dependent variables are estimated using current and lagged input prices, lagged milk price, and lagged quantities.[2]

Data

The model was estimated using annual data for years 1951–82. Data sources were the same and variable construction was similar to that of Howard and Shumway. Differences from their data and some clarification are provided here. The interested reader is referred to the earlier paper for further details.

The rental price of cows was computed as a discounted stream of payments on a replacement heifer kept for three lactations that would make a producer indifferent between paying three annual payments or a cash purchase price.[3] The salvage value was assumed equal to the maintenance cost of the cow.[4]

Both disembodied and embodied technological change were considered in both models. Disembodied technological change is accounted for by time trends. Embodied technical change incorporated genetic improvements in the cow herd and changes in the quality of farm labor, the latter due primarily to improved education of the work force (Gollop and Jorgenson; Ball).

The quantity of labor used on dairies included both family and hired labor. The wage rate index for hired labor was used also as a proxy for the shadow price of family labor. Combining family and hired labor and using the same wage index for hired and family labor are admittedly ad hoc procedures. Aggregate data on the quantity of family and hired labor in the dairy industry do not exist. Dairy labor in this model is constructed from aggregate agricultural labor data. Attempting to separate family and hired labor would likely cause more errors than already exist in this data series. Determining the correct shadow price of family labor can also cause a dilemma. Some analysts argue that the price of family labor should be higher than hired labor since management services are involved. However, when an implicit price for family labor is computed based on distribution of rents, the price is nearly always lower than the hired labor wage rate. Thus, using the hired labor wage rate as a proxy for the shadow price of family labor is a middle ground compromise.

Empirical Results

The parameter estimates for (1) and (2) with symmetry restrictions and a real discount rate of .03 are reported in Table 1.[5] Thirteen of 22 parameter estimates were significant at the 5% level (using asymptotic t-statistics) in the GL model; only three of 25 were significant at the 5% level in the NQ model. Given the nonlinear and simultaneous nature of the system, R^2 values cannot be directly compared, but they do provide an indication of the relative explanatory power of the models (Kvalseth). The R^2 values from the GL and the NQ for milk supply and input demand for feed, labor and cows were, respectively, .14, .97, .99, .98, and .67, .97, .26, and .36. The difference in R^2 between the two functional forms may be because the relationship for milk supply is specified in different ways. The generally low R^2 values for the NQ could be an indication of misspecification.[6]

Although the GL has higher R^2's for most equations and a larger number of significant parameter estimates (5% level), results of both models are examined further for two reasons. First, given the nonlinear and simultaneous nature of the model, a model's theoretical consistency is difficult to judge solely by parameter estimates. Second, the GL

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[2] Estimation was by nonlinear three stage least squares (SYSNLIN, a nonlinear estimation program in SAS).

[3] There is no observable rental price for dairy cows, but amortizing the cash purchase price over the three-year period captures the effect of price changes on the investment decision while permitting use of a reasonable fraction of the price of capital as a proxy for rental price. Dairy cows in the U.S. produce milk for an average of three lactations, or slightly longer than three years.

[4] Dairy cows have a feed maintenance requirement that is much lower than the feed required for maximum milk production. The assumption that the cost of maintaining the cow is recovered through the salvage value and the remaining feed cost is for milk production simplifies the procedure. Sensitivity analysis further revealed that it did not introduce major distortions.

[5] Symmetry conditions in the GL are $K_{12} = K_{21}$ and $G_{12} = G_{21}$. For the NQ symmetry maintains $k_{12} = k_{21}$ and $n_{12} = n_{21}$.

[6] A reviewer pointed out that neither model is completely specified. Costs of seed, fertilizer, machinery, and other operating costs are not included in the models. Data limitations necessitated severe simplification of the model. However, any misspecification due to the omission of those inputs is held constant across both models and should not affect the comparison of robustness of results across functional forms.

Table 1. Parameter Estimates of the Generalized Leontief and Normalized Quadratic Value Functions, Homogeneity and Symmetry Maintained

Leontief		Quadratic	
Parameter	Estimate	Parameter	Estimate
A_{11}	14.47	a_1	−13.01
	(3.610)		(25.60)
A_{12}	1.534	a_2	−0.8506
	(0.4531)		(0.8619)
A_{21}	0.3959	a_3	−7.749
	(1.034)		(7.189)
A_{22}	−0.1121	a_4	47.64
	(0.1149)		(53.62)
B_{11}	−0.1401	a_5	−61.94
	(0.05452)		(68.94)
B_{12}	−0.01008	a_6	−27.92
	(0.01568)		(38.64)
B_{21}	0.003587	b_{11}	0.09007
	(0.3900)		(0.06136)
B_{22}	−0.3688	b_{12}	−0.4302
	(0.1271)		(0.1382)
E_{11}	−8.065	b_{21}	0.6503
	(2.644)		(0.4698)
E_{12}	−4.921	b_{22}	−0.4302
	(4.076)		(0.1382)
E_{21}	−1.122	g	0.03440
	(0.6969)		(0.01973)
E_{22}	0.1588	c_1	0.03852
	(1.551)		(0.1146)
K_{11}	−9.122	c_2	0.7404
	(1.879)		(0.8520)
K_{12}	4.178	e_1	−0.005145
	(1.807)		(0.01405)
K_{22}	−37.21	e_2	−0.006706
	(4.400)		(0.003694)
G_{11}	19.95	k_{11}	−0.009154
	(4.773)		(0.01669)
G_{12}	−0.1029	k_{12}	0.005276
	(0.3639)		(0.004677)
G_{22}	0.7911	k_{22}	−0.000316
	(0.7562)		(0.001472)
H_1	0.1772	n_{11}	−35.76
	(0.04610)		(22.93)
H_2	−0.1651	n_{12}	183.4
	(0.01272)		(120.7)
H_3	0.1139	n_{22}	−914.1
	(0.01746)		(803.9)
H_4	0.3841	h_1	0.01559
	(0.05885)		(0.008205)
		h_2	−0.1611
			(0.01119)
		h_3	0.7046
			(0.8101)
		h_4	0.3051
			(0.4540)

Standard errors of the estimates are in parentheses. MSE = 1.6382 with 106 degrees of freedom for the GL, 1.8400 with 103 degrees of freedom for the NQ. See equations (1) and (2) and subsequent variable descriptions for explanation of parameter notation.

maintains concavity of the value function in quasi-fixed inputs as a byproduct of maintaining linear homogeneity in prices; the NQ allows explicit examination of the concavity conditions. Hence, the theoretical and structural properties of both models were examined.

Tests of Competitive Behavior, Differentiability, and Structure

The models were estimated maintaining the theoretical properties of linear homogeneity in prices and symmetry in both models and concavity in quasi-fixed inputs in the GL. Examinations of monotonicity and convexity in prices (implied by profit maximization for price-taking firms) were conducted. Concavity in quasi-fixed inputs was examined for the NQ.

The necessary monotonicity conditions on the value function, i.e., J( ) increasing in output price and decreasing in input prices, held at all observations for both models.

The tests for convexity of J( ) in prices are reported in Table 2. The test statistic used was the Gallant and Jorgensen T^0, which is approximately Chi-square, with degrees of freedom equal to the number of restrictions. Global convexity of J( ) in prices is satisfied in the GL when $E_{ij} < 0$, i,j = 1,2, and K_{ij}, $G_{ij} < 0$ $i \neq j$. Convexity in the GL was not rejected at the .05 level. Global convexity in the NQ is satisfied when the matrix of price parameters is positive definite. Although a statistical test of convexity in the NQ was not conducted because of the inequality constraints required, a positive definite matrix was achieved by adjusting each of the estimated price parameters less than one standard error.

Global concavity of J( ) in quasi-fixed inputs was maintained by functional form in the GL. A sufficient condition for concavity in the NQ is that

Table 2. Hypothesis Tests for Each Functional Form[a]

Hypothesis	Test Statistic	Critical Value
Convexity		
GL: $E_{ij}<0$, $i,j=1,2$ K_{ij}, $G_{ij}<0$, $i \neq j$	4.870	$X^2_{6,.05} = 12.592$
NQ: $\begin{Bmatrix} g & e_1 & e_2 \\ e_1 & k_{11} & k_{12} \\ e_2 & k_{12} & k_{22} \end{Bmatrix}$ positive definite	Ad hoc — parameters within 1 standard deviation	
Independent Adjustment		
GL: $M_{12}=M_{21}=0$	4.160	$X^2_{2,.05} = 5.991$
NQ: $m_{12}=m_{21}=0$	26.033	$X^2_{2,.05} = 5.991$
Instantaneous Adjustment of Labor		
GL[b]: $M_{22}=-1.0$	140.318	$X^2_{1,.05} = 3.841$
NQ: $m_{22}=-1.0$, $m_{12}=0$	27.035	$X^2_{2,.05} = 5.991$
Instantaneous Adjustment of Cows		
GL[b]: $M_{11}=-1.0$	Did Not Converge	$X^2_{1,.05} = 3.841$
NQ: $m_{11}=-1.0$, $m_{21}=0$	291.987	$X^2_{2,.05} = 5.991$
No Technological Change		
GL[b]: $H_l=0$, $l=1, \ldots 4$	245.858	$X^2_{4,.05} = 9.448$
NQ: $h_l=0$, $l=1, \ldots 4$	220.437	$X^2_{4,.05} = 9.448$
No Unobserved Technological Change in Cows		
GL[b]: $H_3=0$	8.149	$X^2_{1,.05} = 3.841$
NQ: $h_3=0$	1.823	$X^2_{1,.05} = 3.841$
No Unobserved Technological Change in Labor		
GL[b]: $H_4=0$	15.710	$X^2_{1,.05} = 3.841$
NQ: $h_4=0$	2.825	$X^2_{1,.05} = 3.841$

[a] Homogeneity and symmetry in prices were maintained throughout.

[b] Independent adjustment also maintained.

n_{11}, $n_{22} < 0$ and $n_{11}n_{22} - (n_{12})^2 > 0$. Although violated by the estimated parameters, the violation was not statistically significant. A change of less than 0.05 standard deviation in n_{22} was sufficient to obtain concavity of J( ) in quasi-fixed inputs.

A focal point of dynamic models is the rate of adjustment of quasi-fixed inputs. The rate of adjustment of labor was not significantly different in the two models, but the rate of adjustment of cows was very different (M_{22} vs. m_{22}, M_{11} vs. m_{11} for the GL and NQ, respectively). With a real discount rate of 3% (i.e., $r = .03$), the GL estimated that cows adjusted 11% of the difference between current and desired levels per year. This is a stable adjustment, i.e., M_{11} between -1 and 0. The NQ estimated a nonstable adjustment for cows, $m_{11} = 0.12$, which indicates adjustment away from an equilibrium level.

Independent dynamic adjustment of inputs, instantaneous adjustment, and several technological change hypotheses were tested as nested hypotheses while maintaining homogeneity and symmetry of the value functions. These tests are also reported in Table 2.

Independence of adjustment occurs when $M_{12} = M_{21} = 0$ and means that each quasi-fixed input adjusts towards its desired level independently of the other. The null hypothesis of independence was not rejected for the GL but was rejected for the NQ. Since independent adjustment was not rejected in the GL, instantaneous adjustment was tested with this model subject to independent adjustment. If $M_{ii} = -1$ (with $M_{ji} = 0$ in the NQ), the i^{th} quasi-fixed input adjusts instantaneously to its desired level, and should actually be modeled as a variable input. Instantaneous adjustment was tested separately for labor and for cows. It was rejected for the former in both models. It was rejected for the latter in the NQ. Convergence was not attained while maintaining the latter in the GL, so no test statistic is available for that model.

The last set of hypotheses to be tested dealt with technological change. Homogeneity and symmetry were maintained in both models; independence of adjustment was also maintained in the GL. The null hypothesis that technology did not change over the data period 1951–82, i.e., H_l, $h_l = 0$, $l = 1, \ldots, 4$,[7] was soundly rejected in both models. The hypothesis of no disembodied technological change in cows, H_3, $h_3 = 0$, or in labor, H_4, $h_4 = 0$, i.e., the quality indexes fully embodied the technological changes that occurred, was rejected at the .05 level only in the GL.

Short and Long-Run Elasticities

To compare relative magnitudes, short and long-run elasticities obtained from the GL and NQ for 1982 are reported in Table 3. Only homogeneity and symmetry were maintained in both models. Concavity in quasi-fixed inputs was maintained in the GL. Because convexity in prices was not satisfied by either initial model, not all long-run own-price elasticities have the signs expected for competitive behavior. Thus, these elasticities are not presented in any sense either as "best" statistical estimates or as theoretically consistent estimates. Unlike static models, however, dynamic models do not yield testable sign hypotheses on short-run own-price elasticities for competitive behavior (Treadway, pp. 344–345).

The models estimated elasticities with different signs in 11 of the reported 32 pairs of elasticities. Magnitudes of many of the elasticities with the same sign also differed substantially. The larger elasticity (in absolute value) was more than double the smaller elasticity in 23 pairs. Some elasticities from both models changed signs from the short run to the long run.

Conclusions and Implications

The robustness of dynamic dual model results to choice among two functional forms has been examined for the U.S. dairy industry. Robustness of results for modified generalized Leontief and normalized quadratic functional forms was evaluated by examining structural parameters, elasticities, and consistency with competitive behavior. Homogeneity and symmetry were maintained in both models.

Statistical characteristics of the estimated models differed substantially. More than half of the estimated parameters in the GL model were significant at the 5% level; only 12% in the NQ model were. R^2 values differed substantially between models for milk supply and labor demand. Calculated 1982 elasticities also differed substantially with respect to both magnitude and sign. A full third of the elasticities differed in sign between models. Two-thirds of the elasticities differed in absolute value by more than 100%, thus documenting the extreme sensitivity of this important practical empirical result to functional form.

Theoretical properties were not clearly rejected with either model. Monotonicity conditions were satisfied at all observations for both functional forms. Convexity in prices was not rejected in the GL and was not seriously violated in the NQ. Concavity

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[7] Where subscripts 1, 2, 3, and 4 refer respectively to output supply of fluid milk and input demand for feed concentrates, cows, and labor.

Table 3. Output Supply and Input Demand Elasticities for the U.S. Dairy Industry Derived from Each Functional Form, 1982[a]

	Elasticity with Respect to Price of			
Quantity	Milk	Feed	Cows	Labor
Short Run				
Milk				
GL:	−0.121	−0.007	0.098	0.030
NQ:	0.056	0.044	−0.034	−0.066
Feed				
GL:	0.012	−0.048	0.047	−0.011
NQ:	−0.037	−0.027	0.060	0.004
Cows				
GL:	0.127	0.006	−0.075	−0.058
NQ:	0.002	−0.004	−0.005	0.007
Labor				
GL:	0.206	−0.003	−0.305	0.102
NQ:	0.016	−0.023	−0.038	0.045
Long Run				
Milk				
GL:	0.114	0.001	−0.078	−0.037
NQ:	0.057	0.042	−0.044	−0.055
Feed				
GL:	0.007	−0.048	0.043	−0.002
NQ:	−0.035	−0.029	0.060	0.004
Cows				
GL:	1.066	0.057	−0.557	−0.566
NQ:	−0.022	0.055	−0.008	−0.025
Labor				
GL:	0.614	−0.001	−0.909	0.296
NQ:	0.049	−0.055	−0.190	0.196

[a]Homogeneity and symmetry were maintained in both models; concavity in quasi-fixed inputs was maintained in the GL.

in quasi-fixed inputs was maintained in the GL and not rejected in the NQ.

Of five statistical tests of structure completed with both models, however, consistent results were obtained on only two at the 5% level (also at the 1% level). In the GL, independent adjustment was not rejected. In the NQ, fully embodied technical change for cows and labor was not rejected. The remaining structural hypotheses were rejected in both models. Since the only feed input in the model specifications was concentrate and since both the total quantity of concentrate fed per cow and the concentrate:roughage feed ratio increased substantially over the data period, it is not surprising that the hypothesis of "no technological change" was soundly rejected by both models. The time trend may have picked up some of these increases (if not induced by price changes) as well as true technological change. Every structural hypothesis was rejected in at least one model.

The lack of robustness across functional forms has serious implications for policy decisions. For example, the evaluation of proposed programs to reduce the U.S. dairy herd, and thus output, critically depends on which functional form is employed in the evaluation. Pricing policies are regarded more favorably by the GL model as a way to affect herd size and output. Although a 10-year adjustment period is estimated for cows, the GL rate-of-adjustment results are dynamically stable. Labor and cows also adjust independently according to the GL results; dairy labor programs can be implemented separately from herd programs. The NQ model estimates that the U.S. dairy herd is dynamically unstable, and that labor and cows have interdependent adjustments. Hence, pricing policies are estimated by the NQ model to have little effect on herd size. A program such as the dairy herd buyout would be more effective. But, any programs that affect the herd level will also impact on dairy labor adjustment according to the NQ results.

Although only two functional forms were examined, results from this dynamic dual analysis of the U.S. dairy industry documented a serious lack of robustness across functional forms in several important ways. This lack of robustness is consistent with that previously documented in static dual models, but specific areas of nonrobustness differ.

Extreme sensitivity of policy-relevant elasticities to functional form was documented. Robustness across functional forms with respect to theoretical restrictions was found (which was contrary to Swamy and Binswanger) but not with respect to technological change hypotheses (contrary to Baffes and Vasavada). The need for model specification searches previously noted for static dual models applies equally to dynamic dual models.

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