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FACTOR ABUNDANCE THEORY IN VERTICALLY DIFFERENTIATED PRODUCTS*

KIM SANG-HO**

I. Introduction

This paper presents a general equilibrium model of two-country, two-factor and two-commodity in which one commodity is vertically differentiated. Vertically differentiated goods are measured in the total services they generate. The general equilibrium nature of the model enables us to investigate interrelations of factor abundance and factor intensity in relation to the quality of differentiated goods. This paper shows that the capital-abundant country produces lower quality differentiated goods than the labor-abundant country in autarkic equilibrium. This unusual result is due to the assumption that the quality is raised through an increased use of labor. This paper also shows that at free trade, the quality of the differentiated goods becomes equal between countries, and is determined at world trade prices.

The current literature on international trade policy in vertically differentiated goods has been stimulated by the empirical findings[1] that quantitative trade restrictions lead to a shift in the composition of trade toward higher valued, higher-quality products.

The first theoretical models were presented by Rodriguez (1979) and Santoni and Van Cott (1980). These models are limited by

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1 For empirical studies, see Anderson (1985), Aw and Roberts (1988), and Feenstra (1988)

their partial equilibrium and ad hoc nature because they concentrate on explaining the reason behind the hypothesis. Thus, they fail to analyze it thoroughly in a standard H-O-S (Heckscher-Ohlin-Samuelson) economy.

The partial equilibrium models depicted in the literature only emphasize the consumer welfare effects of policy instruments, leaving out social welfare effects resulting from terms of trade effects. Another defect of the literature of ad hoc and partial equilibrium models is that they cannot investigate trade policy within a whole economy, interaction between goods and factors and that between countries.

This paper presents a two-country, two-factor, two-commodity model in which one commodity is vertically differentiated. Vertically differentiated goods are measured in the total services they generate, and total services are determined by a product of unit quality and physical quantity. Firms are assumed to choose optimal quality to minimize their total costs in providing services of differentiated goods (Swan, 1970).

The economic situation of the model is similar to that of the standard H-O-S economy. In the production, Leontief technology is used as a specific example of constant returns to scale technology. Leontief technology is chosen for simplicity, but the basic results are not dependent on this specific functional form. The general equilibrium nature of the model enables us to investigate the interrelations of factor abundance and factor intensity in association with the quality of differentiated goods.

In the next section, the basic model of Leontief technology is set up. The production possibility frontier of the economy is derived on the taking of this technology. In Section 3, autarkic equilibrium and comparative statics of the equilibrium are presented. This section proves the standard theorems of the H-O-S model in the context of the current model. In the final section, brief summaries and conclusions are presented.

II. The Model

Consider an economy made up of two sectors, one consisting of vertically (quality) differentiated goods, and the other of homogenous goods. Leontief (fixed coefficient) technology is used in the production as a specific example of constant return to scale (CRS) technology. The economic situation of the model is similar to that of the standard H-O-S economy.

1. Production

In sector x of vertically differentiated goods, each good is measured by the total services it generates because goods are differentiated by quality, and total services are determined by the quality level and physical units of output as such that :

$$x = q\,Q \tag{1}$$

where x and q are services and the quality of a unit of goods x, respectively, and Q represents physical units of output that can vary by quality. Thus, this equation captures the fact that higher quality grains (q), for example, yield more calories (x) than lower quality grains for a given amount (Q). Q is assumed to have the following fixed coefficient production function:

$$Q = \mathrm{Min}\,\{K_x/\alpha_{kx},\ L_x/(\alpha_{Lx}q^2)\} \tag{2-1}$$

where α_{kx} and $\alpha_{Lx}q^2$ are the capital and labor required to produce one physical unit of Q, respectively. Thus, the physical units of output producible from given factors depend inversely on quality q. The way in which quality enters into a fixed coefficient of labor ($\alpha_{Lx}q^2$) specifies that higher quality requires more labor for a unit production of Q. This amounts to assuming that upgrading quality requires more labor.

Usually quality can be raised through capital investment. In this intuitive case, the production function of Q can be changed to:

$$Q = \mathrm{Min}\,\{K_x/(\alpha_{kx}q^2),\ L_x/\alpha_{Lx}\} \tag{2-2}$$

Whether quality is more closely related to labor or capital is an empirical question, and differs from industry to industry. In some cases of agricultural products, quality can be raised through enhanced labor use. Labor-intensive production in agricultural products is considered to be a way to produce higher quality products than capital-intensive mass production. Also, quality upgrading in agricultural products is considered a way to compete with foreign imports.

The further analysis of this paper can be pursued on either assumption. To consider agricultural products in which quality can be raised through an increase in labor employments, this paper proceeds with the production function (2-1). The results of this paper should be considered in the context of this assumption about quality upgrading. However, the implication of the other case will be also discussed.

From the combination of (1) and (2-1), the production function for x can be derived as (3-2). The production functions for the homogenous good y, and the differentiated good x are:

$$y = \text{Min}\ \{K_y/a_{ky},\ L_y/a_{Ly}\} \qquad (3\text{-}1)$$
$$x = \text{Min}\ \{K_x/a_{kx},\ L_x/a_{Lx}\} \qquad (3\text{-}2)$$
$$\text{with } a_{kx} = \alpha_{kx}/q, \text{ and } a_{Lx} = \alpha_{Lx}q$$

where K_i and L_i represent the capital and labor used in sector i, respectively, and a_{ij} is the factor i required to produce one unit of goods j. The production function for goods y is usual Leontief technology, but that for goods x includes quality variables in the fixed coefficients.

The cost functions for sector x and y can be derived from the production functions as:

$$C_x = (\alpha_{Lx}q)\ w + (\alpha_{kx}/q)\ r \qquad (4\text{-}1)$$
$$C_y = a_{Ly}\ w + a_{ky}\ r \qquad (4\text{-}2)$$

where C_i is the unit (or average) cost function for sector i, and w and r are wages and rents, respectively.

Following Swan (1970), firms in sector x are assumed to choose an optimal quality to minimize their total costs in providing services of x.[2]

[2] In the production of x, firms have to choose optimal quality q* in order to minimize production costs. This optimal quality depends on the market prevailing output price. Once optimal quality is determined, it serves as an fixed coefficient in the production of good x.

From the cost function (4-1), the optimal quality can be derived by partial differentiation with regard to q, and it is a negative function of the wage-rental ratio.

$$q^* = \sqrt{(r\alpha_{kx})/(w\alpha_{Lx})} \tag{5}$$

Intuitively, an increase in quality requires more labor, and excess demand for labor raises the wage-rental ratio. Therefore, optimal quality is inversely related to the wage-rental ratio.

The cost function at an optimal quality can be derived from the substitution of (5) into (4-1).

$$C_x^* = 2\sqrt{\alpha_{Lx}\alpha_{kx}wr} \tag{6}$$

The zero-profit curves in sector x and y can be written as:

$$1 = w\,a_{Ly} + r\,a_{ky} \tag{7-1}$$
$$p = w\,\alpha_{Lx}q + r\,\alpha_{kx}/q \tag{7-2}$$

where good y is used as a numeraire, and p is the relative price of x in terms of y. The slopes of the zero-profit curves which are equal to negative factor intensity ratios can be derived from the differentiation as:

$$dw/dr\,\big|\,\pi_{y=0} = -a_{ky}/a_{Ly} = -k_y\ (= K_y/L_y) \tag{8-1}$$
$$dw/dr\,\big|\,\pi_{x=0} = -a_{kx}/q^2\alpha_{Lx} = -(w/r) = -\ k_x (= K_x/L_x) \tag{8-2}$$

For the derivation of (8-2), the envelope theorem is used, and q* is substituted into q.[3] The zero-profit curves are depicted in Figure 1. The figure shows that there exists a factor intensity reversal in the model because two zero-profit curves intersect twice at E1 and E2.[4]

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[3] Total differentiation of (7-2) yields$(a_{Lx}/q)dw+(a_{kx}q)dr+\{(a_{Lx}w/-q^2)+a_{Lx}r\}dq=0$. The last term of the above equation becomes zero because it is the first order condition of cost-minimizing quality choice, i.e., $dc_x/dq=0$

[4] Two zero-profit curves must intersect at least once in autarky. If there are no intersection points, the zero-profit curve of x lies above that of y at every wage-rental ratio. Thus, production of x is always more profitable than that of good y, and only good x will be produced. In case two goods are to be produced at the same time, there exists an intersection point of the two curves. With an assumption of diversified production, we eliminate the possibility that the two cuves never intersect.

At E_1 the slope of π_x is steeper than that of π_y, and good x is relatively capital-intensive. At E_2 good x is relatively labor-intensive.

The price p can be solved explicitly as a function of w/r from (7) after the substitution of q* into q as:

$$p = \{2p = \{2\sqrt{\alpha_{Lx}\alpha_{kx}}/(a_{Ly}(w/r) + a_{ky})\}\sqrt{(w/r)} \tag{9}$$

Note that

$$(w/r) = 0 \rightarrow p = 0 \text{ and } \lim_{(w/r)\rightarrow\infty} p = 0 \tag{10}$$

also, from the differentiation of p

$$dp/d(w/r) = \{\sqrt{\alpha_{Lx}\alpha_{kx}}/[a_{Ly}(w/r)+a_{ky}]^2\}\{a_{ky}(w/r)^{-1/2}-a_{Ly}(w/r)^{1/2}\} \tag{11}$$

Therefore,

$$dp/d(w/r) \gtrless 0 \rightarrow a_{ky}/a_{Ly} \gtrless w/r \tag{12}$$

Note that the critical point $a_{ky}/a_{Ly}(=k_y) = w/r(=k_x)$ is the point of factor-intensity reversal.

The following relationship between the output price and optimal quality can be derived from the combination of (5) and (12) as:

$$q^* = q(p),\ q' \gtrless 0 \dashrightarrow k_x \gtrless k_y \tag{13}$$

The relationship of the model can be illustrated in the 4-quadrant diagram in Figure 2. Quadrant I represents the inverse relationship between the optimal quality and the wage-rental ratio as in (5). Quadrant II represents the relationship between the wage-rental ratio and the output price p, which is shown in (9), (10), (11), and (12). An increses in w/r leads to an increase in p at a lower w/r ratio, but a further increase of w/r above a_{ky}/a_{Ly} results in a decrease in p (see eq. 12). At the limit, p converges to 0 as w/r approaches infinity. Quadrant IV represents the relationship between output price p and optimal quality q* as in (13). When good x is relatively capital-intensive, an increase in output price p raises the quality of the differentiated good x.[5] In the other case when good x is relatively labor-intensive, a decrease in output price raises the quality of good x.

Intuitively, an increase in the output price of vertically differentiated products which are capital-intensive will lower the wage-rental ratio (the Stolper-Samuelson Theorem), and this lowered wage-rental ratio will make it possible for firms to hire more labor which is required to upgrade quality (optimal quality relationship).

The value of p at the factor-intensity reversal is obtained by substituting $w/r = a_{ky}/a_{Ly}$ into (13):

$$p^m = \sqrt{a_{Ly}\alpha_{kx}/\alpha_{Lx}a_{ky}} \tag{14}$$

Whether the useful equilibrium for the economy is E_1 or E_2 can be detesmined by the relative factor abundancy of the economy. Once the factor endowment is given, the wage-rental ratio of the economy is determined, along with the equilibrium. This can be illustrated by the Harrod-Johnson diagram[6] in Figure 3.

In the model, the factor intensity ratio of good y ($=k_y$) is constant as a fixed parameter of a_{ky}/a_{Ly}, but that of good x ($=k_x$) is w/r which is an increasing function of (w/r). The first quadrant of the diagram represents these factor intensity ratios of the two goods. Note that k represents the constant endowment ratio of the economy K/L. This diagram also shows that there is a factor intensity reversal in the model. The second quadrant of this diagram represents the relationship between output price and w/r.

If the endowment ratio of the economy, k (=K/L) is greater than $k_y(=K_y/L_y)$, $k_x(=K_x/L_x)$ is greater than k_y. The economy, in this case, will have an E_1 type equilibrium. If k is smaller than k_y, the economy will have an E_2 type equilibrium. Therefore, both types of equilibrium

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5 If we assume that quality is upgraded through an enhanced use of capital as represented in (2-2), the relationship of the model changes parallel. Specifically, the relationships in Quadrant I and Quadrant IV are reversed because equations (5) and (13) are changed, but quadrant II remains the same because (9) does not change. Thus, when capital is a vehicle of quality upgrading, the lower price of good x results in higher quality q*.

6 The Harrod-Johnson diagram is a two-quadrant diagram which represents the relationship between the relative factor price and the factor intensity ratio, and that between the relative factor price and the output price.

are not possible at the same time. These relationships can be illustrated as follows:

$$k > k_y \rightarrow k_x > k_y \rightarrow E_1 \qquad (15)$$
$$k < k_y \rightarrow k_x < k_y \rightarrow E_2$$

The model will proceed on the assumption that the economy is relatively capital-abundant, thus good x is relatively capital-intensive. In this case, (13) can be written as:

$$q = q(p), \quad q' > 0 \qquad (16)$$

This fits well with the fact that quality and the output price have a positive relationship. We can specify the production function when good x is relatively labor-intensive with the same result as (16). Thus, the preceding discussions will apply equally to both cases irrespective of the capital-intensity of good x.

2. The Production Possibility Frontier

The production possibility frontier (PPF) can be derived from the production functions in (1) for a given endowment of (K, L) of the economy. The sum of the factors used in both sectors must be equal to the endowment, and this creates the following restrictions:

$$a_{ky}y + (\alpha_{kx}/q)x = K \ \text{(KK)} \qquad (17\text{-}1)$$
$$a_{Ly}y + (\alpha_{Lx}q)x = L \ \text{(LL)} \qquad (17\text{-}2)$$

By rearranging (17), we have:

$$y=k/a_{ky} - (\alpha_{kx}/qa_{ky})x \ \text{(KK)} \qquad (18\text{-}1)$$
$$y=L/a_{Ly} -(\alpha_{Lx}q/a_{Ly})x \ \text{(LL)} \qquad (18\text{-}2)$$

KK curve (18-1) is steeper than LL curve (18-2), since good x is relatively capital-intensive. These two curves are depicted in Figure 4.

The two intercepts K_q/α_{kx}, $L/\alpha_{Lx}q$ change in opposite directions as the price changes in the same direction [see (16)]. For example, as the price increases from the original price, which yields the intersection point E_0, K_q/α_{kx} moves further away from the origin O, whereas $L/\alpha_{Lx}q$ moves closer to the origin. The restrictions (18) change to the dashed line in Figure 4, yielding a new intersection point E_1. Repeating this operation on all prices and connecting the resulting intersection points such as E_1, the PPF of the thick line in Figure 4 can be derived. At these intersection points, the factors are fully employed because two conditions of KK and LL are satisfied at the same time.

From the combination of (13) and the relationship between p and x, we can derive the relationship between x and q. Note that as the production of x increases, so does the quality. This relationship can be written as:

$$dq/dx > 0 \tag{19}$$

The slope of the PPF can be derived from the total differentiation of (17), which is:

$$0 = a_{ky}dy + a_{kx}dx - (a_{kx}x/q)dq \tag{20-1}$$
$$0 = a_{Ly}dy + a_{Lx}dx + (a_{Lx}x/q)dq \tag{20-2}$$

Rearranging (20) into a matrix form, we have:

$$\begin{bmatrix} a_{ky} & a_{kx} \\ a_{Ly} & a_{Lx} \end{bmatrix} \begin{bmatrix} dy/dq \\ dx/dq \end{bmatrix} = \begin{bmatrix} a_{kx}x/q \\ -a_{Lx}x/q \end{bmatrix} \tag{21}$$

Let:

$$\triangle = \begin{vmatrix} a_{ky} & a_{kx} \\ a_{Ly} & a_{Lx} \end{vmatrix} \qquad \triangle_y = \begin{vmatrix} a_{kx}x/q & a_{kx} \\ -a_{Lx}x/q & a_{Lx} \end{vmatrix} = 2a_{kx}a_{Lx}x/q \tag{22}$$

$$\triangle_x = \begin{vmatrix} a_{ky} & a_{kx}x/q \\ a_{Ly} & -a_{Lx}x/q \end{vmatrix} = -(a_{ky}a_{Lx} + a_{kx}a_{Ly})x/q$$

Using Cramer's rule, we have:

$$dy/dq = \triangle_y/\triangle \qquad dx/dq = \triangle_x/\triangle \tag{23}$$

Therefore, the slope of the PPF can be derived from (23) as:

$$dy/dx\,|_{ppf} = \triangle_y/\triangle_x = -2a_{kx}a_{Lx}/(a_{ky}a_{Lx} + a_{kx}a_{Ly}) \quad (24)$$
$$= -2\alpha_{kx}\alpha_{Lx}/(a_{ky}\alpha_{Lx}q+a_{Ly}\alpha_{kx}/q)$$

The PPF as derived in (24) enables us to investigate the relationship between quality and the usual PPF. That is, quality q is included in the PPF as an endogenous variable. This feature enables us to make the comparative static analysis as shown in the next chapter. The partial differentiation of (24) with regard to q gives:

$$(\partial/\partial q)(dy/dx)\,|_{ppf} = 2\alpha_{kx}\alpha_{Lx}[a_{ky}\alpha_{Lx} - a_{kx}a_{Ly}/q]/[a_{ky}\alpha_{Lx}q+a_{Ly}\alpha_{kx}/q]^2 \quad (25)$$

Therefore,

$$(d/dq)(dy/dx)\,|_{ppf} < 0 \text{ if } K_x > K_y \quad (26)$$

The concavity of the PPF is derived from the combination of (26) and (19).

III. Equilibrium and Comparative Statics

1. Autarkic Equilibrium

The indifference curves of the economy are assumed to be downward sloped and convex to the origin. The homotheticity of the consumer's preference is sufficient for this. An autarkic equilibrium can be illustrated with indifference curves, the PPF and a price line as in Figure 5.

The production/consumption point A in Figure 5 yields the highest level of utility in the economy assuming there is no external trade. Not only is A the "optimal" production point, it also represents the "autarkic" equilibrium. The marginal rate of transformation and the marginal rate of substitution at point A are equal to the price ratio p*. The existence of tangency between the price line and the indifference curve is assumed from the well-behaving indifference curves, but the existence of tangency between the price line and the

PPF must be proven because the specific production function is used in the model. It is proved by showing that the price is equal to the negative of the slope of the PPF. The price (9) after the substitution (w/r) from (5) is:

$$\begin{aligned} p &= \{2\sqrt{\alpha_{Lx}\alpha_{kx}}/(a_{Ly}(w/r)+a_{ky})\}\sqrt{(w/r)} \\ &= \{2\sqrt{a_{Lx}a_{kx}}/(a_{Ly}(a_{kx}/a_{Lx})+a_{ky})\}\sqrt{(a_{kx}/a_{Lx})} \\ &= 2a_{kx}a_{Lx}/(a_{Ly}a_{kx}+a_{ky}a_{Lx}) = -dy/dx\,\big|_{ppf} \end{aligned} \tag{27}$$

Now suppose that the economy depicted in Figure 5 is allowed to engage in international trade. The excess demand for or supply of products can be derived for each price. For example, at p1 and the corresponding production-cum-consumption combination (for example, E-cum-C) in Figure 5, there is an offer of exports (FE of y) for an equal market value of imports (FC of x), and this offer is represented by trade triangle EFC. Placing all such triangles in Figure 6 (where triangle TBO represents the equal triangle EFC) generates the offer curve OH.

Note that as we move along the OH further away from the origin the price of x decreases, as does the quality of x.

2. The Comparative Statics of the Equilibrium

Now consider the effects of changing the relative factor endowment on the PPF and the offer curve. These effects can be analyzed by the Rybczynski theorem. The Rybczynski theorem can be proven in this specific production model.

Proposition 1: (The Rybczynski theorem)
At constant prices, an increase in capital will increase by a greater amount the output of the differentiated goods that are intensive in capital and will reduce the output of the homogenous goods.

Proof:
From the total differentiation of (17), we have:

$$a_{ky}dy + a_{kx}dx = dk \ (KK) \quad (28\text{-}1)$$
$$a_{Ly}dy + a_{Lx}dx = dL \ (LL) \quad (28\text{-}2)$$

Note that a_{kx} and a_{Lx} are constant, since q is constant at constant prices. Rearranging (28) in a matrix form after dividing it by dk and letting dL = 0, we have:

$$\begin{bmatrix} a_{ky} & a_{kx} \\ a_{Ly} & a_{Lx} \end{bmatrix} \begin{bmatrix} dy/dk \\ dx/dk \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (29)$$

let,

$$\triangle = \begin{vmatrix} a_{ky} & a_{kx} \\ a_{Ly} & a_{Lx} \end{vmatrix} = a_{ky}a_{Lx} - a_{Ly}a_{kx} \quad (30)$$

$$\triangle_y = \begin{vmatrix} 1 & a_{kx} \\ 0 & a_{Lx} \end{vmatrix} = a_{Lx} \quad \triangle_x = \begin{vmatrix} a_{ky} & 1 \\ a_{Ly} & 0 \end{vmatrix} = -a_{Ly}$$

Therefore, using Cramer's rule we have:

$$dy/dk = \triangle_y/\triangle = a_{Lx}/(a_{ky}a_{Lx} - a_{kx}a_{Ly}) \quad (31)$$
$$dx/dk = \triangle_x/\triangle = -a_{Ly}/(a_{ky}a_{Lx} - a_{kx}a_{Ly})$$

Note that:

$$dy/dk < 0 \text{ and } dx/dk > 0 \text{ since } k_x = (K/L)_x > k_y = (K/L)_y \quad (32)$$

Q.E.D.

As capital increases in a relatively labor-abundant country, the economy will produce relatively more capital-intensive differentiated goods as explained by the Rybczynski theorem, and thus reduce external trade. The increase in capital will neutralize the difference of relative factor endowment between this country and the capital-abundant country. Therefore, trade will decrease.

By the same reasoning, we can analyze the case of decreased capital in a capital-abundant country. This will accentuate the relative factor endowment of the country. As a result, trade will expand, and the offer curve of the country will shift out.

The following theorem on the pattern of the trade can be easily derived from Proposition 1.

Proposition 2: (The Heckscher-Ohlin theorem)
A relatively capital-abundant country has a comparative advantage in relatively capital-intensive differentiated products.

Proof
If a country is relatively capital-abundant, Proposition 1 tells us that:

$$S_x/S_y > S_x^*/S_y^* \tag{33}$$

where Si and Si* are the supply of good i by the capital-abundant and the labor-abundant countries, respectively.

Assuming that two countries have the same homothetic preferences:[7]

$$D_x/D_y > D_x^*/D_y^* \tag{34}$$

where D_i and D_i^* are the demand for good i by the capital-and labor-abundant countries, respectively. The world consumption of each good equals the world supply. Thus:

$$\begin{aligned} S_x/S_y > D_x/D_y &= (S_x + S_x^*)/(S_y + S_y^*) \\ &= D_x^*/D_y^* > S_x^*/S_y^* \end{aligned} \tag{35}$$

The first inequality says that the capital-abundant country exports x and imports y, and the last inequality says the opposite about the labor-abundant country.

Q.E.D.

The next proposition relating the quality of differentiated goods with autarkic equilibrium prices can be derived easily from the patterns of trade.

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7 If different preferences between the countries are assumed, the analysis will lead to a preference-orientated theory of intra-industry trade. This case is beyond the scope of this paper.

Proposition 3: (Quality in Autarky Economy)
In autarkic equilibrium, the capital-abundant country produces lower quality differentiated goods than the labor abundant country.

Proof
Proposition 2, which states the physical version of the Heckscher-Ohlin theorem, can be transformed into the price version of the Heckscher-Ohlin theorem assuming no factor market distortions as:

$$p_A < p_A^* \tag{36}$$

where p_A and p_A^* are the autarkic prices in the capital-and labor-abundant countries, respectively. This together with (16) proves Proposition 3.

Q.E.D.

The above proposition shows that the capital abundant country produces products of lower quality than the labor-abundant country in autarky. This rather unusual result is due to the assumption that quality can be upgraded through increased use of labor. Thus, increased labor is related with high quality products in good x, and the labo-abundant country will produce high-quality products. If we assume that quality is raised through increased capital use, the above proposition will be reversed. That is, the capital-abundant country produces higher-quality differentiated products. See footnote 5 for an explanation of this reasoning.

With free trade the equilibrium price of the differentiated goods is determined by the intersection of the offer curves of the two countries. The law of one price at free trade gives the following proposition.

Proposition 4: (Quality Equalization at Free Trade)
With free trade the quality of the differentiated goods becomes equal between countries, and determined at world trade prices.

Proof
As free trade opens, the price of good x is equalized between the two

countries. Thus, the quality of good x which is determined by output price p also becomes equal.

Q.E.D.

IV. Conclusion

This paper presents a two-country, two-factor and two-commodity general equilibrium model in which one commodity is vertically differentiated. In the model Leontief technology is used in production as a specific example of constant returns to scale technology.

The analysis of the paper based on capital-intensive vertically differentiated goods is equally appropriate to labor-intensive differentiated goods, requiring only minor changes in specification. Quality enters into a fixed coefficient of only one factor of production, and the physical units of output producible from the endowment of the economy depend on quality inversely. Firms choose an optimal quality to minimize their total cost in providing services of the differentiated goods which are measured by a product of a unit quality and physical quality.

In the model, the PPF is derived in association with quality, and the increase of the price and services of the differentiated goods corresponds to higher quality. The Rybczynski theorem and the Heckscher-Ohlin theorem are proven in the context of the model. At equilibrium, the capital-abundant country produces lower quality differentiated goods than the labor-abundant country, assuming that the differentiated goods are labor-intensive. Furthermore, with free trade the quality of the differentiated goods becomes equal between countries and is determined at world trade prices.

The main propositions of this paper are as follows. First, the capital-abundant country produces lower quality differentiated goods than the labor-abundant country in autarkic equilibrium. If we assume that quality is raised through increased capital use, the results of the paper will be reversed. That is, a capital- abundant country produces higher quality differentiated products. Secondly, with free trade the quality of the differentiated goods becomes equal between countries, and is determined at world trade prices.

This model is an attempt to connect partial equilibrium or ad hoc models of the literature to the standard H-O-S economy. Further development of the paper can be pursued by replacing the specific Leontief technology with generalized constant return to scale technology.

FIGURE 1 Zero Profit Curves

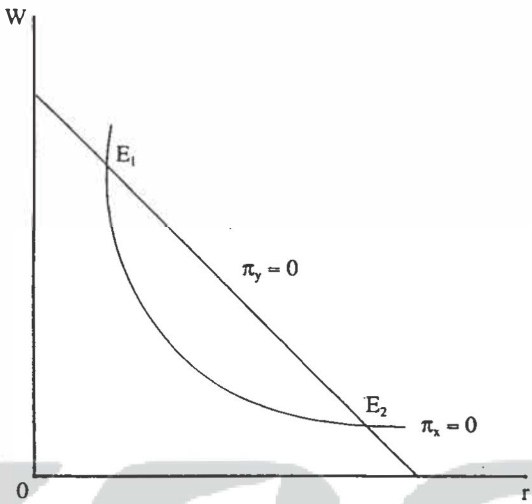

FIGURE 2 Four Quadrant Diagram of the Model

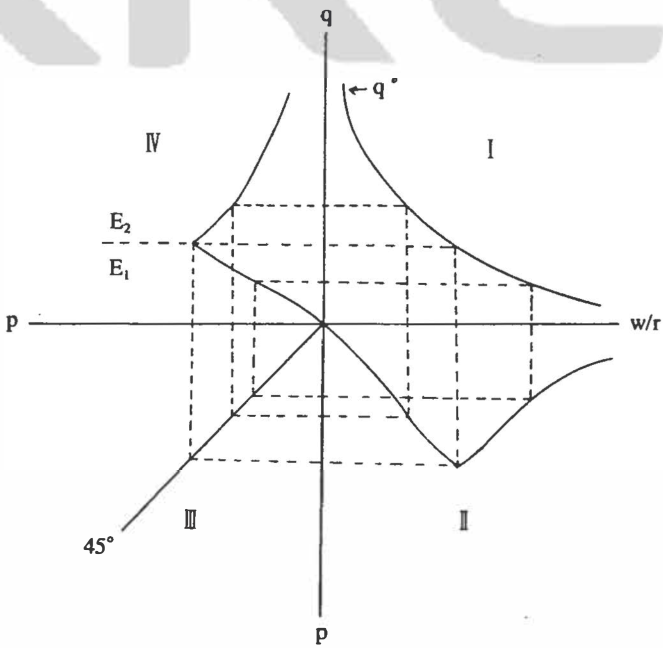

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FIGURE 3 Harrod-Johnson Diagram

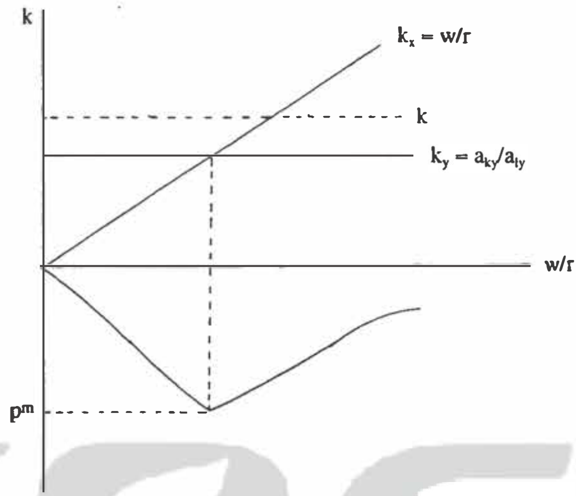

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FIGURE 4 Production Possibility Frontier

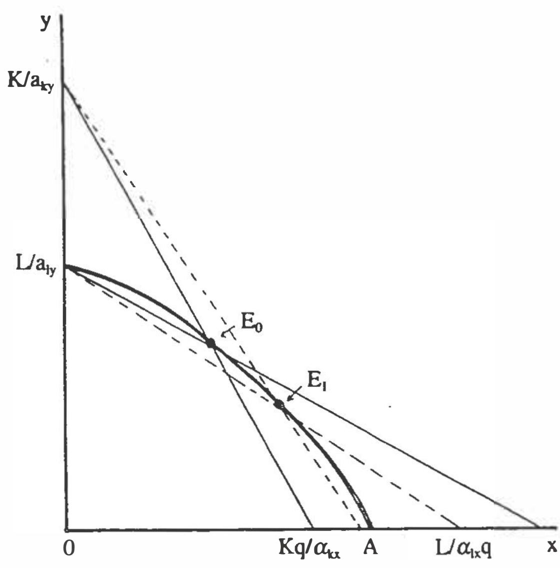


FIGURE 5 Trade Triangle

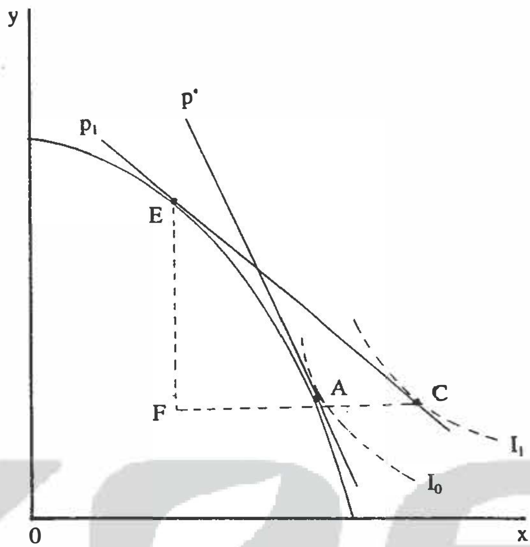
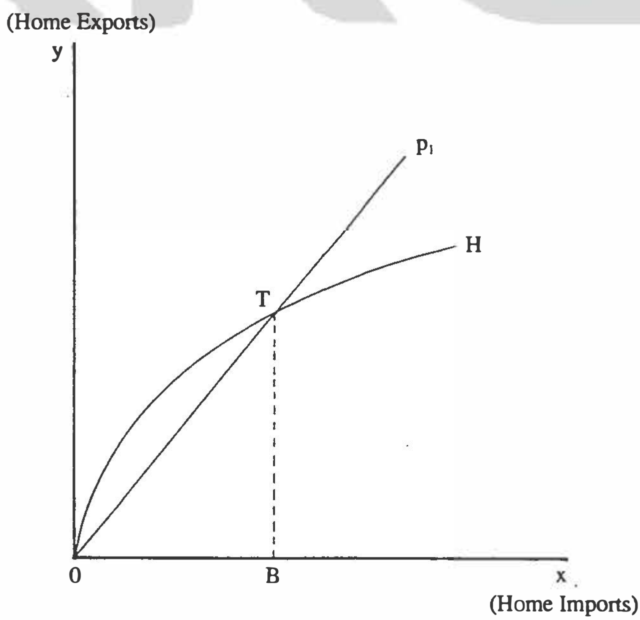


FIGURE 6 Offer Curve



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