A General Welfare Decomposition for CGE Models

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Abstract

Huff and Hertel (2001) derive a welfare decomposition for the equivalent variation in the GTAP model. The derivation appears to be very specific to GTAP. Nevertheless, it contains nearly all the ingredients required for performing welfare decomposition for any CGE model.

In this paper, the approach of Huff and Hertel (2001) is generalised to derive a welfare decomposition that can be applied to most, if not all, CGE models. General production and utility functions are accommodated, as are foreign income flows.

A brief guide to coding the proposed welfare decomposition in GEMPACK is also provided. The decomposition is applied to decomposing the equivalent variation in GTAP.

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General Welfare Decomposition for CGE Models

by Kevin J. HANSLOW

1 Introduction

In the GTAP model, economic welfare is represented as being derived from the allocation of national income between private consumption, government consumption and savings (Hertel 1997). This recognises that households gain benefits from their own current household consumption expenditure. They also benefit from current net national saving, since this increases their future household consumption.[1] Finally, they benefit from the government's provision of public goods and services, as proxied by current government expenditure.[2] National income is allocated between aggregate private consumption, aggregate government consumption and saving to maximise a top-level Cobb-Douglas utility function. With this functional form, successive increases in real household or government expenditure or saving generate equi-proportional increases in economic wellbeing. Aggregate private and government consumption are allocated between particular commodities to maximise constant difference elasticity (CDE) and Cobb-Douglas utility functions, respectively. As the CDE utility function is non-homothetic, this recognises that successive increases in private consumption of *particular* goods or services need not lead to equi-proportional increases in economic wellbeing.

Consequently, given such a definition of economic welfare, how well off a policy change actually makes a region depends on what the change does to its national income. It also depends on the effect of the policy change on prices, and hence the purchasing power of that income. Finally, it depends on how households evaluate the benefits of additional real expenditure. The last item — the marginal utility of real income — is a consequence of the assumed utility functions. National income is nominal net national product (NNP), and is equal to GDP less depreciation less net income payments to foreigners.

One particularly useful feature of GTAP that captures these dependencies is a *welfare decomposition* (Huff and Hertel 2001). This subdivides the overall measure of welfare into components that have a reasonably intuitive interpretation. As just noted, economic well-being depends in part on disposable income, which can be divided into its components — GDP,

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[1] As noted in Hertel (1997), this derives from the work of Howe (1975), who showed that the intertemporal, extended linear expenditure system could be derived from an equivalent, atemporal maximisation problem, in which savings enters the utility function.

[2] As noted in Hertel (1997), this derives from the work of Keller (1980), who showed that if (1) preferences for public goods are separable from preferences for private goods, and (2) the utility function for public goods is identical across households, then a public utility function can be derived. The aggregation of this index with private utility to provide an overall welfare measure requires the further assumption that the level of public goods provided in the initial equilibrium is optimal.

depreciation, and net income payments to foreigners.[3] GDP can be further subdivided into the contributions from primary factors, net indirect taxes and technical changes. Decomposition along these lines leads to the following welfare contributions.

- *Endowment* contributions to welfare arise from changes in the availability of primary factors — for example, increases in the stock of machinery, buildings and agricultural land.
- *Technical efficiency* contributions arise from changes in the use of available inputs in production — for example, improvements in labour productivity.
- *Allocative efficiency* contributions arise when the allocation of resources changes relative to pre-existing distortions.[4]

For any small change in the economy, allocative efficiency contributions are measured as the sum of a number of terms, where each term is the size of an initial indirect tax distortion, multiplied by the policy-induced change in the quantity of goods or services affected by that distortion.[5] The initial indirect tax distortion is the difference between the contribution to output from an additional unit of the good, and the price for which the good could be obtained in the absence of the tax. The product of the distortion and the change in the quantity therefore measures the net contribution to output from the change in the quantity of the good used. The allocative efficiency contribution for a large change to the economy equals the sum of the contributions for a sequence of small changes that are equivalent, in total, to the large change.

There are also contributions to national welfare arising from changes in relative prices (including export relative to import prices, or the terms of trade) as producers and consumers adjust their purchasing and sale patterns in response to policy change. In addition, there are potentially contributions to welfare arising from the likely flow-on effects of production and terms of trade changes on foreign income flows.

The derivation of the welfare decomposition in Huff and Hertel (2001) appears to be very specific to GTAP. It is even expressed in the TABLO notation of the GEMPACK software (Harrison and Pearson 1996) in which GTAP is implemented. Nevertheless, it contains nearly all the ingredients required for performing a welfare decomposition for any CGE model. The derivation uses market clearing conditions for commodities and primary factors, and zero pure profit conditions for industries. These are relationships that would be present in most other CGE models.

In this paper the approach of Huff and Hertel (2001) is generalised to derive a welfare decomposition that can be applied to most, if not all, CGE models. There are six main differences between the approach adopted in this paper and that in Huff and Hertel (2001).

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[3] Net income payments to foreigners are zero in GTAP.

[4] The GTAP welfare decomposition was motivated by the work of Keller (1980), which showed how the aggregate excess burden (the sum across households of compensating variations) was equal to allocative efficiency effects, in a model formulated for examining tax changes.

[5] In multi-step model simulations that correct for linearisation error, this can give an exact measure of the change in the welfare loss 'triangle' associated with a distortion.

First, this paper decomposes the change in utility rather than a money metric measure of the change in welfare (such as the equivalent variation used in GTAP). This decomposition of the change in utility can be used to decompose both money metric measures of welfare and compensation measures based on the balance of trade function. This is important as neither of these welfare measures is clearly superior to the other, and they differ from each other for economies with existing taxes or subsidies (Martin 1996).

Second, this paper includes welfare contributions from foreign income flows.

Third, the decomposition derived in this paper is general enough to cope with multi-product industries with non-separable inputs and outputs, and non-constant returns to scale in production.

Fourth, for each industry, terms measuring the welfare contributions caused by deviations from optimal or price taking behaviour, or from zero pure profits, are derived. They are linear functions of indices of effective inputs and effective outputs for each industry. These terms will be called the *profits effects* (or *effect* if the term for a particular industry, or the economy-wide total of all such terms, is being discussed).[6]

Fifth, whereas in GTAP a nested utility function is assumed — income is allocated between total private and government consumption and savings, and then total consumption is allocated across commodities — there is no requirement for a nested utility function in the current treatment.

Sixth, the welfare decomposition is derived for particular households. This emphasises how each household may have its own relative price effect, since the composition of expenditure may differ with income levels, and the composition of income may vary by household type. By then assuming all households are identical, a welfare decomposition for a representative household, as used in the GTAP model, is obtained.

Another difference between the current paper and the original 1996 version of Huff and Hertel is that the effect of non-homothetic preferences on welfare can be captured in a coefficient by which all the terms, the sum of which equals the change in utility, are multiplied. This is in contrast to the original 1996 version of Huff and Hertel where the effect of non-homothetic preferences on welfare is one of these terms — the variable described as the 'contribution to EV of marginal utility of income'.[7]

Section 2 describes the conceptual economy for which the welfare decomposition is derived. The notation to be used and definitions are introduced, and then the formal derivation is presented.

The profits effects are analysed in section 3. First, it is shown that the profits effects are zero for industries that are revenue maximising, cost minimising and price taking, and have zero pure profits. Second, it is shown that, when the firms in an industry are revenue maximising and cost minimising, the profits effects are sums of terms each of which is the product of a measure of

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[6] This name is chosen since these terms will more usually be non-zero because of non-competitive behaviour, leading to positive pure profits, rather than because of non-optimising behaviour. The choice of this terminology throughout this paper should not, however, be allowed to diminish in the reader's mind the possible usefulness of this term in applications with non-optimising behaviour.

[7] The revised (2001) version of Huff and Hertel eliminates this term.

market power in an output (input) market times the change in the production (usage) of the output (input). Third, for a uniform decrease in market power in all markets, the profits effects are positive.

Section 4 shows how both money metric and compensation based welfare measures can be decomposed using the results of section 2.

Section 5 derives a decomposition of the equivalent variation (EV) for the GTAP model based on the results of sections 2 and 4.

Section 6 concludes with a summary of the current paper and a discussion of issues for future research.

2 *The Formal Derivation of the Welfare Decomposition*

Consider an economy that consists of many activities, each of which uses various inputs. The inputs are divided into two groups — commodities and endowments. The activities are divided into two groups — industries, which produce (possibly multiple) commodities and use both commodities and endowments as inputs, and final demands, which do not produce anything and use only commodities as inputs. Each commodity may be produced by more than one industry. Taxes or subsidies may be levied on all inputs to all activities, and on all outputs from all industries.

The assumption that is fundamental in the derivation of the welfare decomposition is market clearing — the quantity of each commodity produced in the economy equals the total quantity of that commodity used in all activities.

Zero pure profits conditions — that the total cost of all inputs for each industry equals the total value of all commodities produced by that industry — are used later to eliminate some terms in the welfare decomposition (section 3), but are not essential to the derivation.

The market clearing condition applies to both domestic and imported commodities. The following convention is adopted to ensure that the condition applies for imports.[8] A final demand activity 'total imports of each commodity' is included, the inputs to which are imported commodities with negative values, equal in magnitude to the total CIF values of imported commodities used by all other activities. Therefore both the economy-wide production and use of imported commodities is equal to zero; total imports count negatively in final demand; and a market clearing condition can be considered as applying to imported commodities.

Other final demand activities include:

- total exports of each commodity;
- private consumption; and
- government consumption.

[8] This convention is similar to how imports are shown in some input-output tables.

All other final demand activities will be called ‘investment’.[9]

Nominal national income, or NNP, is equal to the returns to all endowments (inclusive of income taxes), minus the value of depreciation of domestic capital, plus all indirect tax revenue, minus all indirect subsidy payments, plus net foreign income flows generated by a range of net foreign assets. Net foreign income flows may be positive or negative. Thus nominal NNP is equal to nominal GDP, minus depreciation, plus net foreign income flows.

Nominal NNP is allocated between purchases of private consumption commodities, government consumption commodities, and savings so as to maximise a utility function.

2.1 Motivation

Before introducing the notation and conventions required for the formal derivation, a brief overview of the derivation is now provided.

The derivation proceeds using linearised equations. Real income (that is, utility) of a household is expressed as a multiple of the difference between nominal household income and an expenditure price index. Nominal household income is expressed as a share of nominal NNP. Then nominal NNP is split into GDP minus depreciation plus foreign income. The latter two items are then decomposed into nominal and real parts. The depreciation terms are written as a sum across industries, but could just as well have been left as a macro aggregate. The GDP index, any price parts of depreciation and foreign income, and the expenditure price index for the household constitute the relative price contributions to welfare of the household. Real GDP is then decomposed, in terms of the industry structure just outlined, into allocative efficiency, technical efficiency and endowment effects. It is at this stage that the market clearing conditions are critical. Finally, a residual term is obtained, which is zero if the conventional assumptions of CGE models — zero pure profits and optimising and price taking behaviour — are satisfied.

Consequently, quite a bit of notation is required to support the formal derivation — both its macro and micro components — and such notation is now introduced.

2.2 Notation

Upper case letters designate levels, lower case percentage changes. Δ means ‘change in’. Superscripts on a symbol indicate to what item the symbol is related. Subscripts indicate a variety of types of the item indicated by the superscript. For example, Π_k^{DK} designates the asset price of domestic capital of type k.

[9] Thus all the final demands usually represented in an IO table and in the definition of GDP are present. Since, in some CGE models (for example, the MONASH model described in Dixon and Rimmer (2000)), there is an investment activity for each industry, it seemed sensible to allow for the possibility of many investment activities in the current treatment. It makes no difference in the formal derivation.

Symbols used are:

- P — tax-inclusive price (rental price for assets), but tax-exclusive price when applied to industry outputs of commodities, that is, with superscript O;
- Π — asset price;
- Q — real quantity;
- V — tax-inclusive value (rental value for assets);
- $\hat{V}$ — tax-exclusive value (rental value for assets);
- R — tax revenue;
- T — *ad valorem* tax rate;
- D — depreciation rate;
- $\Re$ — rate of return (lower case is ρ);
- $U(\cdot)$ — indirect utility function governing the allocation of income;
- N — number of households; and
- Λ — share of a household in national income.

A bar over a symbol indicates effective inputs (outputs). For quantities, these are the input quantities (output quantities) multiplied (divided) by the corresponding technical efficiencies. Effective prices are defined so that:

$$\overline{P}.\overline{Q} = P.Q$$

A bold, non-italicised symbol should be interpreted as a vector. For example, $\mathbf{P}^C$ is the vector of tax-inclusive prices of commodities purchased for private consumption. Multiplication of two vectors should be interpreted as the dot (that is, scalar) product. For example, the value of aggregate private consumption equals the sum of consumption prices times consumption quantities, thus: $V^C = \mathbf{P}^C.\mathbf{Q}^C$.

Superscripts used are:

- NNP — net national product;
- NDP — net domestic product;
- GDP — gross domestic product;
- DK — domestic capital;
- FY — foreign income;
- FA — foreign asset;
- DEP — depreciation;
- C — private consumption;
- G — government consumption;
- I — gross investment;

- X — exports;
- M — imports;
- S — savings;
- I — input into activities; and
- O — output from industries.

Subscripts used are:

- k — to range across types of domestic capital;
- φ — to range across types of foreign assets;
- a — to range over activities;
- f — to range over final demands (a subset of activities);
- j — to range over industries (a subset of activities);
- i — to range over inputs;
- e — to range over endowments (a subset of inputs);
- c — to range over commodities (a subset of inputs);
- h — to range over households; and
- $\bullet$ — total over a dimension.

Where two subscripts occur, the first refers to an element of the set of inputs, while the second refers to an element of the set of activities.

2.3 Definitions and Conventions

Macro Aggregates

The real income of household *h*, Q_h^{NNP},[10] is defined to be the maximised value of utility, that is:

$$Q_h^{NNP} = U_h\left(\mathbf{P}^C, \mathbf{P}^G, P^S, V_h^{NNP}\right) \qquad (1)$$

The percentage changes in aggregates are defined as value-share-weighted averages across all components. For example, national real private consumption is defined by:

$$\begin{aligned} V^C.q^C &= \mathbf{V}^C.\mathbf{q}^C \\ &= \sum_c V_c^C.q_c^C \end{aligned} \qquad (2)$$

Again, if there is a set of investment activities $\Im$ (a subset of the set of final demands), then the investment price index is defined by:

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[10] A superscript *NNP* is used to emphasise that household income is some share of national income.

$$V^{I}.p^{I} = \mathbf{V}^{I}.\mathbf{p}^{I} = \sum_{c,\,f\in\Im} V^{I}_{cf}.p^{I}_{cf} \tag{3}$$

Foreign Income

In reality, some foreign income flows — for example, foreign aid — are not returns to some asset. The convention adopted in such a case is that such foreign income flows are returns on an asset, with the rate of return constant at one, and the asset price equal to the NNP price index, thus:

$$\Pi^{FA}_{\varphi} = P^{NNP} \tag{4}$$

$$\Re^{FA}_{\varphi} = 1 \tag{5}$$

and, consequently, the quantity of the asset is defined to be the foreign income flow divided by the NNP price index. This is a sensible convention, with the foreign income flow being equal to the real foreign income flow times an expenditure price index.

Tax-Inclusive and Tax Exclusive Prices and Values

The next two equations relate the price received by industry *j* for producing commodity *c* (P_{cj}^{O}), the economy-wide uniform output tax inclusive price of commodity *c* (P_{c}^{O}), and the price paid by industry *j* for commodity *c* (P_{cj}^{I}).

$$P^{O}_{c} = P^{O}_{cj}.\left(1+T^{O}_{cj}\right) \tag{6}$$

$$P^{I}_{cj} = P^{O}_{c}.\left(1+T^{I}_{cj}\right) \tag{7}$$

The next four equations clarify the use of V and $\hat{V}$ to denote tax-inclusive and tax-exclusive values, respectively.

$$\hat{V}^{O}_{cj} = P^{O}_{cj}.Q^{O}_{cj} \tag{8}$$

$$V^{O}_{cj} = P^{O}_{c}.Q^{O}_{cj} = \hat{V}^{O}_{cj} + R^{O}_{cj} \tag{9}$$

$$\hat{V}^{I}_{cj} = P^{O}_{c}.Q^{I}_{cj} \tag{10}$$

$$V^{I}_{ij} = P^{I}_{ij}.Q^{I}_{ij} = \hat{V}^{I}_{ij} + R^{I}_{ij} \tag{11}$$

Note that equation (11) uses a subscript of *i* rather than *c*, since we wish to accommodate the possibility of industry-specific taxes on primary factor inputs.

2.4 Derivation

Overview

The derivation proceeds, using linearised equations, as follows. The change in utility for a particular household is expressed as the difference between nominal household income (which is some share of nominal NNP that accrues to the household) and an expenditure price index (ranging over the prices of private and government consumption goods and saving) for the household. The nominal household income is expressed in terms of the change in the share (of the household in NNP) and the change in nominal NNP, the latter being equal to changes in nominal GDP minus depreciation plus foreign income. The endowment and rate of return contributions to welfare from the latter two items are identified. The price index of GDP, the expenditure price index for the household, and any asset price parts of depreciation and foreign income are manipulated to define a relative price contribution to welfare for the household. For an economy consisting of identical households, this collapses to two welfare contribution terms — terms of trade and asset price contributions to welfare. Then it only remains to decompose the percentage change in real GDP. Real GDP is expressed from the expenditure side as a share-weighted sum across commodity inputs into all final demand activities. Allocative efficiency contributions are derived by splitting off indirect tax revenues from the values of inputs and outputs multiplied by percentage changes in quantities. Market clearing conditions are used to eventually yield an expression that is a linear function of share-weighted indices of industries' outputs and inputs. These can be written as a weighted sum of technical efficiency terms — the technical efficiency contribution to welfare — and a difference of weighted sums of effective outputs and effective inputs — the contribution from non-optimising and/or non-price taking behaviour, or from deviations from zero pure profits.

Formal Derivation

Real income (utility) for household h is:

$$Q_h^{NNP} = U_h\left(\mathbf{P}^C, \mathbf{P}^G, P^S, V_h^{NNP}\right) \tag{12}$$

The linearisation of this is:

$$\begin{aligned}
&\Delta Q_h^{NNP} \\
&= \partial U_h / \partial \mathbf{P}^C .\Delta \mathbf{P}^C + \partial U_h / \partial \mathbf{P}^G .\Delta \mathbf{P}^G + \partial U_h / \partial P^S .\Delta P^S + \partial U_h / \partial V_h^{NNP} .\Delta V_h^{NNP} \\
&= \frac{\partial U}{\partial V_h^{NNP}} .\left(\Delta V_h^{NNP} - \mathbf{Q}_h^C .\Delta \mathbf{P}^C - \mathbf{Q}_h^G .\Delta \mathbf{P}^G - Q_h^S .\Delta P^S\right) \\
&= \frac{\partial U}{\partial V_h^{NNP}} .\left(V^{NNP} .\Delta \Lambda_h^{NNP} + \Lambda_h^{NNP} .\Delta V^{NNP} - \mathbf{Q}_h^C .\Delta \mathbf{P}^C - \mathbf{Q}_h^G .\Delta \mathbf{P}^G - Q_h^S .\Delta P^S\right)
\end{aligned} \tag{13}$$

where Roy's identity has been used to derive the third line and the fourth line introduces the share of household h in NNP. If changes are converted to percentage changes (for example, $\Delta Q = Q \cdot q/100$) then:

$$Q_h^{NNP}.q_h^{NNP} = \frac{\partial U_h}{\partial V_h^{NNP}}.\left(V_h^{NNP}.\lambda_h^{NNP} + V_h^{NNP}.v^{NNP} - \mathbf{V}_h^C.\mathbf{p}^C - \mathbf{V}_h^G.\mathbf{p}^G - V_h^S.p^S\right)$$
$$= \frac{\partial U_h}{\partial V_h^{NNP}}.\left(V_h^{NNP}.\lambda_h^{NNP} + V_h^{NNP}.v^{NNP} - V_h^{NNP}.p_h^{NNP}\right) \quad (14)$$

Nominal NNP can be related to NDP and GDP as follows:

$$\begin{aligned} V^{NNP} &= V^{NDP} - V^{FY} \\ &= V^{GDP} - V^{DEP} + V^{FY} \\ &= V^{GDP} - \sum_k V_k^{DEP} + \sum_\varphi V_\varphi^{FA} \\ &= V^{GDP} - \sum_k D_k^{DK}.\Pi_k^{DK}.Q_k^{DK} + \sum_\varphi \Re_\varphi^{FA}.\Pi_\varphi^{FA}.Q_\varphi^{FA} \end{aligned} \quad (15)$$

Linearisation of this yields:

$$\begin{aligned} V^{NNP}.v^{NNP} = & V^{GDP}.\left(p^{GDP} + q^{GDP}\right) \\ & - \sum_k V_k^{DEP}.\left(d_k^{DK} + \pi_k^{DK} + q_k^{DK}\right) \\ & + \sum_\varphi V_\varphi^{FA}.\left(\rho_\varphi^{FA} + \pi_\varphi^{FA} + q_\varphi^{FA}\right) \end{aligned} \quad (16)$$

Finally, we define the NNP and GDP price indices as follows:

$$V^{NNP}.p^{NNP} = V^C.p^C + V^G.p^G + V^S.p^S \quad (17)$$

and

$$V^{GDP}.p^{GDP} = V^C.p^C + V^G.p^G + V^I.p^I + V^X.p^X - V^M.p^M \quad (18)$$

We are now in a position to express the percentage change in real income (utility) for household *h* as a function of the component price and quantity changes using equations (14) and (16)-(18). First, rewrite equation (14) by dividing both sides by Q_h^{NNP} and defining:

$$\Theta_h = \frac{\partial U_h}{\partial V_h^{NNP}}.\frac{V_h^{NNP}}{Q_h^{NNP}} \quad (19)$$

thus yielding:

$$q_h^{NNP} = \Theta_h.\left(\lambda_h^{NNP} + v^{NNP} - p_h^{NNP}\right) \quad (20)$$

Second, divide both sides of equation (16) by V^{NNP} and substitute the expression for v^{NNP} into equation (20) to yield:

$$q_h^{NNP} = \Theta_h \cdot \begin{pmatrix} \lambda_h^{NNP} \\ + V^{GDP}.\left(p^{GDP} + q^{GDP}\right) / V^{NNP} \\ - \sum_k V_k^{DEP}.\left(d_k^{DK} + \pi_k^{DK} + q_k^{DK}\right) / V^{NNP} \\ + \sum_\varphi V_\varphi^{FA}.\left(\rho_\varphi^{FA} + \pi_\varphi^{FA} + q_\varphi^{FA}\right) / V^{NNP} \\ - p_h^{NNP} \end{pmatrix} \tag{21}$$

Third, equation (18) is used to eliminate the expression $V^{GDP} \cdot p^{GDP}$ from equation (21) to yield:

$$q_h^{NNP} = \Theta_h \cdot \begin{pmatrix} \lambda_h^{NNP} \\ + \left(V^C.p^C + V^G.p^G + V^I.p^I + V^X.p^X - V^M.p^M\right) / V^{NNP} \\ + V^{GDP}.q^{GDP} / V^{NNP} \\ - \sum_k V_k^{DEP}.\left(d_k^{DK} + \pi_k^{DK} + q_k^{DK}\right) / V^{NNP} \\ + \sum_\varphi V_\varphi^{FA}.\left(\rho_\varphi^{FA} + \pi_\varphi^{FA} + q_\varphi^{FA}\right) / V^{NNP} \\ - p_h^{NNP} \end{pmatrix} \tag{22}$$

Fourth, the components of the GDP price index in equation (22) are collected into three groups thus:

$$q_h^{NNP} = \Theta_h \cdot \begin{pmatrix} \lambda_h^{NNP} \\ + \left(V^C.p^C + V^G.p^G + V^S.p^S\right) / V^{NNP} \\ + \left(V^I.p^I - V^S.p^S\right) / V^{NNP} \\ + \left(V^X.p^X - V^M.p^M\right) / V^{NNP} \\ + V^{GDP}.q^{GDP} / V^{NNP} \\ - \sum_k V_k^{DEP}.\left(d_k^{DK} + \pi_k^{DK} + q_k^{DK}\right) / V^{NNP} \\ + \sum_\varphi V_\varphi^{FA}.\left(\rho_\varphi^{FA} + \pi_\varphi^{FA} + q_\varphi^{FA}\right) / V^{NNP} \\ - p_h^{NNP} \end{pmatrix} \tag{23}$$

where the expression $V^S \cdot p^S$ has been added to (subtracted from) the first (second) group. Finally, equation (17) is used to replace the first group of prices in equation (23) by the NNP price index. A rearrangement of terms then yields:

$$
\begin{aligned}
& q_h^{NNP} \\
& = \Theta_h . \lambda_h^{NNP} \\
& + \Theta_h . \left[V^X . p^X - V^M . p^M \right] \big/ V^{NNP} \\
& + \Theta_h . \left[\left(V^I . p^I + \sum_{\varphi} V_{\varphi}^{FA} . \pi_{\varphi}^{FA} \right) - \left(V^S . p^S + \sum_k V_k^{DEP} . \pi_k^{DK} \right) \right] \big/ V^{NNP} \\
& + \Theta_h . \left(p^{NNP} - p_h^{NNP} \right) \\
& - \Theta_h . \left(\sum_k V_k^{DEP} . \left(d_k^{DK} + q_k^{DK} \right) \right) \big/ V^{NNP} \\
& + \Theta_h . \left(\sum_{\varphi} V_{\varphi}^{FA} . q_{\varphi}^{FA} \right) \big/ V^{NNP} \\
& + \Theta_h . \left(\sum_{\varphi} V_{\varphi}^{FA} . \rho_{\varphi}^{FA} \right) \big/ V^{NNP} \\
& + \Theta_h . \left(V^{GDP} . q^{GDP} \right) \big/ V^{NNP}
\end{aligned}
\qquad (24)
$$

Note that Θ_h is the elasticity of utility with respect to nominal household income for household *h*. It is equal to one for a homothetic direct utility function.

In the expression for the real income (utility) of household *h* in equation (24), the second line is the impact of changes in the share of national income accruing to household *h*, and is determined by how an economic change affects the particular sources of income of household *h*.[11] The third to fifth lines define the impact of relative prices. The third line is a terms of trade effect for the whole economy. The fourth line is an effect, again for the whole economy, of prices related to assets held or to the creation of assets. The fifth line is the household specific effect of the price paid by the household for its goods relative to the national average price. The sixth line defines the contribution to real income from losses in the nation's endowment of capital arising from depreciation, while the seventh line defines contributions from changes in real foreign assets. The eighth line is the impact of changes in rates of return from foreign assets. The final line, involving real GDP, needs to be decomposed further.

Before proceeding with the decomposition of real GDP, some simplifications for the case where all households are identical will be derived. This provides a decomposition of utility for the representative consumer, as would be required in the current implementation of the GTAP model.

The main simplification is in the relative price contribution. Since all households are now assumed to be identical, the composition of their expenditure across private and government consumption goods is identical to that of the nation as a whole. Hence the household specific expenditure price index is equal to the NNP price index. Formally, designating the number of households by *N*,

[11] The further decomposition of this term is not pursued in this paper, which ultimately has a view to generalising the GTAP welfare decomposition, for which there is only a single representative household.

$$
\begin{aligned}
V^{NNP}.p_h^{NNP} &= N.V_h^{NNP}.p_h^{NNP} \\
&= N.\left(\mathbf{V}_h^C.\mathbf{p}^C + \mathbf{V}_h^G.\mathbf{p}^G + V_h^S.p^S\right) \\
&= N.\left(\frac{\mathbf{V}^C}{N}.\mathbf{p}^C + \frac{\mathbf{V}^G}{N}.\mathbf{p}^G + \frac{V^S}{N}.p^S\right) \\
&= \mathbf{V}^C.\mathbf{p}^C + \mathbf{V}^G.\mathbf{p}^G + V^S.p^S \\
&= V^C.p^C + V^G.p^G + V^S.p^S \\
&= V^{NNP}.p^{NNP}
\end{aligned}
\tag{25}
$$

Consequently, the real income (utility) of the representative household can be decomposed thus:

$$
\begin{aligned}
& q_h^{NNP} \\
= \quad & -\Theta_h.n && (= POP) \\
& +\Theta_h.\left[V^X.p^X - V^M.p^M\right]/V^{NNP} && (= TOT) \\
& +\Theta_h.\left[\left(V^I.p^I + \textstyle\sum_\varphi V_\varphi^{FA}.\pi_\varphi^{FA}\right) - \left(V^S.p^S + \textstyle\sum_k V_k^{DEP}.\pi_k^{DK}\right)\right]/V^{NNP} && (= ASS\_PRI) \\
& +\Theta_h.\textstyle\sum_\varphi V_\varphi^{FA}.\rho_\varphi^{FA}/V^{NNP} && (= RORF) \\
& +\Theta_h.\textstyle\sum_\varphi V_\varphi^{FA}.q_\varphi^{FA}/V^{NNP} && (= FENDW) \\
& -\Theta_h.\textstyle\sum_k V_k^{DEP}.\left(d_k^{DK} + q_k^{DK}\right)/V^{NNP} && (\rightarrow ENDW) \\
& +\Theta_h.V^{GDP}.q^{GDP}/V^{NNP}
\end{aligned}
\tag{26}
$$

The terms in parentheses on the far right-hand side show the correspondence between the algebraic expressions on the right-hand side of the equation and the contributions in the completed welfare decomposition. An equal sign (=) before the name of the welfare contribution indicates that the algebraic expression is equal to this contribution. An arrow (→) indicates that the algebraic expression is one part of this contribution. The welfare contributions are:

- *POP* — population change;
- *TOT* — terms of trade;
- *ASS_PRI* — asset price;
- *RORF* — foreign rate of return;
- *FENDW* — foreign endowment; and
- *ENDW* — (domestic) endowment contributions.

From now on it will be assumed that we are dealing with the case in which all households are identical, that is, the case of the representative household.

It now remains to decompose the real GDP component, which is the final term in equation (26). Real GDP can be expressed, from the expenditure side, as:

$$\begin{aligned}\Theta_h . V^{GDP} . q^{GDP} \big/ V^{NNP} &= \Theta_h . \textstyle\sum_{c,f} V^{I}_{cf} . q^{I}_{cf} \big/ V^{NNP} \\ &= \Theta_h . \textstyle\sum_{c,f} \hat{V}^{I}_{cf} . q^{I}_{cf} \big/ V^{NNP} \\ &\quad + \Theta_h . \textstyle\sum_{c,f} R^{I}_{cf} . q^{I}_{cf} \big/ V^{NNP} \quad (\rightarrow ALLOC)\end{aligned} \tag{27}$$

where equation (11) has been used to split input tax-inclusive values of inputs to final demand into tax-exclusive values and the tax revenue. The sum involving the tax revenues is that part of the allocative efficiency contribution (*ALLOC*) attributable to taxes on final demands. The remaining RHS term of equation (27) can be expressed as:

$$\Theta_h . \textstyle\sum_{c,f} \hat{V}^{I}_{cf} . q^{I}_{cf} \big/ V^{NNP} = \Theta_h . \left[\textstyle\sum_{c,a} \hat{V}^{I}_{ca} . q^{I}_{ca} - \textstyle\sum_{c,j} \hat{V}^{I}_{cj} . q^{I}_{cj} \right] \big/ V^{NNP} \tag{28}$$

where the sum over all final demands on the LHS has been expressed on the RHS as the difference of the sum over all activities and the sum over all industries. In equation (28), the second sum on the RHS, which ranges over all commodity inputs into industries, can be further expressed as a difference of the sum over all inputs and the sum over primary factor inputs thus:

$$\begin{aligned}-\Theta_h . \textstyle\sum_{c,j} \hat{V}^{I}_{cj} . q^{I}_{cj} \big/ V^{NNP} &= -\Theta_h . \textstyle\sum_{i,j} \hat{V}^{I}_{ij} . q^{I}_{ij} \big/ V^{NNP} \\ &\quad + \Theta_h . \textstyle\sum_{e,j} \hat{V}^{I}_{ej} . q^{I}_{ej} \big/ V^{NNP} \quad (\rightarrow ENDW)\end{aligned} \tag{29}$$

the last sum being part of the contribution of changes in domestic endowments to changes in welfare (*ENDW*). The first sum on the RHS of equation (29) involves input tax-exclusive values. It is now expressed as the difference of a sum involving tax-inclusive values and a sum involving tax revenues, using equation (11), thus:

$$\begin{aligned}-\Theta_h . \textstyle\sum_{i,j} \hat{V}^{I}_{ij} . q^{I}_{ij} \big/ V^{NNP} &= -\Theta_h . \textstyle\sum_{i,j} V^{I}_{ij} . q^{I}_{ij} \big/ V^{NNP} \\ &\quad + \Theta_h . \textstyle\sum_{i,j} R^{I}_{ij} . q^{I}_{ij} \big/ V^{NNP} \quad (\rightarrow ALLOC)\end{aligned} \tag{30}$$

where the last sum is that part of the allocative efficiency contribution (*ALLOC*) attributable to all taxes on inputs to industries. Drawing together equations (28)-(30) yields:

$$\begin{aligned}\Theta_h . \textstyle\sum_{c,f} \hat{V}^{I}_{cf} . q^{I}_{cf} \big/ V^{NNP} &= \Theta_h . \left[\textstyle\sum_{c,a} \hat{V}^{I}_{ca} . q^{I}_{ca} - \textstyle\sum_{i,j} V^{I}_{ij} . q^{I}_{ij} \right] \big/ V^{NNP} \\ &\quad + \Theta_h . \textstyle\sum_{i,j} R^{I}_{ij} . q^{I}_{ij} \big/ V^{NNP} \qquad (\rightarrow ALLOC) \\ &\quad + \Theta_h . \textstyle\sum_{e,j} \hat{V}^{I}_{ej} . q^{I}_{ej} \big/ V^{NNP} \qquad (\rightarrow ENDW)\end{aligned} \tag{31}$$

The first term on the RHS of equation (31) is dealt with using the market clearing conditions, which are now derived in linearised form. In levels, for all commodities *c*:

$$\textstyle\sum_a Q^{I}_{ca} = \textstyle\sum_j Q^{O}_{cj} \tag{32}$$

Linearisation yields:

$$\textstyle\sum_a Q^{I}_{ca} . q^{I}_{ca} = \textstyle\sum_j Q^{O}_{cj} . q^{O}_{cj} \tag{33}$$

and, multiplying by the price, P_c^{O}, of commodity *c*:

$$\sum_a \hat{V}^{I}_{ca}.q^{I}_{ca} = \sum_j V^{O}_{cj}.q^{O}_{cj} \tag{34}$$

Substituting equation (34) into the first term on the RHS of equation (31) yields:

$$\Theta_h.\left[\sum_{c,a} \hat{V}^{I}_{ca}.q^{I}_{ca} - \sum_{i,j} V^{I}_{ij}.q^{I}_{ij}\right] \Big/ V^{NNP} = \Theta_h.\left[\sum_{c,j} V^{O}_{cj}.q^{O}_{cj} - \sum_{i,j} V^{I}_{ij}.q^{I}_{ij}\right] \Big/ V^{NNP} \tag{35}$$

The first sum on the RHS of equation (35) involves output tax-inclusive values of commodities produced by each industry. This sum is expressed as the addition of two sums involving tax-exclusive values and tax revenues, using equation (9), thus:

$$\begin{aligned} \Theta_h.\sum_{c,j} V^{O}_{cj}.q^{O}_{cj} \Big/ V^{NNP} \quad = \quad & \Theta_h.\sum_{c,j} \hat{V}^{O}_{cj}.q^{O}_{cj} \Big/ V^{NNP} \\ & + \Theta_h.\sum_{c,j} R^{O}_{cj}.q^{O}_{cj} \Big/ V^{NNP} \quad (\rightarrow ALLOC) \end{aligned} \tag{36}$$

the second term on the RHS of equation (36) being the final part of the allocative efficiency contribution (*ALLOC*), that attributable to output taxes. The final stage in the derivation of the welfare decomposition involves gathering together the terms not yet associated with a particular welfare contribution (the second sum on the RHS of equation (31) and the first sum on the RHS of equation (36)), and replacing percentage changes in quantities by percentage changes in effective quantities and technical efficiencies, thus:

$$\begin{aligned} & \Theta_h.\left[\sum_{c,j} \hat{V}^{O}_{cj}.q^{O}_{cj} - \sum_{i,j} V^{I}_{ij}.q^{I}_{ij}\right] \Big/ V^{NNP} \\ = \quad & \Theta_h.\left[\sum_{c,j} \hat{V}^{O}_{cj}.\bar{q}^{O}_{cj} - \sum_{i,j} V^{I}_{ij}.\bar{q}^{I}_{ij}\right] \Big/ V^{NNP} \quad (= PROFITS) \\ & + \Theta_h.\left[\sum_{c,j} \hat{V}^{O}_{cj}.a^{O}_{cj} + \sum_{i,j} V^{I}_{ij}.a^{I}_{ij}\right] \Big/ V^{NNP} \quad (= TECH) \end{aligned} \tag{37}$$

The replacement of quantities by effective quantities and technical efficiencies yields the technical efficiency contribution (*TECH*) and the profits effect contribution (*PROFITS*).

2.5 Summary of Welfare Decomposition

Combining equations (27)-(31) and (35)-(37), and substituting into equation (26), yields the final welfare decomposition for the representative household. A tabular summary of the welfare decomposition equations is provided in table 1. Table 2 lists the status, in a GEMPACK implementation (Harrison and Pearson 1996), of each of the items introduced in the derivation of the welfare decomposition. It also provides brief descriptions of each item.

Table 1   Summary of the Welfare Decomposition

$$
\begin{aligned}
q_h^{NNP} &= POP + TOT + ASS\_PRI \\
&\quad + RORF + FENDW \\
&\quad + ENDW + ALLOC + TECH + PROFITS \\
\Theta_h &= \frac{\partial U_h\left(\mathbf{P}^C, \mathbf{P}^G, P^S, V_h^{NNP}\right)}{\partial V_h^{NNP}} \cdot \frac{V_h^{NNP}}{Q_h^{NNP}} \\
POP &= -\Theta_h . n \\
TOT &= \Theta_h . \left[V^X . p^X - V^M . p^M\right] / V^{NNP} \\
ASS\_PRI &= \Theta_h . \left[\left(V^I . p^I + \textstyle\sum_\varphi V_\varphi^{FA} . \pi_\varphi^{FA}\right) - \left(V^S . p^S + \textstyle\sum_k V_k^{DEP} . \pi_k^{DK}\right)\right] / V^{NNP} \\
RORF &= \Theta_h . \textstyle\sum_\varphi V_\varphi^{FA} . \rho_\varphi^{FA} / V^{NNP} \\
FENDW &= \Theta_h . \textstyle\sum_\varphi V_\varphi^{FA} . q_\varphi^{FA} / V^{NNP} \\
ENDW &= \Theta_h . \left[\textstyle\sum_{e,j} \hat{V}_{ej}^{\mathbf{I}} . q_{ej}^{\mathbf{I}} - \textstyle\sum_k V_k^{DEP} . \left(d_k^{DK} + q_k^{DK}\right)\right] / V^{NNP} \\
ALLOC &= \Theta_h . \left[\textstyle\sum_{c,f} R_{cf}^{\mathbf{I}} . q_{cf}^{\mathbf{I}} + \textstyle\sum_{i,j} R_{ij}^{\mathbf{I}} . q_{ij}^{\mathbf{I}} + \textstyle\sum_{c,j} R_{cj}^{\mathbf{0}} . q_{cj}^{\mathbf{0}}\right] / V^{NNP} \\
TECH &= \Theta_h . \left[\textstyle\sum_{c,j} \hat{V}_{cj}^{\mathbf{0}} . a_{cj}^{\mathbf{0}} + \textstyle\sum_{i,j} V_{ij}^{\mathbf{I}} . a_{ij}^{\mathbf{I}}\right] / V^{NNP} \\
PROFITS &= \Theta_h . \left[\textstyle\sum_{c,j} \hat{V}_{cj}^{\mathbf{0}} . \bar{q}_{cj}^{\mathbf{0}} - \textstyle\sum_{i,j} V_{ij}^{\mathbf{I}} . \bar{q}_{ij}^{\mathbf{I}}\right] / V^{NNP}
\end{aligned}
$$

Table 2 Welfare Decomposition Terms with Guide to GEMPACK Implementation

Symbol	Description	Status in GEMPACK
q^{NNP}	Percentage change in real net national product	Percentage change variable
Θ_h	Elasticity of utility with respect to nominal net national product	Coefficient (non -parameter)
POP	Population change welfare contribution	Change variable
TOT	Terms of trade welfare contribution	Change variable
ASS_PRI	Asset price welfare contribution	Change variable
RORF	Foreign rate of return welfare contribution	Change variable
FENDW	Foreign endowment welfare contribution	Change variable
ENDW	Domestic endowment welfare contribution	Change variable
ALLOC	Allocative efficiency welfare contribution	Change variable
TECH	Technical efficiency welfare contribution	Change variable
PROFITS	Profits effect	Change variable

3..The Properties of the Profits Effects

The first part of this section shows that under the standard assumptions of CGE models — optimising and price taking behaviour and zero pure profits — the profits effects are zero. Because these assumptions were introduced from the outset in previous welfare decomposition derivations, the residual profits effects terms were not isolated. The advantage of having such a term lies in its possible application in economic models that push beyond the standard assumptions of GE theory, by incorporating features such as imperfect competition.[12] In the second and subsequent parts of this section, the properties of the profits effects when agents are optimising but have market power are examined. The second part of this section shows how the profits effects can be written in terms of quantity changes and the demand (supply) elasticities facing the agent in the output (input) markets in which they wield some power. This is then applied in the third part of this section to examine the behaviour of the profits effects under uniform reductions in market power, as represented by uniform relative decreases in the elasticities.

3.1 The Profits Effects are Zero Under Standard CGE Assumptions

Theorem: If industry *j* maximises revenue, minimises costs, is a price taker and has zero pure profits then

$$\sum_{c,j} \hat{V}^{O}_{cj}.\bar{q}^{O}_{cj} = \sum_{i,j} V^{I}_{ij}.\bar{q}^{I}_{ij} \tag{38}$$

If these conditions are satisfied for all industries then *PROFITS*=0.[13]

Proof: Define revenue and cost functions for industry *j*, thus:

$$\begin{aligned} \mathrm{R}\left(\overline{\mathbf{P}}^{O}_{j}, \overline{\mathbf{Q}}^{I}_{j}\right) &= \max\left\{\overline{\mathbf{P}}^{O}_{j}.\overline{\mathbf{Q}} : \quad \overline{\mathbf{Q}} \in \mathrm{Y}\left(\overline{\mathbf{Q}}^{I}_{j}\right)\right\} \\ \mathrm{C}\left(\overline{\mathbf{P}}^{I}_{j}, \overline{\mathbf{Q}}^{O}_{j}\right) &= \min\left\{\overline{\mathbf{P}}^{I}_{j}.\overline{\mathbf{Q}} : \quad \overline{\mathbf{Q}} \in \mathrm{X}\left(\overline{\mathbf{Q}}^{O}_{j}\right)\right\} \end{aligned} \tag{39}$$

where Y(·) and X(·) denote production possibility and inputs requirement sets, respectively. Note that the revenue and cost functions are expressed in terms of effective prices and effective quantities, which can change because of changes in prices, quantities or technical efficiencies. The derivation to be presented would not be valid if the revenue and cost functions were expressed in terms of actual prices and quantities, as these functions would implicitly embody an

[12] It should be emphasised that although the welfare decomposition derived in the current paper takes into account the market power aspects of imperfect competition, via the profits effect, it does not take into account the welfare effects of changing the number of varieties of a commodity. Although the later derivations in the current section deal with individual firms and their market power, there is no theory governing the number of firms. Hence variety effects cannot be represented. Such effects are derived in the welfare decompositions presented in Swaminathan and Hertel (1996) and Baldwin and Venables (1995).

[13] This theorem is very similar to the result (5.10) of Keller (1980). There it is proved directly from the first order conditions for the maximisation of profit subject to a production function. In the current paper it is proved using revenue and cost functions.

assumption of constant technical efficiency (that is, constant production technology).[14] The zero pure profits condition can be written in one of four ways:

$$
\begin{aligned}
0 &= \mathrm{R}\left(\overline{\mathbf{P}}_j^{\mathbf{O}}, \overline{\mathbf{Q}}_j^{\mathbf{I}}\right) - \overline{\mathbf{P}}_j^{\mathbf{I}}.\overline{\mathbf{Q}}_j^{\mathbf{I}} \\
&= \overline{\mathbf{P}}_j^{\mathbf{O}}.\overline{\mathbf{Q}}_j^{\mathbf{O}} - \mathrm{C}\left(\overline{\mathbf{P}}_j^{\mathbf{I}}, \overline{\mathbf{Q}}_j^{\mathbf{O}}\right) \\
&= \mathrm{R}\left(\overline{\mathbf{P}}_j^{\mathbf{O}}, \overline{\mathbf{Q}}_j^{\mathbf{I}}\right) - \mathrm{C}\left(\overline{\mathbf{P}}_j^{\mathbf{I}}, \overline{\mathbf{Q}}_j^{\mathbf{O}}\right) \\
&= \overline{\mathbf{P}}_j^{\mathbf{O}}.\overline{\mathbf{Q}}_j^{\mathbf{O}} - \overline{\mathbf{P}}_j^{\mathbf{I}}.\overline{\mathbf{Q}}_j^{\mathbf{I}}
\end{aligned}
\qquad (40)
$$

Partial differentiation, with respect to quantities, of each of the first two expressions yields:

$$
\begin{aligned}
\partial \mathrm{R} \big/ \partial \overline{\mathbf{Q}}_j^{\mathbf{I}} &= \overline{\mathbf{P}}_j^{\mathbf{I}} \\
\partial \mathrm{C} \big/ \partial \overline{\mathbf{Q}}_j^{\mathbf{O}} &= \overline{\mathbf{P}}_j^{\mathbf{O}}
\end{aligned}
\qquad (41)
$$

The assumption of price taking ensures that partial derivatives of prices with respect to quantities do not occur in the previous two equations, that is:

$$
\begin{aligned}
\partial \overline{\mathbf{P}}_j^{\mathbf{I}} \big/ \partial \overline{\mathbf{Q}}_j^{\mathbf{I}} &= 0 \\
\partial \overline{\mathbf{P}}_j^{\mathbf{O}} \big/ \partial \overline{\mathbf{Q}}_j^{\mathbf{O}} &= 0
\end{aligned}
\qquad (42)
$$

The assumptions of revenue maximising and cost minimising behaviour allow us to apply Shepherd's lemma to the revenue and cost functions, yielding, respectively:

$$
\begin{aligned}
\partial \mathrm{R} \big/ \partial \overline{\mathbf{P}}_j^{\mathbf{O}} &= \overline{\mathbf{Q}}_j^{\mathbf{O}} \\
\partial \mathrm{C} \big/ \partial \overline{\mathbf{P}}_j^{\mathbf{I}} &= \overline{\mathbf{Q}}_j^{\mathbf{I}}
\end{aligned}
\qquad (43)
$$

Therefore, linearisation of the third expression for zero pure profits yields:

$$
\begin{aligned}
0 &= \Delta \mathrm{R} - \Delta \mathrm{C} \\
&= \partial \mathrm{R} \big/ \partial \overline{\mathbf{Q}}_j^{\mathbf{I}}.\Delta \overline{\mathbf{Q}}_j^{\mathbf{I}} + \partial \mathrm{R} \big/ \partial \overline{\mathbf{P}}_j^{\mathbf{O}}.\Delta \overline{\mathbf{P}}_j^{\mathbf{O}} - \partial \mathrm{C} \big/ \partial \overline{\mathbf{Q}}_j^{\mathbf{O}}.\Delta \overline{\mathbf{Q}}_j^{\mathbf{O}} - \partial \mathrm{C} \big/ \partial \overline{\mathbf{P}}_j^{\mathbf{I}}.\Delta \overline{\mathbf{P}}_j^{\mathbf{I}} \\
&= \overline{\mathbf{P}}_j^{\mathbf{I}}.\Delta \overline{\mathbf{Q}}_j^{\mathbf{I}} + \overline{\mathbf{Q}}_j^{\mathbf{O}}.\Delta \overline{\mathbf{P}}_j^{\mathbf{O}} - \overline{\mathbf{P}}_j^{\mathbf{O}}.\Delta \overline{\mathbf{Q}}_j^{\mathbf{O}} - \overline{\mathbf{Q}}_j^{\mathbf{I}}.\Delta \overline{\mathbf{P}}_j^{\mathbf{I}} \\
&= \left[\mathbf{V}_j^{\mathbf{I}}.\overline{\mathbf{q}}_j^{\mathbf{I}} + \hat{\mathbf{V}}_j^{\mathbf{O}}.\overline{\mathbf{p}}_j^{\mathbf{O}} - \hat{\mathbf{V}}_j^{\mathbf{O}}.\overline{\mathbf{q}}_j^{\mathbf{O}} - \mathbf{V}_j^{\mathbf{I}}.\overline{\mathbf{p}}_j^{\mathbf{I}}\right] \big/ 100
\end{aligned}
\qquad (44)
$$

Linearisation of the fourth expression for zero pure profits yields:

$$
\begin{aligned}
0 &= \overline{\mathbf{P}}_j^{\mathbf{O}}.\Delta \overline{\mathbf{Q}}_j^{\mathbf{O}} - \overline{\mathbf{P}}_j^{\mathbf{I}}.\Delta \overline{\mathbf{Q}}_j^{\mathbf{I}} + \overline{\mathbf{Q}}_j^{\mathbf{O}}.\Delta \overline{\mathbf{P}}_j^{\mathbf{O}} - \overline{\mathbf{Q}}_j^{\mathbf{I}}.\Delta \overline{\mathbf{P}}_j^{\mathbf{I}} \\
&= \left[\hat{\mathbf{V}}_j^{\mathbf{O}}.\overline{\mathbf{q}}_j^{\mathbf{O}} - \mathbf{V}_j^{\mathbf{I}}.\overline{\mathbf{q}}_j^{\mathbf{I}} + \hat{\mathbf{V}}_j^{\mathbf{O}}.\overline{\mathbf{p}}_j^{\mathbf{O}} - \mathbf{V}_j^{\mathbf{I}}.\overline{\mathbf{p}}_j^{\mathbf{I}}\right] \big/ 100
\end{aligned}
\qquad (45)
$$

The difference of the final lines in equations (44) and (45) is equal to:

[14] The current treatment embodies the assumption that production technology only changes via changes in the technical efficiencies associated with industry outputs and inputs.

$$
\begin{aligned}
0 &= \left[\mathbf{V}^{\mathbf{I}}_{j}.\overline{\mathbf{q}}^{\mathbf{I}}_{j} + \hat{\mathbf{V}}^{\mathbf{O}}_{j}.\overline{\mathbf{p}}^{\mathbf{O}}_{j} - \hat{\mathbf{V}}^{\mathbf{O}}_{j}.\overline{\mathbf{q}}^{\mathbf{O}}_{j} - \mathbf{V}^{\mathbf{I}}_{j}.\overline{\mathbf{p}}^{\mathbf{I}}_{j}\right]\Big/100 \\
&\quad - \left[\hat{\mathbf{V}}^{\mathbf{O}}_{j}.\overline{\mathbf{q}}^{\mathbf{O}}_{j} - \mathbf{V}^{\mathbf{I}}_{j}.\overline{\mathbf{q}}^{\mathbf{I}}_{j} + \hat{\mathbf{V}}^{\mathbf{O}}_{j}.\overline{\mathbf{p}}^{\mathbf{O}}_{j} - \mathbf{V}^{\mathbf{I}}_{j}.\overline{\mathbf{p}}^{\mathbf{I}}_{j}\right]\Big/100 \\
&= 2.\left[\mathbf{V}^{\mathbf{I}}_{j}.\overline{\mathbf{q}}^{\mathbf{I}}_{j} - \hat{\mathbf{V}}^{\mathbf{O}}_{j}.\overline{\mathbf{q}}^{\mathbf{O}}_{j}\right]\Big/100
\end{aligned}
\qquad (46)
$$

so, consequently:

$$
\mathbf{V}^{\mathbf{I}}_{j}.\overline{\mathbf{q}}^{\mathbf{I}}_{j} = \hat{\mathbf{V}}^{\mathbf{O}}_{j}.\overline{\mathbf{q}}^{\mathbf{O}}_{j} \qquad (47)
$$

which, when written in summation notation, is:

$$
\sum\nolimits_{c,j} \hat{V}^{\mathbf{O}}_{cj}.\overline{q}^{\mathbf{O}}_{cj} = \sum\nolimits_{i,j} V^{\mathbf{I}}_{ij}.\overline{q}^{\mathbf{I}}_{ij} \qquad (48)
$$

QED.

The previous theorem makes assumptions about the optimising behaviour of an industry, whereas such assumptions are more properly related to the firms constituting an industry. As can be inferred from the first line of equation (54) in the next subsection, the profits effect for an industry is equal to the sum of profits effects across all firms in an industry provided the firms have the same *effective* input and output prices. If this condition does not hold then either the industry could conceptually be split into several industries each with the desired property, or the technical efficiency and profits effect contributions to welfare (*TECH* and *PROFITS*, respectively) would need to be rewritten with a firm dimension.

For the remainder of this section firms in a particular industry (indexed by a subscript of ψ) are assumed to:

1. Maximise profits;
2. Have some market power in all input and output markets;
3. Face negative demand elasticities in all output markets; and
4. Face positive supply elasticities in all input markets.

The distinction between effective and actual prices and quantities is dropped for the remainder of this section. To maintain it would cause notational clutter, and would not lead to any extra insights for the issues of market power discussed.

3.2 The Profits Effects Under Market Power Only

The profit maximisation problem for firm ψ in industry j is:

Choose a vector of input and output quantities

$$
\mathbf{Q}_{\psi j} = \begin{pmatrix} \mathbf{Q}^{\mathbf{O}}_{\psi j} \\ \mathbf{Q}^{\mathbf{I}}_{\psi j} \end{pmatrix} \qquad (49)
$$

to maximise

$$\sum_c \hat{P}^{\mathbf{O}}_{cj}.Q^{\mathbf{O}}_{\psi cj} - \sum_i P^{\mathbf{I}}_{ij}.Q^{\mathbf{I}}_{\psi ij} \tag{50}$$

subject to a production frontier defined by

$$F_{\psi j}\left(\mathbf{Q}_{\psi j}\right) \le 0 \tag{51}$$

The function $F_{\psi j}(\cdot)$ is an increasing (decreasing) function of output (input) quantities. The production technology constraint will be satisfied with equality for a profit maximising firm (a fact that is used below). Taking into account the firm's market power, the first order conditions are:

$$\begin{aligned} &\hat{P}^{\mathbf{O}}_{cj} + \frac{d\hat{P}^{\mathbf{O}}_{cj}}{dQ^{\mathbf{O}}_{\psi cj}}.Q^{\mathbf{O}}_{\psi cj} - \lambda_{\psi j}.\frac{\partial F_{\psi j}\left(\mathbf{Q}_{\psi j}\right)}{\partial Q^{\mathbf{O}}_{\psi cj}} = \hat{P}^{\mathbf{O}}_{cj}.\left(1-\varepsilon^{\mathbf{O}}_{\psi cj}\right) - \lambda_{\psi j}.\frac{\partial F_{\psi j}\left(\mathbf{Q}_{\psi j}\right)}{\partial Q^{\mathbf{O}}_{\psi cj}} = 0 \\ &-P^{\mathbf{I}}_{ij} - \frac{dP^{\mathbf{I}}_{ij}}{dQ^{\mathbf{I}}_{\psi ij}}.Q^{\mathbf{I}}_{\psi ij} - \lambda_{\psi j}.\frac{\partial F_{\psi j}\left(\mathbf{Q}_{\psi j}\right)}{\partial Q^{\mathbf{I}}_{\psi ij}} = -P^{\mathbf{I}}_{ij}.\left(1+\varepsilon^{\mathbf{I}}_{\psi ij}\right) - \lambda_{\psi j}.\frac{\partial F_{\psi j}\left(\mathbf{Q}_{\psi j}\right)}{\partial Q^{\mathbf{I}}_{\psi ij}} = 0 \end{aligned} \tag{52}$$

where $\lambda_{\psi j}$ is the Lagrange multiplier (which will be positive — a fact used below), and the $\varepsilon>0$ are the magnitudes of inverse demand (supply) elasticities faced by the firm in output (input) markets.[15] For any small change in quantities it must be the case that:

$$\begin{aligned} \Delta F_{\psi j}\left(\mathbf{Q}_{\psi j}\right) &= 0 \\ &= \frac{\partial F_{\psi j}\left(\mathbf{Q}_{\psi j}\right)}{\partial \mathbf{Q}'_{\psi j}}.\Delta \mathbf{Q}_{\psi j} \\ &= \sum_c \frac{\partial F_{\psi j}\left(\mathbf{Q}_{\psi j}\right)}{\partial Q^{\mathbf{O}}_{\psi cj}}.\Delta Q^{\mathbf{O}}_{\psi cj} + \sum_i \frac{\partial F_{\psi j}\left(\mathbf{Q}_{\psi j}\right)}{\partial Q^{\mathbf{I}}_{\psi ij}}.\Delta Q^{\mathbf{I}}_{\psi ij} \end{aligned} \tag{53}$$

so

$$\begin{aligned} \sum_c \hat{P}^{\mathbf{O}}_{cj}.\Delta Q^{\mathbf{O}}_{cj} - \sum_{i,j} P^{\mathbf{I}}_{ij}.\Delta Q^{\mathbf{I}}_{ij} &\equiv \left[\sum_c \hat{V}^{\mathbf{O}}_{cj}.q^{\mathbf{O}}_{cj} - \sum_i V^{\mathbf{I}}_{ij}.q^{\mathbf{I}}_{ij}\right]/100 \\ &= \sum_\psi \left[\sum_c \hat{P}^{\mathbf{O}}_{cj}.\varepsilon^{\mathbf{O}}_{\psi cj}.\Delta Q^{\mathbf{O}}_{\psi cj} + \sum_i P^{\mathbf{I}}_{ij}.\varepsilon^{\mathbf{I}}_{\psi ij}.\Delta Q^{\mathbf{I}}_{\psi ij}\right] \\ &\equiv \sum_\psi \left[\sum_c V^{\mathbf{O}}_{\psi cj}.\varepsilon^{\mathbf{O}}_{\psi cj}.q^{\mathbf{O}}_{\psi cj} + \sum_i V^{\mathbf{I}}_{\psi ij}.\varepsilon^{\mathbf{I}}_{\psi ij}.q^{\mathbf{I}}_{\psi ij}\right]/100 \end{aligned} \tag{54}$$

Thus the profits effect for industry j can be expressed as a sum of measures of the market power of individual firms and changes in their input and output quantities. Also, the terms in the profits effect resemble very closely allocative efficiency effects, with the elasticities playing the role of ad-valorem tax rates. This is not altogether surprising because, as can be seen from the first order conditions, the elasticities are the rate of mark-up (markdown) of price over (under) marginal cost (benefit) for each output (input), and mark-ups (markdowns) are distortions — just like taxes.

[15] For ease of expression, the ε shall be referred to as elasticities.

Consistent with these insights, an increase in any input or output quantity for a firm makes a positive contribution to the profits effect, and hence to welfare, ceteris paribus.

3.3 Reductions in Market Power

The value of the profits effect for an industry corresponding to changes in firms' market power is examined in the remainder of this section. Decreases in market power will be represented as decreases in one or more of the inverse demand and supply elasticities. Under a small perturbation in the elasticities, the first order conditions yield:

$$\begin{aligned}
0 &= \Delta\hat{P}^{O}_{cj}.\left(1-\varepsilon^{O}_{\psi cj}\right) - \Delta\lambda_{\psi j}.\frac{\partial F_{\psi j}\left(\mathbf{Q}_{\psi j}\right)}{\partial Q^{O}_{\psi cj}} - \hat{P}^{O}_{cj}.\Delta\varepsilon^{O}_{\psi cj} - \lambda_{\psi j}.\frac{\partial^{2} F_{\psi j}\left(\mathbf{Q}_{\psi j}\right)}{\partial Q^{O}_{\psi cj}\partial\mathbf{Q}'_{\psi j}}.\Delta\mathbf{Q}_{\psi j} \\
&= -\Delta Q^{O}_{\psi cj}.\kappa^{O}_{\psi cj}.\left(1-\varepsilon^{O}_{\psi cj}\right) - \Delta\lambda_{\psi j}.\frac{\partial F_{\psi j}\left(\mathbf{Q}_{\psi j}\right)}{\partial Q^{O}_{\psi cj}} - \hat{P}^{O}_{cj}.\Delta\varepsilon^{O}_{\psi cj} - \lambda_{\psi j}.\frac{\partial^{2} F_{\psi j}\left(\mathbf{Q}_{\psi j}\right)}{\partial Q^{O}_{\psi cj}\partial\mathbf{Q}'_{\psi j}}.\Delta\mathbf{Q}_{\psi j} \\
0 &= -\Delta P^{I}_{ij}.\left(1+\varepsilon^{I}_{\psi ij}\right) - \Delta\lambda_{\psi j}.\frac{\partial F_{\psi j}\left(\mathbf{Q}_{\psi j}\right)}{\partial Q^{I}_{\psi ij}} - P^{I}_{ij}.\Delta\varepsilon^{I}_{\psi ij} - \lambda_{\psi j}.\frac{\partial^{2} F_{\psi j}\left(\mathbf{Q}_{\psi j}\right)}{\partial Q^{I}_{\psi ij}\partial\mathbf{Q}'_{\psi j}}.\Delta\mathbf{Q}_{\psi j} \\
&= -\Delta Q^{I}_{\psi ij}.\kappa^{I}_{\psi ij}.\left(1+\varepsilon^{I}_{\psi ij}\right) - \Delta\lambda_{\psi j}.\frac{\partial F_{\psi j}\left(\mathbf{Q}_{\psi j}\right)}{\partial Q^{I}_{\psi ij}} - P^{I}_{ij}.\Delta\varepsilon^{I}_{\psi ij} - \lambda_{\psi j}.\frac{\partial^{2} F_{\psi j}\left(\mathbf{Q}_{\psi j}\right)}{\partial Q^{I}_{\psi ij}\partial\mathbf{Q}'_{\psi j}}.\Delta\mathbf{Q}_{\psi j}
\end{aligned} \tag{55}$$

where the κ are the absolute values of the derivatives of prices with respect to quantities. If each of these equations is multiplied by the associated changes in quantities, and then summed over all inputs and outputs,

$$\begin{aligned}
&-\left[\sum_{c}\hat{P}^{O}_{cj}.\Delta Q^{O}_{\psi cj}.\Delta\varepsilon^{O}_{\psi cj} + \sum_{i}P^{I}_{ij}.\Delta Q^{I}_{\psi ij}.\Delta\varepsilon^{I}_{\psi ij}\right] \\
&= \sum_{c}\left(\Delta Q^{O}_{\psi cj}\right)^{2}.\kappa^{O}_{\psi cj}.\left(1-\varepsilon^{O}_{\psi cj}\right) + \sum_{i}\left(\Delta Q^{I}_{\psi ij}\right)^{2}.\kappa^{I}_{\psi ij}.\left(1+\varepsilon^{I}_{\psi ij}\right) + \lambda_{\psi j}.\mathbf{Q}'_{\psi j}.\frac{\partial^{2} F_{\psi j}\left(\mathbf{Q}_{\psi j}\right)}{\partial\mathbf{Q}_{\psi j}\partial\mathbf{Q}'_{\psi j}}.\Delta\mathbf{Q}_{\psi j}
\end{aligned} \tag{56}$$

The first term on the right hand side is positive if all demands faced by the firm are elastic (so the ε^{O} are all less than one). The second term is positive. The third term is positive if $F_{\psi j}(\cdot)$ represents a non-increasing returns to scale production technology. However, this may not sit comfortably with the assumption of market power, which might be associated with some economies of scale. If, however, those economies of scale arise from the existence of a fixed cost, then the first order conditions are identical to those derived above with $F_{\psi j}(\cdot)$ determining the variable inputs. In this case $F_{\psi j}(\cdot)$ could quite reasonably be non-IRTS.

This discussion of fixed costs ignores the possibility that these are actually a fixed amount of some aggregate of inputs. For example, the fixed "costs" in Swaminathan and Hertel (1996) are CES composites of primary factors. In these cases the composition of fixed "costs", and hence their actual cost to the industry, will vary with prices and will be determined as part of the profit maximisation decision. But provided that the production function used to produce the fixed "costs" from its component inputs is non-IRTS, the inferences to be drawn below will remain valid. Therefore, we shall persist with the simpler specification of fixed costs assumed in the previous paragraph, to minimise notational clutter.

So for the final theorem of this section we introduce the assumptions:

5. All firms face elastic demands for all outputs; and
6. The production technology implied by the function $F_{\psi j}(\cdot)$ is non-IRTS.

***Theorem**:* Under assumptions 1-6, a uniform reduction in market power across all inputs and outputs for all firms in industry *j*:

$$\frac{\Delta\varepsilon^{\mathbf{O}}_{\psi cj}}{\varepsilon^{\mathbf{O}}_{\psi cj}} = \frac{\Delta\varepsilon^{\mathbf{I}}_{\psi ij}}{\varepsilon^{\mathbf{I}}_{\psi ij}} = -\delta < 0 \quad \forall \psi, c, i \tag{57}$$

leads to a reduction in mark-ups (markdowns) and excess profits, and hence an increase in regional welfare, ceteris paribus.

Proof: By combining equation (57) for the uniform relative changes in elasticities, equation (56) for the impact of changes in market power, and the equation (54) for relating the profits effects to the elasticities and quantity changes, we obtain

$$\begin{aligned}
&\delta.\sum_{\psi}\left[\sum_{c}\hat{P}^{\mathbf{O}}_{cj}.\Delta Q^{\mathbf{O}}_{\psi cj}.\varepsilon^{\mathbf{O}}_{\psi cj} + \sum_{i}P^{\mathbf{I}}_{ij}.\Delta Q^{\mathbf{I}}_{\psi ij}.\varepsilon^{\mathbf{I}}_{\psi ij}\right] = \delta.\left[\sum_{c}\hat{P}^{\mathbf{O}}_{cj}.\Delta Q^{\mathbf{O}}_{cj} - \sum_{i}P^{\mathbf{I}}_{ij}.\Delta Q^{\mathbf{I}}_{ij}\right] \\
&= \sum_{\psi}\left\{\sum_{c}\left(\Delta Q^{\mathbf{O}}_{\psi cj}\right)^2.\kappa^{\mathbf{O}}_{\psi cj}.\left(1-\varepsilon^{\mathbf{O}}_{\psi cj}\right) + \sum_{i}\left(\Delta Q^{\mathbf{I}}_{\psi ij}\right)^2.\kappa^{\mathbf{I}}_{\psi ij}.\left(1+\varepsilon^{\mathbf{I}}_{\psi ij}\right) + \lambda_{\psi j}.\mathbf{Q}'_{\psi j}.\frac{\partial^2 F_{\psi j}\left(\mathbf{Q}_{\psi j}\right)}{\partial\mathbf{Q}_{\psi j}\partial\mathbf{Q}'_{\psi j}}.\Delta\mathbf{Q}_{\psi j}\right\} \\
&> 0
\end{aligned} \tag{58}$$

under assumptions 1-6. QED.

4 Decomposition of Money Metric and Compensation Measures of Welfare Change

Martin (1996) discusses the properties of money metric measures of welfare change, based on the expenditure function, and compensation measures, based on the balance of trade function. The two measures of welfare are not equal in an economy with existing distortions, and neither one is clearly superior to the other. Therefore it is desirable to be able to use both types of welfare measures, and to be able to decompose either one into a sum of components, as has been done for the equivalent variation in GTAP.

Fane and Ahammad (2000) decompose a compensation measure of welfare in a very general context. They represent the welfare measure as a sum of allocative efficiency and terms of trade effects.[16] Because conventional assumptions of CGE models are introduced at an early stage of the derivation (for example, by describing the behaviour of agents' in terms of expenditure functions, thus implying optimising behaviour), no residual profits effects are identified.

[16] Endowment and technical efficiency changes could probably be readily accommodated, but were not of primary interest in the context of the paper.

The decomposition of the change in utility presented in this paper will now be applied to both money metric or compensation measures of welfare.

4.1 Money Metric Welfare Measures

The equivalent variation is the income that must be given to an agent, at some fixed set of prices, to make them as well-off as they would be under some policy change. It can be expressed in terms of the expenditure function thus:

$$\begin{aligned} EV &= E(\mathbf{P}_0, U_1) - E(\mathbf{P}_0, U_0) \\ &= \int_{U_0}^{U_1} \frac{\partial E(\mathbf{P}_0, U)}{\partial U} \cdot dU \end{aligned} \tag{59}$$

In a numerical simulation, the small increments in utility calculated at each step of the solution procedure correspond to the *dU* in the integral, and can be decomposed as shown above (table 1). Thus, all that is required to convert to a decomposition of the equivalent variation is multiplication by the derivative of the expenditure function, with respect to utility, evaluated at the initial prices.[17]

4.2 Compensation Welfare Measures Based on the Balance of Trade Function

A compensation measure of the welfare change attributable to a policy is the value of an income transfer from abroad that must be given to a nation so that its level of welfare remains constant as the policy is reversed.

Fane and Ahammad (2000) describe the calculation of this welfare measure by postulating three states of the economy — base (B), uncompensated (U) and equivalent (E).

In table 3, *U* represents utility, *T* represents all policy variables, and *A* represents an exogenous transfer of income from abroad (as distinct from foreign income flows that may be affected by the policy change through effects such as changes in foreign investment). A change in *A* can be included in the decomposition of utility as an extra foreign income flow not decomposed into its

Table 3 Calculation of Compensation Based Measures of Welfare

Variable type	Base	Uncompensated	Equivalent
Utility	U^B	U^U	$U^E = U^U$
Policy variables	T^B	T^U	$T^E = T^B$
Income transfer from abroad	A^B	$A^U = A^B$	A^E

Source: Adapted from table 1 of Fane and Ahammad (2000).

[17] This derivative for the GTAP model will be derived in section 5.

rate of return, asset price and real parts. It is included as a change rather than a percentage change, since its value may pass through zero. Consequently, considering a small change of the type U→E,

$$\begin{aligned} q_h^{NNP} &= 0 \\ &= POP + TOT + ASS\_PRI + RORF + FENDW \\ &\quad + ENDW + ALLOC + TECH + PROFITS + 100.\Theta_h.\Delta A / V^{NNP} \end{aligned} \tag{60}$$

so

$$-\Delta A = V^{NNP}.\begin{pmatrix} POP + TOT + ASS\_PRI + RORF + FENDW \\ + ENDW + ALLOC + TECH + PROFITS \end{pmatrix} \Big/ \left(100.\Theta_h\right) \tag{61}$$

In contrast to the money metric measure of welfare, the compensation measure does not involve the elasticity of utility with respect to income. Table 1 shows how all the terms in parentheses in the numerator are multiplied by Θ_h, so consequently it cancels out between the numerator and denominator.

5 Application to the GTAP Equivalent Variation Measure of Welfare Change

To obtain a decomposition of the GTAP equivalent variation it is necessary to derive expressions for the elasticity of utility with respect to income (the coefficient Θ_h) and the derivative of the expenditure function with respect to utility (evaluated at initial prices) for the GTAP representative household.

In GTAP, the representative household has a nested utility function with two levels. First, at the upper level, total household income is allocated between private and government consumption goods in total, and saving. Then, at the lower level, the income allocated to each of total private and government consumption is allocated across individual commodities so as to maximise sub-utility functions, of CDE and Cobb-Douglas forms for private and government consumption, respectively. The upper level decision of allocating income between expenditure categories maximises a Cobb-Douglas utility function of the lower level sub-utilities and real saving. Thus, the indirect utility function for the representative household in GTAP is, using an obvious parallel to the notation already established,

$$\ln\left[U_h\left(\mathbf{P}^C, \mathbf{P}^G, P^S, V_h^{NNP}\right)\right] = \sum_{f \in \{C,G,S\}} \alpha^f . \ln\left[U_h^f\left(\mathbf{P}^f, V_h^f\right)\right] \tag{62}$$

where $\mathbf{P}^f$ is interpreted as a vector of prices or a scalar for $f \in \{C,G\}$ or $f=S$, respectively, and α^f are the Cobb-Douglas parameters at the upper level. The aggregate expenditures V_h^f are functions of total income and all prices, and are determined by the optimisation problem:

Choose V_h^C, V_h^G and V_h^S to maximise

$$\sum_{f\in\{C,G,S\}}\alpha^f.\ln\left[U_h^f\left(\mathbf{P}^f,V_h^f\right)\right] \tag{63}$$

subject to

$$V_h^{NNP}=\sum_{f\in\{C,G,S\}}V_h^f \tag{64}$$

The first order conditions from this problem are:

$$\alpha^f.\frac{\partial\ln\left[U_h^f\left(\mathbf{P}^f,V_h^f\right)\right]}{\partial V_h^f}=\lambda_h \tag{65}$$

where λ_h is the Lagrange multiplier.

But

$$\begin{aligned}\frac{\partial\ln\left[U_h\left(\mathbf{P}^C,\mathbf{P}^G,P^S,V_h^{NNP}\right)\right]}{\partial V_h^{NNP}}&=\sum_{f\in\{C,G,S\}}\alpha^f.\frac{\partial\ln\left[U_h^f\left(\mathbf{P}^f,V_h^f\right)\right]}{\partial V_h^f}.\frac{\partial V_h^f}{\partial V_h^{NNP}}\\&=\lambda_h.\sum_{f\in\{C,G,S\}}\frac{\partial V_h^f}{\partial V_h^{NNP}}\\&=\lambda_h\end{aligned} \tag{66}$$

using the first order conditions and the constraint that total expenditure equals total income. This is the familiar result that the Lagrange multiplier in a utility maximisation problem is the marginal utility of income. But a simple expression for λ_h can be found by considering the first order condition for saving, thus:

$$\begin{aligned}\lambda_h&=\alpha^S.\frac{\partial\ln\left[U_h^S\left(P^S,V_h^S\right)\right]}{\partial V_h^S}\\&=\alpha^S.\frac{\partial\ln\left[V_h^S/P^S\right]}{\partial V_h^S}\\&=\frac{\alpha^S}{V_h^S}\end{aligned} \tag{67}$$

So

$$\begin{aligned}\Theta_h&=\frac{\partial\ln\left[U_h\left(\mathbf{P}^C,\mathbf{P}^G,P^S,V_h^{NNP}\right)\right]}{\partial\ln V_h^{NNP}}\\&=\frac{\alpha^S.V_h^{NNP}}{V_h^S}\end{aligned} \tag{68}$$

The derivative of the expenditure function with respect to utility (required for the calculation of the EV) can be derived simply by differentiating the identity:[18]

$$E_h\left[P, U_h(P, Y)\right] = Y \tag{69}$$

to yield:

$$1 = \frac{\partial E_h\left[P, U_h(P, Y)\right]}{\partial U_h} \cdot \frac{\partial U_h(P, Y)}{\partial Y} \tag{70}$$

so

$$\frac{\partial E_h\left[P, U_h(P, Y)\right]}{\partial U_h} = \left[\frac{\partial U_h(P, Y)}{\partial Y}\right]^{-1} \tag{71}$$

To apply this expression in calculating the EV, the prices **P** should be held constant at their initial values, and *Y* should be defined so that:

$$U_h(P, Y) = U_h\left(P^C, P^G, P^S, V_h^{NNP}\right) \tag{72}$$

This can be implemented by incorporating in GTAP a shadow household demand system, in which prices are held constant at their initial values but utility is equal to the value determined in the policy simulation, as described in McDougall (2001).

It should be noted that the derivative of the expenditure function used in converting changes in utility to an EV does not cancel out with Θ_h, since these expressions are evaluated at different values of the prices and nominal income. They are, however, equal at the initial point of a policy simulation. Further, the Cobb-Douglas parameter α^S, which can be arbitrarily chosen to be any positive number, does cancel between the two expressions. Formally expressed, the contribution to the EV for each small step of the policy simulation is:

[18] The price vector **P** and the nominal income *Y* introduced at this point designate the values to be used in the calculation of the EV. They are *not* the same as the prices ($\mathbf{P}^C$, $\mathbf{P}^G$, P^S) and income (V_h^{NNP}) arising in the policy simulation.

$$
\begin{aligned}
\frac{\partial E_h\left[\mathbf{P},U_h\left(\mathbf{P},Y\right)\right]}{\partial U_h}.\Delta U_h &= \frac{\partial E_h\left[\mathbf{P},U_h\left(\mathbf{P},Y\right)\right]}{\partial U_h}.U_h.q_h^{NNP}/100 \\
&= \frac{\partial E_h\left[\mathbf{P},U_h\left(\mathbf{P},Y\right)\right]}{\partial U_h}.\Theta_h.U_h.\xi_h/V_h^{NNP} \\
&= \left[\frac{\partial U_h\left(\mathbf{P},Y\right)}{\partial Y}\right]^{-1}.\Theta_h.U_h.\xi_h/V_h^{NNP} \\
&= \frac{Y}{U_h\left(\mathbf{P},Y\right)}.\left[\frac{\partial \ln U_h\left(\mathbf{P},Y\right)}{\partial \ln Y}\right]^{-1}.\Theta_h.U_h.\xi_h/V_h^{NNP} \\
&= \left(\xi_h/V_h^{NNP}\right)Y.\left(\frac{\alpha^S.V_h^{NNP}}{V_h^S}\right)\Big/\left(\frac{\alpha^S.Y}{Y^S}\right) \\
&= \xi_h.Y^S/V_h^S
\end{aligned}
\tag{73}
$$

where Y^S is the value of nominal savings implied by the shadow household demand system, and ξ_h is a value-weighted linear combination of percentage changes in quantities and relative prices, defined from the welfare decomposition terms listed in table 1 thus:

$$
\begin{aligned}
\xi_h = -n &+ \left[V^X.p^X - V^M.p^M\right] + \left[\left(V^I.p^I + \sum\nolimits_\varphi V_\varphi^{FA}.\pi_\varphi^{FA}\right) - \left(V^S.p^S + \sum\nolimits_k V_k^{DEP}.\pi_k^{DK}\right)\right] \\
&+ \sum\nolimits_\varphi V_\varphi^{FA}.\rho_\varphi^{FA} + \sum\nolimits_\varphi V_\varphi^{FA}.q_\varphi^{FA} \\
&+ \left[\sum\nolimits_{e,j} \hat{V}_{ej}^{\mathbf{I}}.q_{ej}^{\mathbf{I}} - \sum\nolimits_k V_k^{DEP}.\left(d_k^{DK} + q_k^{DK}\right)\right] + \left[\sum\nolimits_{c,f} R_{cf}^{\mathbf{I}}.q_{cf}^{\mathbf{I}} + \sum\nolimits_{i,j} R_{ij}^{\mathbf{I}}.q_{ij}^{\mathbf{I}} + \sum\nolimits_{c,j} R_{cj}^{\mathbf{0}}.q_{cj}^{\mathbf{0}}\right] \\
&+ \left[\sum\nolimits_{c,j} \hat{V}_{cj}^{\mathbf{0}}.a_{cj}^{\mathbf{0}} + \sum\nolimits_{i,j} V_{ij}^{\mathbf{I}}.a_{ij}^{\mathbf{I}}\right] + \left[\sum\nolimits_{c,j} \hat{V}_{cj}^{\mathbf{0}}.\bar{q}_{cj}^{\mathbf{0}} - \sum\nolimits_{i,j} V_{ij}^{\mathbf{I}}.\bar{q}_{ij}^{\mathbf{I}}\right]
\end{aligned}
\tag{74}
$$

It will be observed that the EV is invariant to monotonic transformations of the utility function, as expected.

6 Summary and Outstanding Issues for Future Research

This paper has presented a very general derivation of a welfare decomposition. It is valid for any CGE model in which economic welfare is represented as being derived from the allocation of national income between private consumption, government consumption and savings according to utility maximisation by a representative household. The derivation rests only on the assumption of market clearing in all commodities and, implicitly, whatever differentiability assumptions are required to ensure that relationships can be linearised. Even the assumption of market clearing could probably be relaxed with the explicit representation of changes in stocks.

One of the most innovative aspects of the resulting decomposition is the profits effect. It is a type of residual term that arises because of the absence, in the derivation of the welfare decomposition, of any assumptions about optimising behaviour by firms or zero-pure-profits. The profits effect captures the welfare effects of any market power wielded by some firms, and can be related to the demand and supply elasticities faced by such firms. The latter are related to mark-ups, which lead to extra terms in welfare decompositions derived for models with imperfect competition, such as Swaminathan and Hertel (1996) and Baldwin and Venables (1995). In contrast to these papers it should be noted, however, that while the profits effect captures the welfare contributions of market power, no accounting for welfare gains attributable to increased varieties of a commodity is undertaken in the current paper. This is partly attributable to there being no explicit recognition of the number of firms in the current paper. This is a possible area for further generalisation of the current approach.

The notion of a representative household might seem to limit the usefulness of any welfare analysis for policies where impacts are very different across different households. Another strength of the derivation set forth in this paper is that it does not begin with the notion of a representative household, but rather performs the initial stages of the welfare decomposition distinguishing welfare changes by household. The move to the notion of a representative household is only performed once the household-specific welfare effects have been identified. There are two such effects, one related to the distribution of national income between households, the other related to how the household's price of welfare compares with the average price. Both these terms, even if they are not picked apart further, are plainly of great use to those interested in the distributional impacts of policies. An area for future research may be to decompose these terms further, especially with regard to the sources of income for different households, for example, labour versus capital income.

This paper has also applied the welfare decomposition derived to different definitions of money equivalents of welfare changes — money metric measures of welfare, such as the GTAP equivalent variation, and compensation measures based on the balance of trade function, as described in Martin (1996) and Fane and Ahammad (2000).

McDougall (2001) shows that there is some gain in reorganising the welfare decomposition so that all quantities are expressed in per capita terms. As the terms are defined in the current paper, a uniform increase in all quantities would register allocative efficiency changes merely because of the growth in quantities, even though the composition of the economy has not changed. Plainly, the welfare decomposition should be juggled in whatever way helps the analyst to provide a clearer exposition of their results, and thereby to contribute towards addressing the 'black box' criticism sometimes levelled at CGE modelling.

Finally, welfare decomposition is essentially a phenomenon that inhabits the linearised world. It is based on manipulating the linearised representation of a model or, what is the same thing, partial derivatives. But a large part of the derivation in this paper (from about equation (15) onwards) is built on manipulating definitions and market clearing conditions. Further, the entities being manipulated are changes in values (prices times quantities) that can be expressed exactly by going just one step beyond linearisation to second order terms ($\Delta P \cdot \Delta Q$). A further research question that may help make welfare decomposition more readily available to those modellers that work directly with the levels of variables is: 'How much of the welfare decomposition can be derived using the exact representation of large changes in prices, quantities and values?'

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