Long-term memory in Euronext stock indexes returns: An econophysics approach

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Abstract: The purpose of paper is to assess the long-term memory of stock index returns in the pan-European platform Euronext (CAC-40, AEX, BEL-20 and PSI-20). We find evidence of time dependency in much of the data, suggesting that the series may best be described as fractional Brownian motion. Modified Rescaled-Range Analysis and Detrended Fluctuation Analysis were used to measure the degree of long memory. The global Hurst exponents evidence persistent long memory in the Dutch, Belgian and Portuguese markets. In the French market, evidence of long memory is inconsistent and weak. Fractal structure suggests non-conformity with the Efficient Market Hypothesis, and may compromise the reliability of asset pricing models. Furthermore, time-dependent Hurst exponents show evidence of weakening persistence in these markets, particularly after the international crises of 2000, 2002 and 2010. A possible explanation for those changes is that the markets may have matured over time, becoming more efficient after these severe events.

JEL Classifications: G14, G15, C10

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1. Introduction

The random walk hypothesis with independent and identically distributed (i.i.d.) increments is the basis of the efficient market hypothesis (EMH), according to Fama (1970). This hypothesis states, in a simple way, that (i) the price variation is random, as a result of the activity of traders trying to make gains (arbitrage opportunities), and (ii) the implementation of their strategies induces a dynamic feedback on the market, randomizing the share price (Matos et al., 2004).

A number of recent empirical studies found evidence of time dependency in some stock return price series (Kyaw et al., 2006; Eitelman & Vitanza, 2008; Kristoufek, 2012; Horta et al., 2014). This property was identified by Mandelbrot (1971) and designated as *"long memory or low frequency persistent temporal dependence"*.

The presence of long memory components in prices has controversial implications for the market efficiency and it is inconsistent with continuous stochastic processes employed in

the martingale[*] methods of securities valuation. Therefore, concerns arise with the modelling of the financial asset prices behaviour in the markets and with the technical trading rules used for prediction (Lo, 1991; Sadique & Silvapulle, 2001).

In this scope, the econophysics approach assumes a great importance: it is an area of interdisciplinary research that applies theories and methods developed in statistical physics, in order to contribute for the resolution of problems in economics and finance. The most widely accepted techniques to identify long memory are those that can be used to estimate the Hurst exponent H, which allows to perceive if a given time series follows a (fractional) Brownian motion. Two of the most popular techniques to estimate this exponent are the modified rescaled-range analysis (M-R/S) and the detrended fluctuation analysis (DFA). The M-R/S analysis, proposed by Lo (1991), uses a statistics that records the normalized difference between the maximum and minimum deviations from the mean value in the period of study. A logarithmic regression of this statistics makes it possible to estimate the Hurst exponent. The DFA analysis, proposed by Peng et al. (1994), uses standard statistical techniques to remove possible deterministic trends from the price series, and then analyzes the detrended data to estimate the Hurst exponent by a logarithm regression.

The M-R/S analysis is not exposed to the problems identified in the classical R/S (Lo, 1991; Alvarez-Ramirez et al., 2008) and it is robust to short-term dependence, heteroscedasticity and non-normality (Kristoufek, 2009). The DFA analysis is also robust under non-stationary data. Whereas the M-R/S analysis and the DFA analysis are popularly used in the search for evidence of long-term memory, both techniques were used in this paper.

The objective of this work is the investigation of empirical characteristics of the daily returns of Euronext[†] platform stock indexes (CAC-40, AEX, BEL-20 and PSI-20), placing particular emphasis on measuring the degree of persistence to verify the EMH and modelling the dynamic behaviour of the respective time series. Up to our knowledge, this is the first study involving the simultaneous analysis and comparison of the Euronext indexes.

The dimension and importance of this pan-European market in the international financial context justify this choice. Moreover, the research of persistence is important because (i) it establishes the long-term benchmark models for evaluating assets and derivatives, (ii) provides measures for investment selection and risk management, and (iii) distinguishes the small/local stock markets and the global financial markets.

Given the implications for the theory and practice of financial economics, the scientific work has shown a growing interest regarding the behaviour of stock returns series, with most of the evidence suggesting absence or weak base of any form of long memory (Lo, 1991; Jacobsen, 1996; Lipka & Los, 2002; Kristoufek, 2009, 2012; Braun, Jenkinson & Stoff, 2017) and another part of the evidence suggesting clear fractal structure (Fama & French, 1988; Costa & Vasconcelos, 2003; Assaf & Cavalcante, 2004; Chen & Yu, 2005). However, some results presented mixed findings, depending on the test method, sample period, frequencies of the series and simple or compound rate of returns (Cheung & Lai,

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[*] The martingale theory requires stationarity invariant and independence of the innovations of data sets of historical prices. It is a process of random walk in which *zero* autocorrelation should exist at all lags.

[†] The rules for construction of its four indices are published and can be accessed through the website: https://www.euronext.com/.

1995; Sadique & Silvapulle, 2001; Christodoulou-Volos & Siokis, 2006; Eitelman & Vitanza, 2008; Núñez, Martínez & Villareal, 2017).

The structure of this paper is organized as follows. Section 2 provides a brief review of literature related to long-term memory in the forms of persistence and anti-persistence. The sections 3 and 4 present the data series and describe the methods employed. Section 5 discusses the results of the empirical analysis. Finally, Section 6 describes the main findings, raises some caveats and indicates new directions for future research.

2. Long-term memory

2.1. Persistence and anti-persistence

The statistical analysis of financial time series has exhibited different characteristics from the random walk, which is considered an outcome of the EMH (i.e., stock prices exhibit unpredictable behavior, given available information) (Assaf, 2006). Hence, financial researchers continue to seek for a better understanding of the dynamic nature of stock prices, with more interest in the behaviour of stock returns in the long term. Mandelbrot (1971, 1972) was one of the first to formally recognize the possibility and the implications of long-term persistent statistical dependence. The author indicates that these series are characterized by distinct, but non-periodic, cyclical patterns and relates the processes of long memory with the fractional integration.

The identification of the nature of persistence of financial time series is crucial to decide on the type of modelling diffusion (Lipka & Los, 2003). A succession of persistent or anti-persistent stock returns is characterized by an effect of long-term memory. The existence of long memory indicates that the market will get back to its long-term trend in the future. Theoretically, it means that what happens today will impact the future in a non-linear way.

There are three classifications of dissemination of market prices (Los & Yu, 2008), measured by the Hurst exponent H. The degrees of long-term dependence allow to examine if financial market prices follow a gBm, that is $H = 0.5$, or, alternatively, a persistent fBm, wherein $0.5 < H < 1$, or anti-persistent fBm, wherein $0 < H < 0.5$. A exponent $H = 0.5$ indicates two possible cases: either a process of independent innovations (Beran, 1994), which characterizes a random walk corresponding to efficient markets in the strict sense of Fama (1970), or a dependent process on the short-term (Lillo & Farmer, 2004). If $H > 0.5$, the process is known as long-term dependent, in which case it has positive correlations in every lag (Embrechts & Maejima, 2002). It corresponds to riskier markets and to invest in that persistence allows opportunities for abnormal gains by arbitrage. Existing shocks with persistent impact on financial assets prices arise evidence against the EMH. If $H < 0.5$, the process is said long-term dependent with negative correlations at all lags (Embrechts & Maejima, 2002). It corresponds to markets with fast mean reversion, faster than theoretically postulated by a gBm. Markets that exhibit anti-persistence, or unpredictability, are usually called ultra-efficient markets (Los & Yu, 2008).

In a persistent market, if a change in price was up/down in the last period, then the prospect is that it will continue to be upward/downward in the following period. Often, there are long periods of inertia which can suddenly be wildly interrupted (severe drawdowns or sharp upshifts) (Los & Yu, 2008). The strength of the behaviour of reinforcement, or persistence, increases as the value of the exponent H approaches 1. The

nearer the exponent H is to 0.5, the more choppy the pricing process and the less smooth the apparent trends will be (Robinson, 1994).

In an anti-persistent market, if a price change was upward in the last period, then the prospect is that it will be downward in the following period, and vice-versa. Generally, anti-persistent markets seem to be unpredictable. However, they are more malleable than persistent markets, because their trading ranges remain very limited and the adjustments to the new market balances are very gradual, although usually in quick succession (Kyaw et al., 2006). The strength of the anti-persistence behaviour increases as the value of the exponent H approaches zero. The nearer the exponent H is to zero, the more negative short-term autocorrelations are.

2.2. Empirical evidence from previous studies

In this section, we refer previous studies about the indexes in the pan-European platform Euronext (CAC-40, AEX, BEL-20 and PSI-20). So far, the results of the degree of long-term dependence in the analysed data appear to depend heavily on the analytical methodology used and, therefore, calls into question how and if they can actually be identified by existing procedures and techniques. For example, Christodoulou-Volos & Siokis (2006) examined the evidence of fractional dynamics in the daily returns for 34 international stock indexes, including CAC-40 and BEL-20. Their studies used the semi-parametric method GPH suggested by Geweke & Porter-Hudak (1983) and the Robinson's Gaussian estimator RGSE proposed by Robinson (1995). The results showed strong evidence of long-term dependence in most series in both tests, with the exception of the Dow Jones index.

Eitelman & Vitanza (2008) considered the weekly returns, from 1988 to 2008, of 44 indexes of emerging and industrialized stock markets, including CAC-40, AEX and BEL-20, and used the M-R/S statistics presented by Lo (1991). The authors found insignificant evidence of long-term memory. These results are in accordance with those obtained by Chow et al. (1996) in a study of 22 international stock markets, which includes the French, Dutch and Belgian markets, and in which the modified rescaled range analysis of Lo (1991) and the rescaled variance ratio test of Lo & MacKinlay (1988) are employed to monthly returns from 1962 to 1990. The absence of long-term dependence was also reported by Jacobsen (1996) for the indexes monthly returns of the Netherlands, Germany, United Kingdom, Italy, France, United States and Japan, from 1952 to 1990. The author used the classical R/S and the M-R/S statistics and he compared the results with the process in which the time series were corrected for the short-term dependence through the AR(1), MA (1) and (Autoregressive and Moving Average) ARMA (1,1) models. However, the results contradicted the favourable evidence of long-term dependence detected by Cheung & Lai (1995), when they employed the fractional differencing test of Geweke & Porter-Hudak (1983) on the monthly returns of 18 countries' indexes, including France, the Netherlands and Belgium, from 1970 to 1992.

Oh et al. (2006) pored over the stock indexes of France, Germany, United Kingdom, Hong Kong, Korea, Japan and United States (S&P 500 and Nasdaq), between 1991 and 2005. The property of long-term memory of the daily returns was studied using Hurst exponent calculated from the DFA proposed by Peng et al. (1994). The results revealed no uniform conclusions, but the time series of returns followed the random walk process.

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Matos et al. (2004) modeled the daily returns series of the PSI-20 using a formulation of fBm and determined the Hurst exponent by DFA from 1993 to 2001. They also applied the method to smaller intervals with fixed size of 100, 200 and 400 points, corresponding to approximately half a year, one and two years of trading, respectively. The results exhibited persistent short-term behavior, although degenerating over long periods. In particular, for intervals of 400 observations the exponent H showed a gradual decline to anti-persistent behavior.

Later, Matos et al. (2008) analyzed the daily returns of stock indexes of Japan, Canada, Brazil and Portugal, from the early 1990s until 2005. The authors developed a combination of DFA with the dependence of scale and time, which they designated by time and scale Hurst exponent (TSH). As in the previous study, they found a decrease of the Hurst exponent H in all these indexes over time.

3. Sample and data series

The Euronext was founded on September 22, 2000 through the merger of the stock exchanges of Paris, Amsterdam and Brussels, in order to benefit from the harmonization of financial markets in the European Union. Later, in September 2002, the group expanded with the entry of the stock exchanges of Lisbon and Porto. In 2015, the Euronext stock market reached a capitalization of EUR 3 012 102 millions, corresponding to 457 939 934 trades with shares of 868 listed companies, becoming one of the largest global markets. Table 1 presents a simple characterization of the Euronext stock indexes studied in this work.

TABLE 1. CHARACTERISTICS OF THE EURONEXT STOCK INDEXES (CAC-40, AEX, BEL-20, PSI-20)

COUNTRY	DESIGNATION	INDEX	DATE BASIS - LEVEL BASIS	NUMBER LISTED COMPANIES	TURNOVER (Eur millions) (Ac. May 2015)
France	Euronext Paris	CAC-40	1987/12/31 – 1000	40	1 245 793
Netherlands	Euronext Amsterdam	AEX	1983/01/03 - 100	25	533 503
Belgium	Euronext Brussels	BEL-20	1991/01/01 - 1000	20	131 038
Portugal	Euronext Lisbon	PSI-20	1992/12/31 - 3000	20	28 053

Source: Euronext, May/2016, available on website: https://www.euronext.com/pt-pt/reports-statistics/factbooks?archive=this_year.
Note: The number of companies listed on the BEL-20 index has become equal to 20 from June 2011.

These four indexes are fully comparable across countries, since they are built on a consistent value-weighted basis. The start date of this study coincides with the opening of the latest index (PSI-20), in order to allow the temporal comparison of results in all markets. For reasons of space, the paper only presents the figures for the most important index (CAC-40), but the figures for the other indexes will be made available upon request to the authors.

The original data refers to the series of daily closing price stock indexes, provided by Euronext, and cover a period of over 24 years (since January 1, 1993 until February 8, 2016), providing 6 026 observations, during which it includes several significant events. The data used in the empirical study consist of the simple transformation of the stock indexes through the first log difference of their levels. In practical terms, the returns

(excluding dividends) compounded continuously X_t at time t are calculated from the consecutive daily prices P_t index, as follows:

$$X_t \equiv \log P_t - \log P_{t-1} = \log \left(\frac{P_t}{P_{t-1}} \right) \qquad (1)$$

The growth of dispersion with the increasing index value is typical of this kind of financial data and, therefore, the adoption of logarithms in the original series is normally necessary to stabilize the variance (Matos et al., 2004).

4. Methodology

In order to pursue the objective of the empirical study related to the dynamic nature of the stock indexes returns, we will:

1) examine whether the innovations of the stock indexes returns are ergodic;

2) examine whether the innovations of the stock indexes returns are stationary;

3) examine whether the time series of indexes returns exhibits long-term memory;

4) identify whether the possible long-term memory is persistent or anti-persistent;

5) identify what are the most suitable theoretical benchmark models for the series of returns examined.

Complementarily, other additional issues related to the risk management of portfolios will be studied:

6) examine whether price diffusion models empirically identified suggest inefficiency of stock markets, and thus, call into question the adequacy of pricing models;

7) examine whether price diffusion models empirically identified can assist financial market participants to earn abnormal returns;

8) examine whether price diffusion models empirically identified can help to classify the stage of maturity of the markets.

4.1. Stationarity

Kyaw et al. (2006) state that a time series is said strictly stationary if the joint distribution of any set of n observations $X_{t_1}, X_{t_2}, ..., X_{t_n}$ is the same as the joint distribution of $X_{t_{1+k}}, X_{t_{2+k}}, \cdots, X_{t_{n+k}}$ for all n time points and for all lags k. In other words, it refers to a stochastic process in which the joint probability distribution remains invariant over time.

To test stationarity, one can calculate the sliding windows for the first four moments of the stock market indexes. If constant moments are not observed, then there is an indication of possible fractality of the time series, concluding that the series are not stationary (Lipka & Los, 2003).

4.2. Rescaled-range analysis

4.2.1. Modified rescaled-range statistics

An approach to detect long-term dependence is to use the range over standard deviation or rescaled-range (R/S) statistics, originally developed by Hurst (1951), later improved by Mandelbrot & Wallis (1969a; 1969b) and Mandelbrot & Taqqu (1979) and popularized in the financial context by Mandelbrot (1972, 1975).

The classical R/S is given by *"range of partial sums of deviations of a time series from its mean, rescaled (divided) by its standard deviation"* (Lo, 1991, p. 1287). Specifically, being X_j the return of a stock in period j, for $\{X_1, X_2, \dots, X_n\}$ it may be defined as:

$$(R/S)_n = S_n^{-1} \left[\underset{1 \leq k \leq n}{Max} \sum_{j=1}^{k} (X_j - \bar{X}_n) - \underset{1 \leq k \leq n}{Min} \sum_{j=1}^{k} (X_j - \bar{X}_n) \right] \qquad (2)$$

where $\bar{X}_n = (1/n) \sum_{j=1}^{n} X_j$ is the sample mean and $S_n = \left[(1/n) \sum_{j=1}^{n} (X_j - \bar{X}_n)^2 \right]^{1/2}$ is standard deviation. The time series is divided by an integer number of adjacent non-overlapping sub-intervals of equal size. The first term (in brackets) is the maximum (in k) of the partial sums of the first k deviations of X_j from the sample mean. Since the sum of all n deviations of X_j from its mean is *zero*, that maximum is always non-negative. The second term is the minimum (in k) of the same sequence of partial sums; so it is always non-positive. Thus, the difference between the two parties, called range (R_n), is always non-negative and, therefore, the statistics $(R/S)_n \geq 0$, as indicated by Lo (1991).

Although it has been shown that the R/S statistics is able to detect long-term memory, later studies have found that it is sensitive to short-term memory. Therefore, Lo (1991, p. 1289) defined a modified rescaled-range (M-R/S) statistics wherein the short-term dependency is incorporated into its denominator, which becomes the square root of a consistent estimator of the variance of partial sums until the lag q in expression [2], presented as:

$$(M-R/S)_{n,q} = S_q^{-1} \left[\underset{1 \leq k \leq n}{Max} \sum_{j=1}^{k} (X_j - \bar{X}_n) - \underset{1 \leq k \leq n}{Min} \sum_{j=1}^{k} (X_j - \bar{X}_n) \right] \qquad (3)$$

where S_q^2 is a heteroskedasticity and autocorrelation consistent variance estimator (Andrews, 1991) and includes the usual sample variance S_n^2 and autocovariance $\hat{\gamma}_j$ estimators of X:

$$S_q^2 = S_n^2 + 2\sum_{j=1}^{q} \omega_j(q)\hat{\gamma}_j \tag{4}$$

where the weighting function was suggested by Newey & West (1987) and given by $\omega_j(q) = 1 - j/(q+1)$, $q < n$ with the truncation lag suggested by Andrews (1991) and given by $q = \text{Int}\left[(3n/2)^{1/3} \times \left(2\hat{\rho}_1/(1-\hat{\rho}_1^{\,2})\right)^{1/3}\right]$, being $\hat{\rho}_1 = \hat{\gamma}_1/\hat{\gamma}_0$ the first-order autocorrelation, and the estimator of autocovariance is given by $\hat{\gamma}_j = (1/n)\sum_{i=1}^{n-j}(X_i - \bar{X}_n)(X_{i+j} - \bar{X}_n)$.

4.2.2. Graphical rescaled-range analysis and Hurst exponent

It is possible to impose specific models of short-term dependence in time series to analyse the influence of this kind of dependence on the estimation of the Hurst exponent H, which is a related measure of long-term dependence. This analysis (sometimes also called R/S analysis) uses a graphical technique, firstly suggested by Mandelbrot & Wallis (1969a; 1969b) to estimate the Hurst exponent, that is closely related to the R/S statistics.

If the time series exhibits positive or negative long-term dependence, the exponent H should converge in values larger (persistence) or smaller (anti-persistence) than 0.5, respectively. Such scaling reflects a trend of strengthening of deviations from the mean and it is also characteristic of the time series models known as fractional Gaussian noise (Mandelbrot & Wallis, 1969b) and as fractionally integrated ARMA (Granger, 1980). In these processes, the long-term dependence is identified in a slow (hyperbolic) decay of the ACF, based on the asymptotic scaling relationship (Lux, 1996, p. 702):

$$(R/S)_t \sim at^H \tag{5}$$

where a is a finite positive constant independent of t (Di Matteo, 2007) and H is the Hurst exponent. The linear relationship in log-log scale indicates the power scaling (Weron, 2002). To discover this scaling law and estimate the exponent H one can employ a simple linear least-squares regression on the logarithms of each side of the expression [5] in a sample of increasing time horizons ($s = t_1, t_2, ..., t_n$) (Lux, 1996, p. 702) as $\log(R/S)_s = \log(a) + H\log(s)$.

The graphical process of rescaled-range analysis involves calculating the mean of the rescaled-range for several values up to n for a given value of s. Being this mean represented by R/S, the limit of the ratio $\log(R/S)/\log(s)$ is often referred to as exponent H. Mandelbrot & Wallis (1969a) suggested the technique of representing $\log(R/S)$ as a function of $\log(s)$ for different values of s. The slope of that representation, estimated using ordinary least-squares, reflects an estimate of the Hurst exponent H.

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4.3. Detrended fluctuation analysis

Compared to the M-R/S analysis, the DFA method, proposed by Peng et al. (1994) as a modification of the standard variance analysis, has the advantage of being able to detect long-term dependence on non-stationary time series. The basis of the DFA is to subtract the possible deterministic trends from the original time series and then analyse the fluctuation of detrended data.

Firstly, after subtracting the mean, one integrates the original time series $\{X_j\}$ to obtain the cumulative time series $Y(t)$ as follows (Oh et al., 2006, p. 2):

$$Y(t) = \sum_{j=1}^{t}(X_j - \bar{X}) \qquad ; \qquad t = 1, \dots, n \tag{6}$$

This accumulation process is what transforms the original data into a self-similar process, where $\bar{X} = \frac{1}{n}\sum_{j=1}^{n}X_j$ represents the mean.

Secondly, the series $Y(t)$, of length n, is divided by an integer equal to n/τ non-overlapping boxes, each containing τ points. Then, the linear local trend $z(t) = at + b$ in each box is defined as the standard linear least-squares[*] fit of the data points (Grech & Mazur, 2005). Subtracting $z(t)$ to $Y(t)$ in each box the trend is removed[†]. This process is applied to all the boxes, and the detrended fluctuation function F is defined by the square root of the mean deviation of $Y(t)$ from the trend function $z(t)$ (Kristoufek, 2010, p. 317):

$$F_k^2(\tau) = \frac{1}{\tau}\sum_{t=k\tau+1}^{(k+1)\tau}|Y(t) - z(t)|^2 \qquad k = 0, \dots, \qquad \frac{n}{\tau} - 1 \tag{7}$$

The calculation of the average of $F_k^2(\tau)$ over the n/τ intervals provides the definition of the fluctuation function $F(\tau)$ defined by (Matos et al., 2008):

$$F^2(\tau) = \frac{\tau}{n}\sum_{k=0}^{n/\tau-1}F_k^2(t) \tag{8}$$

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[*] The a and b coefficients represent the linear least-squares fit of $Y(n)$ in each box.
[†] This method is called DFA-1, wherein the trend is modelled as a first order polynomial; similarly, DFA-n corresponds to a trend modeled by an n-order polynomial.

$$F(\tau) = \sqrt{\frac{\tau}{n} \sum_{k=0}^{n/\tau-1} F_k^2(t)} \qquad (9)$$

Thirdly, if the observable $X(t)$ are uncorrelated random variables, the expected behaviour should be a power-law (Matos et al., 2004), and the previous fluctuation function has the following scaling relation (Peng et al., 1994):

$$\langle F(\tau) \rangle \sim (const)\tau^H \qquad (10)$$

Returning to run a linear least-squares regression over the relationship represented by log-log scale in the expression [10] produces a straight line, whose slope is the Hurst exponent H. So, from a linear (in log-log scale) regression of data corresponding to $F(\tau)$ the empirical value for exponent H can be estimated to define the degree of polynomial trend (Costa & Vasconcelos, 2003, p. 237), as occurred for the R/S analysis, as $\log \langle F(\tau) \rangle = \log(const) + H \log \tau$.

5. Empirical results

5.1. Preliminary analysis

Figure 1 shows the daily time series of price levels (in the first panel) and the CAC-40 index returns (in the second panel), as well as the peaks (i.e., the log-returns to the cube, in the third panel) to evidence the most relevant events over time.

FIGURE 1. DAILY SERIES OF THE CAC-40 INDEX:
PRICE LEVELS, INDEX-RETURNS AND LOG-RETURNS
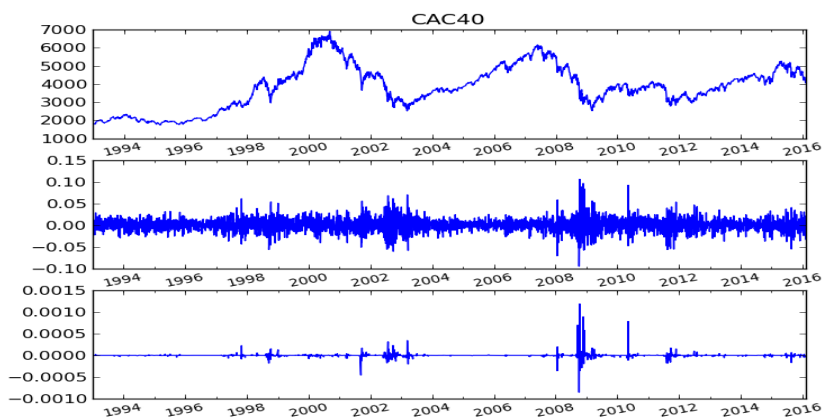
In general, the graphical behaviour shows different periods of stock indexes. Until the crash of the *Asian Tigers*, in July 1997, there was a relatively steady phase. Then, the markets had a period of strong growth and increased volatility until the fall noted in mid-1998, before the signs of a global recession. After a new impulse, even at the end of that year, with historic highs in 2000 (except in BEL-20), a more volatile regime arose characterized by successive losses until the beginning of 2003, marked by the explosion of the *dot com* bubble in the Nasdaq in April 2000, the terrorist attack of September 11 in 2001 and the global crash after March 2002. Posteriorly, the markets returned to consolidate gains until August 2007, when the first news emerged about the problem of the American mortgage market and, thus, reversing the trend and reinforcement of the variability, which were compounded with the failure of Lehman Brothers and remained until the beginning of 2009. During the following three years, the indexes showed signs of high fluctuation, with alternating movements and short-term trends. Since then, they have evolved favorably to recover the losses inflicted by the financial crisis, with the exception of the PSI-20 which reversed the trend at the beginning of 2015.

5.2. General characteristics of temporal frequency

The first four moments[*] of daily returns series of the CAC-40 index are calculated using time windows of increasing size, in Figure 2, and sliding time windows of fixed size equal to 50 days (about 2 months of trading), in Figure 3. These procedures allow testing the invariant nature of trading.

FIGURE 2. MOMENTS IN INCREASING WINDOWS
FOR DAILY RETURNS SERIES OF THE CAC-40 INDEX

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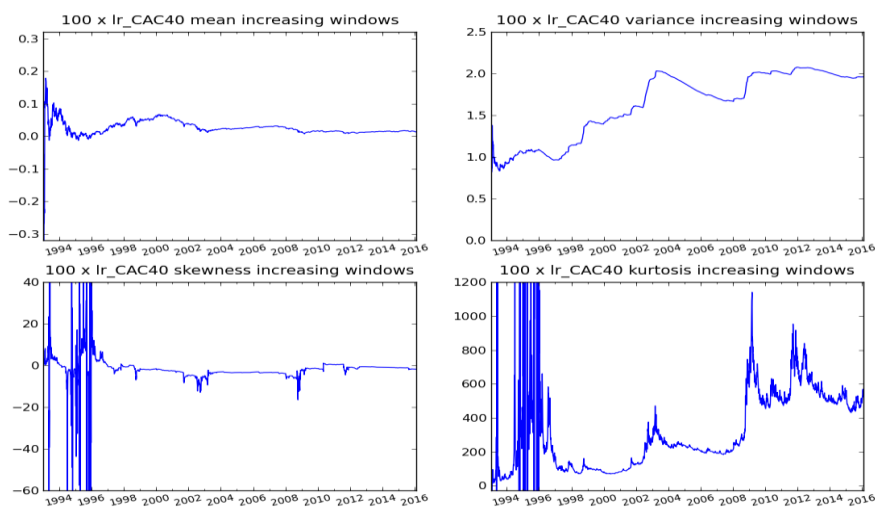
[*] The values of the moments of first, second, third and fourth order were multiplied by 100 for readability.

The generality of representations reveal the occurrence of several sharp discontinuities (singularities) and the absence of points of gradual convergence to a flat line parallel to the abscissa time (i.e., for a constant value). The less pronounced situation concerns to the evolution of the mean. This signifies that the values of the moments (in increasing windows) change as the number of observations increases. Such behaviour occurs in all markets and indicates possible fractality in the data set, concluding that the series of Euronext stock indexes are not ergodic.

The changes in the skewness and kurtosis of the returns reflect the volatility of stock exchanges, being more pronounced from mid-2001 to mid-2004 and since 2008. Certainly, much of the influence came from the permeability of European markets to the crash of 2002 and the current global financial crisis. The changes in the structure of market participation, evidenced by the unrest of the asymmetry, suggest the existence of strong upward or downward pressure; while the leptokurtic manifestation, evidenced by the gradual increase of kurtosis, suggests that each market is classified into a group of hedging noise traders (Los & Yu, 2008).

FIGURE 3. MOMENTS IN SLIDING WINDOWS OF FIXED SIZE (50 DAYS)
FOR DAILY RETURNS SERIES OF THE CAC-40 INDEX
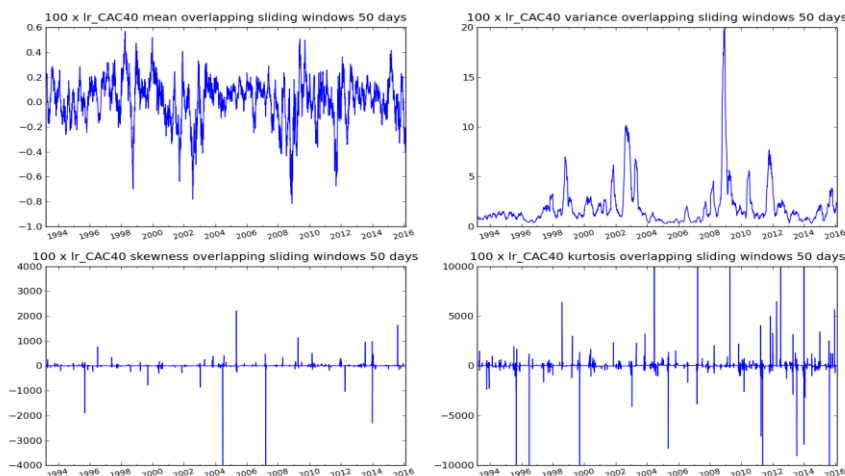
Again, the visual inspection of representations reveals varying moments (in sliding windows) over time. This occurs in all markets and indicates possible fractality in the data set, concluding that the series of Euronext stock indexes are not stationary, neither in the strict sense, nor in the broad sense. This analysis shows that stock returns are not i.i.d. and, therefore, are not processes integrated driven by white noise and the random walk model is immediately adulterated. Furthermore, the findings of ergodicity and stationarity also demonstrate that the series do not meet the assumptions of conventional diffusion models of log-normal price prevailing in the theoretical finance literature.

5.3. Estimation of long-term memory under fractional Brownian motion approach

The general empirical characteristics of temporal frequency reveal that the series of stock indexes returns cannot be well described by the gBm. Verifying that the time series have long-term dependence and, therefore, long memory, they can be better represented by fBm. When long-term memory exists, one can identify whether the series are persistent or anti-persistent through the estimation of the Hurst exponent H, considering the procedure of the M-R/S analysis and the procedure of DFA.

5.3.1. Estimation of the Hurst exponent by modified rescaled-range analysis

In an experimentation for dependency the simple linear regression was applied to a sample of restricted time horizons $\log(R/S)_s = \log(a) + H \log(s)$, commonly used to estimate the exponent H in the asymptotic scaling regime $(R/S)_t \sim at^H$ with lags s.

Figure 4 shows the diagram of the linear least-squares regression of $\log(R/S)_s$ as a function of $\log(s)$, with logarithms base 10, for the complete daily returns series of the CAC-40 index.

FIGURE 4. HURST EXPONENT *H* VIA M-R/S ANALYSIS
FOR DAILY RETURNS SERIES OF THE CAC-40 INDEX
Procedure: linear regression of $\log(R/S)_s$ over $\log(s)$
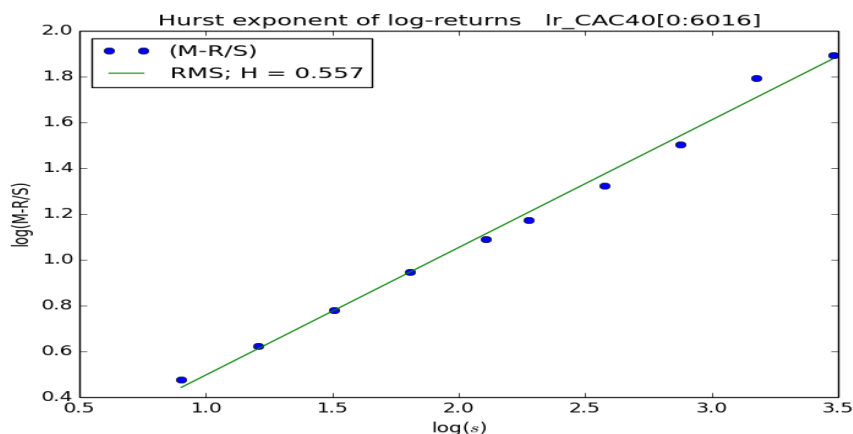
Note: The complete series for log-returns has a length of 6 026 observations, but given the need of entire divisibility in R/S procedure (Mandelbrot & Wallis, 1969a), we considered only the first 6 016 closing prices. Specifically, the decimation for the $(R/S)_t$ ratio established the lags $s = 8, 16, 32, 64, 128, 188, 376, 752, 1504, 3008$. The $\log(R/S)_s$ was calculated as the mean of a fixed number of non-overlapping intervals.

Table 2 presents the estimates of the Hurst exponent H for Euronext indexes, obtained by similar regression and the coefficients of determination (R^2).

TABLE 2. HURST EXPONENT H VIA M-R/S ANALYSIS AND COEFFICIENT R^2
FOR DAILY RETURNS SERIES OF THE EURONEXT INDEXES
Procedure: linear regression of $\log(R/S)_s$ over $\log(s)$

ESTIMATES		CAC-40	AEX	BEL-20	PSI-20
Hurst exponent (via M-R/S)	H	0.557	0.571	0.586	0.596
Coefficient of determination	R²	0.993	0.997	0.995	0.999

Note: Insofar as the asymptotic distribution of the Hurst exponent is "intractable" (Cheung, 1990), the standard errors are not given in this table.

During the entire period of analysis, the statistics H calculated by the technique M-R/S is slightly above the benchmark $H = 0.5$ in the CAC-40 index and higher up in the BEL-20, AEX and PSI-20 indexes, indicating the existence of long memory in the form of persistence. This suggests that the markets are more subjected to predictability *("Joseph effect")*, but also trends can be unexpectedly disrupted by discontinuities *("Noah effect")* and, therefore, they tend to be more risky for trading and investing.

An important feature is the excellent fit of the regression $(R/S)_s$, given by R^2 close to unity. This means that the scaling law $(R/S)_t \sim at^H$ with estimated exponent H appears to depict very accurately the rescaled range behaviour of the time series.

5.3.2. Estimation of the Hurst exponent by detrended fluctuation analysis

In another experimentation for dependence, the simple linear regression was applied on a sample of restricted time horizons $\log \langle F(\tau) \rangle = \log(const) + H \log \tau$, commonly used to estimate the exponent H in the scaling regime with power law $F(\tau) \sim (const) \, \tau^H$, in which the local trend is quadratic and given by $z(t) = at^2 + bt + c$. The DFA with local quadratic trend is called DFA-2. The $\log F(\tau)$ was calculated as the average of a fixed number of sliding overlapping intervals, wherein the minimum lag τ is equal to 20 (about one month of trading).

Table 3 presents the estimates of Hurst exponent H for the complete daily returns series of the Euronext indexes, obtained by linear least-squares regression of $\log F(\tau)$ as a function of $\log(\tau)$, the standard deviations and the coefficients of determination (R^2).

TABLE 3. HURST EXPONENT H VIA DFA ANALYSIS, STANDARD DEVIATION AND COEFFICIENT R^2
FOR DAILY RETURNS SERIES OF THE EURONEXT INDEXES
Procedure: linear regression of $\log F(\tau)$ over $\log(\tau)$

ESTIMATES		CAC-40	AEX	BEL-20	PSI-20
Hurst exponent (via DFA)	H	0.473	0.502	0.511	0.566
Standard deviation	S	0.004	0.004	0.006	0.004
Coefficient of determination	R²	0.996	0.996	0.992	0.996

Note: Given the sample size of 6 026 observations, the central limit theorem allows to consider an approximation to the normal distribution (for $n > 30$) and use the standard deviation to indicate the variability of the estimated Hurst exponent H.

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The result of the DFA technique for CAC-40 fails to confirm the evidence of persistence from the M-R/S technique, and even constitutes weak evidence of possible anti-persistence, where $0 < H < 0.5$, suggesting that the market has fast mean reversion, faster than theoretically postulated by a gBm, where $H = 0.5$. For the AEX and BEL-20 indexes the estimated exponents are slightly greater than 0.5, but considerably weaker than the results from the M-R/S procedure. The evidence for Portugal constitutes stronger evidence of persistence, consistent with the findings under the M-R/S procedure.

The comparison of the results in both approaches shows that in three of the four countries the findings are in the same direction, although with different intensity, and for Portugal they are rather strong.

5.3.3. Time-varying Hurst exponent estimated by modified rescaled-range analysis

The estimates of the Hurst exponent calculated for the entire samples hide the complexity inherent in the time series of returns, due to periods of different behaviour observed. In a complementary study we applied the R/S analysis (using the modified rescaled-range statistics) to smaller intervals of fixed length, calculating for each sub-interval its Hurst exponent, in order to investigate if H varies over time. The choice of sub-intervals of 200 and 400 points corresponds approximately to one year and two years of negotiation, respectively.

Figure 5 shows the estimates of time-varying Hurst exponents $H(t)$, obtained by adjusting the asymptotic scaling regime to $(R/S)_t$ calculated for moving window sizes equal to 200 and 400 days, and the coefficients R^2 at each point.

FIGURE 5. TIME-VARYING HURST EXPONENTS H VIA M-R/S ANALYSIS (200 DAYS AND 400 DAYS WINDOWS) AND COEFFICIENTS R^2 FOR DAILY RETURNS SERIES OF THE CAC-40 INDEX
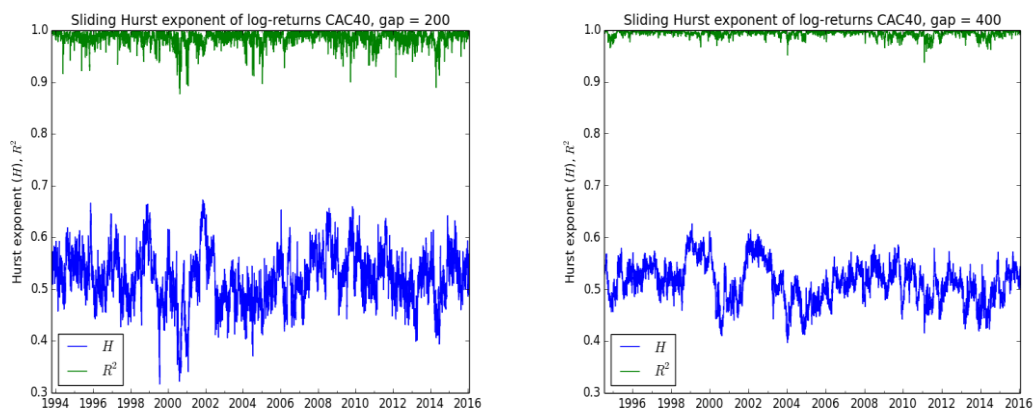
Note: In the graphical analysis of sliding windows each point represents the end of a window, moved over the series, and its corresponding exponent H. The decimation for the M-R/S ratio established the lags $s = 10, 20, 40, 50, 100$ for the 200 days windows and $s = 8, 16, 25, 50, 100, 200$ for the 400 days windows.

The most localized study (in time) of the French index, based on the calculation of the Hurst exponent "local" $H(t)$ in sliding windows by the R/S analysis, shows that H varies considerably over time, indicating the existence of non-stationary fluctuations. There are several periods of distinct behaviour with the characteristic of dependence on two time scales. These results, which are common to all Euronext stock indexes, corroborate the first signals of visual inspection of the returns distribution, as shown in Figure 1 (second panel), and the sliding windows moments, as shown in Figure 3.

The quality of the adjustments, measured by R^2 close to unity, ensures that the results are reliable and the time series are well described by a fBm. This is most evident in the 400-day window. In this upper time scale, softer variations of the exponent H can be found, because it has more data points and, thus, it is expected greater robustness to sudden changes of the data.

The empirical evidence shows that in the period subsequent to 2000, 2002 and 2010 the Hurst exponent had sharp breaks in the CAC-40 index. The other markets have also become less persistent, the AEX in the period subsequent to 2000, 2003 and 2010, the BEL-20 in the period subsequent to 2003 and 2010, and the PSI-20 in the period subsequent to 2011 and 2014. Nevertheless, the Euronext indexes do not correspond to the EMH in the strict sense of Fama (1970), although they approached this theoretical ideal during the periods coinciding with worsening of international crises. Becoming more efficient and neutral, the markets do not allow abnormal gain opportunities by arbitrage.

5.3.4. Time-varying Hurst exponent estimated by detrended fluctuation analysis

In another further study, the DFA methodology was applied under the same conditions as the R/S procedure and forthe same sub-intervals of fixed size. Figure 6 shows the estimates of time-varying Hurst exponents $H(t)$, obtained by adjusting a power law to the DFA function $F(\tau)$ calculated for moving window length equal to 200 and 400 days, and the coefficients R^2 at each point.

FIGURE 6. TIME-VARYING HURST EXPONENTS H VIA DFA ANALYSIS (200 DAYS AND 400 DAYS WINDOWS) AND COEFFICIENTS R^2 FOR DAILY RETURNS SERIES OF THE CAC-40 INDEX
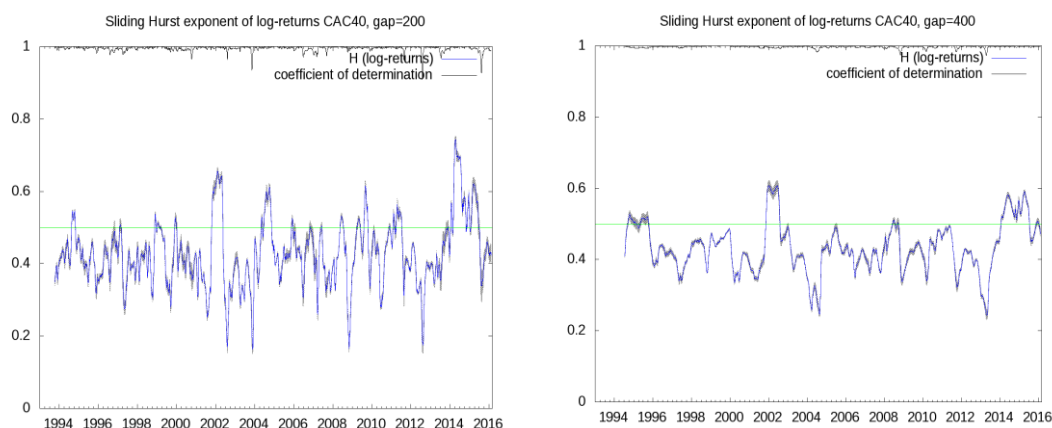
Note: In the graphical analysis of sliding windows each point represents the end of a window, moved over the series, and its corresponding exponent H. The decimation for the relationship $\langle F(\tau) \rangle$ established a minimum lag τ equal to 20 for both windows.

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Again, the R^2 coefficients close to unity ensure that the results are reliable and the CAC-40 is well described by a fBm. These results are common to all Euronext stock indexes.

The use of DFA to study the Hurst exponent indicates that the French financial series exhibit anti-persistent short-term behaviour (with more pronounced extremes in the smaller scale). The others indexes of Euronext exhibit behavior in a persistent way, although degenerating over long periods. For a larger scale, the configuration of the Hurst exponent "local" $H(t)$ undergoes a sudden change in the character, from persistent to anti-persistent behaviour, in all markets by the years 2002 and 2015. Much of this is due to the erratic period of the index (marked by volatility), as shown in Figure 1, and coincided with the dissemination of effects of the global crash and the current financial crisis. A possible explanation for the change in the Hurst exponent is that the efficiencies in these markets may have improved as the markets matured.

6. Conclusion

The series of daily returns of the Euronext stock indexes are studied through the first four moments, calculated in time windows of increasing size and sliding time windows of fixed size equal to 50 days. Evidence of non-stationarity, non-ergodicity, and dependency was found in the series. Therefore, the empirical innovations of the returns follow correlated Gaussian processes and cannot be compared with benchmark theoretical models of white noise (independent), such as gBm. Consequently, the stock market returns are described under the theoretical benchmark model characterized by fBm, which is more general and covers the gBm.

In the search for evidence for the long memory property in the markets, we modelled the time series using a formulation of fBm to obtain Hurst exponents H estimated by the M-R/S technique and the DFA technique, with different window sizes. The linear regression over the full sample data found different values for the Hurst exponent in the two methodologies, with nonexclusive results in the CAC-40 index and with exclusive and convergent results in the AEX, BEL-20 and PSI-20 indexes. These empirical evidences suggest the use of additional methodologies to test the real or spurious long memory property in the French market. This will be a topic for future research.

The most localized analysis (in time) of the returns series, based on the calculation of the Hurst exponent "local" $H(t)$ in moving windows, provides a map of the development of the Euronext indexes to maturity. This makes it easier to identify the main events that affect the international markets and analyse how the effects propagate through each of the scales (with lags of 200 and 400 days). The consistent decline in long-term dependence degrees (from 2000, 2003 and 2010 in the M-R/S analysis, and from 2002 and 2015 in the DFA analysis) can mean a maturing market with the extension of the international crisis. This conclusion is an important contribution of the manuscript. Those changes of the Hurst exponent in periods of extreme events are in line with those obtained by Costa & Vasconcelos (2003), Los & Yu (2008), Kristoufek (2012) and Ferreira (2018), but they contradict those of Horta et al. (2014).

Moreover, the Hurst exponent behavior for multiple periods and two scale intervals provides a finer detail on data series, compared to the estimate of the exponent for the entire sample. The substantial changes in the Hurst exponent over time suggest that the fBm can be a model somehow restrictive, less able to fully capture the complex dynamics of stock indexes and, possibly, recommend the use of the multi-fractional Brownian

motion (mfBm) to describe better the temporal scaling of the exponent H. This will be another topic for future research.

The fractal dynamics, more pronounced in the M-R/S analysis, refutes the random walk hypothesis with i.i.d. increments in the Euronext indexes, which is the basis of the EMH in its weak form. Consequently, many of the paradigms used in the modern financial theory will be violated, namely the validity of the martingale methods of derivatives instruments (such as the Black & Scholes model) and the adequacy of the asset pricing models (such as the capital asset pricing model - CAPM and the arbitrage pricing theory - APT).

According to the classification proposed by Matos et al. (2008), the French market seems mature, given the consistency of the Hurst exponent around 0.5. However, the representation of this index returns show distinct periods and leptokurtic moments, typical of markets that are not fully developed. The Dutch and Belgian markets are tendentiously mature, with Hurst exponents slightly above 0.5. The Portuguese market has a higher Hurst exponent, although its performance over the last 8 years suggests a transitional phase from persistent to anti-persistent behavior.

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