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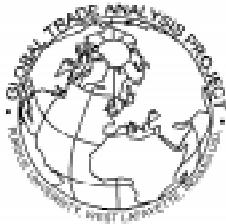
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***Implementing Quotas in GTAP using  
GEMPACK or  
How to Linearize an Inequality***

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# ***Implementing Quotas in GTAP using GEMPACK or How to Linearize an Inequality***

## ***Abstract***

This document describes how explicit import and export quotas can be implemented and solved in the GTAP CGE trade model. The techniques described here apply equally well to other general and partial equilibrium models implemented and solved using the GEMPACK software. They also generalize to procedures for handling other inequalities in models solved via GEMPACK (even though GEMPACK does not allow explicit inequalities in the algebraic representation of models).

This document describes some recent applications of GTAP in which explicit import and export quotas have been modelled, and discusses how important it was for these applications to have explicit quotas modelled.

Accompanying this document are various computer files containing the ingredients of examples that readers can carry out for themselves while reading this paper. These files can also be used as a starting point for those who wish to explicitly model quotas in their own models.

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# ***Implementing Quotas in GTAP using GEMPACK -or How to Linearize an Inequality***

## ***1. Introduction***

Quantitative trade restrictions have been implemented in a number of Computable General Equilibrium models. However, modellers from the linearized school (that is, those working with a linearized representation of the equations of their model) have been reluctant to move towards explicit modelling of quota constraints as this inevitably requires implementing an inequality in a linearized representation. This technical paper demonstrates how these problems can be overcome, and how quotas can prove useful in a number of everyday policy problems. Binding volume quotas are introduced in the GTAP model [Hertel, 1996] and solved via a linearized representation using GEMPACK [Harrison and Pearson, 1994].

In section 2 we introduce the policy problem which spurred the current formulation of bilateral export quotas in GTAP, and present the levels versions of the export quota equations. In section 3 we take a detailed look on the actual implementation of bilateral export quotas in the GTAP model (which is implemented via a linearized representation). In section 4 we tell you how you can work through several examples of simulations involving bilateral export quotas with a 3-commodity, 3-region version of GTAP. These sections are subsequently replicated but with bilateral import quotas in sections 5, 6 and 7. Solving a model with quotas via GEMPACK involves a two-part process (as explained in the examples in sections 3.6 and 6.6). Ways of automating the second part of this process are described in section 8. In section 9 we briefly discuss ways in which other inequalities can be modelled using GEMPACK. In section 10 we make a few concluding remarks. Section 11 contains references.

Some more technical material is presented in the various appendices. In Appendix 1 we tell you how you can install and use the computer files associated with this document on your PC. You can then carry out the examples as you read this document. Appendix 2 contains the TABLO statements which are added to the standard TABLO Input file GTAP94.TAB for GTAP to implement bilateral export quotas. Appendix 3 contains the details of the linearization of the key levels equation used to describe quotas.

Quotas and other inequalities can, of course, also be modelled directly using other software, including GAMS [Brooke, 1988] and GAMS/MPSGE [Rutherford, 1995].

We assume that readers of this document are familiar with the standard computer version of the GTAP model [as documented in Hertel, 1996] and are also familiar with the procedure for implementing and solving models using GEMPACK [Harrison and Pearson, 1994]. In particular we frequently use notation from the TABLO Input file GTAP94.TAB for GTAP without explicit cross referencing.

We are grateful to Mark Horridge who worked out the procedures described here for handling inequalities using GEMPACK [Horridge, 1993]. We are also grateful to Jill Harrison, Tom Hertel and Michael Malakellis for helpful discussions about various aspects of this paper.

### ***1.1 Bilateral Quotas - The Basic Idea***

In this section we introduce the essential concepts involved in the way we have modelled explicit bilateral export and import quotas, and also introduce the corresponding relevant GTAP notation for those not familiar with GTAP.

Consider exports of some commodity, say food, from one region, say USA, to another region, say the European Union EU. Firstly there is the volume exported, say **QXS**. Clearly the volume of USA food exported to the EU is the same as the volume of food from the USA imported into EU.

There are several prices associated with this volume. Firstly there is

**PM**                    the basic or market price in the exporting region  
(this is usually the cost of producing the commodity)

There may be export taxes or subsidies added to this to give

**PXS**                    the price including export taxes or subsidies

[If there are taxes,  $PXS > PM$  while  $PXS < PM$  if there are subsidies.] Then there is

**PFOB**                    the f.o.b. price

Without export quotas,  $PFOB = PXS$ . But if there is a binding export quota we expect PFOB to be larger than PXS.

Then this volume is transported to the importing region where we have

**PCIF**                    the c.i.f. import price

PCIF has costs such as freight and insurance added to PFOB. There may be import tariffs or subsidies added to this to give

**PIS** the price of imports including any import tariff or subsidy (but not including the tariff equivalent of any quota)

There is also

**PMS** the basic or market price in the importing region

Without import quotas, PIS=PMS. But if there is a binding import quota we expect PMS to be larger than PIS.

With this notation, we have

$PXS \leq PFOB$   
 $PFOB = PXS$  if the quota is not binding  
 $PFOB > PXS$  if the quota is binding

$PMS \geq PIS$   
 $PMS = PIS$  if the quota is not binding  
 $PMS > PIS$  if the quota is binding

We consider the export quota responsible for any difference between PXS and PFOB and the import quota responsible for any difference between PIS and PMS. These simple ideas are the key to our implementation of these quotas.

### *GTAP Notation*

Above we have used the usual GTAP notation for those prices which exist in standard GTAP (that is, the standard version without export or import quotas). The prices denoted above by PXS and PIS are not present in the standard GTAP model since there  $PXS=PFOB$  and  $PIS=PMS$ . Of course each of the above prices really needs indices after it to account for the different commodities, exporting and importing regions in the model: thus we really should be writing  $PM(i,r)$ ,  $PXS(i,r,s)$ ,  $PFOB(i,r,s)$  etc.

The table below summarises these different prices and their associated values in GTAP.

GTAP price	GTAP value
PM(i,r)	VXMD(i,r,s) ] Differences due to "normal" export taxes/subsidies
PXS(i,r,s)	VXQD(i,r,s) ] Differences due to export quotas
PFOB(i,r,s)	VXWD(i,r,s) ] Differences due to transport costs, insurance etc
PCIF(i,r,s)	VIWS(i,r,s) ] Differences due to "normal" import tariffs/subsidies
PIS(i,r,s)	VIQS(i,r,s) ] Differences due to import quotas
PMS(i,r,s)	VIMS(i,r,s)

## 2. *Bilateral Export Quotas*

The need for bilateral export quotas grew out of an actual trade policy, the Multi-Fibre Agreement. To protect domestic producers, governments in the EU and the US have persuaded a number of developing countries to curb their exports of textiles and wearing apparel. Traditionally these restrictions have been modelled by endogenizing the export tax rate, TXS\_L in GTAP, and exogenizing the quantity variable, QXS\_L, for the bilateral flow of textiles and wearing apparel from most developing countries to the EU and the US. This approach is appealing in simplicity, but in two cases it may prove inadequate. The first and most important case is when the export quota shifts status from binding to non-binding. The second case is if the actual export tax on the flow has two elements: a tax equivalent of the export quota and a "normal" export tax imposed by the exporting country government.

To model export quotas, we need to have a notation for the levels value of the quotas. This is

**QXS\_QUOTA\_L(i,r,s)** The volume export quota on exports of i from r to s.

where i is in TRAD\_COMM (the set of tradeable commodities) and r and s are in REG (the set of regions). QXS\_QUOTA\_L(i,r,s) needs to be distinguished from

**QXS\_L(i,r,s)** The actual volume of exports of i from r to s.

Clearly  $QXS_L \leq QXS\_QUOTA\_L$  for all (i,r,s). The quota on i from r to s is said to be

(a) **Binding** if  $QXS\_L(i,r,s) = QXS\_QUOTA\_L(i,r,s)$ .

(b) **Non-binding** if  $QXS\_L(i,r,s) < QXS\_QUOTA\_L(i,r,s)$ .

$QXS\_L$  is in the standard GTAP model but  $QXS\_QUOTA\_L$  is not.<sup>1</sup>

For each  $i$  in TRAD\_COMM,  $r$  and  $s$  in REG, we distinguish between

<b>PXS_L(i,r,s)</b>	The price of exports inclusive of "normal" export taxes (but excluding the tax equivalent of the quota)
---------------------	---

and

<b>PFOB_L(i,r,s)</b>	The fob export price. (This includes any "normal" export taxes and the tax equivalent of the quota.)
----------------------	--

Clearly  $PXS\_L \leq PFOB\_L$ . These will be equal if the quota is not binding. If the quota is binding we expect  $PXS\_L < PFOB\_L$ .  $PFOB\_L$  is in the standard GTAP model but  $PXS\_L$  is not.

Associated with these two different prices are the associated values, namely

$$VXQD(i,r,s) = PXS\_L(i,r,s) * QXS\_L(i,r,s),$$

$$VXWD(i,r,s) = PFOB\_L(i,r,s) * QXS\_L(i,r,s).$$

$VXQD$  (Value of eXports before Quota by Destination) is not in the standard GTAP model but  $VXWD$  (Value of eXports at World prices by Destination) is. Assigning  $VXQD$  data depends very much on the application. We say more about this in section 3.4 below. The next paragraph says how the  $VXQD$  data was set up for this MFA application.

In this MFA application, explicit quotas were only modelled for exports from MFA-exporting regions to MFA-importing regions. The standard GTAP data contains  $VXWD$  values and also  $VXMD$  values where  $VXMD(i,r,s) = QXS\_L(i,r,s) * PM(i,r)$  is the value of exports valued at domestic market prices. When  $VXWD$  is greater than  $VXMD$ , we assumed that there is no "normal" export tax but that all of this difference is due to an export quota and so set  $VXQD = VXMD$ . When  $VXWD$  is less than or equal to  $VXMD$  we assumed that this difference was due to an export subsidy and that the quota is not binding, so set  $VXQD = VXWD$ . That is, in all cases  $VXQD$  was determined via the formula

---

<sup>1</sup>In describing the levels equations, here we often add "\_L" to the end of some linearized GTAP variable to indicate the associated levels value. For example, in standard GTAP,  $qxs(i,r,s)$  denotes the percentage change in the volume of exports of  $i$  from  $r$  to  $s$ . We use  $QXS\_L(i,r,s)$  to denote the actual volume of these exports.

$$VXQD(i,r,s) = \text{MIN}(VXMD(i,r,s), VXWD(i,r,s)).$$

In general, if  $VXWD > VXMD$  and the difference is due partly to a "normal" export tax and partly the tax equivalent of a quota, a split of this difference is necessary. In this case  $VXQD$  must be equal to the value inclusive of the "normal" export taxes but exclusive the tax equivalent of the quota (in version 3 of the GTAP database this split is easy as the information is available in separate headers).

Continuing the general case, it is also convenient to have a notation for two associated ratios, namely

$$QXS\_RATIO\_L(i,r,s) = QXS\_L(i,r,s)/QXS\_QUOTA\_L(i,r,s)$$

and

$$TQXS\_L(i,r,s) = VXQD(i,r,s)/VXWD(i,r,s).$$

$QXS\_RATIO\_L$  effectively says how many times larger than the present export volumes ( $QXS\_L$ ) the export quota is. The initial value of this variable must be calibrated and read in from a separate file. If, for example, you put  $QXS\_RATIO\_L$  equal to 0.1 it is equivalent to saying that the quota is 10 times the existing export volume.

In this MFA application the initial values of  $QXS\_RATIO\_L$  were generated with the statement:

$$QXS\_RATIO\_L(i,r,s) = 1 \text{ if } VXQD(i,r,s) < VXWD(i,r,s) \quad (\text{Binding quota})$$

$$QXS\_RATIO\_L(i,r,s) = 0.1 \text{ if } VXQD(i,r,s) \geq VXWD(i,r,s) \quad (\text{Non-binding quota})$$

The 0.1 value here is arbitrary, and should be set so as to assure that the quota does not become binding in those cases where no quota is supposed to exist. In simulations with very large changes in trade flows the value 0.1 could be too large and values of 0.001 or smaller should be chosen instead.

In general,  $TQXS\_L$  is one plus the tax equivalent of the quota. Note that, since  $TQXS\_L$  is the ratio of values with a common volume  $QXS\_L(i,r,s)$ , we can also write

$$TQXS\_L(i,r,s) = PXS\_L(i,r,s)/PFOB(i,r,s).$$

Clearly neither  $QXS\_RATIO\_L(i,r,s)$  nor  $TQXS\_L(i,r,s)$  can exceed 1. The cases of binding and non-binding quotas can be distinguished from a knowledge of the values of these ratios.

(a) If the quota is binding then

**QXS\_RATIO\_L=1** and **TQXS\_L <= 1** (usually  $TQXS_L < 1$ ).

This is because  $QXS_L = QXS_{QUOTA\_L}$  and  $PXS_L \leq PFOB_L$  (and usually  $PXS_L < PFOB_L$ ).

(b) If the quota is not binding then

**QXS\_RATIO < 1** and **TQXS\_L = 1**.

This is because  $QXS_L < QXS_{QUOTA\_L}$  and  $PXS_L = PFOB_L$ .

This introduces the important equation

$$\text{MAX} [ QXS_{RATIO}(i,r,s), TQXS_L(i,r,s) ] = 1 \quad \text{for all } (i,r,s).$$

This equation says that, for each  $(i,r,s)$ , the maximum of these two ratios must equal 1. Since each ratio is at most one, one of the two ratios must equal 1 for each  $(i,r,s)$ . (Usually the other is less than 1.)

To summarize, the main new levels variables are,

**QXS\_QUOTA\_L, PXS\_L, QXS\_RATIO\_L, TQXS\_L, VXQD.**

Of these, usually  $QXS_{QUOTA\_L}$  is exogenous and the rest are endogenous. The new levels equations are

$$QXS_{RATIO\_L}(i,r,s) = QXS_L(i,r,s)/QXS_{QUOTA\_L}(i,r,s) \quad (L1)$$

$$TQXS_L(i,r,s) = PXS_L(i,r,s)/PFOB(i,r,s) \quad (L2)$$

$$VXQD(i,r,s) = PXS_L(i,r,s) * QXS_L(i,r,s) \quad (L3)$$

$$\text{MAX} [ QXS_{RATIO\_L}(i,r,s), TQXS_L(i,r,s) ] = 1 \quad (L4)$$

As you can see, there are 5 lots of new levels variables and 4 lots of new levels equations. This is consistent with just one lot of these (usually the  $QXS_{QUOTA\_L}$  values) being set exogenously. The model should then determine the rest of these.

It is equation L4 above which causes the problems for GEMPACK whose solution algorithms are not well suited to handling such non-smooth levels equations. We refer to this equation by the name **Exp\_Quo\_Ratios**, which is the name given to the associated linearized version in the TABLO Input file GTAP33XQ.TAB as set out in Appendix 2.

In addition to the above new equations the equation EXPRICES connecting the fob price of exports  $pfo(i,r,s)$  to the domestic price  $pm(i,r)$  and "normal" export taxes  $tx(i,r)$  and  $txs(i,r,s)$

must now be rewritten. It now connects, in the levels,  $PXS\_L(i,r,s)$  to these rather than  $PFOB\_L(i,r,s)$ . Thus the levels equation previously in the model, namely

$$PFOB\_L(i,r,s) = PM\_L(i,r) * TX\_L(i,r) * TXS\_L(i,r,s)$$

must be replaced by

$$PXS\_L(i,r,s) = PM\_L(i,r) * TX\_L(i,r) * TXS\_L(i,r,s).$$

## 2.1 Results of this MFA Application

The MFA policy scenario involved projections from 1992 to 2005 with shocks to factor endowments, total factor productivity and the quota level. The question is how these projections affect the status of the quotas. It turned out that on only one trade flow (textiles from the Rest of the World to the US) the quota changed status from binding to non-binding as a result of the simulation from 1992 to 2005 (see Table 1). In this case the export tax equivalent became zero ( $TQXS\_L = 1$ ) and the quota ratio  $QXS\_RATIO\_L$  shifted from one to less than one.

If we had limited ourselves to the conventional procedure of exogenizing  $QXS\_L$  and endogenizing the export tax level  $TXS\_L$  the latter would have changed from 1.05 to 0.94. This is equivalent to a subsidy on exports of textiles to the US, which is not a likely outcome. In two other cases the export quotas became less binding (Latin America to the US and to the EU), in one case it became neither more nor less binding (ROW to the EU), while all the remaining quotas become more binding from 1992 to 2005. This is so even though the quota levels are increased over the period according to the pre-Uruguay Round agreement [see Hertel et al, 1996].

**Table 1. Export tax equivalents for MFA exporters, 1992 and 2005**

from	to	US		EU	
		1992	2005	1992	2005
South Korea		10	14	10	16
Taiwan		8	25	12	28
Hong Kong		7	17	8	22
China		19	36	27	44
Indonesia		13	18	17	26
Malaysia		10	16	12	22
Philippines		9	12	10	24
Thailand		9	16	13	25
Latin America		10	5	13	12

South Asia	19	24	27	36
Rest of the World	5	0	6	6

Note: Calculated as  $100 * [1/TQXS\_L(i,r,s) - 1] = 100 * [VXWD(i,r,s)/VXQD(i,r,s) - 1]$

Thus, in the actual application the explicit formulation did not have a large impact on the final result. We could have managed without explicit quotas (and simply shock the export tax for the specific flow to zero). However, if several trade flows change status the quota methodology will increase the transparency and aid in designing the simulations. In section 5 we present a case where the explicit quotas are more vitally needed and increase our understanding of the policy choices.

The TABLO file used to calibrate the initial data (including values for VXQD and QXS\_RATIO\_L) is XQ-DATA.TAB, which can be found amongst the associated files (see Appendix 1). The additional statements added to the standard GTAP TABLO file for the equations modelling export quotas in this example are shown in the associated file MFA-XQ.TAB.

### ***3. Implementing Bilateral Export Quotas in GTAP***

In this section we describe in more detail how bilateral export quotas can be modelled in GTAP.

Several export quota examples which you can carry out on your computer while reading this document are described in section 4 below. [See Appendix 1 below for information about installing the associated files on your PC.]

#### ***3.1 Implementation as Additions/Modifications to GTAP.TAB***

The new levels equations for export quotas must be linearized when added to the GTAP TABLO Input file. The levels variables and their associated linear variables are as follows.

<b>Levels variable</b>	<b>Associated linear variable</b>
QXS_RATIO_L(i,r,s)	qxs_ratio(i,r,s)
TQXS_L(i,r,s)	tqxs(i,r,s)
QXS_QUOTA_L(i,r,s)q	qxs_quota(i,r,s)
PXS_L(i,r,s)	pxs(i,r,s)

VXQD is present just a COEFFICIENT (which is really a levels variable).

The new linearized equations corresponding to the new levels equations L1 and L2 in section 2 above are easily written down. They are

$$\begin{aligned} \text{qxs\_ratio}(i,r,s) &= \text{qxs}(i,r,s) - \text{qxs\_quota}(i,r,s) \\ \text{tqxs}(i,r,s) &= \text{pxs}(i,r,s) - \text{pfob}(i,r,s) \end{aligned}$$

The linearized equation corresponding to L3 is the UPDATE statement

$$\begin{aligned} \text{UPDATE (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)} \\ \text{VXQD}(i,r,s) = \text{pxs}(i,r,s) * \text{qxs}(i,r,s) ; \end{aligned}$$

The linearized equation corresponding to L4 is somewhat trickier. This levels equation L4 is not a smooth equation of the kind normally handled by GEMPACK. Its linearization involves the rather special variable called

`del_Newton`

We give a discussion of the linearization of this equation and the significance of `del_Newton` in Appendix 2. For the present we merely note that `del_Newton` is a CHANGE variable which is normally exogenous and given a shock of 1. The linearized version of equation L4 is called `Exp_Quo_Ratios`.

The linearized version `Exp_Quo_Ratios` of L4 has another term in it, namely the variable

`xq_slack(i,r,s)`

This is a slack variable so that we can turn off certain of these equations in certain circumstances. Normally the variables `xq_slack(i,r,s)` are exogenous for all  $(i,r,s)$ . To turn off this equation for a selected  $(i,r,s)$ , we set `xq_slack(i,r,s)` endogenous. Setting selected `xq_slack(i,r,s)` endogenous is the key to getting accurate results from quota simulations, as we explain in sections 3.6 and 8 below.

### 3.2 Quota Rents

The quota rent associated with commodity  $i$  from region  $r$  to region  $s$  is

$$\text{QXS\_L}(i,r,s) * [\text{PFOB\_L}(i,r,s) - \text{PXS\_L}(i,r,s)].$$

It is the value associated with a volume  $\text{QXS\_L}(i,r,s)$  multiplied by the difference between the two relevant prices. In the implementation of bilateral export quotas we also report, for each  $(i,r,s)$ , this quota rent as a ratio of the total value  $\text{VXQD}(i,r,s) = \text{QXS\_L}(i,r,s) * \text{PXS\_L}(i,r,s)$  of the associated exports. We refer to this ratio as the **quota rent ratio**. Simulation results report the change `c_qxs_rent_rat(i,r,s)` in this ratio for each  $(i,r,s)$ .

For example, if PFOB\_L is 1.2 and PXS\_L is 1 then the associated quota rent is QXS\_L(1.2 - 1.0) and so the associated quota rent ratio is 0.2. If the pre-simulation value of this quota rent ratio was zero (meaning that the quota was not binding), then the simulation result for variable c\_qxs\_rent\_rat would be 0.2 (a 0.2 increase from 0.0 to 0.2).

### 3.3 TABLO Statements for Export Quotas

The equations above for bilateral export quotas need to be added to the standard TABLO Input file for GTAP. Appendix 2 shows the TABLO statements for implementing such export quotas. Those shown are the ones used in the examples described in section 4 below. [For the MFA example described in section 2 above, slightly different TABLO statements were added because there quotas were only implemented for a subset of the commodities and regions - see the note near the start of Appendix 2 about this.]

### 3.4 Assigning Initial QXS\_RATIO\_L and VXQD Values

For each (i,r,s), we need to set up suitable QXS\_RATIO\_L(i,r,s) and VXQD(i,r,s) values. Different (i,r,s) triples will probably need different treatments.

If the quota is thought to be not binding,

- VXQD(i,r,s) must be set equal to VXWD(i,r,s);
- the QXS\_RATIO\_L(i,r,s) value put on the GTAPQUOTA file must not be larger than one. The QXS\_RATIO\_L(i,r,s) tells what fraction of the quota you believe the current export volume represents so the value assigned will reflect your best information as to how far the quota is from being binding. If it is nearly binding, you will want a value just less than one. If it is a long way from binding you will want a smaller value.

If the quota is thought to be binding,

- QXS\_RATIO\_L(i,r,s) must be set equal to one;
- the VXQD(i,r,s) value must be no larger than VXWD(i,r,s). The more binding the quota is thought to be, the greater will be the difference between VXWD and VXQD. If you think that the quota is just binding you could set VXQD=VXWD. If you think that the quota is very restrictive you would set VXQD significantly less than VXWD. In some cases you might look at the VXMD=PM\*QXS values for guidance; if you have VXMD < VXWD and you think that there are no normal export taxes but that all the difference between VXMD and VXWD is due to the export quota, you could set VXQD equal to VXMD [This is essentially what was done in the MFA example described in section 2 above.]

### 3.5 Closures for the Model

The normal (or standard) closure for the GTAP model with export quotas has the usual GTAP variables exogenous. In addition, all components of the linear variables

`qxs_quota` and `xq_slack` are exogenous

and the variable

`del_Newton` is exogenous.

All other new linear variables (namely `qxs`, `pxs`, `qxs_ratio` and `tqxs`) are endogenous in this closure.

To obtain accurate solutions of a simulation with a version of GTAP which includes export quotas involves solving the model twice. The first calculation is done to see which quotas change from binding to non-binding or vice versa. In the second calculation we modify the closure to set `qxs_ratio(i,r,s)` exogenous and `xq_slack(i,r,s)` endogenous for those  $(i,r,s)$  for which the quota has changed from non-binding to binding, and to set `tqxs(i,r,s)` exogenous and `xq_slack(i,r,s)` endogenous for those  $(i,r,s)$  for which the quota has changed from binding to non-binding. [See section 3.6 below for more detail.]

### 3.6 Accurate Export Quota Solutions

To obtain accurate solutions of a model implemented and solved using GEMPACK usually involves extrapolating after 3 different multi-step calculations (see, for example, section 4 of Harrison and Pearson (1994)). The theory underpinning extrapolation relies on the underlying levels equations being smooth (Pearson,1991). However the `Exp_Quo_Ratios` equation (equation L4 in section 2 above) is not smooth and so we need a slightly different strategy with a model which has explicit export (or import) quotas. The strategy is as follows.

- (a) First do an **approximate simulation** using a single Euler multi-step calculation (for example, a 10-step Euler calculation). Here the closure and shocks are those for the application you are carrying out, and a shock of one should be given to the variable `del_Newton`. The sole purpose of this is to find which quotas change their binding/non-binding status (that is, which change from binding to non-binding and which change from non-binding to binding). The number of Euler steps must be sufficient to get this correct. [The examples in section 4 below say more about this.] Always use Euler's method for this simulation, never Gragg's or the midpoint method.
- (b) Then do an **accurate simulation** (extrapolating from 3 multi-step calculations, using Gragg or any other method as appropriate). In this calculation, a closure swap is used to set exogenous `qxs_ratio(i,r,s)` for all  $(i,r,s)$  for which the quota changes in (a) from non-

binding to binding and to set exogenous  $tqxs(i,r,s)$  for all  $(i,r,s)$  for which the quota changes in (a) from binding to non-binding (in each case, setting the corresponding slack variable  $xq\_slack(i,r,s)$  in the `Exp_Quo_Ratios` equations endogenous). No shock need be given to the variable `del_Newton` in this accurate simulation.

The details of this will become much clearer when you carry out some of the examples in section 4 below. We suggest that you at least skim the rest of this section before doing these examples. We also suggest that you re-read this section carefully after carrying out the examples: the details here will probably seem much more straightforward after you have done a few of the examples.

Exact shocks for the components of `qxs_ratio` and `tqxs` set exogenous in the accurate calculation in (b) can be calculated from a knowledge of the pre-simulation `QXS_RATIO_L(i,r,s)` and `TQXS_L(i,r,s)` levels values. Indeed,

- if the quota on exports of commodity  $i$  from region  $r$  to region  $s$  changes from non-binding to binding, you know that the post-simulation value of `QXS_RATIO_L(i,r,s)` must be one (this is exactly what binding means). The pre-simulation value of `QXS_RATIO_L(i,r,s)` is on the `GTAPQUOTA` data file. Hence it is easy to work out the percentage change in `QXS_RATIO_L(i,r,s)`: this is the shock to give `qxs_ratio(i,r,s)` in the accurate simulation.
- if the quota on exports of commodity  $i$  from region  $r$  to region  $s$  changes from binding to non-binding, you know that the post-simulation value of the price ratio `TQXS_L(i,r,s)` must be 1 (since the post-simulation value of `QXS_RATIO_L(i,r,s)` is less than one and equation `Exp_Quo_Ratios` must hold). The pre-simulation value of `TQXS_L(i,r,s)` can be obtained from the `Display` file produced by calculation (a) and so it is easy to work out the percentage change in `TQXS_L(i,r,s)`: this is the shock to give to `tqxs(i,r,s)` in the accurate simulation.

A TABLO Input file `XQ-BIND.TAB` supplied with the associated files calculates the `qxs_ratio` and `tqxs` shocks required for the accurate simulation.

The closure used in the accurate simulation is that from (a) with the swaps indicated above. We discuss automating the setting up of this closure in section 8.3 below.

### 3.6.1 Checking the Accurate Simulation

It is important to check that the accurate simulation (the one in (b) above) has worked correctly. As well as checking the convergence, you should check that the same quotas are binding in the post-simulation data from the accurate simulation as were binding after the approximate simulation. You must also check that the post-simulation values of the quantity and price ratios `QXS_RATIO_L` and `TQXS_L` are as expected [that is, not larger than one

and having maximum one for each relevant (i,r,s)]. The file TABLO Input file XQCHK.TAB (see the examples in section 4 below) can be used to check this. This checking of QXS\_RATIO\_L and TQXS\_L values is necessary to ensure that the initial (approximate) simulation was sufficiently accurate to correctly identify the quotas which change their binding/non-binding nature.

If this check fails, it indicates that the approximate simulation in (a) did not have enough Euler steps. Thus the remedy is to redo the approximate calculation with more Euler steps and then redo the accurate simulation and check it again.

## ***4. Export Quotas in a 3x3 Version of GTAP***

This section introduces the export quota examples associated with this document. Detailed instructions for carrying out these are also given in the file **XQ.TXT**.

The relevant TABLO Input file is GTAP33XQ.TAB which can be found in the associated examples in the file QUOTA-EX.ZIP (see Appendix 1 below to see how to install these examples on your PC). This file is based on the standard TABLO Input file GTAP94.TAB for GTAP. The maximum set sizes have been reduced so that at most 3 tradeable commodities and 3 regions are allowed. An additional section containing the TABLO implementation of the export quota equations is added at the end. The original EXPRICES equation is modified as indicated in section 3 above. The export quota part of this file GTAP33XQ.TAB is shown in full in Appendix 2.

As indicated above in sections 2 and 3.4, supplementary data are required for these quotas. We have added the VXQD data, which are set equal to the usual VXWD data, at header "VXQD". The resulting GTAPDATA file is the file **DAT201XQ.HAR** which is identical to the standard 3-tradeable-commodity, 3-region file DAT2-01.HAR with the VXQD data added. By setting VXQD=VXWD we are assuming that none of the export quotas are binding in the initial data.

The other supplementary data needed are the **QXS\_RATIO\_L(i,r,s)** values. These are held on the text file **XQ2-01.DAT** which is the actual file corresponding to the logical file called GTAPQUOTA in GTAP33XQ.TAB. Since no quotas are binding, all QXS\_RATIO\_L values on this file are less than 1. We have set these ratios

- equal to 0.98 for exports of manufactures "mnfcs" from the Rest of the World "ROW" to both the United States "USA" and the European Union "EU". This value means that these two quotas are nearly binding.
- equal to 0.2 in all other cases, which means that these other quotas are far from binding.

The first examples **A1-A7** are ones in which the only shocks to the model are to change the level of various quotas. Detailed instructions for carrying out these examples are given in the file **XQ.TXT**. The second examples **P1** and **P2** are more realistic in that there the export quotas are not shocked - rather the example looks at the effect of the existing quotas on a projection simulation (a stylized version of that described in section 2 above) aimed at taking the economy from 1992 to 2005.

We encourage you to install the relevant files on your PC (following the instructions in Appendix 1 below) and then to carry out these examples before reading further in this paper. This should give you a clear understanding of what is involved in solving GTAP with explicit export quotas. [If you don't have a version of GEMPACK on your PC, you can obtain the Demonstration Version of GEMPACK at no cost from the World-Wide Web - see Appendix 1 for details - and use this to carry out the export quota examples.]

## ***5. Bilateral Import Quotas***

The need for bilateral import quotas grew out from research on China and grain imports. Chinese agricultural policies are at an important turning point these years as production of agricultural commodities fails to keep pace with consumption. The pressure for agricultural protection and support is increasing. The second Chinese offer to the World Trade Organization (WTO) within the ongoing negotiations for Chinese membership displays this concern. The offer contains tariff quotas on wheat and coarse grains with above-quota tariff levels being so high that they will effectively block imports [WTO, 1994].

For wheat the proposed tariff quota will fix imports at 9.76 million tons and for barley at 0.785 million tons between 1995 and 2004. Compared with the base period quantity of imports it is evident that this will essentially fix import quantities at the 1992 level. In the quota scenario it is therefore assumed that the quota equals the 1992 import level and remains unchanged to the year 2005. We extend this assumption to rice as well (for rice the base year import quantities are very low but if high import quantities should appear, a logical consequence would be an import quota on rice as well).

Once again these quotas could have been modelled by simply fixing the quantity flows (QXS\_L) and endogenizing the tariff rate (TMS\_L). In this case, however, the approach is inadequate, as we simultaneously need to cut the tariff on coarse grains. Moreover, we are still stuck with the problem, that if we simply use the bilateral trade flow and endogenize the tariff, the quota could change status from binding to non-binding with potentially misleading results.

Finally, it is attractive from a policy perspective to trace the specific tariff equivalents of the quotas.

Thus, explicit modelling of quotas is a logical outcome. Explicit import quotas are modelled, quite equivalently to the export quotas, by introducing two new levels variables in the model:

QIS_QUOTA_L	The volume of the quota
PIS_L	Price of imports inclusive of tariffs but exclusive of the tariff equivalent of the quota

and adding a header in the base data with the value of imports (VIQS) valued at this price PIS\_L.

### 5.1 Import Quota Variables and Equations

The status of the quota can then be determined from the values of the two ratios

$$QIS\_RATIO\_L(i,r,s) = QXS\_L(i,r,s) / QIS\_QUOTA\_L(i,r,s)$$

$$TQIS\_L(i,r,s) = PIS\_L(i,r,s) / PMS\_L(i,r,s)$$

(similarly to export quotas). Here QIS\_RATIO\_L is the ratio between actual imports and the quota level and TQIS\_L is one plus the tariff equivalent of the quota. It will then be the case that

if  $QIS\_RATIO\_L < 1$  and  $TQIS\_L = 1$ , the quota is not binding

if  $QIS\_RATIO\_L = 1$  and  $TQIS\_L \leq 1$  (usually  $TQIS\_L < 1$ ), the quota is binding

To summarize, the main new levels variables are

**QIS\_QUOTA\_L, PIS\_L, QIS\_RATIO\_L, TQIS\_L, VIQS.**

Of these, usually QIS\_QUOTA\_L is exogenous and the rest are endogenous. The new levels equations are

$$QIS\_RATIO\_L(i,r,s) = QXS\_L(i,r,s) / QIS\_QUOTA\_L(i,r,s) \quad (L5)$$

$$TQIS\_L(i,r,s) = PIS\_L(i,r,s) / PMS\_L(i,r,s) \quad (L6)$$

$$VIQS(i,r,s) = PIS\_L(i,r,s) * QXS\_L(i,r,s) \quad (L7)$$

$$\text{MAX} [ QIS\_RATIO\_L(i,r,s), TQIS\_L(i,r,s) ] = 1 \quad (L8)$$

Once again, there are 5 lots of new levels variables and 4 lots of new levels equations. This is consistent with just one lot of these (usually the QIS\_QUOTA\_L values) being set exogenously. The model should then determine the rest of these.

In addition to the above new equations, the equation MKTPRICES connecting the cif price of imports pcif(i,r,s) to the domestic price pms(i,r,s) and "normal" import tariffs tm(i,r) and tms(i,r,s) must now be rewritten. It now connects, in the levels, PIS\_L(i,r,s) to these rather than PMS\_L(i,r,s). Thus the levels equation previously in the model, namely

$$PMS\_L(i,r,s) = PCIF\_L(i,r,s) * TM\_L(i,r) * TMS\_L(i,r,s)$$

must be replaced by

$$PIS\_L(i,r,s) = PCIF\_L(i,r,s) * TM\_L(i,r) * TMS\_L(i,r,s)$$

## 5.2 Calibrating VIQS and QIS\_RATIO\_L Values for this Application

In this application we have a situation where the base data does not contain any quotas or quota tax equivalents. The policy scenario is to implement quotas from a quota free (but low import) situation in 1992. Thus, all quotas are equal to the exact imports but no tariff equivalents have yet developed. The new header with the quota information is simply added with the statement

$$VIQS(i,r,s) = VIMS(i,r,s)$$

Essentially, we wish to calibrate the model so that all quotas are just binding in the base data (QIS\_RATIO\_L = 1). However, it turns out that if we do this in those cases where the initial trade volume is zero (and remains zero) the matrix becomes singular during a simulation. In those cases we must assume that the quota is not-binding and we end up having a mix of binding and non-binding quotas given by:

$$QIS\_RATIO\_L(i,r,s) = 1 \quad \text{if } VIMS(i,r,s) \text{ is larger than zero,}$$

$$QIS\_RATIO\_L(i,r,s) = 0.9 \quad \text{if } VIMS(i,r,s) \text{ is zero.}$$

Once again the value 0.9 is arbitrarily chosen. But in this case it makes no difference as the model code assures that trade flows that are initially zero remains zero.

## 5.3 Results of this Application

Once again the simulations involve projections from the year 1992 to 2005. Three different scenarios are presented. The first is a baseline with shocks to factor endowments, total factor productivity and implementation of the trade reforms agreed upon in the GATT Uruguay Round. The second scenario is the baseline plus liberalization in China according to the second

schedule submitted to the WTO, but without the quotas. The third scenario adds the import quotas on grain.

Fortunately the quotas do not change status as a result of the simulation. In terms of understanding the impact of the quotas the formulations are very useful. The tariff equivalents of the import quotas are increasing to quite significant levels as shown in Table 3 [Bach, 1996]. For instance, the tariff equivalent of the quota on rice from the USA and Canada reaches 78%. On those flows where the trade flow is zero there is no tax equivalent (e.g., wheat from Japan).

**Table 3. Import tax equivalents for grain imports to China, 2005.**

	Import tax equivalents		
	Rice	Wheat	Other grains
USA and Canada	78	86	82
EU	69	49	45
Japan	95	00	31
Australia and NZ	67	81	77
NICs	36	00	64
Other ASEAN	60	54	85
South Asia	68	00	00
Sub-Saharan Africa	00	00	00
Latin America	00	97	93
Rest of the World	79	92	93

Calculated as  $100 * [1/TQIS\_L(i,r,s) - 1] = 100 * [VIMS(i,r,s)/VIQS(i,r,s) - 1]$

The results for output, imports and welfare are shown in Table 4. The quotas leads to higher production of grains in China and fix imports at 9 million tons. Without the quotas net-import of grains to China increases to 68 million tons in the baseline and 60 million tons in the scenario with liberalization in China.

The welfare effect of quotas is relatively low as the lower imports leads to less negative terms of trade effects on the export side (the trade balance is almost fixed as we use the GTAP closure where investments are allocated as a fixed share of global investments). The total welfare gain from liberalization in China is US\$ 54 billion without quotas and US\$ 50 billion with.

**Table 4. Output, import and welfare effects in China, 1992-2005.**

	Grain production mn tons	Net-import of grains mn tons	Welfare effect bn US\$
BASELINE	535	68	-
China lib. schedule	514	60	54
China lib.+ quotas	578	9	50

In this case the quota formulation gives us additional insights and allows us to introduce quotas while simultaneously changing the import tariffs.

The TABLO file used to calibrate the initial data for this application (including values for the arrays VIQS and QIS\_RATIO\_L) is IQ-DATA.TAB, which can be found amongst the associated files (see Appendix 1). The additional statements added to the standard GTAP TABLO file for the equations modelling import quotas in this example are shown in the associated file CHINA-IQ.TAB.

## 6. *Bilateral Import Quotas in GTAP*

In this section we describe how import quotas can be modelled in GTAP. As this is quite similar to section 3 for export quotas, the discussion is more brief.

### 6.1 *Implementation as Additions/Modifications to GTAP.TAB*

The new levels equations must be linearized when added to the GTAP TABLO Input file. The levels variables and their associated linear variables are as follows.

<b>Levels variable</b>	<b>Associated linear variable</b>
QIS_RATIO_L(i,r,s)	qis_ratio(i,r,s)
TQIS_L(i,r,s)	tqis(i,r,s)
QIS_QUOTA_L(i,r,s)q	qis_quota(i,r,s)
PIS_L(i,r,s)	pis(i,r,s)

VIQS is present just a COEFFICIENT (which is really a levels variable).

The new linearized equations corresponding to the new levels equations L5 and L6 are easily written down. They are

$$\begin{aligned} qis\_ratio(i,r,s) &= qxs(i,r,s) - qis\_quota(i,r,s) \\ tqis(i,r,s) &= pis(i,r,s) - pms(i,r,s) \end{aligned}$$

The linearized equation corresponding to L7 is the UPDATE statement

$$\begin{aligned} \text{UPDATE (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)} \\ \text{VIQS(i,r,s) = pis(i,r,s) * qxs(i,r,s) ;} \end{aligned}$$

Once again it is the MAX equation corresponding to L8 which causes problems. The linearization of this Imp\_Quo\_Ratios equation is quite similar to the export quota case described in Appendix 3, so we will only give the result:

$$\begin{aligned} \text{IF[ QIS\_RATIO\_L(i,r,s) >= TQIS\_L(i,r,s),} \\ \text{ QIS\_RATIO\_L(i,r,s)/100*qis\_ratio(i,r,s) ] +} \\ \text{IF[ QIS\_RATIO\_L(i,r,s) < TQIS\_L(i,r,s),} \\ \text{ TQIS\_L(i,r,s)/100*tqis(i,r,s) ] +} \\ [\text{MAX}\{QIS\_RATIO\_L(i,r,s), TQIS\_L(i,r,s)\} - 1] * \text{del\_Newton} + \\ \text{iq\_slack(i,r,s) = 0 ;} \end{aligned}$$

As in the export quota case, the equation contains the variable del\_Newton to make sure that the two ratios QIS\_RATIO\_L and TQIS\_L do not get larger than 1, and the slack variable iq\_slack which allows us to turn off the equation for those flows where we do not need it.

## 6.2 Quota rents

The quota rents associated with the import quotas on commodity i from region r to region s are in this case the difference between the value of imports outside the border and the value inside the country after the quota constraint but before the “normal” import tariffs. Specifically, we are interested in the quota rent as a ratio of the total value of imports inclusive the “normal” import tariffs:

$$\begin{aligned} \text{QIS\_RENT\_RATIO\_L} &= [\text{QXS\_L} * (\text{PMS\_L} - \text{PIS\_L})] / [\text{QXS\_L} * \text{PIS\_L}] \\ &= [\text{PMS\_L} / \text{PIS\_L}] - 1 \\ &= [1 / \text{TQIS}] - 1 \end{aligned}$$

Linearized this equation becomes:

$$\begin{aligned} \text{c\_qis\_rent\_rat(i,r,s)} &= -\{1 / [100 * \text{TQIS\_L}(i,r,s)]\} * \\ \text{tqis(i,r,s)} \end{aligned}$$

where  $c_{qis\_rent\_rat}$  is the change in the quota rate ratio (a change variable).

### ***6.3 TABLO Statements for Import Quotas***

The entire block of declarations, coefficients and equations for modelling import quotas can be found in the file GTAP33IQ.TAB. Again these quotas are written to make it possible to introduce quotas on all bilateral flows in the model. In the application described in section 5, import quotas were only introduced on grain imports to China with the use of appropriate SET definitions. This reduces the amount of variables and the size of the model, which can be convenient or even necessary in some cases.

### ***6.4 Assigning Initial QIS\_RATIO\_L and VIQS Values***

For each  $(i,r,s)$ , we need to set up suitable  $QIS\_RATIO\_L(i,r,s)$  and  $VIQS(i,r,s)$  values. Different  $(i,r,s)$  triples will probably need different treatments.

If the import quota is thought to be not binding,

- $VIQS(i,r,s)$  must be set equal to  $VIMS(i,r,s)$ ;
- the  $QIS\_RATIO\_L(i,r,s)$  value put on the GTAPQUOTA file must not be larger than one. The  $QIS\_RATIO\_L(i,r,s)$  tells what fraction of the quota you believe the current import volume represents so the value assigned will reflect your best information as to how far the quota is from being binding. If it is nearly binding, you will want a value just less than one. If it is a long way from binding you will want a smaller value.

If the quota is thought to be binding,

- $QIS\_RATIO\_L(i,r,s)$  must be set equal to one;
- the  $VIQS(i,r,s)$  value must be no larger than  $VIMS(i,r,s)$ . The more binding the quota is thought to be, the greater will be the difference between  $VIMS$  and  $VIQS$ . If you think that the quota is just binding you could set  $VIQS=VIMS$ . If you think that the quota is very restrictive you would set  $VIQS$  significantly less than  $VIMS$ . In some cases you might look at the  $VIWS=PCIF*QXS$  values for guidance; if you have  $VIWS < VIMS$  and you think that there are no normal import tariffs but that all the difference between  $VIWS$  and  $VIMS$  is due to the import quota, you could set  $VIQS$  equal to  $VIMS$ .

### ***6.5 Closures for the Model***

Beyond the normal (or standard) closure of the GTAP model the following variables must be added to the list of exogenous variables in the Command file:

qis\_quota

iq\_slack

del\_Newton

All other new linear variables (qis, pis, pis\_ratio and tqis) are endogenous. If you wish to run a simulation without any quota constraints just endogenize iq\_slack and exogenize tqis. This is a convenient way to test that the model still behaves exactly as the standard GTAP model without quotas.

When solving the model accurately after the first approximate run, qis\_ratio(i,r,s) must be swapped with iq\_slack(i,r,s) for those bilateral flows where the quota changes status from non-binding to binding and tqis(i,r,s) must be swapped with iq\_slack(i,r,s) for those flows where the quota changes status from binding to non-binding. [See section 6.6 below for more details.]

## ***6.6 Accurate Import Quota Solutions***

As with export quotas, this involves first doing an approximate (Euler) simulation to find which quotas change their binding/non-binding status, and then doing an accurate simulation using this knowledge. In the accurate simulation, a closure swap is used to set exogenous qis\_ratio(i,r,s) for all (i,r,s) for which the quota changes from non-binding to binding and to set exogenous tqis(i,r,s) for all (i,r,s) for which the quota changes from binding to non-binding (in each case, setting the corresponding slack variable iq\_slack(i,r,s) in the Imp\_Quo\_Ratios equations endogenous). A shock of one is given to variable del\_Newton in the approximate simulation but no shock needs to be given to this variable in the accurate simulation.

Exact shocks for the components of qis\_ratio and tqis set exogenous in the accurate simulation can be calculated from a knowledge of the pre-simulation QIS\_RATIO\_L(i,r,s) and TQIS\_L(i,r,s) levels values. Indeed,

- if the quota on imports of commodity  $i$  from region  $r$  to region  $s$  changes from non-binding to binding, you know that the post-simulation value of QIS\_RATIO\_L(i,r,s) must be one (this is exactly what binding means). The pre-simulation value of QIS\_RATIO\_L(i,r,s) is on the GTAPQUOTA data file. Hence it is easy to work out the percentage change in QIS\_RATIO\_L(i,r,s): this is the shock to give qis\_ratio(i,r,s) in the accurate simulation.
- if the quota on imports of commodity  $i$  from region  $r$  to region  $s$  changes from binding to non-binding, you know that the post-simulation value of the price ratio TQIS\_L(i,r,s) must be 1 (since the post-simulation value of QIS\_RATIO\_L(i,r,s) is less than one and equation Imp\_Quo\_Ratios must hold). The pre-simulation value of TQIS\_L(i,r,s) can be

obtained from the Display file produced during the approximate simulation and so it is easy to work out the percentage change in  $TQIS\_L(i,r,s)$ : this is the shock to give to  $tqis(i,r,s)$  in the accurate simulation.

A TABLO Input file IQ-BIND.TAB supplied with the associated files calculates the  $qis\_ratio$  and  $tqis$  shocks required for the accurate simulation.

The closure used in the accurate simulation is that from the approximate simulation with the swaps indicated above. We discuss automating the setting up of this closure in section 8.3 below.

#### *6.6.1 Checking the Accurate Simulation*

As with export quotas, it is important to check that the accurate simulation has worked correctly. As well as checking the convergence, you should check that the same quotas are binding in the post-simulation data from the accurate simulation as were binding after the approximate simulation. You must also check that the post-simulation values of the quantity and price ratios  $QIS\_RATIO\_L$  and  $TQIS\_L$  are as expected [that is, not larger than one and having maximum one for each relevant  $(i,r,s)$ ]. The file TABLO Input file IQCHK.TAB (see the examples in section 7 below) can be used to check this. This checking of  $QIS\_RATIO\_L$  and  $TQIS\_L$  values is necessary to ensure that the initial (approximate) simulation was sufficiently accurate to correctly identify the quotas which change their binding/non-binding nature.

If this check fails, it indicates that the approximate simulation did not have enough Euler steps. Thus the remedy is to redo the approximate calculation with more Euler steps and then redo the accurate simulation and check it again.

## **7. Import Quotas in a 3x3 Version of GTAP**

We now turn to the associated import quota examples, which involve a 3x3 version of the GTAP model and database. Detailed instructions are contained in the file **IQ.TXT**.

The relevant TABLO Input file is GTAP33IQ.TAB, which can be found in the associated examples in the file QUOTA-EX.ZIP (see Appendix 1). The quota section is found at the very end of the file. When starting with a standard GTAP database we first need to,

- 1) add the header VIQS.
- 2) create a file with the values of  $QIS\_RATIO\_L$ .

As in section 5 we assume that initially all quotas are just binding but that no quota rents are present. That is  $QIS\_RATIO\_L$  and  $TQIS\_L$  are both equal to one and the new VIQS data

are set equal to the VIMS values. In the standard 3x3 GTAP data, there are no zeros in the VIMS data since very small values are shown even for imports from both USA and EU to themselves.

These two steps are conveniently done with the TABLO file IQ-DATA.TAB (see the file IQ.TXT for details).

Again we encourage you to work through the associated import quota examples on your PC before you read further in this document. There are three examples called B1, B2 and B3. They all involve shocks to import tariffs with import quotas on food in place.

A noteworthy feature of these import quota examples is that the quota constraints are limited to the commodity FOOD: this is done by endogenizing the variable iq\_slack for the other commodities (MNFCS and SVCES) and exogenizing tqis for these commodities. This means that, when checking the post-simulation values of QIS\_RATIO\_L(i,r,s) and TQIS\_L(i,r,s) after a hopefully accurate import quota simulation, only the values for i=FOOD need to be checked.

## ***8. Automating the Second Part of the Two-Part Solution Procedure***

In this section we set out the procedure for the bilateral export quota case. The procedure for bilateral import quotas is very similar - we leave the details to our readers.

The second more accurate simulation in the bilateral export quota case may require a closure change from the first (approximate) simulation and may require shocks to be given to some components of variables qxs\_quota and tqxs. The procedure to use with Release 5.1 (April 1994) of GEMPACK is set out in sections 8.1 and 8.2 below. A simpler procedure that only works with Release 5.2 (September 1996) of GEMPACK is set out in section 8.3 below.

### ***8.1 Automating the Shocks***

Automating calculation of the shocks is straightforward and uses the familiar technique of calculating all possible values via a data-manipulation TABLO Input file and selecting those required via statements of the form

```
shock qxs_ratio("mnfcs","ROW","USA") = select from file <FILE1> ;  
shock tqxs("mnfcs","ROW","EU") = select from file <FILE2> ;
```

on your Command file. [This technique is similar to the GTAP method of calculating shocks to remove distortions via SHOCKS.TAB.]

The TABLO Input file to calculate shocks for bilateral export quotas is called **XQ-BIND.TAB**. [This is one of the files associated with this document.] If you examine it, you will see that it relies on reading both the pre-simulation data (GTAPDATA and GTAPQUOTA files) and also the post-simulation data files produced by the first approximate version of the simulation. It looks at both to first identify which quotas have changed their binding/non-binding status and then writes two text files intended to be used as shocks files for the variables qxs\_ratio and tqxs respectively. In each case, nonzero values are only written for those components for which the binding/non-binding status changes. [The qxs\_ratio shocks file only has a nonzero value for (i,r,s) when the export quota for this changes from non-binding to binding. The tqxs shocks file only has a nonzero value for (i,r,s) when the export quota for this changes from binding to non-binding.]

Carrying out the calculation of these two shocks files is straightforward. All you need is a Command file with the appropriate files associated with each logical file. The file BIND-A1.CMF shows how to do this. In the shocks statements above, FILE1 is the name of the actual file with logical name **QXS\_RAT\_SH** produced from XQ-BIND.TAB and FILE2 is the name of the actual file with logical name **TQXS\_SH** produced from XQ-BIND.TAB.

Using these shocks files in the accurate simulation is also quite straightforward. You merely need to insert one shock statement (same format as the example above) for each relevant component. Specifically, you need one such qxs\_ratio shock statement for each (i,r,s) for which the quota changes from non-binding to binding, and you need one such tqxs shock statement for each (i,r,s) for which the quota changes from binding to non-binding. [See, for example, the Command file XQ-A6.CMF used in Example A6.]

The need to do this separately for the different components is removed with Release 5.2 of GEMPACK, as explained in section 8.3 below.

Note that amongst the associated files is a file **IQ-BIND.TAB** which does for import quotas what XQ-BIND.TAB does for export quotas.

## ***8.2 Automating the Closure Change***

The closure changes required are swap statements of the form

```
swap qxs_ratio("mnfc", "ROW", "USA") = xq_slack ("mnfc", "ROW", "USA");
```

One such statement is needed for each (i,r,s) for which the quota changes from non-binding to binding. And one similar statement swapping tqxs with xq\_slack is required for each component which changes from binding to non-binding.

With Release 5.1 of GEMPACK, you will need to insert these statements by hand. The shocks files produced by running XQ-BIND tell you which components are relevant in each case (since these only contain nonzero values when such a closure swap is indicated).

### ***8.3 Automating Shocks and Closure Swaps with Release 5.2 of GEMPACK***

Release 5.2 of GEMPACK has new statements allowed in Command files which make it very easy to specify the closure swaps and shocks required for the second accurate calculation.

For the closure change, the following statements

```
exogenous qxs_ratio = nonzero value on file <FILE1>;  
endogenous xq_slack = nonzero value on file <FILE1>;  
exogenous tqxs      = nonzero value on file <FILE2>;  
endogenous xq_slack = nonzero value on file <FILE2>;
```

are all that is required in every case. Here FILE1 is the name of the actual file with logical name **QXS\_RAT\_SH** produced from XQ-BIND.TAB and FILE2 is the name of the actual file with logical name **TQXS\_SH** produced from XQ-BIND.TAB (see section 8.1 above). The first of the statements above is an instruction to set exogenous those components of variable qxs\_ratio for which the associated value on FILE1 is nonzero; similarly for the other statements.

The required shocks to qxs\_ratio and tqxs can be specified by the two statements

```
shock qxs_ratio = select from file <FILE1>;  
shock tqxs      = select from file <FILE2>;
```

where FILE1 and FILE2 are as above.<sup>2</sup> [The only time when these statements need to be altered is when no components of qxs\_ratio or tqxs are exogenous; then the shock statement for this variable must be omitted or commented out.] Note that the same files FILE1 and FILE2 serve to specify the closure changes and the extra shocks.

Thus the second accurate simulation is very easy to set up and run under Release 5.2 of GEMPACK. After the first approximate version is run, you need to compute the shocks files by running XQ-BIND (from XQ-BIND.TAB) and to modify the Command file by putting in the statements above (and making the other changes to the Command file as described earlier).

---

<sup>2</sup>The astute reader will notice that these statements should work with Release 5.1 of GEMPACK. However a bug in that version of the software means that they will almost certainly not work. This is why we recommended individual shock statements for each relevant (i,r,s) in section 8.1 above.

You can then run the simulation required to produce accurate results in the presence of the quotas.

The files XQ-A22.CMF and XQ-A62.CMF in the accompanying files are examples using this automation.

#### **8.4 An Alternative Closure for the Accurate Simulation**

In some cases it may be better to use an alternative closure for the second accurate simulation. In this alternative closure,

- `xq_slack` is set endogenous for all  $(i,r,s)$  - this turns off equation `Exp_Quo_Ratios` in all cases,
- `qxs_ratio` is set exogenous for all  $(i,r,s)$  for which you believe the quota will be binding (as indicated by the first approximate simulation),
- `tqxs` is set exogenous for all other  $(i,r,s)$  (that is, for which you believe that the quota will not be binding).

There may be some simulations in which a quota changes status during the simulation (for example, changes from binding to non-binding during step 6 of a 10-step calculation) and then changes back to its original status by the end. In such a case the results are likely to converge better if the associated non-smooth `Exp_Quo_Ratios` plays no role.

### **9. Handling Other Inequalities using GEMPACK**

We mention very briefly some other inequalities which can arise in CGE models and ways in which they can be handled with GEMPACK.

#### *Kuhn-Tucker Conditions*

Inequalities often enter GE models in the form of Kuhn-Tucker conditions attached with an inequality or sign constraint. Consider, for example, the Kuhn-Tucker conditions

$$P - MC \leq 0, \quad X \geq 0, \quad (P - MC)X = 0.$$

These can be rewritten as

$$\text{MIN} ( X, MC - P ) = 0$$

*(which is equivalent to the three Kuhn-Tucker conditions). This levels equation can then be linearized and treated as in the quota cases dealt with above.*

### *Non-negative Investment*

Investment in each sector of a GE model must be non-negative in each period. Michael Malakellis has used Mark Horridge's method (essentially as set out in this document - see also Horridge, 1993) in his intertemporal model ORANI-INT of the Australian economy to ensure that investment does indeed stay non-negative for each sector in each period. See Malakellis (1994) for details.

### *Other Quotas*

Other treatments of quotas are easy to imagine. For example, instead of bilateral import quotas by commodity and source, each region may impose a quota on the total imports of each commodity (from all other regions). This could be modelled explicitly following similar methods to those described in this paper.

## **10. Conclusion**

The purpose of this technical paper has been to demonstrate that bilateral export and import quotas (and other inequalities) can be modelled explicitly in GTAP and GEMPACK. If you need to do so, we hope that this paper and the associated computer files will provide sufficient detail for you to be able to model these in your application.

The examples in sections 2 and 5 above also make the point that when quotas are relevant to your application, modelling them explicitly is not necessarily the most appropriate way to proceed.

Modelling such inequalities explicitly using GEMPACK is relatively new. We expect that modifications to some of the procedures described here will be suggested as they are used more.

## ***References***

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## ***Appendix 1: Installing and Using the Associated Files***

The associated files are in the PKZIP file QUOTA-EX.ZIP. This contains TABLO, data and Command files required for the examples described in this paper. To use them you need an 80386/80486/pentium PC running DOS, Microsoft Windows or Windows 95 which has either a DOS-Lahey source-code version, or the Executable-image version, or the Demonstration Version of Release 5.1 or later of GEMPACK installed. If you don't have any of these, you can obtain the Demonstration Version at no cost from the GEMPACK World-Wide Web site (the address of which is given later in this section).

[If you have a source-code version of GEMPACK for an operating system other than DOS/Windows, you could easily convert the files in QUOTA-EX.ZIP for use with GEMPACK on your machine.]

To install the files we suggest that you make a new directory or subdirectory (perhaps C:\QUOTAS) and copy the file QUOTA-EX.ZIP into this directory. You will also need the file PKUNZIP.EXE in order to unzip the files in QUOTA-EX.ZIP. [The files QUOTA-EX.ZIP and PKUNZIP.EXE can be obtained from the World-Wide Web at the address shown below.] Then unzip the files by first changing into this directory and then issuing the command

**pkunzip quota-ex**

You should then be able to carry out the examples described in sections 4 and 7 above by working in this directory. The files XQ.TXT and IQ.TXT contain detailed instructions for carrying out export quota examples (section 4) and import quota examples (section 7) respectively.

### ***Obtaining QUOTA-EX.ZIP and/or the Demonstration Version of GEMPACK***

These will normally be available from the World-Wide Web on the

- GEMPACK Web site <http://www.monash.edu.au/policy/gempack.htm> and/or
- the GTAP Web site <http://www.agecon.purdue.edu/gtap/>

In order to use the Demonstration Version of GEMPACK for the quota examples, you will need two more .ZIP files besides QUOTA-EX.ZIP, namely XQ-TG.ZIP and IQ-TG.ZIP. These are also normally available from the addresses above.

## Appendix 2: Additions to GTAP94.TAB for Bilateral Export Quotas

This shows the additions to the standard GTAP94.TAB file for bilateral export quotas. You can find these additions at the end of the file GTAP33XQ.TAB used in Examples A1-A6 and P1-P2. The only part of GTAP94.TAB which is changed is the EXPRICES equation, the original version of which is replaced by the version shown below. [More specifically, GTAP33XQ.TAB is based on version 2a August 1995 of GTAP94.TAB.]

[In the export quota example above in section 2 of the paper, bilateral export quotas were only added for certain commodities (the MFA commodities) from certain regions (the MFA-exporting regions) to other specified regions (the MFA-importing regions). This requires slightly more complicated additions to distinguish these different sets. You can see the TABLO statements added to GTAP94.TAB for the example in section 2 of the paper by looking at the file MFA-XQ.TAB which you should find amongst the associated computer files -see Appendix 1.]

```

! ----- !
!   Additions for bilateral volume export quotas           !
! ----- !
!
! Export price ratios
-----
!

VARIABLE (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)           pxs(i,r,s)
  # price of commodity i supplied from r to s inclusive of export taxes # ;
  ! May be less than pfob(i,r,s) in the levels if the export quota
  on i from r to s is binding !

VARIABLE (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)           tqxs(i,r,s)
  # import quota tax in r on good i bound for region s # ;
  ! In the levels, TQXS_L(i,r,s) = PXS_L(i,r,s)/PFOB_L(i,r,s) <= 1.
  TQXS_L < 1 if export quota binds. TQXS_L=1 if export quota does not bind. !

EQUATION E_tqxs    (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)
  tqxs(i,r,s) = pxs(i,r,s) - pfob(i,r,s) ;

COEFFICIENT (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)           VXQD(i,r,s)
  ! exports of commodity i from region r to destination s valued
  prior to any quota rent (tradeables only). This value
  includes export taxes ! ;
UPDATE (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)
  VXQD(i,r,s) = pxs(i,r,s) * qxs(i,r,s) ;

READ

```

```

(all,i,TRAD_COMM)(all,r,REG)(all,s,REG)           VXQD(i,r,s)
      FROM FILE GTAPDATA HEADER "VXQD" ;

COEFFICIENT (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)           TQXS_L(i,r,s)
      # import quota tax in r on good i bound for region s # ;

! If both VXQD and VXWD are zero, set TQXS_L to 1.0 !
ZERO_DIVIDE DEFAULT 1.0 ;

FORMULA (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)
      TQXS_L(i,r,s) = VXQD(i,r,s)/VXWD(i,r,s) ;
! In the levels, TQXS_L(i,r,s) also = PXS_L(i,r,s)/PFOB_L(i,r,s) <= 1.
      TQXS_L < 1 if export quota binds. TQXS_L=1 if export quota does not bind. !

ZERO_DIVIDE OFF ;
!
Change from usual GTAP94.TAB
-----
!

! Next equation is modified from its usual GTAP94.TAB version !
EQUATION EXPRICES
! This equation links agent's and pre-quota prices. In addition to tx we have ts
which embodies both production taxes (all s) and export taxes (r not equal
to s) (HT#27) !
(all,i,TRAD_COMM)(all,r,REG)(all,s,REG)
! (GTAP94.TAB version) pfob(i,r,s) = pm(i,r) - tx(i,r) - txs(i,r,s) ;!
! Now pxs is on LHS since pfob also includes possible quota price change !
pxs(i,r,s) = pm(i,r) - tx(i,r) - txs(i,r,s) ;

!
Export volume ratios
-----
!

VARIABLE (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)           qxs_ratio(i,r,s)
# Ratio between volume of exports of i from r to s and export quota # ;

VARIABLE (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)           qxs_quota(i,r,s)
# Volume export quota on exports of i from r to s # ;

COEFFICIENT (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)           QXS_RATIO_L(i,r,s)
# Ratio between volume of exports of i from r to s and export quota # ;
UPDATE (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)
      QXS_RATIO_L(i,r,s) = qxs_ratio(i,r,s) ;

File (text) gtapquota # File containing export quota data # ;
READ
      (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)           QXS_RATIO_L(i,r,s)
      FROM FILE gtapquota ;

EQUATION E_qxs_ratio (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)
      qxs_ratio(i,r,s) = qxs(i,r,s) - qxs_quota(i,r,s) ;

DISPLAY VXMD ; VXWD ; VXQD ;
DISPLAY TQXS_L ; QXS_RATIO_L ;

!
Link between export volume and export price ratios

```

```

!
VARIABLE (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)           xq_slack(i,r,s)
! Exogenous in first multi-step run.
Endogenized in the second run where tqxs or qxs_ratio is
exogenized and shocked. !
# Slack variable for equation Exp_Quo_Ratios # ;

VARIABLE (LINEAR, NO_SPLIT) del_Newton
# Shock this by one to do Newton corrections at each step # ;

EQUATION Exp_Quo_Ratios (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)
! This is linearisation of MAX( QXS_RATIO_L(i,r,s), TQXS_L(i,r,s) ) = 1 !
! The linearisation below includes the Newton correction term !
! Variable xq_slack can be used to turn selected ones of these off. !
IF[ QXS_RATIO_L(i,r,s) >= TQXS_L(i,r,s),
    QXS_RATIO_L(i,r,s)/100*qxs_ratio(i,r,s) ] +
IF[ QXS_RATIO_L(i,r,s) < TQXS_L(i,r,s),
    TQXS_L(i,r,s)/100*tqxs(i,r,s) ] +
[ MAX {QXS_RATIO_L(i,r,s), TQXS_L(i,r,s)} - 1 ] * del_Newton
+ xq_slack(i,r,s) = 0 ;

!
Quota rent
-----
!

VARIABLE (Change)
(all,i,TRAD_COMM)(all,r,REG)(all,s,REG)           c_qxs_rent_rat(i,r,s)
# Ratio of quota rent to pre-quota value of imports for each commodity # ;
! In the levels, QXS_RENT_RATIO_L= [QXS_L*(PFOB_L-PXS_L)]/[QXS_L*PXS_L]
= [PFOB_L/PXS_L] - 1
= [1/TQXS] - 1 !

EQUATION E_qxs_rent      (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)
c_qxs_rent_rat(i,r,s) = -{1/[100*TQXS_L(i,r,s)]} * tqxs(i,r,s) ;

!
-----
!
!   End of additions for bilateral volume export quotas
!
-----
```

## **Appendix 3: Linearizing the Exp\_Quo\_Ratios Levels Equations for Quotas**

Here we show how this is done in the bilateral export quotas case. Other cases are similar.

The relevant equation is the **Exp\_Quo\_Ratios** equation (see equation L4 in section 2 above), namely

$$\text{MAX} [ \text{QXS\_RATIO}(i, r, s)_{-L}, \text{TQXS\_L}(i, r, s) ] = 1 \quad (\text{L4})$$

In this section we abbreviate

- **QXS\_RATIO\_L(i,r,s)** as **QR** (quantity ratio) and
- **TQXS\_L(i,r,s)** as **PR** (price ratio - it is the ratio of PXS and PFOB - see section 2).

### ***Linearizing MAX(QR,PR)=1***

The equation "MAX(QR,PR)=1" must be put as a linearized EQUATION on the TABLO Input file since, at present, TABLO Input files do not allow the function MAX to appear in LEVELS EQUATIONS.

When considering MAX(QR,PR), there are two cases.

- (a) If  $QR \geq PR$ , then  $\text{MAX}(QR,PR)=QR$  so that the change in  $\text{MAX}(QR,PR)$  is equal to the change in  $QR$ .
- (b) If  $PR > QR$ , then  $\text{MAX}(QR,PR)=PR$  so that the change in  $\text{MAX}(QR,PR)$  is equal to the change in  $PR$ .

Combining these two into one equation using "IF" we can say that

$$\text{change in MAX}(QR,PR) = \text{IF}(QR \geq PR, c_{QR}) + \text{IF}(QR < PR, c_{PR})$$

where  $c_{QR}$  and  $c_{PR}$  denote the changes in  $QR$  and  $PR$ . Thus we can use

$$\text{IF}(QR \geq PR, c_{QR}) + \text{IF}(QR < PR, c_{PR}) = 0$$

as the linearization of the equation  $\text{MAX}(QR,PR)=1$ .

If  $QR$  and  $PR$  have associated percent change linear variables  $p_{QR}$  and  $p_{PR}$  in the model, this becomes

$$\text{IF}(QR \geq PR, QR/100*p_{QR}) + \text{IF}(QR < PR, PR/100*p_{PR}) = 0.$$

In the GTAP case,

$QR = QXS\_RATIO\_L(i,r,s)$  so that  $p\_QR$  is just the linear variable  $qxs\_quota(i,r,s)$ .

$PR = TQXS\_L(i,r,s)$  so that  $p\_PR$  is just the linear variable  $tqxs(i,r,s)$ .

Thus, the linearized equation `Exp_Quo_Ratios` might be written as

```
EQUATION Exp_Quo_Ratios (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)
! This is linearization of MAX( QXS_RATIO_L(i,r,s), TQXS_L(i,r,s) ) = 1 !
IF[ QXS_RATIO_L(i,r,s) >= TQXS_L(i,r,s),
    QXS_RATIO_L(i,r,s)/100*qxs_ratio(i,r,s) ] +
IF[ QXS_RATIO_L(i,r,s) < TQXS_L(i,r,s),
    TQXS_L(i,r,s)/100*tqxs(i,r,s) ] = 0 ;
```

In fact the actual equation has two other terms (see Appendix 2 above). One of these terms is the slack variable `xq_slack(i,r,s)`. This term is added so that, for selected  $(i,r,s)$ , these variables can be set endogenous, which effectively turns this equation off for these  $(i,r,s)$ . This is done in the second accurate simulation (see sections 3.6 and 8 above).

The other term in the `Exp_Quo_Ratios` equation shown in Appendix 2 involves the variable **del\_Newton**. This is introduced to implement the so-called **Newton correction** when running a multi-step calculation. Roughly, the idea of this Newton correction term is to make sure that the two ratios  $QR$  and  $PR$  return to no larger than 1.0 if they ever get larger than that (as they sometimes will in a multi-step calculation). More details about this are given below.

#### *Linearizing $\text{MAX}(QR,PR)=1$ with Newton Correction*

Although the linearization above will produce fairly accurate multi-step solutions provided a very large number of steps is taken, more accurate solutions with fewer steps can be obtained using Newton corrections, which are described below, first in general terms, and then as they apply to our  $\text{MAX}(QR,PR)=1$  equation.

For an equation

$$A = 0$$

the Newton correction is to handle the case where (during a multistep calculation, say) we have wandered off the curve so that the current levels value of  $A$  is not zero but has the value  $E$  (error) say. Thus we rewrite the equation as

$$A = E.$$

The change linearization of this is

$$c_A = c_E.$$

The Newton correction comes by taking  $c_E$  equal to  $-A$  (or  $-E$ ). Imposing this change on  $E$  will hopefully return  $E$  to zero (as desired). Thus the Newton correction linearization of  $A=0$  is

$$c_A = -A \quad \text{or} \quad c_A + A = 0.$$

Since this is not a legal GEMPACK linearized equation (the constant term  $-A$  is not allowed) we introduce a special linear variable **del\_Newton** whose value is set exogenously equal to 1 in every step of a multi-step calculation. This gives us the Newton correction linearization of  $A=0$  as

$$c_A + A * \text{del\_Newton} = 0.$$

Hence the Newton correction linearization of  $\text{MAX}(QR, PR)=1$  is

$$\text{IF}(QR \geq PR, c_QR) +$$

$$\text{IF}(QR < PR, c_PR) + [\text{MAX}(QR, PR) - 1] * \text{del\_Newton} = 0.$$

or

$$\text{IF}(QR \geq PR, QR/100 * p_QR) + \text{IF}(QR < PR, PR/100 * p_PR) +$$

$$[\text{MAX}(QR, PR) - 1] * \text{del\_Newton} = 0.$$

Note that the Newton correction is designed for use with Euler's method only. It will almost certainly produce numerically poor results if used with Gragg's method or the midpoint method.

Thus the linearized version of `Exp_Quo_Ratios` in `GTAP33XQ.TAB` (see Appendix 2) is

```

EQUATION Exp_Quo_Ratios (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)
  ! This is linearization of MAX( QXS_RATIO_L(i,r,s), TQXS_L(i,r,s) ) = 1
!
  ! The linearization below includes the Newton correction term !
  ! Variable xq_slack can be used to turn selected ones of these off. !
  IF[ QXS_RATIO_L(i,r,s) >= TQXS_L(i,r,s),
      QXS_RATIO_L(i,r,s)/100*qxs_ratio(i,r,s) ] +
  IF[ QXS_RATIO_L(i,r,s) < TQXS_L(i,r,s),
      TQXS_L(i,r,s)/100*tqxs(i,r,s) ] +
  [ MAX {QXS_RATIO_L(i,r,s), TQXS_L(i,r,s)} - 1 ] * del_Newton
  + xq_slack(i,r,s) = 0 ;

```

Note that the variable `del_Newton` is declared in the TABLO Input file via the statement

```
VARIABLE (LINEAR, NO_SPLIT)  del_Newton
  # Shock this by one to do Newton corrections at each step #
;
```

The qualifier "NO\_SPLIT" is a previously-undocumented VARIABLE qualifier. It means that any shock given to the variable will be given in each step of a multi-step calculation (rather than being spread out across all steps as usual). Variable `del_Newton` should always be set as exogenous. Usually it is shocked by one (always in the first approximate quota simulation). However it does not need to be shocked in the second (accurate) quota simulations. (Shocking it should make no noticeable difference to the results, except for any components of `xq_slack` which are endogenous.)