Scale Economies and Imperfect Competition in the GTAP Model

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Abstract

The universe of existing CGE models can be divided into 3 broad categories. The first class of models (of which the standard GTAP model is a classic example) emphasizes the static effects of policy related to general equilibrium resource reallocation. The second involves scale economics and imperfect competition and the third involves dynamic accumulation effects. Development of the second class of models has followed a long period during which many of the basic tenants of modern industrial organization theory were integrated into the core of mainstream trade theory. The resulting class of applied models emphasizes procompetitive effects. This paper presents techniques for the incorporation of several stylized representations of scale economies and imperfect competition into the GTAP modeling framework. A numerical example is also provided

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1. Introduction

The universe of existing CGE models can be divided into 3 broad categories. The first class of models (of which the standard GTAP model is a classic example) emphasizes the static effects of policy related to general equilibrium resource reallocation. The move to incorporate scale economies and imperfect competition into such models has followed a long period during which many of the basic tenets of modern industrial organization theory were integrated into the core of mainstream trade theory. The result of these extensions is a second class of models that emphasize not only reallocation effects, but also procompetitive effects. Recently, a third class of models has emerged that involves extensions to include classic and new-growth related accumulation effects.[1]

This paper presents techniques for the incorporation of several stylized representations of scale economies and imperfect competition into the GTAP modeling framework. The theoretical discussion draws heavily from Francois and Roland-Holst (1997). The following important <u>warning</u> is posted at the beginning of this endeavor. The representations developed here are stylized, and are not offered as the "correct" way to model industrial structure in the GTAP model or any other applied model. They demand information that will sometimes be unavailable. In addition, given the level of aggregation often employed in CGE models, it may be fully inappropriate to take small group stories about firm interaction as literally correct representations. Even so, it is hoped that the techniques spelled out here, when employed with appropriate caution, may provide some insight into the potential importance of scale economies and imperfect competition for the qualitative assessments drawn from GTAP-based policy assessments.

The paper is organized as follows. Section 2 discusses scale economies. This is followed by a discussion of oligopoly and monopoly under Armington preferences in Section 3. Section 4 covers monopolistic competition, while section 5 provides an empirical example using a 3 sector, 2 region aggregation of the GTAP data base.

2. Firm-level Costs

In simulation models, the cost structure of firms, and hence of industry, follows from the choice of modeling technique and the observed data to which it is calibrated. One aspect which has received intense scrutiny in recent years is returns to scale. Beginning with a study by Harris (1984), a large

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[1] See Francois, McDonald, and Nordstrom (1996) for one such extension that includes accumulation effects in the GTAP model.

literature on simulation modeling arose to evaluate trade liberalization under various specifications of returns to scale. This numerically based research initiative was abetted by the intense parallel interest among trade theorists in applying concepts from industrial organization to trade theory. Both strains of work on firm-level scale economies confirm a basic conclusion of the earlier literature on trade with industry-wide scale economies -- the results of empirical and theoretical work grounded in classical trade theory can be contradicted, in magnitude and/or direction, when scale economies or diseconomies play a significant role in the adjustment process.

The most common departure from constant returns to scale (CRTS) incorporates unrealized economies of scale in production. Increasing returns to scale (IRTS) often takes the form of a monotonically decreasingly average cost function, calibrated to some simple notion of a fixed cost intercept. In other words, one assumes that marginal costs are governed by the preferred CRTS production function (usually CES), but that some subset of inputs are committed *a priori* to production and their costs must be covered regardless of the output level. The total cost function may be homothetic (i.e. fixed costs involve the same mix of inputs as marginal costs), or alternatively fixed costs may be assumed to involve a different set of inputs. In either case, average costs are given by a reciprocal function of the form

$$AC = \frac{FC}{X} + MC \tag{1}$$

As an alternative, scale economies can also be specified as deriving from costs that enter multiplicatively, with an average cost function like the following:

$$AC = X^{\theta-1} f(\omega) \quad where\ 0 < \theta < 1 \tag{2}$$

where $f(\omega)$ represents the cost function for a homogenous bundle of primary and intermediate inputs. This type of reduced form structure can be derived, for example, from scale economies due to returns from specialization (i.e. an increased division of labor) inside firms. (See Francois, 1990). In reduced form, it can also represent returns to specialization on an industry-wide basis of intermediate inputs, resulting in industry-wide scale effects. (See Markusen 1990).

With scale economies as in equation (1) (i.e. with fixed costs), the cost disadvantage ratio (CDR) as defined below, will vary with the scale of output. Alternatively, with a cost function like (2), the *CDR* remains fixed. Under either approach, one "only" needs to calibrate the cost function from engineering estimates of the distance between average and marginal cost. With fixed costs, this also requires some idea about how to impute fixed costs to initial factor and/or intermediate use. In practice, it has become customary to appeal to the concept of a cost disadvantage ratio. This measure of unrealized scale economies is generally defined as

$$CDR = \frac{AC - MC}{AC} \tag{3}$$

For homothetic technologies, output elasticities at the margin with respect to inputs are equal to (1/(1-*CDR*)).

In practice, calibration of either (1) or (2) can be problematic. At a conceptual level, estimated *CDR*s may be based on one level of "typical" production, while the benchmark data set we are working with corresponds to another. If we model scale economies with fixed costs and variable *CDR*s (i.e. equation 1), then the *CDR* estimates can be inappropriate and even misleading. At a more basic level, the pattern of citations in the empirical literature employing scale economies is suspiciously circular. It converges on a set of engineering studies on scale elasticities, many of which are surveyed by Pratten (1988), and many of which date from the 1950s, 1960s, and early 1970s. Given technical change over this period, including the introduction of numerically controlled machinery, computerization of central offices, and the shift toward white-collar workers and away from production workers in the OECD countries these estimates appear somewhat stale. Clearly, this is an important area for future research.

To implement industry-wide scale economies along the lines of equation (2) in the GTAP model, the GEMPACK code presented in Table 1 is sufficient. (Note that all of the code discussed here is available, in a functioning set of applications, from the GTAP Web site in the companion zip file for this technical paper). Assuming such scale economies at the industry level implies either that you are working with (i) increasing returns, monopoly, and average cost pricing forced by free entry (discussed below), or (ii) industry-wide external scale economies.[2]

This specification is based on the observation that, for homothetic technologies, percentage changes in output of *X* with respect to percentage changes in inputs *Z* depend on the output elasticity, which equals (1/(1-*CDR*)).

(4)
$$\hat{X} = \left[\frac{1}{1-CDR} \right] \hat{Z}$$
$$= \left[1 + \frac{CDR}{1-CDR} \right] \hat{Z}$$

The GTAP implementation offered here provides an exact solution for cost functions like equation (2), and a linear approximation for cost functions like equation (1). An exact solution for equation (1) can be obtained by the addition of an update for the parameter $s = CDR/(1-CDR)$, where $\hat{s} = -\hat{x}$.

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[2] As will become evident later in the paper, industry-wide external scale economies can result from a number of underlying microeconomic stories, including regional/national returns from increasing returns due to specialization. In fact, the qualitative behavior of these models is representative of both the national and international returns to specialization models at the heart of the new literature on economic geography and the location of industry. (See Francois and Nelson 1998). The exact behavior of the generic reduced form (i.e. which of these theoretical stories we adopt) is reflected in assumptions about underlying parameters. This also means that, under almost all of the specifications discussed in this paper, multiple equilibria and local instability are potentially present. This is one cost of introducing a further dose of reality to the analytical mix.

Table 1. GTAP Code For Scale Economies
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<u>Code</u>:

```
VARIABLE (Levels) (all, i, TRAD_COMM)(all, r, REG)                    SCALE(i, r)
 ! SCALE is a CDR-based parameter for sectors to be modeled
         as being characterized by various specifications
         of output scaling.  The actual specification is controlled
         through values entered in the parameter file.  The CDR is the
         inverse elasticity of scale, or (AC-MC)/AC ! ;


VARIABLE (all, i, TRAD_COMM)(all, r, REG)                             OSCALE(i, r)
         # switch for output scaling # ;


EQUATION O_SCALE  (all, i, TRAD_COMM)(all, r, REG)
         ! computes output scaling effect for various specifications of increasing returns
         for value added in sector i in region r !
         OSCALE(i,r)  =  [SCALE(i,r)] *  qva(i,r)  - ao(i,r);
```

<u>To Implement in GTAP</u>:
  (1) The variable SCALE is defined as CDR/(1-CDR), and must be read in as a parameter.

  (2) Through closure swaps, scale economies are turned on in an otherwise standard GTAP
       specification by declaring the variable OSCALE exogenous, and the variable ao endogenous, for
       the relevant sectors.
_____

3. *Market Power With Armington Preferences*

3.1 *Perfect Competition*

The standard starting point for market structure in applied trade models, and our reference point for the
discussion in this section, is a competitive industry that can be described in terms of a representative firm
facing perfectly competitive factor markets and behaving competitively in its relevant output markets.
Under these assumptions, the representative firm takes price as given, and the cost structure of the
industry then determines output at a given price. Formally, we have:

$$P = AC \qquad\qquad\qquad\qquad (5)$$

Under increasing returns to scale at the firm level, equation (5) can be motivated by contestability, with
real or threatened entry forcing economic profits to zero. Demand for primary and intermediate inputs

will then depend on the specific cost structure that is assumed. If we assume constant instead of increasing returns, average cost pricing then also implies pricing at marginal cost.

$$P = MC \tag{6}$$

3.2 Monopoly

Our first departure from the competitive paradigm is the case of monopoly. The monopoly specification is a straightforward extension of perfect competition. In terms of equations (5) and (6), we still have a representative firm in the sector under consideration. The difference lies in the firm's pricing behavior. In particular, the monopolist does not take price as given, but rather takes advantage of her ability to manipulate price by limiting supply. This means that the pricing equation (5) is then replaced by the following equation:

$$\frac{P - MC}{P} = \frac{1}{\varepsilon} \tag{7}$$

where the market elasticity of demand is given by

$$\varepsilon = -\frac{\partial Q}{\partial P}\frac{P}{Q} \tag{8}$$

The relationship of price to average cost depends on our assumptions about the cost and competitive structure of the industry. For example, with contestability and scale economies, entry may still force economic profits to zero, such that the monopolist prices according to equations (5) and (7). This is the approach taken in models with monopolistic competition. Alternatively, we may instead have price determined by equation (6) in isolation from (4), such that demand quantities at the monopoly price also then determine average cost. Equation (5) is then replaced by a definition of economic profits.

$$\pi = (P - AC)Q \tag{9}$$

3.3 Between Perfect Competition and Monopoly: Oligopoly

Between the perfect competition and monopoly paradigms lies a continuum of possible firm distributions. When the number of firms is small enough for them to influence one another, complex strategies can arise. One vehicle often used to explore oligopoly interactions is the so-called Cournot conjectural variations model. Under this approach, we assume that each firm produces a homogeneous product, faces downward sloping demand and adjusts output to maximize profits, with a common market price as the equilibrating variable. We further assume, following Frisch (1933), that firms anticipate or conjecture the output responses of their competitors. Consider an industry populated by n identical firms producing collective output $Q = nQ_i$. When the ith firm changes its output, its conjecture with respect to the change in industry output is represented by

$$\Omega_i = \frac{dQ}{dQ_i} \qquad (10)$$

Which equals a common value Ω under the assumption of identical firms. Combined with a representative profit function this yields the first-order condition, and also the oligopoly pricing rule.

$$\frac{d\,\Pi_i}{dQ_i} = P + Q_i \frac{dP}{dQ}\frac{dQ}{dQ_i} - \frac{dTC_i}{dQ_i} = P^D - \frac{Q_i}{n\varepsilon}\frac{P}{Q_i}\Omega - MC = 0 \qquad (11)$$

$$\Pi_i = PQ_i - TC_i \qquad (12)$$

$$\frac{P - MC}{P} = \frac{\Omega}{n\varepsilon} \qquad (13)$$

The above expression encompasses a variety of relevant cases. The standard Cournot-Nash equilibrium corresponds to $(\Omega/n)=(1/n)$, where each firm believes that the others will not change their output, and industry output changes coincide with its own. Price-cost margins vary inversely with the number of firms and the market elasticity of demand, as logic would dictate. In the extreme cases, a value of $\Omega=0$ corresponds to perfectly competitive, average cost pricing, while $\Omega=n$ is equivalent to a perfectly collusive or monopolistic market. The range of outcomes between these extremes, as measured by $1 \geq (\Omega/n) \geq 0$, can provide some insight into the significance of varying degrees of market power. For this reason, in the econometric industrial organization literature, the value of (Ω/n) is used as a relatively general measure of the degree of competition.[3]

The critical endogenous term needed for equation (13) is the elasticity of demand. Recall that in Armington models, goods are differentiated by country of origin, and the similarity of goods from different regions is measured by the elasticity of substitution. Formally, within a particular region, we assume that demand goods from different regions are aggregated into a composite good according to the following CES function:

$$q_{j,r} = \left[\sum_{i=1}^{R} \alpha_{j,i,r} X_{j,i,r}^{\rho_j} \right]^{1/\rho_j} \qquad (14)$$

In equation (14), $X_{j,i,r}$ is the quantity of X_j from region i consumed in region r. The elasticity of substitution between varieties from different regions is then equal to σ_j , where $\sigma_j = 1/(1-\rho_j)$. For tractability, we focus here on the non-nested case, where σ_j is identical across regions, and is equal to the

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[3] The Cournot model is criticized for being overly simplistic. However, it can be used to represent the ultimate outcome of more complex underlying interactions. In particular, where repeated games can yield tacit collusion. (Tirole 1988; Shapiro 1989), the simple Cournot strategy emerges as the Nash equilibrium. However, this strategy does not maximize profits for the industry as a whole or for individual firms. The same is true of Bertrand competition.

degree of substitution between imports, as a class of goods, and domestic goods. Within a region, the price index for the composite good $q_{j,r}$ can be derived from equation (14):

$$P_{j,r} = \left[\sum_{i=1}^{R} \alpha_{i,r}{}^{\sigma_j} P_{i,r}{}^{1-\sigma_j} \right]^{[1/(1-\sigma_j)]} \tag{15}$$

At the same time, from the first order conditions, the demand for good $X_{j,i,r}$ can then be shown to equal

$$X_{j,i,r} = [\, \alpha_{j,i,r} / P_{j,i,r} \,]^{\sigma_j} [\, \sum_{i=1}^{R} \alpha_{j,i,r}{}^{\sigma_j} P_{j,i,r}{}^{1-\sigma_j} \,]^{-1} E_{j,r}$$

$$\tag{16}$$

$$= [\, \alpha_{j,i,r} / P_{j,i,r} \,]^{\sigma_j} P_{j,r}{}^{\sigma_j - 1} E_{j,r}$$

where E_j represents economy-wide expenditures in region r on the sector j Armington composite. From equation (16), the elasticity of demand for a given variety of good X_j, produced in region i and sold in region r, will then equal

$$\varepsilon_{j,i,r} = \sigma_j + (1 - \sigma_j) \left[\sum_{k=1}^{R} \left(\frac{\alpha_{j,k,r}}{\alpha_{j,i,r}} \right)^{\sigma_j} \left(\frac{P_{j,k,r}}{P_{j,i,r}} \right)^{1-\sigma_j} \right]^{-1} \tag{17}$$

The last term measures market share.

At this stage, there are a number of ways to introduce imperfectly competitive behavior. For example, for a monopolist or oligopolist in each region that can price discriminate between regional markets, the regional elasticity of demand (and hence the relevant mark-up of price over marginal cost) is determined in each market by equation (16). This implies, potentially, $n \times R^2$ sets of elasticity and price mark-up equations for a model with R regions and N sectors. In models where different sources of demand can potentially source imported inputs in different proportions (like the GTAP model), we then have a potential for $(n+k) \times n \times R^2$ elasticity and mark-up equations, where k is the number of final demand sources in each region. Hence, in large multiregion models, full regional price discrimination for each product in each region can add a great deal of numerical complexity to the model.

A greatly simplifying assumption, and one that will be adopted here, involves assuming a monopolist or oligopolist that does not price discriminate, but assumes he is operating in a single market. (With trade, this could mean that traded goods are first sold domestically to exporters.) He hence charges a single mark-up. From equation (16), the aggregate elasticity of demand will then be determined by a combination of σ_j and a weighting of $(1-\sigma_j)$ determined by regional market shares. This involves the weighting parameter ζ. For a monopolist in region i producing j, we then have:

$$\varepsilon_{j,i} = \sigma + (1 - \sigma) \zeta_{j,i} \tag{18}$$

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$$\zeta_{j,i} = \sum_{r=1}^{R} \frac{X_{j,i,r}}{X_{j,i}} \left(\sum_{k=1}^{R} \left(\frac{\alpha_{j,k,r}}{\alpha_{j,i,r}} \right)^{\sigma_j} \left(\frac{P_{j,k\,r}}{P_{j,i,r}} \right)^{1-\sigma_j} \right)^{-1} \tag{19}$$

For oligopoly we assume that firms are identical. Demand for the regional product is downward sloping, as defined by equations (18) and (19). We further assume that firms adjust output to maximize profits, with a common market price as the equilibrating variable, and that firms anticipate or conjecture the output responses of their competitors. This leaves us with a variation of the basic oligopoly pricing rule presented in (13).

$$\frac{P - MC}{P} = \frac{\Omega}{n\varepsilon} = \frac{\Omega}{n} \left[\sigma + (1 - \sigma)\zeta \right]^{1} \tag{20}$$

Implementation of this type of market structure in the GTAP framework involves added equations for calculating and endogenizing the markups of prices over marginal costs. The necessary GEMPACK code (for non-nested Armington preferences) is provided in the annex, and implementation is demonstrated with the empirical example that accompanies this technical paper. The basic approach involves calculating the appropriate markup based on equation (13) and the elasticity of demand as defined by equations (18) and (19). It also involves insertion of margins into the benchmark data, as the standard GTAP data set does not include oligopoly markups. This is discussed at more length in section 5 of this paper.

4. Firm-Level Product Differentiation

Next, we turn to firm-level product differentiation. This approach builds on the theoretical foundations laid by Ethier (1979, 1982), Helpman and Krugman (1985), Krugman (1979, 1980), and Venables (1987). Arguments for following this approach, where differentiation occurs at the firm level, have been offered by Norman (1990) and Brown (1987). The numeric properties of this type of model have been explored in a highly stylized model by Brown (1994). Generic properties (including multiple equilibria and non-convexities) are examined in Francois and Nelson (1998).

4.1 General Specification of Monopolistic Competition

Formally, within a region r, we assume that demand for differentiated intermediate products belonging to sector j can be derived from the following CES function, which is now indexed over firms or varieties instead of over regions. We have

$$q_{j,r} = \left[\sum_{i=1}^{n} \alpha_{j,i,r} X_{j,i,r}^{\rho_j} \right]^{1/\rho_j} \tag{21}$$

where $\alpha_{j,i,r}$ is the demand share preference parameter, $X_{j,i,r}$ is demand for variety i of product j in region r, and $\sigma_j = 1/(1-\rho_j)$ is the elasticity of substitution between any two varieties of the good. Note that we can interpret q as the output of a constant returns assembly process, where the resulting composite product enters consumption and/or production.[4] Equation (21) could therefore be interpreted as representing an assembly function embedded in the production technology of firms that use intermediates in production of final goods, and alternatively as representing a CES aggregator implicit in consumer utility functions. In the literature, both cases are specified with the same functional form. For this exercise, we assume both. While we have technically dropped the Armington assumption by allowing firms to differentiate products, the vector of α parameters still provides a partial geographic anchor for production.

In each region, industry j is assumed to be monopolistically competitive. This means that individual firms produce unique varieties of good j, and hence are monopolists within their chosen market niche. Given the demand for variety, reflected in equation (21), the demand for each variety is less than perfectly elastic. However, while firms are thus able to price as monopolists, free entry drives their economic profits to zero, so that pricing is at average cost. The joint assumptions of average cost pricing and monopoly pricing imply the following conditions for each firm f_i in region i:

$$\frac{P_{f_i} - MC_{f_i}}{P_{f_i}} = \frac{1}{\varepsilon_{f_i}} \tag{22}$$

$$P_{f_i} = AC_{f_i} \tag{23}$$

The elasticity of demand <u>for each firm f_i</u> will be defined by the following conditions.

$$\varepsilon_{j,f_i} = \sigma_j + (1 - \sigma_j) \zeta_{j,f_i} \tag{24}$$

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[4] An approach sometimes followed involves monopolistic competition within regions, with trade only involving composite goods. Trade then is not based on firm level differentiation (i.e. monopolistic competition) per se. Rather, trade is then based on the Armington assumption regarding regional composite goods. The basic difference between this approach and the one developed in the text is the relaxation of the linkage between upper-tier substitution elasticities and measures of market power for regional firms. We leave it to the reader to verify that this implies a model exhibiting, in reduced form, external scale economies at the regional level.

$$\zeta_{j,f_i} = \sum_{r=1}^{R} \frac{X_{j,f_i,r}}{X_{j,f_i}} \left(\sum_{k=1}^{n} \left(\frac{\alpha_{j,k,r}}{\alpha_{j,f_i,r}} \right)^{\sigma_j} \left(\frac{P_{j,k,r}}{P_{j,f,r}} \right)^{1-\sigma_j} \right)^{-1} \tag{25}$$

In a fully symmetric equilibrium, $\zeta = n^{-1}$. Under more general conditions, it is a quantity weighted measure of market share. To close the system for regional production, we index total resource costs for sector j in region i by the resource index Z. Full employment of resources hired by firms in the sector j in region i then implies the following condition.

$$Z_{j,i} = \sum_{f=1}^{n\_i} TC_{j,i,f} \tag{26}$$

In models with regionally symmetric firms (so that $Z_{ji} = n_{ji} \times TC_{ji}$), equations (22) - (26), together with the definition of $AC=AC(x)$, define a subsystem that determines six sets of variables: $x, \varepsilon, \zeta, P, n$, and the cost disadvantage ratio $CDR= (1- MC/AC)$.

These equilibrium conditions are represented graphically in Figure 1. The full employment of resources at level Z in the regional sector implies, from equation (26), possible combinations of n and x mapped as the curve FF. At the same time, demand for variety, combined with zero profit pricing (equations (22) and (23)), imply demand-side preference for scale and variety mapped as the curve ZZ. Equilibrium is at point E_0. Holding the rest of the system constant, expansion of the sector means the FF curve shifts out, yielding a new combination of scale and variety and point E_1. The exact pattern of shifts in n and x depends on the assumptions we make about the cost structure of firms, and about the competitive conditions of the sector. It may also be affected by general equilibrium effects.

Figure 1. Equilibrium under Monopolistic Competition

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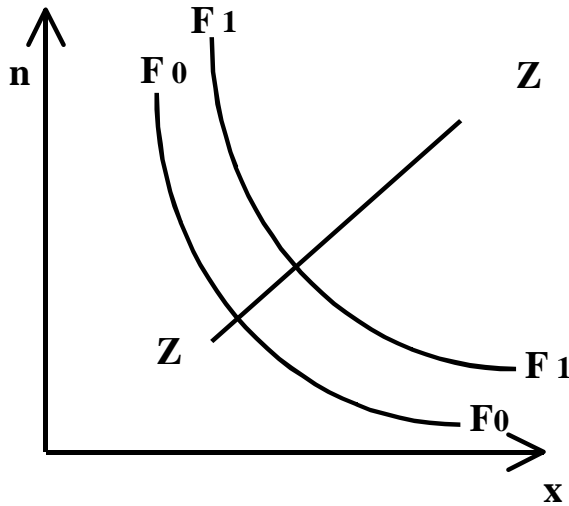
4.2. A Simplification: Variety Scaling

To simplify the system of equations somewhat, symmetry can be imposed on the cost structure of firms within a region. Regional symmetry means that, in equilibrium, regional firms will produce the same quantity of output and charge the same price. Under variety scaling, we further assume that the CES weights applied to goods produced by sector j firms from region i, when consumed in a particular region r, are equal. This means we can rewrite equation (21) as follows.

$$q_{j,r} = \left[\sum_{i=1}^{R} n_{j,i}\, \alpha_{j,i,r}\, \bar{x}_{j,i,r}{}^{\rho_j} \right]^{\frac{1}{\rho_j}} \tag{27}$$

Where $x_{j,i,r}$ is the identical consumption in region r of each variety produced in region i. Upon inspection of equations (27) and (14), it should be evident that the Armington assumption and firm level product differentiation, in practice, bear a number of similarities. The primary difference is that, in equation (27), the CES weights are now endogenous, as they include both variety scaling effects and the base CES weights. We can make a further modification to equation (27):

$$q_{j,r} = \left[\sum_{i=1}^{R} \gamma_{j,i,r}\, \tilde{x}_{j,i,r}{}^{\rho_j} \right]^{\frac{1}{\rho_j}}$$

$$\gamma_{j,i,r} = \alpha_{j,i,r}\, n_{j,i\,0}{}^{1-\rho_j} \tag{28}$$

$$\tilde{x}_{j,i,r} = \left(\frac{n_{j,i}}{n_{j,i\,0}} \right)^{(1-\rho_j)/\rho_j} X_{j,i,r}$$

Where $\tilde{x}$ is variety-scaled output, and where $n_{j,i\,0}$ is the benchmark number of firms. Note that $X_{j,i} = \tilde{x}_{j,i}$ in the initial equilibrium.

When we specify the system of equations for monopolistic competition using a variation of equation (28), the final set of equations for producing sector j composite commodities is then almost identical to that employed in standard, non-nested Armington models. The key difference is that the relevant CES weights are endogenous as defined by equation (28). In fully symmetric equilibria, the reader should be able to verify that complete firm exit from particular regions is possible, since the regional CES weights are simply equal to the number of firms, which collapse to zero with full exit. Depending on the specification of the structure of monopolistically competitive markets, as detailed below, the combination of output and variety scaling can then be specified as part of the regional production function for $\tilde{x}_{j,i}$.

4.3. Scale Economies from Fixed Costs

We will focus on common specifications of increasing returns. This is a variation of equation (1), in which we assume that the cost function, while exhibiting increasing returns due to fixed costs, is still homothetic. In particular, for a firm in region i, we have:

$$C(x_{j,i}) = (\alpha_{j,i} + \beta_{j,i}\, x_{j,i})\, P_{Z_{j,i}} \tag{29}$$

where $\alpha_{r,i}$ and $\beta_{r,i}$ represent fixed and marginal costs, and P_{Zji} represents the price for a bundle of primary and intermediate inputs Z_{ji}, where the production technology for Z_{ji} is assumed to exhibit constant returns to scale.

Substituting equation (29) into (22), (23), and (26), the system of equations (22) through (26), along with the definition of average cost, can be used to define general conditions for equilibrium in a monopolistically competitive industry. Starting from equations (22) and (23), the elasticity of demand can be related directly to the cost disadvantage ratio.

$$\frac{AC - MC}{AC} = \frac{\alpha_{j,i}}{\alpha_{j,i} + \beta_{j,i}\, x_{j,i}} = \frac{1}{\varepsilon_{j,i}} \tag{30}$$

The remainder of the system is as follows:

$$Z_{j,i} = n_{j,i}\,(\alpha_{j,i} + \beta_{j,i}\, x_{j,i}) \tag{31}$$

$$\zeta_{j,i} = \sum_{r=1}^{R} \frac{\tilde{x}_{j,i,r}}{\tilde{x}_{j,i}} \left(\sum_{k=1}^{R} n_{j,k} \left(\frac{\alpha_{j,k,r}}{\alpha_{j,i,r}} \right)^{\sigma_j} \left(\frac{P_{j,k,r}}{P_{j,i,r}} \right)^{1-\sigma_j} \right)^{-1} \tag{32}$$

$$\varepsilon_{j,i} = \sigma_j + (1 - \sigma_j)\,\zeta_{j,f_i} \tag{33}$$

Given the resources allocated to sector j in region i, as measured by the index Z_{ji}, equations (30) through (33) define a subsystem of 4 sets of equations and 4 sets of unknowns: n_{ji}, x_{ji}, ζ_{ji}, and ε_{ji}. In addition, the value of $\tilde{x}_{j,i}$ is then determined by equation (28), while producer price is set at average cost. Note that the price terms in equation (32) are internal prices, and will hence reflect trade barriers and other policy and trade cost aspects of the general equilibrium system, implying still more equations linking producer and consumer prices.

A special case of this specification involves "large group" monopolistic competition. In large group specifications, we assume that n is arbitrarily large, such that $\zeta_{j,i}$ is effectively zero, and hence, through equations (30) and (31), the elasticity of demand and the scale of individual firms are also fixed. In this case, changes in the size of an industry involve entry and exit of identically sized firms. The full set of equations then collapses to the following single equation:

$$\tilde{X}_{j,i} = \left(\frac{Z_{j,i\,1}}{Z_{j,i\,0}} \right)^{(1-\rho_j)/\rho_j} X_{j,i} \tag{34}$$

Here, X_{ji} is produced subject to constant returns to scale, given entry and exit of identical firms of fixed size, which follows from our assumptions about the cost function for Z_{ji}. At the same time, changes in variety are directly proportional to changes in Z_{ji}. Note that, for calibration, we have arbitrarily rescaled quantities of X to that $X = \tilde{X}$ in the benchmark.

More generally, proportional changes in $\tilde{X}_{j,i}$ relate to proportional changes in $Z_{j,i}$ as follows:

$$\hat{\tilde{X}}_{i,j} = [\sigma_j / (\sigma_j - 1)] \hat{Z}_{j,i} + \left(\frac{(\sigma_j - \varepsilon_{j,i}) \zeta_{j,i}}{(\sigma_j - 1)(1 - \zeta_{j,i})} \right) CDR \, \hat{\zeta}_{j,i} \qquad (35)$$

What does equation (35) tell us? The first term is clearly positive, and relates to the impact of increased resources on the general activity level of the sector, given its structure. The second term relates to changes in the condition of competition. Controlling for changes in market share for the entire regional industry, changes in $\zeta_{j,i}$ are proportional to changes in the inverse number of firms in the industry. Hence, we expect the last term to have a negative sign, but also to become smaller as the sector expands. In particular, as the sector expands, the value $(\sigma - \varepsilon)$ converges on zero, as does $\zeta_{j,i}$, so that this last term becomes less important. This follows from the procompetitive effects of sector expansion. As the sector expands, new entrants intensify the conditions of competition, forcing existing firms down their cost curves and hence squeezing the markup of price over marginal cost. As the sector becomes increasingly competitive, the marginal benefits of devoting more resources to the sector are greater, until at the limit the output elasticity for variety-scaled output converges on $(1 / \rho)$ This is the large group case, where $(\sigma = \varepsilon)$ such that the second term vanishes.

To implement this specification in GTAP, we adopt the large group assumption. This means that variety scaling effects are represented by equation (35), where the last term equals zero. This is identical, in form though not interpretation, to equation (4), once we are able to drop the last term. This means that we can use the same set of GEMPACK code discussed in the context of Table 1. The key difference is that we must make one additional change. We must make the values for ESUBD=ESUBM (due to the non-nested CES assumption) and ESUBM=SCALE=[CDR/(1-CDR)] due to (22). Again, this is demonstrated in the empirical example that follows.

5. *Empirical Illustrations*

We now turn to a specific application involving scale economies and imperfect competition. We work with an aggregation of version 3 of the GTAP database that has two regions (Japan, ROW) and three sectors (primary, manufactures, agriculture). (The files for replicating these experiments are available in the zip file accompanying this technical paper.) We limit ourselves to a single policy experiment, involving global free trade. This is modeled as elimination of import and export measures. While the closure is basically the same as in the "standard" GTAP models, some differences should be highlighted. First, I hold the current account balance fixed in all simulations. In addition, the Armington structure is a non-nested one (ESUBD=ESUBM), in keeping with the discussion on Cournot-Nash and monopolistically competitive equilibria above. Finally, as described below, the assumption of constant returns to scale (CRTS) and perfect competition is replaced by various specifications of increasing returns to scale (IRTS) and imperfect competition. Table 2 presents scale elasticities and estimated markups employed in the numeric assessments. The CDR estimate is taken as being representative of typical reported CDR values, as presented in various sources (like Pratten 1988).

Table 2. Manufacturing Scale Economies And Markups In The Benchmark

	CRTS	IRTS	IRTS	CRTS	IRTS
	AC pricing	AC pricing	Monop.Comp	Cournot	Cournot
CDR					
Japan	*	0.15	0.15	*	0.15
ROW	*	0.15	0.15	*	0.15
Markups over MC					
Japan	*	0.18	0.18	0.63	0.78
ROW	*	0.18	0.18	0.00	0.00

Note: For the Cournot applications, it has been assumed that $\Omega/n = 0.5$, which corresponds to two firms in the classic Cournot case, or to a relatively high degree of collusion otherwise.

When specifying oligopolistic competition, we limit ourselves to imperfect competition in the Japanese manufacturing sector. In this case, we work with Cournot conjectural variations, as defined earlier in the paper, assuming the $(\Omega/n) = 0.5$. This value is consistent with classic Cournot competition with 2 firms. In general, with Cournot competition and identical firms, the markup of price over average cost is defined as follows:

$$P_{j,i} = AC_{j,i} \left(1 - CDR_{j,i}\right)\left(1 - [\Omega_{j,i} / (n_{j,i}\varepsilon_{j,i})]\right)^{1} \tag{36}$$

Upon inspection of equation (36), it should be clear that, with scale economies, Cournot behavior can be inconsistent with positive profits. In particular, with a large enough CDR or highly elastic demand, pricing such that MR=MC will imply setting P<AC. This is implemented in GTAP, in the code provided in the annex, through the variable MARKUP.

```
VARIABLE (Levels)  (all,i,TRAD_COMM)(all,r,REG)              MARKUP(i,r)

#The monopoly markup for i, prod in r, if pref are NON-NESTED Armington # ;

FORMULA (Initial) & EQUATION (Levels) E_MARKUP (all,i,TRAD_COMM)(all,r,REG)

MARKUP(i,r) = (1/(1+SCALE(i,r)))/(1-(CV_RATIO(i,r)/DELAST(i,r)));
```

This employs the definition SCALE=(*CDR*/1-*CDR*). The term DELAST defines the quantity weighted composite of regional demand elasticities, as discussed in relation to equations (18) and (19). The difference between the MC and AC markup is the term (1/(1+SCALE)).

The estimated oligopoly markups in Table 2 are based on equation (41), and are derived from the benchmark 1992 data set. These markups are reported as part of the output produced by the markup insertion process (demonstrated as part of the available GTAP code for this exercise). They are a function of market shares, and of the Armington substitution elasticities. Home market shares, and hence the implicit markups, will in part be a direct result of import protection. This becomes evident

when we examine the output effects of trade liberalization, which exhibit significant procompetitive features under the Cournot structure.

Note that, in this example, calibration involves inferring markups given our assumption about competition. One could, of course, follow the opposite approach, and infer the degree of competition given markups (or given markups and scale economies). A similar "re-adjustment" of the benchmark data would be required.

Table 3 presents estimated macroeconomic and output effects for our free trade experiment. The first set of simulation results involves CRTS and perfect competition, and serves as a reference experiment. The next column in the table corresponds to IRTS and average cost pricing. This involves scale economies with fixed CDRs.[5] We then move to monopolistic competition and on to Cournot competition. Note that welfare effects for Japan increase monotonically as we move across the columns. Not coincidentally, the distortions due to price markups also increase monotonically, as indicated in Table 2. This is an indication of the potential importance of scale effects when evaluating trade liberalization.

Consider the results under the Cournot specifications (the last two columns Table 3). Evidence of the procompetitive effects of our experiment can be seen if we compare these results with those in the first two columns. Recall from Table 2 that manufacturing had particularly high estimated markups under these scenarios. Because trade liberalization erodes the market power derived from protection, these markups are reduced and output increased significantly in the Cournot sector. The result is output effects roughly twice as great as those estimated under CRTS and perfect competition. Welfare effects (proxied here by consumption) are correspondingly higher as well .

Finally, the second and third columns provide a contrast of the implications of scale economies under national product differentiation (the Armington assumption) with those given scale economies under firm level product differentiation (large group monopolistic competition.) The result is a magnification of estimated welfare benefits for Japan and ROW (1.99 percent vs. 1.16 percent in columns 3 and 2 for Japan). In this example, firm level product differentiation clearly implies greater pro-competitive benefits then those estimated under Armington preferences. However, this assertion does not hold once we introduce imperfect competition in an Armington structure (columns 4 and 5).

---

[5] The estimated effects are almost identical to those that follow from scale economies from fixed costs. See Francois and Roland-Holst (1997). To implement scale economies from fixed costs, one simply needs to add an update term for SCALE.

Table 3. Consumption, Production, And Real Factor Income: Percent Changes From Global Free Trade

	CRTS AC pricing	JRTS AC pricing	IRTS Monop.Comp	CRTS Cournot	IRTS Cournot
Welfare					
Japan	0.84	1.16	1.99	2.28	2.86
ROW	0.37	0.34	0.61	0.46	0.41
Manufacturing output					
Japan	2.30	2.14	1.38	5.71	5.64
ROW	-0.25	-0.24	-1.55	-0.37	-0.31
Real wages					
Japan	2.37	2.66	3.41	13.34	14.75
ROW	2.13*	2.10	2.32	2.15	2.10
Real returns to capital					
Japan	-0.45	0.18	0.61	46.82	50.81
ROW	5.54*	0.18	6.11	4.70	4.89

Source: Author calculations. These are available as part of the implementation example files that accompany this paper

References

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Annex: Add-in GEMPACK code to be placed at bottom of standard GTAP model

(see the example files distributed with this paper, and available on the GTAP web site).

```
!---Part (2):  Add-on module for imperfect competition--------------------!

!------------------------------------------------------------------------!
!   MODIFICATIONS TO BASIC GTAP MODEL CODE TO ADD FEATURES OF            !
!   SCALE ECONOMIES AND IMPERFECT COMPETITION.                          !
!   prepared by J. Francois, Tinbergen Institute and CEPR, February 1998. !
!------------------------------------------------------------------------!
!------------------------------------------------------------------------!
!   This .TAB file can be added to the bottom of the standard GTAP.TAB   !
!   file. When you do this, you must comment out the SUPPLYPRICES        !
!   equation in the standard GTAP.TAB (since this IRTS module has        !
!   a modified version of this equation).                               !
!------------------------------------------------------------------------!

!------------------------------------------------------------------------!
!   This code supports incorporation of the following                   !
!   features for individual sectors:                                    !
!                                                                       !
!                  -  scale economies with average cost pricing         !
!                  -  cournot behavior with or without scale economies  . !
!                  -  small group monopolistic competition              !
!                  -  large group monopolistic competition              !
!------------------------------------------------------------------------!
!
This code is for the implementation of
increasing returns to scale and imperfect competition in the GTAP model.
Supporting documentation is available from the following:

1.     Francois, J.F., "Increasing Returns to Scale and Imperfect
       Competition in the GTAP Model,"  GTAP consortium technical
       paper, 1998.

2.     Francois, J.F. and D.W. Roland-Holst, "Scale
       Economies and Increasing Returns," in J.F.
       Francois and K.A. Reinert eds.,  APPLIED METHODS
       FOR TRADE POLICY ANALYSIS:  A HANDBOOK, Cambridge
       University Press, July 1997.

3.     Chapter applications from J.F. Francois and K.A. Reinert eds.,
       APPLIED METHODS FOR TRADE POLICY ANALYSIS:  A HANDBOOK, Cambridge
       University Press, July 1997, available for download on the world
       wide web at
       http://www.intereconomics.com/handbook.

4.     The technical paper and further model updates (including the example
       Discussed in the technical paper, with GEMPACK-based model and data set)
```

```
          are available at the GTAP world wide web site:

             http://www.agecon.purdue.edu/gtap.
!
!-------------------------------------------------------------------------------!
!    WARNING   WARNING   WARNING   WARNING   WARNING   WARNING   WARNING   WARNING !
!                                                                                 !
!    If you are using scale economy features, you will probably run into         !
!    solution problems with GEMPACK.  Maybe not with every application,          !
!    but certainly with some of them.  This is particularly true if you          !
!    have large policy shocks, or large scale effects.  Under monopolistic       !
!    competition, whole sectors (almost) may decide to try                       !
!    to move from one region to another.  There are dampeners built into         !
!    the theory to avoid corner solutions of this type (i.e. complete shut       !
!    down of sectors due to increasing returns.)  HOWEVER, this does not         !
!    preclude multiple equilibria or convergence problems even for local         !
!    equilibria.  In addition, corner solutions can cause                        !
!    problems for GEMPACK. When this happens, you may want try to break the      !
!    problem up into lots of substeps, and use Euler, NOT Gragg.  Also,          !
!    extrapolation, given the relative nonconvexity of IRTS specifications,      !
!    can lead to nonsense results (like negative quantities and prices)          !
!    under GEMPACK, so always be sure to check the quantity values when          !
!    you use this type of model structure.  If GEMPACK has a particularly        !
!    difficult time, an alternative is to settle for an approximate              !
!    solution involving one set of passes at the data:                           !
!                    Euler 7                                                      !
!    or alternatively something time intensive like                              !
!                    Euler 3                                                      !
!                    Subintervals =  250                                         !
!    The worst problems with convergence seem to follow from                     !
!    3 interval solutions, like Euler 3 5 7, and relate to the extrapolation     !
!    routines employed in GEMPACK. These same problems rear their ugly heads     !
!    in a different way, with optimization packages like GAMS, where they        !
!    appear as convergence problems.)                                            !
!                                                                                 !
!    WARNING   WARNING   WARNING   WARNING   WARNING   WARNING   WARNING   WARNING !
!-------------------------------------------------------------------------------!
!-------------------------------------------------------------------------------!
!                       imperfect competition variables                          !
!-------------------------------------------------------------------------------!

VARIABLE (all,i,TRAD_COMM)(all,r,REG)                           CDRSCALE(i,r)
        # switch for allowing a change in scale effects under fixed costs #
        ! This is implemented through the CMF file ! ;

VARIABLE (all,i,NSAV_COMM)(all,r,REG)                             mu(i,r)
        # monopoly or oligopoly markup on output in region r # ;

VARIABLE (all,i,TRAD_COMM)(all,r,REG)                           qo_lrge(i,r)
        # real industry output in large group variety-scaled models # ;


!-------------------------------------------------------------------------------!
! The following coefficients relate to imperfect competition.                   !
!-------------------------------------------------------------------------------!

VARIABLE (Levels) (all,i,TRAD_COMM)(all, r, REG)          SCALE(i,r)
        ! SCALE is a CDR-based parameter for sectors to be modeled
          as being characterized by various specifications
          of output scaling.  The actual specification is controlled
          through values entered in the parameter file.  The CDR is the
          inverse elasticity of scale, or (AC-MC)/AC ! ;
VARIABLE (Levels, Change) (all,i,TRAD_COMM)(all, r, REG)       CV_RATIO(i,r)
        ! CV_RATIO is the ratio of the Cournot conjectural
          variation to the number of firms in the sector.
          Under monopoly, this is 1, while under pure
          Cournot oligopoly this is (1/n).  With perfect competition,
          it is equal to zero  ! ;


!-------------------------------------------------------------------------------!
```

```
!    The following additional data are needed                         !
!------------------------------------------------------------------------!

File (Text) IRTS_DATA # Additional data for IRTS simulations # ;
READ SCALE    FROM FILE IRTS_DATA ;
READ CV_RATIO FROM FILE IRTS_DATA ;

COEFFICIENT (Parameter) (all,i,NSAV_COMM)(all,r,REG)        MRKUP_ON(i,r)
        # switch for oligopoly markup insertion in i in r #
        ! This is implemented through the MARKUP_ON data file  ! ;
FILE (Text) MARKUP_ON # Contains data to tell if markups are on or not # ;
Read MRKUP_ON from file MARKUP_ON ;

! Here need to know that ESUBD is a COEFFICIENT(Parameter)
  So that this module can be added to the bottom of GTAP.TAB, make
  a "copy" of it here !
COEFFICIENT (Parameter) (all,i,TRAD_COMM)  CESUBD(i) # Copy of ESUBD # ;
FORMULA (Initial) (all,i,TRAD_COMM) CESUBD(i) = ESUBD(i) ;

  !------------------------------------------------------------------------!
  !      ADD THE FOLLOWING TO THE SECTION "DERIVATIVES OF THE BASE DATA"   !
  !                                                                        !
  !      Note that the equations for Cournot behavior are based on a       !
  !      non-nested Armington structure.  In terms of GTAP, this involves  !
  !      ESUBD=ESUBM for the relevant sectors.  Working with a nested      !
  !      Armington structure will require modification to the definition   !
  !      of ZETA and DELAST, though the rest of the model remains unaffected. !
  !      (Time permitting, a more general specification that covers        !
  !      both nested and non-nested specifications will be made available.) !
  !------------------------------------------------------------------------!

! Make this a LEVELS submodel !

! Introduce LEVELS variables with the same values as VXMD etc !
VARIABLE (Levels)
  (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)  VXMD_L(i,r,s) # Equals VXMD # ;
FORMULA (Initial)
  (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)  VXMD_L(i,r,s) = VXMD(i,r,s) ;
Equation (Linear) E_p_VXMD_L (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)
        p_VXMD_L(i,r,s) = pm(i,r) + qxs(i,r,s) ;
VARIABLE (Levels)
  (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)  VIMS_L(i,r,s) # Equals VIMS # ;
FORMULA (Initial)
  (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)  VIMS_L(i,r,s) = VIMS(i,r,s) ;
Equation (Linear) E_p_VIMS_L (all,i,TRAD_COMM)(all,r,REG)(all,s,REG)
        p_VIMS_L(i,r,s) = pms(i,r,s) + qxs(i,r,s) ;

VARIABLE (Levels) (all,i,NSAV_COMM)(all,r,REG)   VOM_L(i,r) # Equals VOM # ;
FORMULA (Initial)
  (all,i,NSAV_COMM)(all,r,REG)   VOM_L(i,r) = VOM(i,r) ;
Equation (Linear) E_p_VOM_L (all,i,NSAV_COMM)(all,r,REG)
        p_VOM_L(i,r) = pm(i,r) + qo(i,r) ;

VARIABLE (Levels) (all,i,TRAD_COMM)(all,r,REG)  VDM_L(i,r) # Equals VDM # ;
FORMULA (Initial)
  (all,i,TRAD_COMM)(all,r,REG)   VDM_L(i,r) = VDM(i,r) ;
Equation (Linear) E_p_VDM_L (all,i,TRAD_COMM)(all,r,REG)
        p_VDM_L(i,r) = pm(i,r) + qds(i,r) ;

VARIABLE (Levels) (all,i,TRAD_COMM)(all,r,REG)  VIM_L(i,r) # Equals VIM # ;
FORMULA (Initial)
  (all,i,TRAD_COMM)(all,r,REG)   VIM_L(i,r) = VIM(i,r) ;
Equation (Linear) E_p_VIM_L (all,i,TRAD_COMM)(all,r,REG)
        p_VIM_L(i,r) = pim(i,r) + qim(i,r) ;

VARIABLE (Levels) (all,i,TRAD_COMM)(all,r,REG)                ZETA(i,r)
        ! The weighted average market share of goods
          produced in r, in global expenditure on i ! ;
```

```
FORMULA (Initial) & EQUATION (Levels) E_ZETA (all,i,TRAD_COMM)(all,r,REG)
        ZETA(i,r) = sum(s,REG,((VXMD_L(i,r,s)/VOM_L(i,r))*
        (VIMS_L(i,r,s) / (sum(k,REG,VIMS_L(i,k,s)) + VDM_L(i,s)))))
        + (VDM_L(i,r)/VOM_L(i,r))*(VDM_L(i,r)/(VIM_L(i,r)+VDM_L(i,r)))) ;

VARIABLE (Levels) (all,i,TRAD_COMM)(all,r,REG)          DELAST(i,r)
    # The composite global demand elasticity for i, produced in r # ;
FORMULA (Initial) & EQUATION (Levels) E_DELAST (all,i,TRAD_COMM)(all,r,REG)
! Use CESUBD here !
!      DELAST(i,r) = ESUBD(i) + ((1-ESUBD(i))*ZETA(i,r));!
       DELAST(i,r) = CESUBD(i) + ((1-CESUBD(i))*ZETA(i,r));

VARIABLE (Levels)  (all,i,TRAD_COMM)(all,r,REG)          MARKUP(i,r)
! The cournot markup over average cost for i, prod in r, if pref are    !
! NON-NESTED Armington                                                  !;
FORMULA (Initial) & EQUATION (Levels) E_MARKUP (all,i,TRAD_COMM)(all,r,REG)
     MARKUP(i,r) = (1/(1+SCALE(i,r)))/(1-(CV_RATIO(i,r)/DELAST(i,r)));

!--------------------------------------------!
!  This is a modification to the supply      !
!  price equation to include markups         !
!--------------------------------------------!

EQUATION SUPPLYPRICES
! This equation links pre- and post-tax supply prices for all industries.
  This captures the effect of output taxes. TO(i,r) < 1 in the case of a
  tax. (HT#15)  In addition, the term mu(i,r) represents markups over average
  cost in the Cournot specification.  Because all non-factor income goes
  directly to the household, oligopoly markups are represented as a tax !
(all,i,NSAV_COMM)(all,r,REG)
ps(i,r) = to(i,r) + pm(i,r) - mu(i,r);

! ------------------------------------------------------------------------- !
!                          VALUE OF OUTPUT                                   !
! This is added to the model to allow recalibration of the data set with    !
! markups under Cournot behavior.                                           !
! ------------------------------------------------------------------------- !

VARIABLE (all,i,PROD_COMM)(all,r,REG)                        voutput(i,r)
      # value of merchandise regional production, by commodity # ;

EQUATION OUTPUT  (all,i,PROD_COMM)(all,r,REG)
! change in production values !
voutput(i,r) = pm(i,r) + qo(i,r) ;

! ------------------------------------------------------------------------- !
!    THESE ARE ADDED AT THE END FOR IMPERFECT COMPETITION FEATURES          !
!                                                                           !
!    NOTE: SOME EFFECTS ARE CONTROLLED THROUGH CLOSURE SWITCHES             !
!    (LIKE THE USE OF OSCALE), WHILE OTHERS INVOLVE THE USE OF              !
!    SHOCK SWITCHES, WHERE A SHOCK TURNS ON THE RELEVANT EQUATION           !
!    (AS IN THE VARIABLE mu).                                               !
! ------------------------------------------------------------------------- !

VARIABLE (all,i,TRAD_COMM)(all,r,REG)                        SCALE(i,r)
     # switch for output scaling                             # ;

EQUATION O_SCALE  (all,i,TRAD_COMM)(all,r,REG)
! computes output scaling effect for
various specifications of increasing returns
for value added in sector i in region r !
OSCALE(i,r)  =  [SCALE(i,r)] *  qva(i,r)
           - ao(i,r);

EQUATION CDR_SCALE  (all,i,TRAD_COMM)(all,r,REG)
! computes changes in intensity of scale effects.
This is used with fixed-cost based scale economies,
and updates the scale elasticity !
CDRSCALE(i,r) = p_SCALE(i,r)  +  qo(i,r);
```

```
EQUATION QO_LARGE  (all,i,TRAD_COMM)(all,r,REG)
! computes physical output for large group
monopolistic competition sectors !
qo_lrge(i,r)  =  qo(i,r) - ao(i,r);

EQUATION MRK_UP  (all,i,TRAD_COMM)(all,r,REG)
! computes changes in markup over average cost
under Cournot competition with conjectural
variations !
mu(i,r) =  IF( MRKUP_ON(i,r) NE 0, p_MARKUP(i,r)) ;

!  THAT IS IT.  REMEMBER TO MODIFY YOUR PARAMETER FILES                    !

! --------------------------------------------------------------------------- !
!              END SCALE ECONOMIES AND IMPERFECT COMPETITION FILE             !
! --------------------------------------------------------------------------- !
```