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by

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Abstract

See attached.

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Pollution Abatement Under Learning by Doing with Heterogeneous Costs *

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April 2002

Abstract

Many environmental policies including the Kyoto Protocol, the Acid Rain Program, the Montreal Protocol and the phase-out of leaded gas are designed to achieve a target level of abatement within a specified period of time. This paper examines how pollution abatement should be allocated over time using heterogeneous technologies characterized by learning by doing. In the presence of learning by doing, conventional economic wisdom regarding the allocation of pollution abatement must be modified. This paper derives the appropriate generalization of the principle that marginal abatement costs are equalized and shows how learning by doing alters the optimal allocation of abatement. The paper considers three classes of abatement cost functions: linear costs, convex costs and constant elasticity costs. The optimal solution is derived for linear and constant elasticity costs. In the case of convex costs sufficient conditions are given for abatement to be shared across all technologies and a bound is placed on the optimal cumulative abatement that occurs over any interval $[0, t]$. The results are used to provide insight into the effects of technological learning on pollution abatement policy, the tradeoffs involved in allocating abatement between mature and infant technologies, and the role played by abatement costs and discounting in determining how pollution abatement should be allocated over time and across technologies.

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1 Introduction

For many pollutants the goal of environmental policy is to achieve a target level of abatement within a specified period of time. Prominent examples include the treatment of global warming gases under the Kyoto Protocol, the reduction of SO₂ and NO_x under the Acid Rain Program, the phase-out of CFC products under the Montreal Protocol, and the phase-out of leaded gasoline. Typically, the desired level of abatement can be achieved by a variety of different agents and an efficient abatement strategy must determine how to allocate emission reductions among them. Depending on the context, agents can be firms, industries, countries or technologies. In this paper the focus is primarily on technologies.

In conventional economic analysis of environmental problems it is typical to assume that technologies are static, or that technological progress is exogenous. In such circumstances cost minimization is achieved by equating marginal abatement costs between technologies at each point in time. Technologies with lower marginal costs abate more than those with higher marginal costs. In reality, technological change is an endogenous and dynamic process influenced by government policy and other factors that affect the acquisition of knowledge. Recent research has begun to recognize that the relationship between technological change and the environment has very important implications for environmental problems. In their survey of this literature Jaffe, Newell and Stavins [2001, p. 1] argue that we should improve our understanding of the process of technological change for two reasons. First, "because many environmental problems and policy responses are evaluated over time horizons of decades or centuries, the cumulative effect of technological change on environmental problems is likely to be large;" and second, because environmental policies alter the process of technological change itself. In particular, there is a need to develop a better understanding of how the dynamics of technological change affects the design of good environmental policy and vice versa.

One important source of technological progress is learning by doing [Arrow, 1962], the process through which costs decline as firms gain experience utilizing a technology. For empirical purposes this relationship is often characterized by a learning curve or progress ratio in which each doubling of experience leads to a fixed percentage reduction in unit

costs [Argote and Epple, 1990]. Industry studies indicate that, in some cases, learning by doing contributes more to technological progress than the initial process development itself [Kline and Rosenberg, 1986]. At the same time, a cross section of over one hundred learning curve studies suggests that the rate of learning by doing varies substantially across technologies, with reductions in marginal cost ranging between minus ten and fifty percent with a doubling of accumulated experience [Dutton and Thomas, 1984]. McDonald and Schrattenholzer [2001] present a compilation of learning rates from 26 studies of energy technologies. As in the case of manufacturing firms, they find a wide range of learning rates with a median rate of 16-17%.

Gains from learning may vary across technologies for two reasons. First, technologies will generally be at different stages of their learning curves.¹ The potential cost reductions associated with learning are typically much higher for infant technologies that are at the beginning of their use, than for mature technologies where much of the learning opportunities have already been exploited. Second, even when technologies have the same degree of maturity, differences in product cycles, the organizational structure of producing firms, and other factors may result in inherent differences in their learning rates.

The presence of learning by doing and differences in learning opportunities across technologies have important implications for pollution abatement policy. For example, any policy that equalizes the instantaneous marginal costs of abatement between technologies is not an optimal abatement policy under learning by doing. Since increased abatement today lowers costs at all future dates, the optimal allocation of abatement across technologies does not depend solely on current marginal costs. Rather, it depends on the marginal effect abatement today has on the entire time path of abatement costs. It may be optimal to utilize a technology with high marginal costs of abatement today, if the added experience with the technology results in sufficiently lower marginal costs in the future. This provides one rationale for why it might be optimal to utilize a new, emerging technology whose current costs are not competitive with those of an existing, mature technology. If the

¹As an example, chemical sequestration can be considered an infant technology for CO₂ abatement while reducing emissions from the burning of fossil fuels represents a more mature technology.

new technology offers high learning opportunities, social costs may be reduced by abating more with this technology, even if its initial marginal costs of abatement is higher than the alternatives.

This paper examines the efficient abatement of pollution using multiple technologies when there is technical change characterized by learning by doing. Technologies are heterogeneous with respect to both the costs of abatement and the cost reducing effects of experience. The main question analyzed in this paper is: How should emission reductions be allocated across technologies to achieve a desired level of overall abatement in the presence of learning by doing? Our approach divides this question in two parts; one that solves for the optimal abatement if each technology were to be used in isolation, and the second that solves for the optimal division of cumulative abatement between technologies.

The central lesson we draw from our analysis is that optimal abatement policies are determined by the interplay of three competing incentives that we refer to as ‘discounting’, ‘abatement’, and ‘learning.’ First, discounting provides an incentive to delay abatement. Second, if the instantaneous marginal cost of abatement is increasing in abatement this tends to even out the abatement paths through time and it encourages greater use of technologies that have lower abatement costs. This is the abatement effect. Third, learning by doing provides an incentive to shift abatement to technologies that have higher learning opportunities, i.e., those that are initially less mature or have a higher learning rate. This is the learning effect. When a technology is initially more expensive, but with high learning opportunities, the abatement and learning effects act in opposite directions. In this case the discount rate determines the type of technology that is utilized. High discount rates diminish the importance of learning and favor mature technologies, whereas low discount rates increase the importance of learning and favor infant technologies.

We first study the case where marginal abatement costs depend only on experience, i.e., costs are linear in abatement and decreasing and convex in experience. In this case the instantaneous intertemporal elasticity of substitution for abatement is infinite and the abatement effect or the incentive to smooth abatement across time is absent. When there is only one technology available, the effect of learning is independent of its timing and

abatement is delayed until the latest possible date from which it is still feasible to reach the target. Discounting determines the outcome. When more than one abatement technology exists there is a trade-off between discounting and learning in the allocation of cumulative abatement to different technologies. We show that, when the discount rate is low, the learning effect dominates and a unique technology is used to reach the target. This technological winner is the technology that has the lowest undiscounted cumulative cost. For a high discount rate, we show that it is optimal to share abatement between all technologies. If only one technology is used then abatement must begin earlier in order to reach the target. As the discount rate increases, the opportunity costs of time increasingly dominate the benefits from learning. As a consequence, abatement is shared across technologies in order to delay the costs of abatement from any technology as much as possible.

We then analyze the general case where marginal abatement costs are increasing in abatement. In this general analysis the abatement effect comes into play. This has the implication that, if marginal abatement costs are small at low rates of abatement, then abatement is strictly positive for each technology at all points in time. Thus, there exist mild conditions under which a technological winner never emerges. On the other hand, when the discount rate is high, we show that the abatement effect is dominated by the discounting effect and the basic insight from the linear case generalizes. The optimal policy under a high discount rate is to delay abatement towards the end of the time horizon and it is possible to place an upper bound on the optimal cumulative abatement that occurs over any interval $[0, t]$. More importantly, we derive the appropriate generalization of the rule of equalization of marginal costs. In doing so, we characterize precisely how an optimal abatement policy should reflect the incremental reduction in the discounted stream of future costs and not just the instantaneous marginal costs of each technology. This generalized rule embodies in an intuitive way the interaction between discounting, abatement, and learning.

Finally, we illustrate the general analysis for the important class of constant elasticity costs. We are able to obtain closed form analytical solutions when costs have constant elasticity with respect to both abatement and experience. This allows additional insights into properties of the optimal solution including the trade-offs involved in the allocation of

abatement between mature and infant technologies.

Our analysis is related to previous research that examines the significance of technological change for energy use and the abatement of global warming gases. A relatively recent review of this literature is provided by Azar and Dowlatabadi [1999]. Much of the research uses simulations to examine how various policies and technologies can be used to achieve desired reductions in greenhouse gases (e.g. Grubler, Nakicenovic, and Victor [1999]). Our work is most closely related to research by Goulder and Mathai [2000] that uses an aggregate model with a single abatement technology to examine the relation between CO₂ abatement and policy induced accumulation of knowledge through research and development and learning by doing. Goulder and Mathai find that induced technical change in the form of R&D lowers the initial optimal carbon tax and optimal abatement while the effects are ambiguous under learning by doing. Our analysis differs from theirs in several respects. First, since the type of environmental policies we analyze are often stated in terms of desired levels of emission reductions, as opposed to target ambient concentrations, we do not explicitly model ambient pollution concentrations. Instead, our approach is consistent with the view expressed by Wigley, Richels and Edmonds [1996] who suggest that “for each stabilization level there is, roughly, a fixed allowable amount of CO₂ to be released. The basic choice is, therefore, how this budget is to be allocated over time.” Second, we focus solely on how learning by doing affects pollution abatement. This is the case where the existing analytical results are less complete. Most importantly, we examine how to allocate abatement across more than one technology. This adds a significant policy dimension to our model since a controversial aspect of many environmental policies is how to allocate abatement across different agents.

The policy relevance of our paper depends in part on the extent to which learning by doing plays a role in reducing pollution abatement costs. To our knowledge there are no empirical estimates of learning curves for pollution abatement. The best evidence is a study by Bellas [1998] that investigates whether progress has occurred in flue gas desulfurization (FGD) technology employed by coal burning power plants. Bellas examines the relation between FGD abatement costs and two variables that act as a proxy for technological

progress: the year each FGD unit began operation and the age of each FGD unit. If R&D is an important source of technological progress one would expect there to be a negative relation between abatement costs and the year in which FGD units began operation, while if there is learning by doing one would expect lower abatement costs the longer a unit has been operating. Bellas finds little evidence to support the first hypothesis, but he finds a significant negative relation between abatement cost and the age of FGD units. To the extent that the age of the unit is a proxy for experience, these findings suggest that learning by doing has been a more important source of technological progress than R&D and that learning by doing has played a role in reducing the costs of SO₂ abatement in the U.S.

2 The model

This section develops a model of pollution abatement across different technologies to achieve a target level of abatement by some specified future date. Each technology is characterized by learning by doing where the costs of abatement decline as experience with the technology increases. Technologies are heterogeneous in two ways. First, the costs of abatement may differ between technologies even if each has the same level of experience. Second, the rate at which costs decline with experience may differ.

Technologies are indexed by $i = 1, 2$. All of our results generalize to the case of N technologies. The rate of pollution abatement by technology i at time t is denoted $a_i(t)$. Experience using technology i is measured by the cumulative abatement from time 0 to t and is given by $z_i(t) = z_{i0} + \int_0^t a_i(s)ds$, where z_{i0} denotes the initial level of experience. This assumption is consistent with empirical studies of learning by doing, in which experience is generally measured as cumulative production, (see Argote and Epple [1990]).² Instantaneous abatement costs, $c_i(a_i, z_i)$, $i = 1, 2$, depend on both the rate of abatement and the experience using a technology. The more a technology is utilized, the greater are the improvements in cost and efficiency. Hence, learning by doing is synonymous with the

²We could equivalently measure experience as a certain transform of cumulative abatement, i.e., $f(z_i)$, under suitable assumptions on the transform f . However, our setting does not allow the effect of instantaneous abatement on experience to depend on the level of experience itself, as assumed in Goulder and Mathai [2000].

reductions in cost that occur through experience gained by utilizing a technology. Future costs are discounted at the rate $\delta \geq 0$. We shall use the following conventions regarding derivatives. Time derivatives will be indicated by $\dot{z} = dz/dt$, while partial derivatives will be denoted using subscripts as in $dc/da = c_a$. Technology subscripts shall be omitted for notational convenience when there is little possibility of confusion and when we focus on the case of a single technology. For the case where there are two technologies the cost functions are always allowed to differ even if the technology index is suppressed.

The objective is to allocate abatement across technologies over time to meet a target level of cumulative abatement, A , by some specified date, T , in the least cost manner. Formally the problem can be written as:

$$\begin{aligned} & \min_{a_1(t), a_2(t)} \int_0^T e^{-\delta t} [c_1(a_1(t), z_1(t)) + c_2(a_2(t), z_2(t))] dt \\ & \text{subject to : } z_i(t) = z_{i0} + \int_0^t a_i(s) ds, \quad a_i(t) \geq 0, \quad i = 1, 2, \quad \int_0^T [a_1(s) + a_2(s)] ds = A. \end{aligned} \quad (2.1)$$

This is a parsimonious specification designed to capture the essential elements of allocating abatement to achieve a target. As such it abstracts from issues related to what happens after the target has been achieved. We shall return to this point in the conclusion.

To analyze how each of the abatement technologies is utilized over time it is useful to decompose this problem into two parts. The first solves the optimal time path for abatement, for each technology in isolation, taking the total abatement for that technology over the interval $[0, T]$ as given. The second part of the decomposition solves for the optimal allocation of cumulative abatement across technologies. We state this decomposition formally in the following principle.

Fundamental Decomposition Principle

Problem (2.1) is equivalent to the following two-stage optimization:

Stage 1. $C_i(A_i) = \min_{a_i(t)} \int_0^T e^{-\delta t} c_i(a_i(t), z_i(t)) dt$

subject to $z_i(t) = z_{i0} + \int_0^t a_i(s) ds, \quad a_i(t) \geq 0, \quad \int_0^T a_i(s) ds = A_i.$

Stage 2. $C(A) = \min_{A_1, A_2} C_1(A_1) + C_2(A_2) \quad \text{subject to } A_1, A_2 \geq 0, \quad A_1 + A_2 = A.$

This Decomposition Principle relies on the fact that the instantaneous abatement cost is additively separable between (a_1, z_1) and (a_2, z_2) .³ One advantage of this decomposition is that the problem of finding the optimal time path for abatement is simplified to a one-dimensional optimal control problem, while the problem of how to allocate abatement between technologies is reduced to a static optimization.

In what follows we shall refer to $C_1(A_1)$ and $C_2(A_2)$ as **cumulative cost functions**. They represent the minimum discounted stream of costs from using each technology to achieve a given level of cumulative abatement, A_i . This decomposition makes it clear that the allocation of abatement between technologies depends on the properties of the cumulative cost functions for each technology. Under learning by doing it is not sufficient to compare technologies based on their marginal costs of abatement at any point in time. In order to capture the effects of experience on future abatement costs, each technology must be evaluated based on the discounted stream of abatement costs that accrue over the entire time horizon. However, once costs are expressed in cumulative form, standard insights from economic theory can be used to evaluate the allocation of abatement between technologies. If cumulative cost functions are convex, abatement will generally be shared between both technologies. This is analogous to the usual outcome in the static case where marginal abatement costs are increasing in abatement. If cumulative costs are concave so that marginal cumulative costs are decreasing in cumulative abatement, then one technology is dominant and is the sole technology used to achieve the target.

3 Linear Abatement Costs

In this section we analyze the case where costs are linear in abatement. The study of this case helps to illuminate the conditions under which only one technology will be used to achieve the target. When there is only one period and technologies are fixed, linear abatement costs always leads to a corner solution where the technology with the lowest

³Additive separability does not hold when there are spillovers between the two technologies, e.g., if the experience acquired by using technology 1 lowers the abatement cost of technology 2. We note that, in any case, the study of abatement allocation without spillovers constitutes a necessary first step of the analysis with spillovers.

marginal cost of abatement abates everything. An examination of the linear case shows how learning by doing affects this logic.

Throughout this section the abatement cost functions are assumed to take the form

$$c_i(a_i, z_i) = a_i c_i(z_i)$$

The marginal cost of abatement depends on experience, but not on abatement. It is assumed to satisfy the following assumptions.

- A.3.1. $c_i(z)$ is continuously differentiable from $\mathbb{R}_+ \rightarrow \mathbb{R}_+$.
- A.3.2. $c_i(z)$ is strictly decreasing.
- A.3.3. $c_i(z)$ is strictly convex.

In this setting $c_i(z_0)$ represents the initial marginal cost of abatement. Assumption A.3.2 implies that marginal costs decrease as experience increases, while A.3.3 indicates that there are diminishing returns to learning.

To obtain well-defined solutions, we assume there exists an exogenous upper bound on the instantaneous rate of abatement for each technology.

$$a_i \leq \bar{a}_i$$

One interpretation of this constraint is that marginal abatement costs are infinite if $a_i > \bar{a}_i$. It is also assumed that $A \leq \bar{a}_i T$, so that sole reliance on either technology is a feasible way to achieve the target.⁴

We base our analysis on the decomposition principle presented in the previous section. First, we solve the intertemporal cost minimization problem for each technology in isolation. With a linear abatement cost, it is possible to derive an analytical expression for the cumulative cost functions. These cumulative cost functions can then be used to analyze how to allocate cumulative abatement between the two technologies. The discount rate

⁴If cost minimization requires to abate with a technology more than is physically feasible, the solution will simply be to abate at full capacity with this technology and to abate the remainder with the other technology.

turns out to play a crucial role in the solution. Low discount rates lead to an optimal solution where one technology abates everything, whereas high discount rates lead to an interior solution where both technologies are used to meet the target. As the discount rate increases, considerations of delay eventually outweigh the cost reductions from experience.

To represent this formally we introduce the strategy of **maximal delay**. This strategy postpones abatement until the latest date from which it is still feasible to meet the target. With two technologies, the strategy of maximal delay is to delay abatement until the date $\tau = T - \frac{A}{\bar{a}_1 + \bar{a}_2}$ and then to abate at full capacity with both technologies. With a single technology the strategy of maximal delay is to wait until $\tau_i = T - \frac{A}{\bar{a}_i}$ and then abate at the full capacity of technology i .

3.1 Optimal Abatement with One Technology

For one technology, the cost minimization problem becomes

$$\begin{aligned} \min_{a(t)} \int_0^T e^{-\delta t} a(t) c(z(t)) dt \quad (3.1) \\ \text{subject to } 0 \leq a(t) \leq \bar{a}, \quad z(t) = z_0 + \int_0^t a(s) ds, \quad \int_0^T a(s) ds = A, \end{aligned}$$

where the technology index is omitted for notational convenience. The solution to this problem is described in the following Proposition.

Proposition 3.1. *If $\delta = 0$, the solutions of (3.1) are all the feasible abatement paths. If $\delta > 0$, the solution of (3.1) is to follow the strategy of maximal delay. For any $\delta \geq 0$, the cumulative cost is equal to*

$$C(A) = e^{-\delta \tau} \int_{z_0}^{z_0 + A} c(z) e^{-\delta \frac{z - z_0}{\bar{a}}} dz.$$

Proposition 3.1 means that, when the discount rate is strictly positive, abatement is delayed as long as possible, and then occurs at full capacity until the target is reached. As expected, the cumulative cost function is increasing in the target A and decreasing in the level of initial experience z_0 .

3.2 Optimal Abatement with Two Technologies

We now solve the second stage of the decomposition, that determines how to allocate cumulative abatement between two technologies with linear abatement costs. To obtain a first idea of the main result of this section, consider the case where $\delta = 0$. When the discount rate is equal to zero, cumulative costs are $C_i(A_i) = \int_{z_0}^{z_0+A_i} c_i(z_i) dz_i$, which is the undiscounted cumulative cost of any feasible abatement path.⁵ Since $C_i''(A_i) = c_i'(A_i) < 0$, the cumulative cost C_i is concave. When future costs are not discounted, increasing returns in experience make marginal cumulative cost decreasing in cumulative abatement. As a consequence, reliance on a single technology is the best way to reap the benefits of learning and the optimal allocation is that one technology abates everything.

With a strictly positive discount rate, starting to abate early is costly. Reliance on one technology incurs these costs since such a policy induces an earlier starting date than using both technologies. This implies, that in general, one has to balance the benefits from learning and the costs of starting early.

Proposition 3.2 *There exist $\bar{\delta} > \underline{\delta} > 0$ such that (a) if $\delta < \underline{\delta}$ the solution is to abate everything with the technology possessing the lowest undiscounted cumulative cost, (b) If $\delta > \bar{\delta}$ the solution relies on the use of both technologies, and as δ increases, the solution tends to the strategy of maximal delay.*

Proposition 3.2 clarifies the circumstances under which reliance on a single technology is optimal and the circumstances under which diversification across technologies minimizes costs.

When the discount rate is low, there is a technological winner. Since cumulative costs are determined by cumulative abatement, a slight change in the target level of cumulative abatement may induce a switch of this technological winner. Specifically, assume that marginal cost curves cross once as depicted in Figure 1.⁶ Let z_1 denote the experience level

⁵Effectively, since $a_i = \dot{z}_i$, for any path such that $\int_0^T a_i = A_i$, we have $\int_0^T c_i(a_i, z_i) dt = \int_0^T c_i(z_i) \dot{z}_i dt = \int_{z_0}^{z_0+A_i} c_i(z_i) dz_i$.

⁶If marginal cost curves never cross, one technology has a lower undiscounted cumulative cost for all

at which the marginal cost curves cross and let $z_2 > z_1$ be the experience level at which the undiscounted cumulative cost curves cross. Proposition 3.2 tells us that the technological winner for low discount rates is technology 1 if $A < z_2$ and technology 2 if $A > z_2$. When $z_1 < A < z_2$, technology 2 is a more promising technology. Yet, it is not used because the cumulative cost to reach its final and lower marginal cost is too high. The long-term potentialities of feasible options should not make us neglect the cost to reach them. As obvious as this may seem, it is sometimes forgotten in the debate about technical change.

The effects of learning on the optimal abatement strategy can also be derived in the linear model. When there is no learning by doing, $c(z) = c$ is constant and abatement cost is simply equal to ca . When $\delta = 0$, cumulative cost is simply equal to $C(A) = cA$. When $\delta > 0$, cumulative costs are given by

$$C(A) = \int_{\tau}^T c\bar{a}e^{-\delta t} dt = c\frac{\bar{a}}{\delta}[e^{-\delta\tau} - e^{-\delta T}] = c\frac{\bar{a}}{\delta}e^{-\delta T}[e^{\delta\frac{A}{a}} - 1]$$

and the derivative of the cumulative cost is now

$$C'(A) = ce^{-\delta T}e^{\delta\frac{A}{a}}$$

The optimal allocation involves diversification across technologies if and only if $C'_1(0) < C'_2(A)$ and $C'_2(0) < C'_1(A)$. This is equivalent to:

$$c_1 < c_2e^{\delta\frac{A_2}{a_2}} \text{ and } c_2 < c_1e^{\delta\frac{A_1}{a_1}}.$$

From these inequalities it is easily seen that there exists a sufficiently large discount rate $\bar{\delta} > 0$, such that if $\delta > \bar{\delta}$, the optimal allocation is to diversify abatement across both technologies.

When there is no learning C is linear if $\delta = 0$ and convex if $\delta > 0$. In contrast, learning by doing induces concavity of the cumulative abatement cost function when the discount

experience levels. In this case, the technological winner is always the same. Notice that if marginal cost curves cross several times, so might do undiscounted cumulative cost curves, and the technological winner might switch several times as well.

rate is low. This suggests that, in general, learning increases the prospect of having a “technological winner”.

Proposition 3.2 expresses a fundamental trade-off between learning from experience and the incentives discounting provides to delay abatement cost. Focusing on one technology generates more gains from learning, but at the expense of starting earlier. When marginal abatement cost depends only on experience, this trade-off has simple consequences for the properties of cumulative costs. A low discount rate results in concave cumulative costs, whereas a high discount rate leads to convex cumulative costs. In the next section, we show that this trade-off plays a similar crucial role for more general abatement costs, even though its analytic expression is more complex.

4 Convex Abatement Costs

In this section we consider the case of general abatement costs, $c_i(a_i, z_i)$, that are assumed to satisfy the following assumptions:

- A.4.1. $c_i(a, z)$ is twice continuously differentiable from $\mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$.
- A.4.2. $c_i(a, z)$ is strictly increasing in a and strictly decreasing in z .
- A.4.3. $c_i(a, z)$ is strictly convex in (a, z) .
- A.4.4. $c_i(0, z) = 0$ for all $z \in \mathbb{R}_+$.

The assumptions on how costs vary with abatement and experience are traditional. Costs are increasing and convex in abatement and decreasing and convex in experience. Learning reduces abatement cost at a decreasing rate and the gains from experience are higher when experience is low.

4.1 Optimal Abatement with One Technology

In this section we examine the properties of optimal abatement with one technology. We introduce the Hamiltonian and derive necessary and sufficient conditions for the optimal path. We focus on two issues. First, we examine the conditions under which all available

technologies are utilized at all points in time. Second, we study how the optimal abatement path behaves as the discount rate increases. As in the previous section, we will first analyze properties of the optimal abatement path for a technology in isolation and then seek to understand how abatement is allocated between two technologies.

The intertemporal cost minimization problem is

$$\begin{aligned} C(A) &= \min_{a(t)} \int_0^T e^{-\delta t} c(a(t), z(t)) dt \\ \text{subject to } z(t) &= z_0 + \int_0^t a(s) ds, \quad a(t) \geq 0, \quad \int_0^T a(s) ds = A. \end{aligned}$$

The current value Hamiltonian for this problem is

$$H(a, z, \mu) = -c(a(t), z(t)) + \mu(t)a(t),$$

where the variable $\mu(t)$ represents the current value shadow price of a unit increase in experience using the technology. This Hamiltonian measures the direct cost of abatement as well as the indirect benefit associated with technological learning. Under assumptions A.4.1-A.4.4 the current value Hamiltonian is strictly concave, optimal abatement is unique, and the optimal abatement path is characterized by the following conditions (e.g. Leonard and Long [1992, Theorem 6.5.2]).

$$a(t) \geq 0, c_a(a(t), z(t)) - \mu(t) \geq 0, [c_a(a(t), z(t)) - \mu(t)]a(t) = 0 \quad (4.1)$$

$$\dot{\mu} = \delta\mu(t) + c_z(a(t), z(t)) \quad (4.2)$$

$$z(t) = z_0 + \int_0^t a(s) ds, \quad z(T) = z_0 + A.$$

The optimality conditions have the following economic interpretation. The marginal cost of using a technology for abatement is c_a while the marginal benefit from using a technology is reflected by μ , the shadow value of experience. Hence, (4.1) is simply a benefit

cost condition that says a technology is not used if the marginal costs exceed the marginal benefits, and that when a technology is in use, it is utilized to the point that equates marginal benefits and marginal costs. Condition (4.2) is the dynamic optimality condition that governs the evolution of the shadow value of experience. The two components of the the right hand side reflect the tradeoffs created by discounting and learning by doing. The first indicates that μ increases at the rate of discount, to offset the opportunity cost of time. The second component on the right hand side of (4.2) is associated with learning by doing and has a negative effect on the rate of change in μ . If the learning by doing effect is large enough relative to the discounting effect, it is possible for the shadow value of experience to decrease. When a technology is utilized, (4.2) implies that marginal abatement costs increase at the rate of discount less the direct marginal effect learning by doing has on costs. Integrating the differential equation yields $\mu(t) = e^{-\delta(T-t)}\mu(T) - \int_t^T e^{-\delta(s-t)}c_z(a(s), z(s))ds$. This expression shows that $\mu(t)$ represents the discounted present value of an incremental unit of experience at time T less the discounted stream of cumulative effects learning by doing has on costs over the interval $[t, T]$. In other words, the shadow value of experience at time t is the present value of terminal marginal abatement costs plus the discounted stream of future cost reductions associated with an incremental increase in experience at t .

We now state some well known properties of the cumulative cost.

Lemma 4.1. *The cumulative cost $C(A)$ is strictly increasing and strictly convex in A with marginal cumulative cost given by $C'(A) = \mu(T)e^{-\delta T}$.*

With convex costs the cumulative cost function is always convex. This is in contrast to the linear case where it was possible for cumulative costs to be concave. The marginal cumulative cost is directly related to the final shadow value of experience.

We now consider conditions under which abatement is always positive. It is optimal not to abate if and only if the shadow value of experience is lower than the marginal cost of abatement. This might happen when the marginal cost of abatement is always sufficiently high. This leads us to introduce the following additional assumption:

$$\text{A.4.5.} \quad c_a(0, z) = 0 \text{ for all } z \in \mathfrak{R}_+.$$

Assumption A.4.5 implies that the marginal cost of abatement is small for low levels of abatement. For one technology assumption A.4.5 is a natural condition ensuring interiority of the optimal abatement path. We will see in section 4.2 that assumption A.4.5 ensures interiority for two technologies as well. Let $a^*(t)$ denote an optimal abatement path.

Proposition 4.2. *Under assumption A.4.5, $a^*(t) > 0$ for all t .*

When assumption A.4.5. does not hold, it might be optimal not to abate at certain times. However if $\delta > 0$, this can only happen at the beginning. Either $a^*(t) > 0$ for all t or there exists some τ such that optimal abatement is zero on $[0, \tau]$ and strictly positive on $[\tau, T]$.⁷ The latter is possible when the initial shadow value of experience $\mu(0)$ is lower than the initial marginal abatement cost $c_a(0, z_0)$. In that case, abatement is postponed while the shadow value of experience increases. Once the shadow value of experience reaches $\mu(\tau) = c_a(0, z_0)$ abatement starts and never stops.

We now investigate how the optimal abatement path varies when the discount rate increases. The next results characterize the manner in which increases in the discount rate provide an incentive to postpone cumulative abatement.

Proposition 4.3. *For any $\varepsilon > 0$,*

$$\text{if } \delta > \frac{c(\frac{A}{T-t}, z_0)}{tc(\frac{\varepsilon}{t}, z_0 + A)} \text{ then } \int_0^t a^*(s)ds < \varepsilon.$$

This proposition states that for any ε , there is a sufficiently high discount rate such that the cumulative abatement that occurs over the time interval 0 to t is smaller than ε . One implication of this result is that cumulative abatement over any finite time interval approaches zero as the discount rate increases without limit.

Corollary to Proposition 4.3. *For every $t < T$, $\int_0^t a^*(s)ds \rightarrow 0$ as $\delta \rightarrow +\infty$.*

⁷If abatement is strictly positive on $[\tau_1, \tau_2]$ and zero on $[\tau_2, \tau_3]$, it is less costly to postpone some abatement on $[\tau_2, \tau_3]$.

High discount rates provide an incentive to postpone abatement toward the end of the time horizon. This result is similar to that obtained in Proposition 3.2 and shows that the optimality of delaying abatement as the discount rate increases is a general property of our model.

4.2 Optimal Abatement with Two Technologies

Now consider the issue of how to allocate abatement between two technologies. Using the fundamental decomposition principle, the allocation of cumulative abatements between the two technologies is determined by solving

$$\min_{\substack{A_1 + A_2 = A \\ A_1, A_2 \geq 0}} C_1(A_1) + C_2(A_2)$$

where C_1 and C_2 are the cumulative abatement costs of each technology.

Let A_i^* denote the optimal cumulative abatement from technology i and let $a_i^*(t)$ and $z_i^*(t)$ denote the associated time paths for optimal abatement and experience. By Lemma 4.1., we know that C_1 and C_2 are strictly convex. This means that there is a dominant technology if and only if the marginal cumulative abatement cost of one technology at zero is greater than the marginal cumulative abatement cost of the other technology at A . Formally, $A_1^* = 0$ if and only if $C_1'(0) \geq C_2'(A)$ and $A_1^* = A$ if and only if $C_2'(0) \geq C_1'(A)$. The solution to the cumulative abatement allocation problem is interior and $0 < A_1^*, A_2^* < A$ if, and only if, $C_1'(0) < C_2'(A)$ and $C_2'(0) < C_1'(A)$. The next result establishes that Assumption A.4.5. is a sufficient condition for this outcome.

Proposition 4.4. *If technologies 1 and 2 satisfy assumption A.4.5., then $A_i^* > 0$.*

By Proposition 4.2, assumption A.4.5 also implies that $a_i^*(t) > 0$. As a consequence, the optimal solution involves positive abatement from both technologies at all points in time. This means that if, for all technologies, the marginal costs of abatement are small at low levels of abatement, then an optimal strategy is to utilize all available technologies. In other words, no matter what the relative learning rates are of the different technologies, there is

no dominant technology under assumption A.4.5.

We now analyze the arbitrage equation.

Proposition 4.5. (a) Assume that $A_i^* > 0$ for $i=1,2$. Then the allocation of abatement between the two technologies is determined by equalization of marginal cumulative abatement costs: $C'_1(A_1^*) = C'_2(A_2^*)$.

(b) Assume that $a_i^*(t) > 0$ for $i = 1, 2$. Then the allocation of abatement between the two technologies is determined by

$$c_{1a}(a_1^*(t), z_1^*(t)) + \int_t^T c_{1z}(a_1^*(t), z_1^*(t))e^{-\delta(s-t)}ds = c_{2a}(a_2^*(t), z_2^*(t)) + \int_t^T c_{2z}(a_2^*(t), z_2^*(t))e^{-\delta(s-t)}ds$$

In addition, $c_{ia}(a_i^*(t), z_i^*(t)) + \int_t^T c_{iz}(a_i^*(t), z_i^*(t))e^{-\delta(s-t)}ds$, $i = 1, 2$, grows exponentially over time.

These conditions are analogous to the standard rule for allocating abatement across technologies in a static model. The difference here is that the allocation is determined by the cumulative cost functions which represent the discounted stream of abatement costs over the entire time horizon. The condition in part (b) indicates that abatement is allocated to equalize the marginal costs of abatement less the cumulative (marginal) cost reductions attributed to learning by doing. The net cost of an additional unit of abatement today is the marginal cost of abatement adjusted for the incremental effect of the gain in experience on future costs. Along the intertemporal optimum this must equal the discounted marginal cost of a unit of abatement at any other point in time adjusted for the effects of experience. Hence, the adjusted marginal abatement cost grows exponentially at the rate of discount. If $a_i^*(t) > 0$ for all t for $i = 1, 2$ then the condition in (a) is equivalent to the condition in (b) evaluated at $t = T$. When there is no learning, $c_{1z} = c_{2z} = 0$, and the condition in (b) reduces to the standard rule of equalization of instantaneous marginal costs and marginal abatement costs increase at the rate of discount along the optimal intertemporal allocation. When there is learning, however, the condition in (b) implies that an optimal policy equalizes the instantaneous marginal costs corrected for the cumulative benefits from learning. Current abatement costs are traded off against their learning potential. In addition, learning has

more importance when the discount rate is lower.

5 Constant Elasticity Abatement Costs

In this section we focus on the case where costs have constant elasticity with respect to both abatement and experience. This case serves to illustrate our general results and to gain further analytical insights.

Definition *The abatement cost function exhibits constant elasticity if there are three real numbers θ, α, β with $\theta > 0$ such that*

$$c(a, z) = \theta a^\beta z^{-\alpha}$$

A constant elasticity abatement cost function satisfies assumptions A.4.1-A.4.4 if and only if the following restrictions on the parameters are satisfied⁸

$$\beta > 1, \alpha > 0, \text{ and } \beta > \alpha + 1$$

These restrictions are assumed to hold for the remainder of the paper. It is also assumed that the level of initial experience z_0 is strictly positive. Under these assumptions, the abatement cost function satisfies assumption A.4.5 as well.

The specification of constant elasticity cost provides a useful parametric representation of the tradeoffs involved in abatement when there is learning by doing. It is multiplicatively separable in abatement and experience. For values of a and z close to 1 any abatement cost function can be approximated by a constant elasticity cost function. Finally, the constant elasticity specification is well suited to econometric estimation.

The three parameters have natural interpretations:

- α measures the opposite of the elasticity of abatement cost with respect to experience. In fact, α represents a real learning rate in the sense that a doubling of experience reduces the

⁸ c is increasing and convex in a if and only if $\beta > 1$, c is decreasing and convex in z if and only if $\alpha > 0$. c is jointly convex in (a, z) if and only if $\alpha > 0$ and $\beta > \alpha + 1$.

cost by $2^{-\alpha}$ (at all abatements levels). If $z > 1$, then $\alpha_2 > \alpha_1$ implies $c_2(a, z) < c_1(a, z)$ and $c_{2a}(a, z) < c_{1a}(a, z)$ so that both absolute and marginal abatement costs are lower for higher learning rates. The case where $\alpha = 0$ models the limiting case where there is no learning by doing.

- β measures the elasticity of abatement cost with respect to abatement. If $a > 1$, then $\beta_2 > \beta_1$ implies $c_2(a, z) > c_1(a, z)$ and $c_{2a}(a, z) > c_{1a}(a, z)$ so that higher values of β are associated with higher absolute and marginal abatement costs. In the limiting case, where $\beta \rightarrow 1$, marginal abatement cost only depends on experience.

- θ is a scaling factor, determined by the choice of the units in which abatement, experience, and cost are measured.

The case of constant elasticity abatement costs is especially valuable because we are able to solve for a closed form expression for the optimal abatement path. To simplify notation, we introduce four parameters γ, η, k_1 , and k_2 as follows:

$$\begin{aligned}\gamma &= \frac{\beta}{\beta - \alpha} \\ \eta &= \frac{\delta}{\beta - 1} \\ k_1 &= \frac{1}{e^{\eta T} - 1} [-(z_0 + A)^{\frac{1}{\gamma}} + e^{\eta T} z_0^{\frac{1}{\gamma}}] \\ k_2 &= \frac{1}{e^{\eta T} - 1} [(z_0 + A)^{\frac{1}{\gamma}} - z_0^{\frac{1}{\gamma}}]\end{aligned}$$

Notice that $\gamma > 1$ and $\eta, k_2 > 0$ under the assumptions of the model.

Proposition 5.1. *If $\delta > 0$, the optimal time paths for cumulative abatement and abatement are given by⁹*

$$\begin{aligned}z^*(t) &= [k_1 + k_2 e^{\eta t}]^\gamma \\ a^*(t) &= \eta \gamma k_2 e^{\eta t} [k_1 + k_2 e^{\eta t}]^{\gamma-1}.\end{aligned}$$

⁹The solution when $\delta = 0$ has a different shape and is given in Appendix. It can be checked that as $\delta \rightarrow 0$, $a(t)$ and $z(t)$ tend to the solutions obtained when $\delta = 0$.

The cumulative cost and the final marginal cost are given by

$$\begin{aligned} C(A) &= \theta \eta^{\beta-1} \gamma^\beta k_2^\beta [e^{\eta T} - 1] \\ \mu(T)e^{-\delta T} &= c_a(a^*(T), z_0 + A)e^{-\delta T} = \frac{dC}{dA} = \theta \beta \eta^{\beta-1} \gamma^{\beta-1} k_2^{\beta-1} (z_0 + A)^{-\frac{\alpha}{\beta}}. \end{aligned}$$

This optimal solution has a number of interesting properties that we now describe. First, $a^*(t)$ is increasing in time. When the discount rate δ increases, $z^*(t)$ decreases. As predicted by Proposition 4.3, when the discount rate tends to infinity, $z^*(t) \rightarrow z_0$ and $a^*(t) \rightarrow 0$ if $t < T$. Abatement is increasingly postponed as the discount rate increases. When the elasticity of abatement β tends to infinity, optimal abatement tends to the average abatement $\frac{A}{T}$ at every time. This means that when β becomes high, abatement tends to even out. Considerations of pure abatement costs eventually dominate those associated with learning and discounting.

We now focus on the learning rate and on its effect on $a^*(t)$, $z^*(t)$, and $\mu(t)$. Figure 2 depicts representative optimal paths for three values of the learning rate $\alpha_1 < \alpha_2 < \alpha_3$ such that α_1 is low, α_2 is intermediate and α_3 is high.¹⁰ Abatement is convex in time for α_1 and α_3 , but concave for α_2 . In general, $z^*(t)$ decreases as α increases. The shadow value of experience is increasing in time for α_1 , decreasing then increasing for α_2 , and decreasing for α_3 .

When the learning rate is small, the optimal solution is close to the optimal solution obtained when there is no learning. Notably, abatement is proportional to $e^{\eta t}$ and the shadow value of experience is proportional to $e^{\delta t}$. Considerations associated with the opportunity cost of time dominate. The introduction of learning modifies the qualitative features of these optimal paths. First, learning has an ambiguous effect on the curvature of the abatement path. The abatement path might be concave for intermediate learning rates, even though it becomes convex again for high learning rates. This especially implies that the level of instantaneous abatement at $T/2$ is in general non monotonic in the learning rate.¹¹

¹⁰In the figures with 3 cases, Case 1 is represented by a plain line, Case 2 by the long dashed line, and Case 3 by the short dashed line.

¹¹ $a(T/2) > A/T$ if a is concave and $a(T/2) < A/T$ if a is convex. In general, $a(T/2)$ will first increase

Second, learning affects the shadow value of experience in a unambiguous way. Specifically, one can show that there exists a time threshold $\tau \in [0, T]$ such that μ is decreasing on $[0, \tau]$ and increasing on $[\tau, T]$. Additionally, this threshold τ increases when the learning rate α increases. This clearly expresses the opposite effects of learning and discounting that, in general, drive the evolution of μ .

Finally, we study how the optimal path behaves when the initial level of experience is large. Since the benefits from learning exhibit diminishing returns from experience, one would expect learning to play a diminishing role as initial experience increases. That is exactly what we find. Specifically, when $z_0 \rightarrow \infty$, the abatement and cumulative abatement path tend to the solution obtained when there is no learning (i.e. when $\alpha = 0$). In terms of abatement paths, a low learning capability is synonymous with having a high level of initial experience.

We summarize these findings in the following corollary.

Corollary to Proposition 5.1. *With one technology, the optimal solution for the constant elasticity cost satisfies the following properties:*

- a. $dz^*(t)/d\delta \leq 0$ and $\lim_{\delta \rightarrow \infty} z^*(t) = z_0$ if $t < T$
- b. $\lim_{\beta \rightarrow \infty} a^*(t) = A/T$
- c. If $\dot{\mu}(t) > 0$ then $\dot{\mu} > 0$ on $[t, T]$
- d. $\lim_{\alpha \rightarrow 0} a^*(t) = \lim_{z_0 \rightarrow \infty} a^*(t) = \frac{\eta}{e^{\eta T} - 1} A e^{\eta t}$.

We now consider the question of allocation of cumulative abatement across technologies. Since the constant elasticity cost satisfies assumption A.4.5, we know that solution is interior, and the allocation of cumulative abatement is given by solving

$$C'_1(A_1) = C'_2(A - A_1)$$

Hence properties of the derivative of the cumulative cost determine how cumulative abatement is allocated.

and then decrease as a function of the learning rate.

The effects conform to intuition. Everything else held constant, technologies with higher learning rates or lower abatement elasticity have lower cumulative derivatives (at all abatement levels), hence are allocated more cumulative abatement. Additionally, Proposition 5.1 allows us to study abatement allocation between a mature and an infant technology. Mature technologies typically have lower initial abatement costs, but smaller prospects of learning, whereas infant technologies have larger initial abatement costs, but high learning opportunities. To model these ideas, we first assume that mature and infant technologies have the same elasticities β and α . The mature technology has a high level of initial experience z_0 , while the infant technology has a low level of initial experience. The scaling factors c are then adjusted so that the initial abatement cost of the infant technology is greater than the initial abatement cost of the mature technology by a fixed proportion (equal to 5 in figures 3 and 4). Figure 3 depicts how cumulative abatement is allocated as a function of the learning rate. When learning is low, the mature technology is allocated the larger share of abatement. As the learning rate increases, more cumulative abatement is allocated to the infant technology. A higher target for cumulative abatement amplifies the importance of learning, resulting in greater reliance on the infant technology.

Next, we study the influence of the elasticity of abatement β on this allocation. The first part of figure 4 depicts the allocation as one function of the learning rate for different values of β . Higher β 's induce flatter allocation curves, but do not modify which technology is allocated the greatest share of abatement. When β increases, considerations of pure abatement cost become more important, which explains the flattening of the allocation curve. Finally, we allow the infant and mature technologies to have different β . The second part of figure 4 compares configurations where the mature technology has a lower, equal, or higher elasticity of abatement than the infant technology. As expected, if the mature technology has a lower β , allocation towards the infant technology is postponed, whereas if the mature technology has a higher β , allocation to the infant technology occurs earlier.

6 Implementation

We now consider the implementation of the optimal abatement paths via Pigouvian taxes on emissions or subsidies of abatement. For purposes of comparison it is useful to recall that in the traditional static case the optimal policy is one that equalizes marginal abatement costs across agents or technologies. The static optimum can be implemented through Pigouvian taxes that tax emissions from different agents at the same rate or through abatement subsidies that subsidize abatement from different agents at the same rate. In response, each agent abates to the point where marginal abatement costs equal the tax / subsidy rate. This yields the cost effective allocation of abatement. In contrast, under learning by doing, the design of an optimal policy mechanism must account for the intertemporal effects of experience on costs. The following policy mechanisms are conceivable: (a) an instantaneous tax on emissions or an instantaneous subsidy on abatement, and (b) a final tax on cumulative emissions or a final subsidy on cumulative abatement. It is assumed that a policy is credible, so that agents take it as given when determining their optimal response, and that there is no strategic interaction between the government and agents. In the case of an instantaneous tax / subsidy it is assumed that a regulatory agency precommits to a time-dependent policy over the interval $[0, T]$. The optimal policy design depends on the degree of rationality assigned to agents. We consider two cases. In the first, agents are myopic and their objective is to minimize instantaneous costs. In the second case, agents are assumed to be perfectly rational. Such agents are forward looking dynamic optimizers who fully anticipate the cost reducing effects of learning by doing and have the same discount rate as the regulator. The main question we address is: What is the optimal emission tax / abatement subsidy for each agent in the presence of learning by doing?

To analyze this question it is convenient to define a transformation from abatement to emissions. We assume that there are two technologies that emit pollution. Let $s_i(t)$ denote the instantaneous rate of pollution emission from technology i and let \bar{s}_i represent baseline emissions when there is no abatement. Then, instantaneous abatement is given by $a_i(t) = \bar{s}_i - s_i(t)$. First, consider an instantaneous emission tax on technology i ,

denoted by $\tau_i(t)$. As a function of abatement, instantaneous costs including the tax are $\tau_i \cdot (\bar{s}_i - a_i) + c_i(a_i, z_i)$. Alternatively, let $\tau_i(t)$ be an instantaneous subsidy on abatement from technology i . Instantaneous costs including the abatement subsidy are $-\tau_i \cdot a_i + c_i(a_i, z_i)$. In each case abatement must satisfy the constraints $0 \leq a_i(t) \leq \bar{s}_i(t)$. Since $\tau_i \cdot \bar{s}_i$ does not effect the optimal choice of abatement under an emission tax, agent i faces the same decision problem in the presence of a tax on emissions or in the presence of a subsidy on abatement. Thus, the instantaneous policy instrument, $\tau_i(t)$, can be equivalently interpreted as an emission tax or an abatement subsidy. For the sake of simplicity, suppose in what follows that optimal abatement paths are interior so that $0 < a_i^*(t) < \bar{s}_i$.

The following proposition characterizes optimal emission taxes / abatement subsidies for both myopic and perfectly rational agents. The discounted final marginal abatement cost in the optimal program is denoted by $\tau^* = \mu_i(T)e^{-\delta T} = c_{ia_i}(a_i^*(T), z_i^*(T))e^{-\delta T} = C'_i(A_i^*)$, $i = 1, 2$. The optimal tax / subsidy rates for myopic and perfectly rational agents are designated by $\tau_i^m(t)$ and $\tau_i^r(t)$, respectively.

Proposition 6.1. *If agent i is myopic, the optimal emission tax / abatement subsidy of agent i is equal to*

$$\tau_i^m(t) = \tau^* e^{\delta t} - \int_t^T e^{\delta(t-s)} c_{iz_i}(a_i^*(s), z_i^*(s)) ds.$$

If agent i is perfectly rational, the optimal emission tax / abatement subsidy of agent i is equal to

$$\tau_i^r(t) = \tau^* e^{\delta t}.$$

If agent i is perfectly rational, an equivalent mechanism to obtain the optimal abatement path is a terminal tax on cumulative emissions or subsidy of cumulative abatement equal to $\tau^ e^{\delta T}$.*

A perfectly rational agent correctly accounts for the cost reducing effects of learning by doing. In this case the intertemporal incentives for an agent are aligned with those of the regulating agency and the emission tax / abatement subsidy plays its classical role of

internalizing the environmental externality. Since one unit of abatement from two different agents contributes equally toward pollution reduction, the tax / subsidy rate is identical across agents. In addition, the tax / subsidy grows exponentially at the rate of discount through time. This induces the optimal intertemporal allocation of abatement where the discounted present value of a unit of abatement, adjusted for the consequences of learning, is equalized at all points in time. In addition, since a perfectly rational agent internalizes the cost reducing effects of learning, the optimal tax is less than the instantaneous marginal cost of abatement.

In contrast, a myopic agent equates the instantaneous marginal cost of abatement to the emission tax. Further, with a myopic agent there are two externalities. There is the classic pollution externality and there is also a dynamic externality that arises because a myopic agent ignores the effect that current emissions have on future costs. The second term of the optimal Pigouvian tax / abatement subsidy for a myopic agent acts to internalize this externality as well. This second term is null, and the optimal policy when agents are myopic is the same as when agents are perfectly rational, only when there is no learning by doing. When there is learning by doing and agents are myopic, the optimal tax / subsidy rates are different across agents. The reason for this is that when agents are myopic, optimal policies must account for differences in learning across agents. Since myopic agents do not account for learning in their decisions, the optimal policy is weighted towards technologies with high learning possibilities in order to induce greater abatement from those agents. This can be done in two ways: through higher taxes on pollution emissions or higher subsidies on abatement.¹²

When agents are neither myopic nor perfectly rational, the dynamic externality is still present, although to a lesser extent. Therefore, the optimal policy will still be differentiated in order to induce greater abatement from the agents with higher learning opportunities. Only when the intertemporal incentives of the agents and of the regulator are perfectly

¹²In some cases, it might not be possible to apply differentiated taxes on emissions. For example, sequestration of CO₂ in forests acts independently of the origin of the CO₂. In this case abatement via sequestration can be differentiated from other sources of abatement, but differentiated Pigouvian taxes on emissions might not be feasible.

aligned (i.e. when the agents are perfectly rational) does the dynamic externality disappear. Thus, learning by doing provides an important rationale for why the social value of one unit of abatement might differ between two different technologies and for why promising new technologies should be encouraged, especially when the regulatory agency has a better understanding of their long-term potential than the agents.

7 Conclusions

This paper has analyzed the effect of learning by doing on the allocation of pollution abatement over time and across technologies when the objective is to minimize the discounted costs of achieving a target level of abatement. The analysis has studied the optimal time path for abatement with one technology as well as the allocation of abatement between two technologies. The results show that learning by doing has important implications for the allocation of abatement. When there is no learning, cumulative abatement is increasing over time and is allocated to equalize marginal abatement costs across technologies at every point in time. With learning, equalizing marginal abatement costs does not lead to the cost-effective solution, because it does not take into account the effects that current actions have on future costs. The optimal solution involves an interaction between three factors: pure costs of abatement, the opportunity cost of time, and the intertemporal effects of learning.

We derived the appropriate generalization of the marginalist principle: abatement is allocated by equalizing at every point in time marginal costs of abatement corrected for the cumulative marginal savings that current abatement induces on future costs. We found that for any specification of the cost function, and consequently any specification of learning by doing, when the discount rate becomes large abatement is increasingly postponed towards the end of the time horizon.

We studied two general cases in depth: the case where abatement cost is linear in abatement and the case where abatement cost has constant elasticity with respect to both abatement and experience. The first case constitutes the polar case which expresses most clearly the trade-off between learning and discounting. Learning implies that the cumulative

cost function becomes concave for low discount rates, in which case a technological winner emerges, and the optimal policy relies solely on this technology to achieve the target. The second case provides a general parametric formulation that turns out to be analytically tractable. The shadow value of experience is generally decreasing when learning is high, especially at the beginning, when learning opportunities are high. This case allowed us to study how the learning rate and the elasticity of abatement affect the optimal allocation. It also provided insight into issues such as the allocation of abatement between mature and infant technologies.

Two of our main assumptions are that the target is exogenous and that technical change is modelled as learning by doing. The alternative to having an exogenous target would be to endogenize it through a benefit-cost maximization. In practice, however, objectives are often the result of political negotiations among institutional actors as much as they are the result of detailed benefit-cost considerations. For example, Stavins [1998, p. 77] notes that "many of the economists involved in the deliberations regarding the SO₂ allowance system took the approach of accepting – implicitly or otherwise – a political goal of reducing SO₂ emissions by 10 million tons. Rather than debating the costs and benefits of that goal, they simply focused on the cost-effective means of achieving it." Equally important, the analysis of a benefit-cost maximization would not bring additional insights into the logic of allocation between different technologies. For example, if benefit depends on cumulative abatement or if instantaneous benefit is linear in abatement, then the allocation of abatement across technologies can be equivalently determined through a cost minimization. Since our main objective is to understand the mechanisms driving the allocation of effort and not to understand which level of overall abatement should be pursued, we focused on cost-effectiveness.

As described above, learning by doing is a crucial source of technical change. Another important source of technical change is research and development, defined as specific investments directed to the invention of new technologies or the improvement of existing ones. Conceptually, learning by doing and R&D differ on a fundamental point. With learning by doing, improvement in technologies occurs as a byproduct of their use. With R&D, agents

have an additional control variable at their disposal. Nevertheless, the distinction between learning by doing and R&D may not be as clear cut as it may seem. Learning by doing aggregates numerous purposive efforts and investments intended to improve efficiency. In addition, it seems to be much easier to estimate empirically the effects of learning by doing on cost, than it is to estimate the effects of R&D. The learning curve provides a simple and empirically robust formulation, while more complex R&D production functions are notoriously hard to come by. Finally, we conjecture that many of the core principles emerging from our analysis would not change in the presence of R&D. For example, the trade-offs between infant and mature technologies and the role of the discount rate would certainly be qualitatively similar.

The analysis in this paper is potentially relevant for a broader set of problems. Our model is fundamentally a model of the allocation of effort between different agents, where the effort of each agent contributes to a common goal. The results of this paper may shed light on the effects of learning by doing on other allocation problems that share these characteristics such as some labor allocation and production planning problems.

There are a number of issues that can be examined in further research. The allocation of abatement after the target has been reached could be made endogenous. This will affect the precise allocation at the target, but should not alter the qualitative nature of our results in a significant way. Another interesting question is how learning spillovers across technologies influences the optimal allocation of abatement both over time and between technologies. Further, the theoretical model developed here could be used as the foundation for empirical research on the effect of learning by doing on pollution abatement. Such an analysis would provide a stronger foundation for more precise policy recommendations. Finally, it would be interesting to introduce uncertainty and examine the effects of stochastic learning on optimal pollution abatement.

8 Appendix

8.1 Proof of Proposition 3.1.

First assume that $\delta = 0$. Since $a_i = \dot{z}_i$, for any feasible path such that $\int_0^T a = A$, we have $\int_0^T ac(z)dt = \int_0^T c(z)\dot{z}dt = \int_{z_0}^{z_0+A} c(z)dz$. All feasible abatement paths have the same cumulative cost, hence all are solutions of the minimization problem.

Now assume that $\delta > 0$ and recall that the maximum time abatement can be delayed is given by $\tau = T - \frac{A}{\bar{a}}$. Define

$$z^*(t) = \begin{cases} z_0 & \text{if } t < \tau \\ z_0 + (t - \tau)\bar{a} & \text{if } t \geq \tau \end{cases}.$$

Let $z(t)$ be cumulative abatement along any other feasible policy. From the definition of $z^*(t)$ it follows that $z(t) \geq z^*(t)$ for all t (see Figure 5). Let $J^* = \int_0^T e^{-\delta t} c(z^*(t))dt$ and let $J = \int_0^T e^{-\delta t} c(z(t))dt$. Let Γ be the closed curve ABCA and let Ω be the region enclosed by Γ . Then $J^* - J$ can be written as the following line integral:

$$J^* - J = \oint_{\Gamma} e^{-\delta t} c(z)dz = \iint_{\Omega} -\delta e^{-\delta t} c(z)dzdt = -\delta \int_0^T e^{-\delta t} \int_{z^*(t)}^{z(t)} c(z)dz dt,$$

by Green's theorem.

Define $\tilde{C}(z) = \int_{z_0}^z c(s)ds$. $\tilde{C}(z)$ represents the accumulation of the marginal abatement costs incurred over the range $[z_0, z]$. Note that $\tilde{C}(z)$ is increasing in z . The cost differential between the two policies can now be expressed as:

$$J^* - J = -\delta \int_0^T e^{-\delta t} \left(\tilde{C}(z(t)) - \tilde{C}(z^*(t)) \right) dt \leq 0. \quad (7.1)$$

The inequality follows from the fact that $z^*(t) \leq z(t)$ implies $\tilde{C}(z(t))$ is always greater than $\tilde{C}(z^*(t))$. Equation (7.1) implies that J exceeds J^* by the annuity value of the lifetime discounted difference in accumulated marginal abatement costs, hence $z^*(t)$ must be optimal.

Note that when $\delta = 0$ it follows immediately that $J^* - J = 0$, which is another way of expressing that any feasible policy from z_0 to $z_0 + A$ is optimal with a zero discount rate.

Under a policy of maximal delay the cumulative cost of the optimal abatement path is

$$C(A) = \int_{\tau}^T e^{-\delta t} \bar{a} c(z_0 + \bar{a}(t - \tau)) dt.$$

A change of variables $z = z_0 + \bar{a}(t - \tau)$ leads to

$$C(A) = e^{-\delta \tau} \int_{z_0}^{z_0 + A} c(z) e^{-\delta \frac{z - z_0}{\bar{a}}} dz,$$

since $dz = \bar{a} dt$ for $t > \tau$. This completes the proof.

8.2 Proof of Proposition 3.2.

For simplicity of exposition, the proof is given under the assumption that the levels of initial experience are zero, i.e., $z_{i0} = 0$. This is without loss of generality, since the instantaneous cost functions can always be rescaled, e.g., $\hat{c}(a, z) = c(a, z_0 + z)$.

First, we show that for a technology in isolation, the cumulative cost $C_i(A_i)$ is concave on the interval $[0, A]$ if δ is sufficiently low and convex if δ is sufficiently high. From Proposition 3.2 and through successive derivations, we obtain:

$$\begin{aligned} e^{\delta T} C_i(A_i) &= e^{\delta \frac{A_i}{\bar{a}_i}} \int_0^{A_i} c_i(z_i) e^{-\delta \frac{z_i}{\bar{a}_i}} dz_i \\ e^{\delta T} C'_i(A_i) &= \frac{\delta}{\bar{a}_i} e^{\delta \frac{A_i}{\bar{a}_i}} \int_0^{A_i} c_i(z_i) e^{-\delta \frac{z_i}{\bar{a}_i}} dz_i + c(A) \\ e^{\delta T} C''_i(A_i) &= \left(\frac{\delta}{\bar{a}_i}\right)^2 e^{\delta \frac{A_i}{\bar{a}_i}} \int_0^{A_i} c_i(z_i) e^{-\delta \frac{z_i}{\bar{a}_i}} dz_i + \frac{\delta}{\bar{a}_i} c_i(A_i) + c'_i(A_i) \end{aligned}$$

with $c_i(A_i) > 0$ and $c'_i(A_i) < 0$, which imply that $C'_i(A_i) > 0$ and $C''_i(A_i)$ is ambiguous. We now derive a lower bound and an upper bound for C''_i . Since c_i is decreasing, we have for all $z_i \in [0, A_i]$, $c_i(A_i) \leq c_i(z_i) \leq c_i(0)$.

Since $\int_0^{A_i} e^{-\delta \frac{z_i}{\bar{a}_i}} dz_i = \frac{\bar{a}_i}{\delta} [1 - e^{-\delta \frac{A_i}{\bar{a}_i}}]$, this leads to

$$\frac{\delta}{\bar{a}_i} c_i(A_i) [e^{\delta \frac{A_i}{\bar{a}_i}} - 1] \leq \left(\frac{\delta}{\bar{a}_i}\right)^2 e^{\delta \frac{A_i}{\bar{a}_i}} \int_0^{A_i} c_i(z_i) e^{-\delta \frac{z_i}{\bar{a}_i}} dz_i \leq \frac{\delta}{\bar{a}_i} c_i(0) [e^{\delta \frac{A_i}{\bar{a}_i}} - 1]$$

and, since c'_i is increasing, for all $A_i \in [0, A]$, we obtain

$$\frac{\delta}{\bar{a}_i} c_i(A) + c'_i(0) \leq e^{\delta T} C''_i(A_i) \leq \frac{\delta}{\bar{a}_i} c_i(0) e^{\delta \frac{A}{\bar{a}_i}} + c'_i(A)$$

where $c'_i(0), c'_i(A) < 0$ and $c_i(0), c_i(A) > 0$.

Now introduce $\check{\delta}_i = \min(\frac{\bar{a}_i}{A}, \frac{-c'_i(A)\bar{a}_i}{c_i(0)e}) > 0$. If $\delta < \check{\delta}_i$ then $\delta \leq \frac{\bar{a}_i}{A}$ and $e^{\delta \frac{A}{\bar{a}_i}} \leq e$. Thus,

$$\frac{\delta}{\bar{a}_i} c_i(0) e^{\delta \frac{A}{\bar{a}_i}} + c'_i(A) \leq \frac{\delta}{\bar{a}_i} c_i(0) e + c'_i(A) < 0$$

and C_i is concave. On the other hand, define $\hat{\delta}_i = \frac{-c'_i(0)\bar{a}_i}{c_i(A)}$. If $\delta > \hat{\delta}_i$ then $\frac{\delta}{\bar{a}_i} c_i(A) + c'_i(0) > 0$ and C_i is convex. Hence, if $\delta < \min(\check{\delta}_1, \check{\delta}_2)$ then C_1 and C_2 are concave, and the solution to the second stage of the decomposition is at a corner. Similarly, if $\delta > \max(\hat{\delta}_1, \hat{\delta}_2)$ then C_1 and C_2 are convex. In this case, to insure that the solution is interior, one has to check that $C'_1(0) < C'_2(A)$ and $C'_2(0) < C'_1(A)$. We have

$$\begin{aligned} e^{\delta T} [C'_1(0) - C'_2(A)] &= c_1(0) - c_2(A) - \frac{\delta}{\bar{a}_2} \int_0^A c_2(z_2) e^{\delta \frac{A-z_2}{\bar{a}_2}} dz_2 \\ &\leq c_1(0) - c_2(A) - c_2(A) [e^{\delta \frac{A}{\bar{a}_2}} - 1] \end{aligned}$$

which tends to $-\infty$ as δ tends to $+\infty$, and the same is true for $e^{\delta T} [C'_2(0) - C'_1(A)]$.

If δ is high enough, the solution is unique and involves some abatement from each technology. Let $A_1(\delta)$ and $A_2(\delta) = A - A_1(\delta)$ denote the levels of cumulative abatement corresponding to this solution. In the following, the dependence of A_1 and A_2 on δ is implicitly assumed. We now show that as δ tends to $+\infty$, the solution tends to the strategy of maximal delay. That is, A_1 and A_2 satisfy $\frac{A_1}{\bar{a}_1} = \frac{A_2}{\bar{a}_2}$. Suppose that this is not the case, i.e., $\frac{A_1(\delta)}{\bar{a}_1} - \frac{A_2(\delta)}{\bar{a}_2}$ does not tend to 0 as δ tends to infinity. Without loss of generality, this means that there exists an $\varepsilon > 0$ and a sequence δ_n tending to infinity such that for all

$k, \frac{A_1(\delta_n)}{\bar{a}_1} > \frac{A_2(\delta_n)}{\bar{a}_2} + \varepsilon$. In the following, subscripts n are omitted for simplicity. We know that if δ is high enough, the solution is characterized by

$$C'_1(A_1) = C'_2(A_2).$$

This is equivalent to

$$-c_1(A_1) + c_2(A_2) = \delta \left[\int_0^{\frac{A_1}{\bar{a}_1}} c_1(A_1 - \bar{a}_1 s) e^{\delta s} ds - \int_0^{\frac{A_2}{\bar{a}_2}} c_2(A_2 - \bar{a}_2 s) e^{\delta s} ds \right].$$

The left hand side is bounded. We will show that the right hand side is unbounded, which will imply a contradiction. The term in square brackets on the right can be rewritten as

$$\int_0^{\frac{A_2}{\bar{a}_2}} [c_1(A_1 - \bar{a}_1 s) - c_2(A_2 - \bar{a}_2 s)] e^{\delta s} ds + \int_{\frac{A_2}{\bar{a}_2}}^{\frac{A_1}{\bar{a}_1}} c_1(A_1 - \bar{a}_1 s) e^{\delta s} ds.$$

We now show that the second component of this expression strictly dominates the first. To show this, notice first that the the first term is in the order of $e^{\delta \frac{A_2}{\bar{a}_2}}$. Effectively,

$$\left| \int_0^{\frac{A_2}{\bar{a}_2}} [c_1(A_1 - \bar{a}_1 s) - c_2(A_2 - \bar{a}_2 s)] e^{\delta s} ds \right| \leq M \int_0^{\frac{A_2}{\bar{a}_2}} e^{\delta s} ds \leq M e^{\delta \frac{A_2}{\bar{a}_2}}$$

with, for example, $M = c_1(0) + c_2(0)$. To evaluate how the second term evolves, we use the following result, based on the mean value theorem for increasing and convex functions.

Lemma

If f is continuous, increasing and convex, $a < b \in \mathbb{R}$ then there exists a $c \in [\frac{a+b}{2}, b]$ such that

$$\int_a^b f(x) dx = (b - a) f(c).$$

Since $c_1(A_1 - \bar{a}_1 s) e^{\delta s}$ is increasing and convex, we can apply the Lemma to the second term. This leads to

$$\int_{\frac{A_2}{\bar{a}_2}}^{\frac{A_1}{\bar{a}_1}} c_1(A_1 - \bar{a}_1 s) e^{\delta s} ds = \left(\frac{A_1}{\bar{a}_1} - \frac{A_2}{\bar{a}_2} \right) c_1(A_1 - \bar{a}_1 \hat{s}) e^{\delta \hat{s}}$$

for some $\hat{s} > \frac{1}{2}(\frac{A_1}{\bar{a}_1} + \frac{A_2}{\bar{a}_2})$. By assumption, $\frac{A_1(\delta)}{\bar{a}_1} > \frac{A_2(\delta)}{\bar{a}_2} + \varepsilon$, which yields

$$\int_{\frac{A_2}{\bar{a}_2}}^{\frac{A_1}{\bar{a}_1}} c_1(A_1 - \bar{a}_1 s) e^{\delta s} ds > \varepsilon c_1(A) e^{\delta \frac{A_2}{\bar{a}_2}} e^{\delta \frac{\varepsilon}{2}}.$$

This implies that

$$\frac{\int_{\frac{A_2}{\bar{a}_2}}^{\frac{A_1}{\bar{a}_1}} c_1(A_1 - \bar{a}_1 s) e^{\delta s} ds}{e^{\delta \frac{A_2}{\bar{a}_2}}} \rightarrow +\infty$$

as δ tends to infinity, which completes the proof.

8.3 Proof of Proposition 4.2.

By Lemma 4.1, cumulative cost is increasing and strictly convex. Hence, $C'(A) > 0$ for $A > 0$. The optimality conditions (4.1) then imply

$$c_a(a^*(T), A) \geq \mu(T) = C'(A) e^{-\delta T} > 0$$

From this it follows that $a^*(T) > 0$. For $t < T$, $e^{-\delta t} \mu(t) = e^{-\delta T} \mu(T) - \int_t^T e^{-\delta s} c_z(\cdot) ds > 0$ since $\mu(T) > 0$ and $c_z(\cdot) \leq 0$. Hence, $a^*(t) > 0$ for all t .

8.4 Proof of Proposition 4.3.

Take $t < T$ and suppose that $\int_0^t a^*(s) ds$ does not tend to 0 as δ tends to ∞ . This means that there exists an $\varepsilon > 0$ and a sequence $\delta_n \rightarrow +\infty$ such that for every δ_n , the optimal abatement path satisfies $\int_0^t a^*(s) ds \geq \varepsilon$. Let \hat{a} be an alternative abatement path defined as follows:

$$\hat{a}(s) = 0 \text{ if } 0 \leq s < t \text{ and } \hat{a}(t) = \frac{A}{T-t} \text{ if } t \leq s \leq T.$$

Since $\int_0^T \hat{a} = A$, \hat{a} is a feasible abatement path. We now show that δ is high enough, $\int_0^T c(\hat{a}, \hat{z}) e^{-\delta s} ds < \int_0^T c(a^*, z^*) e^{-\delta s} ds$, which will imply that a^* cannot be the optimal abatement path. We proceed by computing an appropriate lower bound for $\int_0^T c(a^*, z^*) e^{-\delta s} ds$

and an appropriate upper bound for $\int_0^T c(\hat{a}, \hat{z})e^{-\delta s} ds$. First, we have

$$\int_0^T c(\hat{a}, \hat{z})e^{-\delta s} ds \leq \int_t^T c\left(\frac{A}{T-t}, z_0\right)e^{-\delta s} ds \leq \frac{c\left(\frac{A}{T-t}, z_0\right)}{\delta} e^{-\delta t},$$

where the first inequality comes from the fact that c is decreasing in z , and the second is obtained by integrating. Second, we have

$$\int_0^T c(a^*, z^*)e^{-\delta s} ds \geq \int_0^t c(a^*, z^*)e^{-\delta s} ds \geq \int_0^t c(a^*, z_0 + A)e^{-\delta s} ds \geq e^{-\delta t} \int_0^t c(a^*, z_0 + A) ds,$$

since $z^* \leq z_0 + A$. By Theorem 204 in Hardy, Littlewood, and Polya [1934, p. 150], we know that for any convex and twice differentiable function f and any integrable function g

$$\int_0^t f(g(s)) ds \geq t f\left(\frac{1}{t} \int_0^t g(s) ds\right).$$

Applying this result to $c(a^*, z_0 + A)$ leads to

$$\int_0^T c(a^*, z^*)e^{-\delta s} ds \geq t e^{-\delta t} c\left(\frac{1}{t} \int_0^t a^*(s) ds, z_0 + A\right) \geq t e^{-\delta t} c\left(\frac{\varepsilon}{t}, z_0 + A\right),$$

since c is increasing in a . Combining both inequalities yields

$$\frac{\int_0^T c(a^*, z^*)e^{-\delta s} ds}{\int_0^T c(\hat{a}, \hat{z})e^{-\delta s} ds} \geq \delta \frac{t c\left(\frac{\varepsilon}{t}, z_0 + A\right)}{c\left(\frac{A}{T-t}, z_0\right)}.$$

The right hand side tends to $+\infty$ as $\delta \rightarrow +\infty$.

As a consequence, if

$$\delta > \bar{\delta} = \frac{c\left(\frac{A}{T-t}, z_0\right)}{t c\left(\frac{\varepsilon}{t}, z_0 + A\right)}$$

then $\int_0^T c(\hat{a}, \hat{z})e^{-\delta s} ds < \int_0^T c(a^*, z^*)e^{-\delta s} ds$ and a^* cannot be the optimal abatement path. Hence, for $\delta > \bar{\delta}$ it must be the case that $\int_0^t a^*(s) ds < \varepsilon$. For a given $\varepsilon > 0$, $\bar{\delta}$ provides an explicit threshold (possibly not the tightest one) insuring that $\int_0^t a^*(s) ds < \varepsilon$. Note that $\bar{\delta}$ increases when A increases, T increases, or ε decreases.

8.5 Proof of Proposition 4.4.

Suppose that it is optimal to rely on a single technology and without loss of generality suppose $A_2^* = 0$. Then $a_2^*(t) = 0$ for all t . From this we get

$$0 = c_{2a}(0, 0) \geq \mu_2(T) = e^{-\delta T} C_2'(0) \geq e^{-\delta T} C_1'(A) > 0$$

The first inequality follows from the optimality conditions (4.1). The second inequality is due to $A_2^* = 0$, while the third (strict) inequality holds since C_1 is strictly increasing and strictly convex in A . Combining these yields a contradiction. Identical arguments imply that A_1^* cannot be zero.

8.6 Proof of Proposition 4.5.

(a) The proof follows from the Fundamental Decomposition Principle, Proposition 4.4, and the strict convexity of the cumulative cost functions.

(b) The equation of motion for μ_i is $\mu_i - \delta\mu_i = c_{iz}(\cdot)$. Multiplying by $e^{-\delta t}$ it leads to

$$\frac{d}{dt}[\mu_i e^{-\delta t}] = c_{iz}(\cdot) e^{-\delta t}.$$

Integrating between t and T implies

$$\mu_i(t) e^{-\delta t} = \mu_i(T) e^{-\delta T} - \int_t^T c_{iz}(\cdot) e^{-\delta s} ds. \quad (7.2)$$

Since $A_i^* > 0$, the allocation of cumulative abatement is given by $C_1'(A_1^*) = C_2'(A_2^*)$. By Lemma 4.1, this implies $\mu_1(T) = \mu_2(T)$. Since $a_i^*(t) > 0$, we know that $\mu_i = c_{ia}(\cdot)$, which leads to

$$c_{1a}(\cdot) + \int_t^T c_{1z}(\cdot) e^{-\delta(s-t)} ds = c_{2a}(\cdot) + \int_t^T c_{2z}(\cdot) e^{-\delta(s-t)} ds.$$

The final part is obtained by using (7.2) and substituting $c_{ia} = \mu_i$.

8.7 Proof of Proposition 5.1.

For simplicity we suppress the arguments of $c(a, z)$ and its derivatives. Differentiating (4.1) with respect to t and substituting for μ and $\dot{\mu}$ in (4.2) yields the following differential equation:

$$\dot{a}c_{aa} + ac_{az} = \delta c_a + c_z. \quad (7.4)$$

The derivatives of the constant elasticity cost function are:

$$\begin{aligned} c_a &= \beta a^{-1} c \\ c_z &= -\alpha z^{-1} c \\ c_{aa} &= \beta(\beta - 1)a^{-2} c \\ c_{az} &= -\alpha\beta a^{-1} z^{-1} c. \end{aligned}$$

The differential equation (7.4) then becomes

$$\beta(\beta - 1)\dot{a}a^{-2}c - \alpha\beta z^{-1}c = \delta\beta a^{-1}c - \alpha z^{-1}c.$$

If we divide by c and multiply by a^2z , we obtain

$$\beta(\beta - 1)\dot{a}z - \alpha(\beta - 1)a^2 - \delta\beta az = 0.$$

Substituting \dot{a} by \ddot{z} and a by \dot{z} leads to

$$\beta(\beta - 1)\ddot{z}z - \alpha(\beta - 1)\dot{z}^2 - \delta\beta\dot{z}z = 0.$$

This is a second order nonlinear differential equation in z , with exogenous initial and terminal conditions $z(0) = z_0$ and $z(T) = z_0 + A$. If we divide by $z\dot{z}$, we obtain

$$\beta(\beta - 1)\frac{\ddot{z}}{\dot{z}} - \alpha(\beta - 1)\frac{\dot{z}}{z} - \delta\beta = 0.$$

We can now integrate with respect to t . There is a constant k such that

$$\beta(\beta - 1) \ln(\dot{z}) - \alpha(\beta - 1) \ln(z) - \delta\beta t - k = 0$$

which leads to

$$\dot{z} z^{-\frac{\alpha}{\beta}} = e^{\frac{k}{\beta(\beta-1)}} e^{\frac{\delta t}{\beta-1}}.$$

Integrating once more and defining $k_2 = \frac{\beta-1}{\delta} \frac{\beta-\alpha}{\beta} e^{\frac{k}{\beta(\beta-1)}}$, there is another constant k_1 such that

$$\begin{aligned} z^{\frac{\beta-\alpha}{\beta}} &= k_1 + k_2 e^{\frac{\delta t}{\beta-1}} \\ \text{or } z^*(t) &= [k_1 + k_2 e^{\frac{\delta t}{\beta-1}}]^{\frac{\beta}{\beta-\alpha}}. \end{aligned}$$

The two constants k_1 and k_2 are determined by the initial and terminal conditions

$$\begin{aligned} [k_1 + k_2]^{\frac{\beta}{\beta-\alpha}} &= z_0 \\ [k_1 + k_2 e^{\frac{\delta T}{\beta-1}}]^{\frac{\beta}{\beta-\alpha}} &= z_0 + A. \end{aligned}$$

The optimal abatement path is then simply obtained by differentiating $z^*(t)$ with respect to time to obtain

$$a^*(t) = \dot{z}^* = k_2 \frac{\delta}{\beta-1} \frac{\beta}{\beta-\alpha} e^{\frac{\delta t}{\beta-1}} [k_1 + k_2 e^{\frac{\delta t}{\beta-1}}]^{\frac{\beta}{\beta-\alpha}}.$$

When $\delta = 0$, the same reasoning leads to

$$z^*(t) = [k_1 + k_2 t]^{\frac{\beta}{\beta-\alpha}}$$

with

$$\begin{aligned} [k_1]^{\frac{\beta}{\beta-\alpha}} &= z_0 \\ [k_1 + k_2 T]^{\frac{\beta}{\beta-\alpha}} &= z_0 + A \end{aligned}$$

which is equivalent to

$$\begin{aligned} k_1 &= z_0^{\frac{\beta-\alpha}{\beta}} \\ k_2 &= \frac{1}{T}[(z_0 + A)^{\frac{\beta-\alpha}{\beta}} - z_0^{\frac{\beta-\alpha}{\beta}}]. \end{aligned}$$

8.8 Proof of the Corollary to Proposition 5.1.

Let us first show that $z^*(t)$ decreases as δ increases. We have

$$z^*(t) = [k_1 + k_2 e^{\eta t}]^\gamma = [z_0^{\frac{1}{\gamma}} + \frac{e^{\eta t} - 1}{e^{\eta T} - 1}[(z_0 + A)^{\frac{1}{\gamma}} - z_0^{\frac{1}{\gamma}}]]^\gamma.$$

Define an auxilliary function φ as

$$\varphi(\eta) = \frac{e^{\eta T} - 1}{e^{\eta T} - 1}.$$

If φ is decreasing in η then $z^*(t)$ decreases as δ increases. Evaluate the derivative of φ :

$$\varphi'(\eta) = \frac{e^{\eta(t+T)}}{(e^{\eta T} - 1)^2} [t(1 - e^{-\eta t}) - T(1 - e^{-\eta T})]$$

which means that

$$\varphi'(\eta) \leq 0 \text{ if and only if } \frac{t}{T} \leq \frac{1 - e^{-\eta t}}{1 - e^{-\eta T}}.$$

Now consider the second inequality. As functions of t , both terms are positive, increasing, and take the same value at $t = T$. The left hand side is lower than the right hand side at $t = 0$, the left hand side is linear, and the right hand side is concave, hence the inequality is always satisfied and $\varphi'(\eta) \leq 0$.

Let us now evaluate the limit of $a^*(t)$ as $\beta \rightarrow \infty$. When $\beta \rightarrow \infty$, $\eta \rightarrow 0$ and $\gamma \rightarrow 1$. Since $e^{\eta t} - 1 \sim \eta t$ when $\eta \rightarrow 0$, it follows that

$$k_1 + k_2 e^{\eta t} = z_0^{\frac{1}{\gamma}} + \frac{e^{\eta t} - 1}{e^{\eta T} - 1}[(z_0 + A)^{\frac{1}{\gamma}} - z_0^{\frac{1}{\gamma}}] \rightarrow z_0 + \frac{t}{T}A.$$

Hence

$$[k_1 + k_2 e^{\eta t}]^{\gamma-1} \rightarrow 1$$

and

$$\eta \gamma k_2 e^{\eta t} \sim \frac{\eta}{e^{\eta T} - 1} A \rightarrow \frac{A}{T}.$$

When $\alpha \rightarrow 0$, $\gamma \rightarrow 1$, which implies

$$k_1 + k_2 e^{\eta t} = z_0^{\frac{1}{\gamma}} + \frac{e^{\eta t} - 1}{e^{\eta T} - 1} [(z_0 + A)^{\frac{1}{\gamma}} - z_0^{\frac{1}{\gamma}}] \rightarrow z_0 + \frac{e^{\eta t} - 1}{e^{\eta T} - 1} A.$$

Hence

$$[k_1 + k_2 e^{\eta t}]^{\gamma-1} \rightarrow 1$$

and

$$\begin{aligned} a^*(t) &\rightarrow \frac{\eta A}{e^{\eta T} - 1} e^{\eta t} \\ z^*(t) &\rightarrow z_0 + \frac{e^{\eta t} - 1}{e^{\eta T} - 1} A. \end{aligned}$$

Now consider the following expressions when the initial experience z_0 tends to infinity:

$$\begin{aligned} k_1 + k_2 e^{\eta t} &= z_0^{\frac{1}{\gamma}} + \frac{e^{\eta t} - 1}{e^{\eta T} - 1} [(z_0 + A)^{\frac{1}{\gamma}} - z_0^{\frac{1}{\gamma}}] \\ &= z_0^{\frac{1}{\gamma}} [1 + \frac{e^{\eta t} - 1}{e^{\eta T} - 1} [(1 + \frac{A}{z_0})^{\frac{1}{\gamma}} - 1]]. \end{aligned}$$

Using the fact that $(1 + \frac{A}{z_0})^{\frac{1}{\gamma}} - 1 \sim \frac{1}{\gamma} \frac{A}{z_0}$ this leads to

$$z^*(t) - z_0 = [k_1 + k_2 e^{\eta t}]^{\gamma} - z_0 \sim z_0 [(1 + \frac{e^{\eta t} - 1}{e^{\eta T} - 1} \frac{1}{\gamma} \frac{A}{z_0})^{\gamma} - 1]$$

and

$$z^*(t) - z_0 \rightarrow \frac{e^{\eta t} - 1}{e^{\eta T} - 1} A.$$

Next, examine the shadow value of experience $\mu(t)$. We know that $\mu(t) = c_a =$

$\theta\beta a^*(t)^{\beta-1}(z_0 + z^*(t))^{-\alpha}$. Substituting the expressions for $a^*(t)$ and $z^*(t)$, we obtain

$$\mu(t) \sim e^{\delta t} [k_1 + k_2 e^{\frac{\delta}{\beta-1}t}]^{-\frac{\alpha}{\beta-\alpha}}.$$

Differentiating, and rearranging, we obtain

$$\text{sgn}(\dot{\mu}) = \text{sgn}[k_1 + (1 - \frac{\alpha}{(\beta-\alpha)(\beta-1)})k_2 e^{\eta t}]$$

which implies that if $\dot{\mu}(t) > 0$ then $\dot{\mu} > 0$ on $[t, T]$.

8.9 Proof of Proposition 6.1

A myopic agent chooses a_i to satisfy $c_{ia}(\cdot) = \tau_i^m$. Therefore, the optimal tax rate should equal the marginal abatement cost of the optimal abatement path, $c_{ia}(a_i^*(t), z_i^*(t)) = \mu_i(t)$. The optimal tax then follows from the expression of the shadow value of experience derived in section 4.1.

A perfectly rational agent chooses $a_i(t)$ by solving the following optimization problem:

$$\begin{aligned} \min_{a_i(t)} & \int_0^T [\tau_i^r(t)(\bar{s}_i - a_i(t)) + c(a_i(t), z_i(t))] e^{-\delta t} dt \\ \text{subject to } 0 & \leq a_i(t) \leq \bar{s}_i \text{ and } \dot{z} = a_i(t) \end{aligned}$$

The Hamiltonian is $H = -[\tau_i^r(t)(\bar{s}_i - a_i(t)) + c(a_i(t), z_i(t))] + \lambda_i(t)a_i(t)$, where $\lambda_i(t)$ is the shadow value of experience. The necessary and sufficient first order conditions are

$$\begin{aligned} c_{ia}(\cdot) &= \tau_i^r + \lambda_i \\ \text{and } \dot{\lambda}_i - \delta\lambda_i &= c_{iz}(\cdot). \end{aligned}$$

The terminal condition is given by

$$c_{ia}(a_i(T), z_i(T)) = \tau_i^r(T).$$

At the last period the agent does not expect subsequent cost reducing effects of experience

and just sets marginal abatement cost equal to the tax.

Consider now the corresponding first order conditions of the optimal abatement path

$$\begin{aligned} c_{ia}(a_i^*, z_i^*) &= \mu_i \\ \text{and } \dot{\mu} - \delta\mu &= c_{iz}(a_i^*, z_i^*). \end{aligned}$$

Since we want that $a_i(t) = a_i^*(t)$, it must be that

$$\tau_i^r(t) = \lambda_i(t) - \mu_i(t)$$

Differentiating and using the dynamic optimality condition yields

$$\dot{\tau}_i^r - \delta\tau_i^r(t) = 0.$$

Therefore the tax rate grows exponentially through time. Since we know that $\tau_i^r(T) = \tau_i^*$, we have $\tau_i^r(t) = \tau_i^* e^{\delta(t-T)}$.

Finally, suppose we have a final tax rate τ_i on cumulative emissions. An agent chooses cumulative abatement in order to minimize the cumulative cost $\tau_i(\int_0^T \bar{s}_i - A_i)e^{-\delta T} + C(A_i)$, which leads to $\tau_i = C'(A_i)e^{\delta T} = \mu_i(T)$.

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Figure 1: Linear abatement costs

$$c_1(z) < c_2(z) \text{ iff } z < z_1 \quad \text{and} \quad \int_0^z c_1(s) ds < \int_0^z c_2(s) ds \text{ iff } z < z_2$$

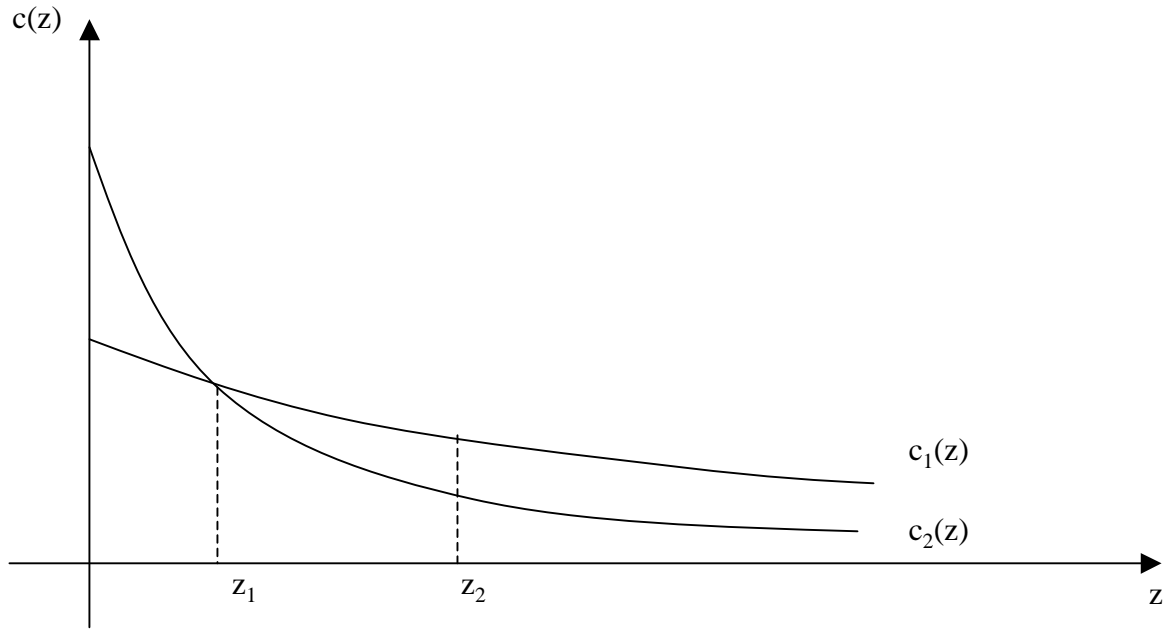
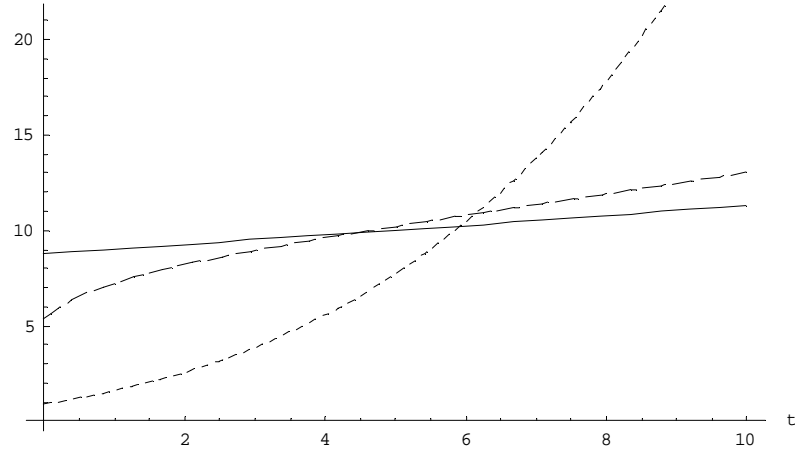


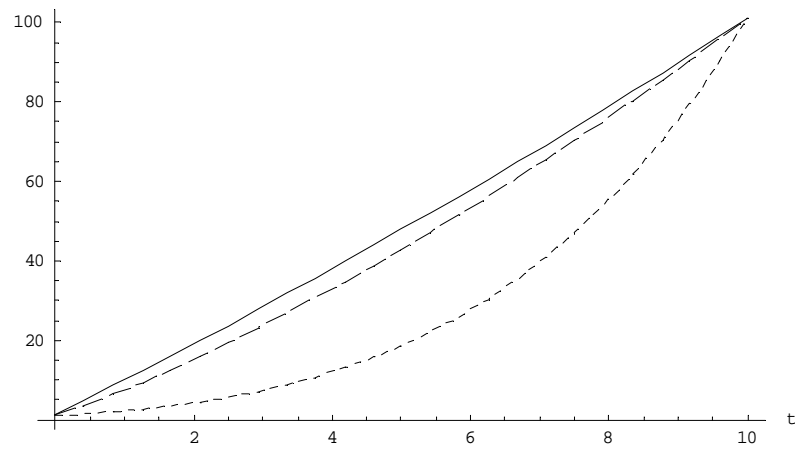
Figure 2: optimal solutions for various learning rates

Case 1: $\alpha_1=10^{-4}$ Case 2: $\alpha_2=0.7$ Case 3: $\alpha_3=3.5$ $A=100$ $T=10$ $\delta=0.1$ $\beta=5$ $z_0=1$

Abatement $a(t)$



Cumulative abatement $z(t)$



Shadow value of experience $\mu(t)$

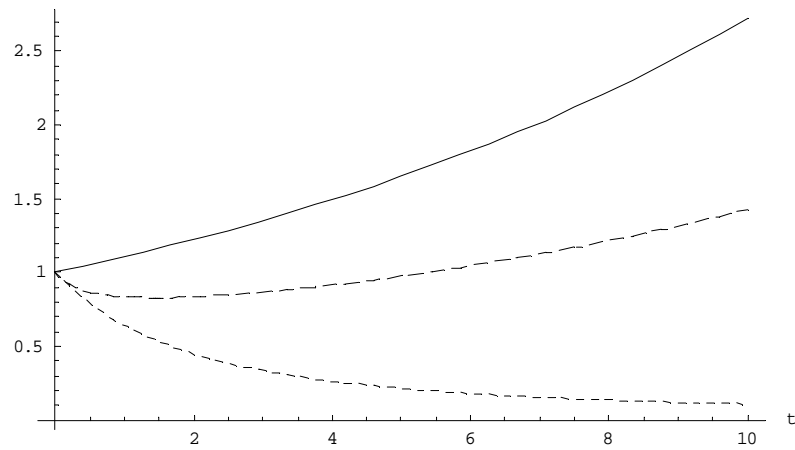
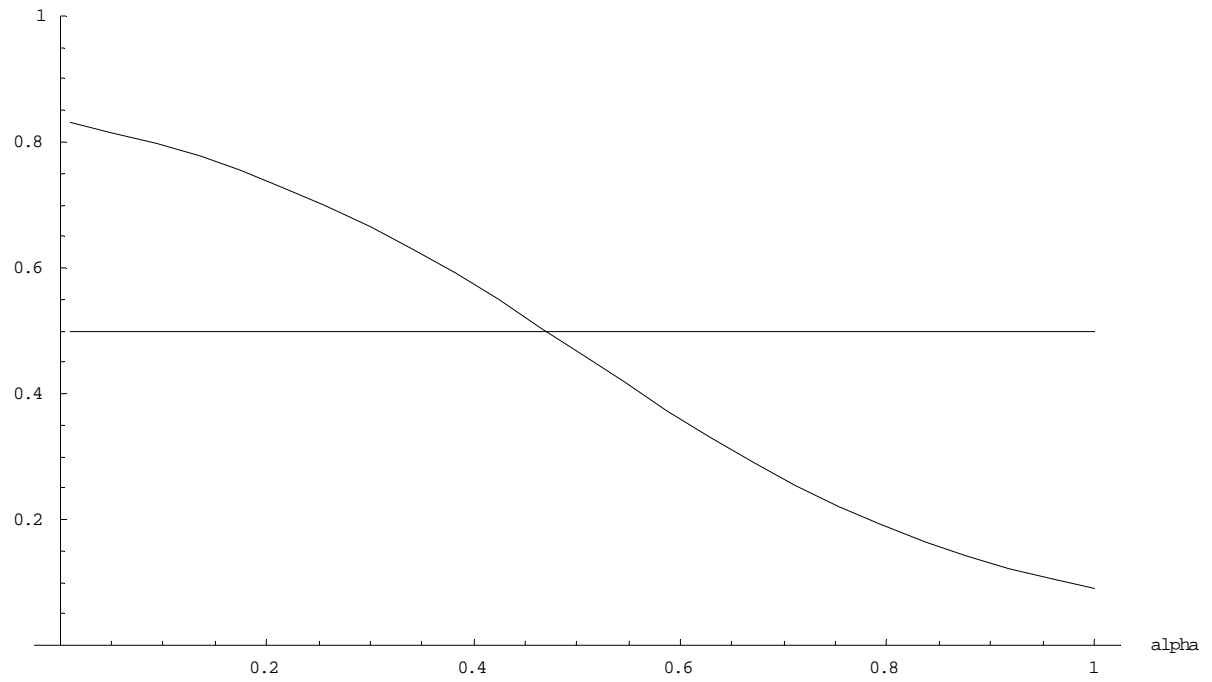


Figure 3: share of cumulative abatement allocated to the mature technology
 $z_0(\text{infant})=1$ $z_0(\text{mature})=10^6$ $c_0(\text{infant})=5*c_0(\text{mature})$ $T=10$ $\delta=0.1$ $\beta=2$

Cumulative abatement $A=100$



Cumulative abatement $A=10^5$

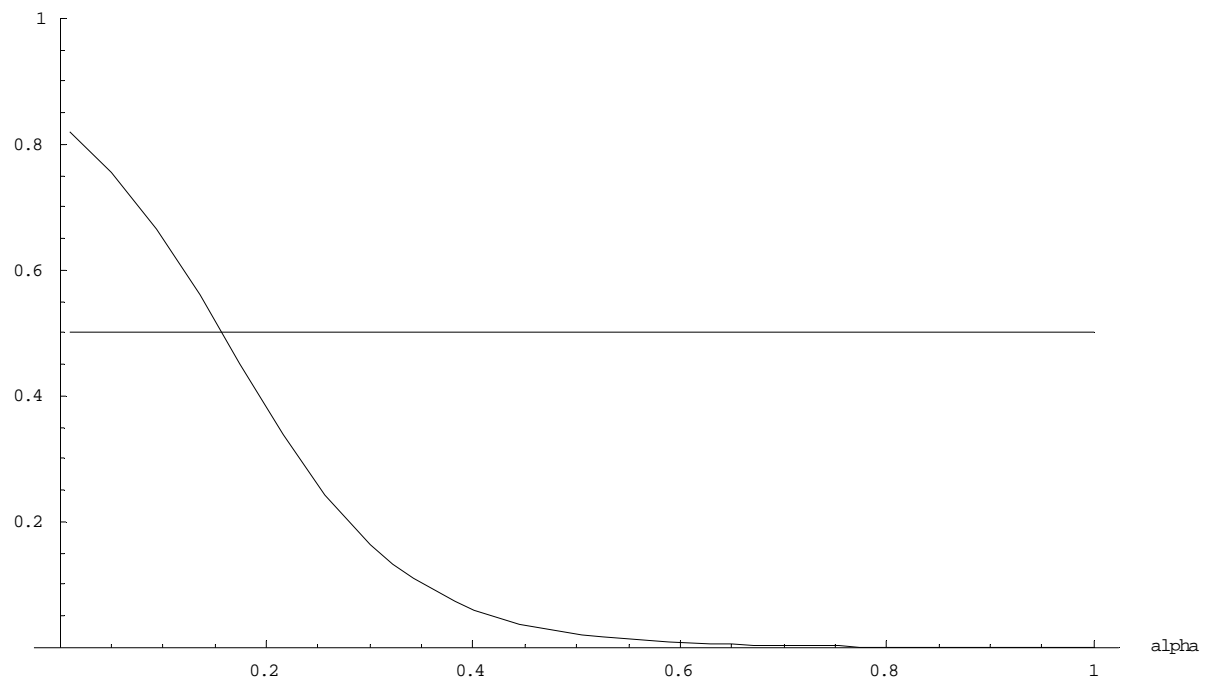
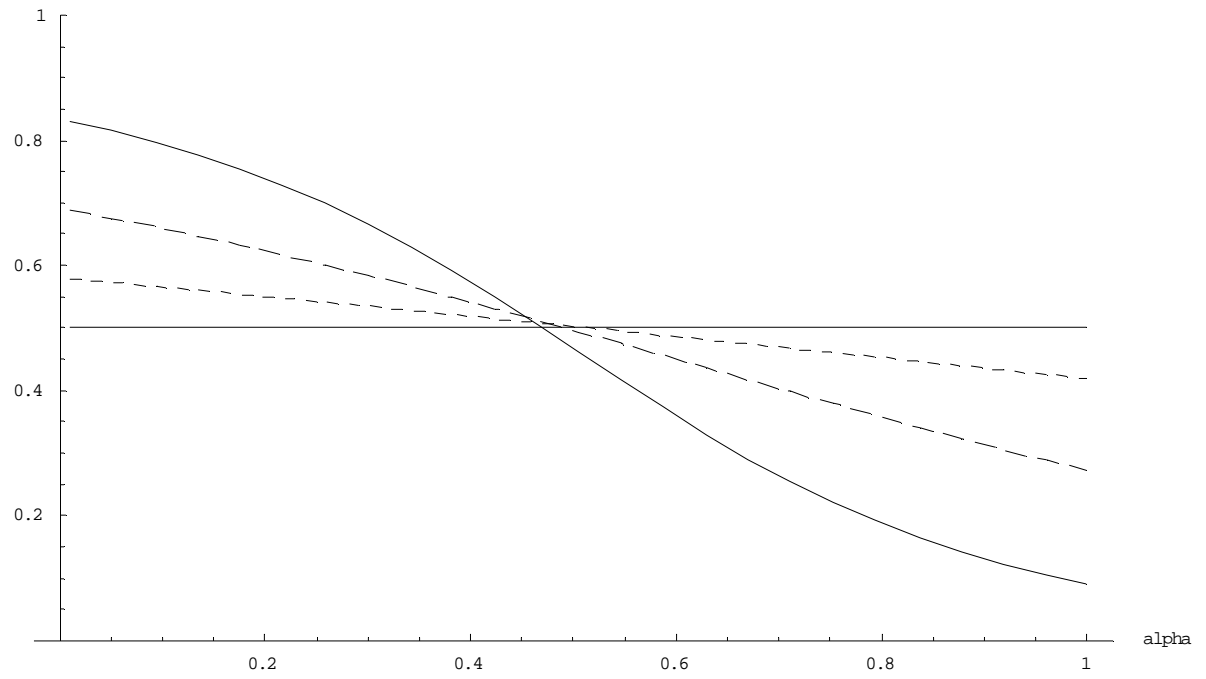


Figure 4: effect of the abatement elasticity

Case 1: $\beta_I=2$ Case 2: $\beta_2=3$ Case 3: $\beta_3=6$ $T=10$ $\delta=0.1$ $A=100$



Case 1: $\beta_I=\beta_M=2$ Case 2: $\beta_M=2 < \beta_I=2.5$ Case 3: $\beta_M=2.5 > \beta_I=2$

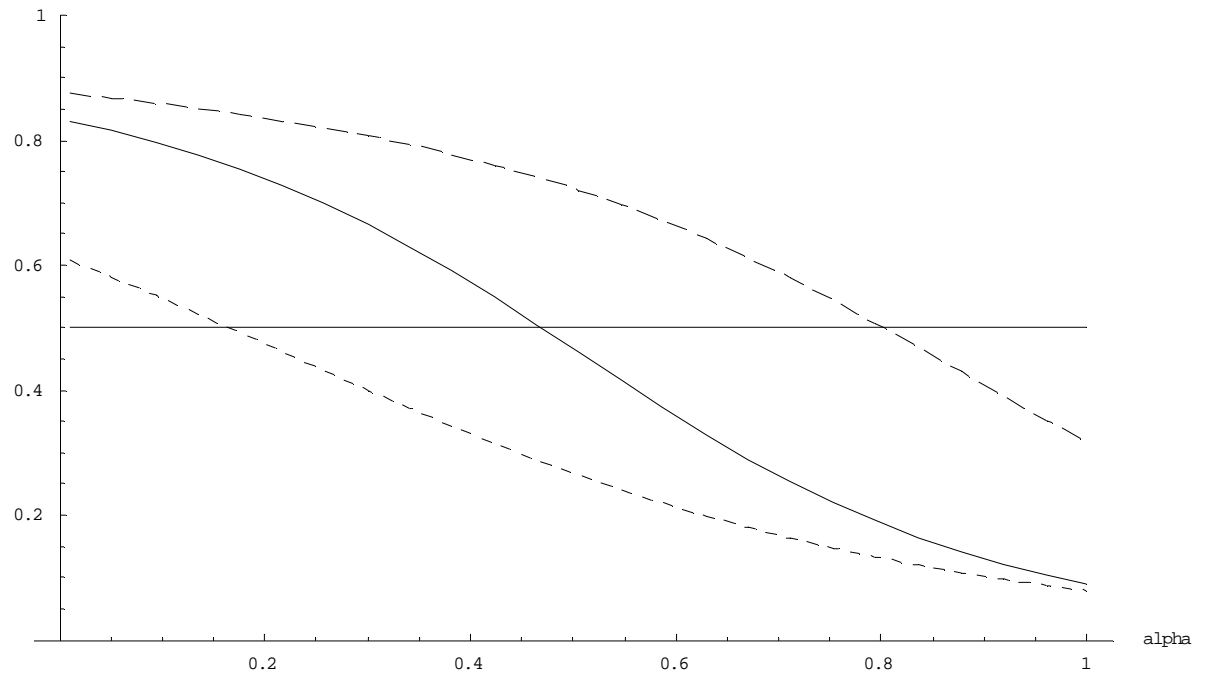


Figure 5

Optimality of maximal delay via Green's theorem

