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Exploring Underlying Distributional Assumptions of Livestock Gross Margin Insurance for Dairy

Practitioner's Abstract: *Livestock Gross Margin Insurance for Dairy Cattle (LGM-Dairy) is a recently introduced tool for protecting average income over feed cost margins in milk production. In this paper we examined the assumptions underpinning the rating method used to determine premium charged for LGM-Dairy insurance contract. The first test relates to assumption of lognormality in terminal futures prices. Using high-frequency data for futures and options for milk, corn and soybean meal we estimate implied densities with flexible higher moments. Simulations indicate there is no strong evidence that imposing lognormality introduces bias in LGM-Dairy premiums. The rest of the paper is dedicated to examining dependency between milk and feed marginal distributions. LGM-Dairy rating method imposes the restriction of zero conditional correlation between milk and corn, as well as milk and soybean meal futures prices. Using futures data from 1998-2011 period we find that allowing for non-zero milk-feed correlations considerably reduces LGM-Dairy premiums for hedging profile with substantial feed amounts declared. Further examination of the nature of milk-feed dependencies reveals that Spearman's correlation coefficient is mostly reflecting tail dependence. Using empirical copula approach we find that non-parametric method of modeling milk-feed dependence decreases LGM-Dairy premiums more than a method that allows only for linear correlation. Unlike all other situations in portfolio risk assessment where extremal dependence increases risk, in agricultural margins, tail dependence between inputs and outputs may actually decrease insurance risk, and reduce actuarially fair premiums.*

Keywords: LGM-Dairy, margin insurance, generalized lambda distribution, tail dependence, empirical copula

Introduction

Agricultural crop insurance has been around for many decades. These insurance products originated with the authorization of the Federal Crop Insurance Corporation, FCIC, under Title V of the Agricultural Act of 1938. It is the stated "purpose of this subtitle to promote the national welfare by improving the economic stability of agriculture through a sound system of crop insurance and providing the means for the research and experience helpful in devising and establishing such insurance." Each new agricultural act or farm bill reauthorizes the FCIC and this is true of the current Food, Conservation, and Energy Act of 2008, Public Law 110-246.

The LGM-Dairy margin insurance product was submitted to the Federal Crop Insurance Corporation (FCIC) for consideration and approval for sale in July of 2006.[1] This product received final approval from the FCIC in 2007 and was made available for sale starting with the August 2008 sales month. Initially the product was made available to dairy farmers in 32 states

[1] The LGM-Dairy Insurance product was developed by Iowa Agricultural Insurance Innovations Consortium, L.L.C.(IAII). Section 508(h) of the FCIA provides for the submission of new insurance products to the FCIC, which, if found to be actuarially sound, can be offered for sale by certified agricultural crop insurance agencies. The product itself is owned by IAII, now dba Farm Risk. IAII has created other livestock insurance products, such as LGM-Swine and LGM-Cattle prior to LGM-Dairy.

and later this was extended to all contiguous 48 states. A few of the initial provisions for the LGM-Dairy product have been modified, such as now providing for a premium subsidy, extending the sales period, making premium payment at the end of the contract period, and increasing the maximum amount of the deductible. These changes became effective with the December 2010 contract month.

A key feature of these insurance products is the calculation of the premium as being actuarially fair. An actuarially fair premium is one for which the calculated premium equals the expected loss on the contract. To make this calculation a number of assumptions must be made regarding the distributional characteristics of pertinent prices. Get these correct, and the calculated premium is actuarially fair, yet if these assumptions are incorrect, then the premium is not fair.

In this article, we take a close look at the assumptions underpinning the calculations of the premium structure for LGM-dairy insurance product. We investigate the assumption of lognormal distributions for these prices and the assumed zero-correlation structure between milk price and feed prices.

Anticipating our conclusions, we find that log-normality is a reasonable simplification while the zero correlation between milk and feed price is not. This leads to the conclusion that LGM-dairy insurance is currently over-priced, and substantially so, with the current method for premium calculation. We propose a more robust method for premium calculation which results in actuarially fair but substantially lower premiums for the LGM-dairy insurance product. Applying our premium calculation method we show that LGM-dairy can be a net revenue stabilizing management tool for dairy farmers, and is substantially less expensive relative to market based options on milk and feed prices.

This paper has the following sections. First, we provide a short review of the earlier work on LGM insurance by Hart, Babcock and Hayes (2001). Next, we review LGM-dairy and its current method for calculating the premium. In section three, we consider each of the assumptions and the impact these have on the premium calculation. In the fourth section we show how our method, incorporating more appropriate assumptions based on empirical data, for premium calculation results in substantially lower premiums and provide empirical evidence of the magnitude of these savings. Finally, in the last section we provide our summary and conclusions.

Past Literature

In their 2001 paper, Hart, Babcock and Hayes introduce the concept of a livestock net revenue insurance product for swine and cattle production. Livestock net revenue insurance is designed as an Asian style basket put option. By this design, LGM-Dairy is an insurance product which, in exchange for a premium paid by the buyer, provides a payment in the event that actual gross margin (realized price) is less than expected gross margin (strike price) at the end of the insurance contract period. Gross margin is calculated as the difference between expected gross revenue and expected purchased feed cost. If, at the end of the contract period the realized price is less than the strike price less the stated deductible, an indemnity is paid to the policy holder, otherwise the put expires out of the money.

There are three key assumptions underpinning the Hart, Babcock and Hayes approach: 1) marginal distributions of input and product prices at the contract expiration are lognormal; 2) futures prices at contract sales time are efficient and unbiased expectations of terminal futures prices; and 3) the use of Iman-Conover (1982) method with historical rank correlations and the use of van der Waerden scores, is a reasonable approach to model dependency among the price variables of interest. Distribution of realized gross margin is then calculated numerically, payouts are calculated as difference, if positive, between expected and simulated gross margins, and, finally, actuarially fair insurance premium is set as the expected value of payouts. In their 2006 paper, Hart, Hayes and Babcock, suggest that livestock gross margin insurance can be treated as a "whole-farm" insurance, whereby an insurance product is designed that accounts for the diversified farm at which crops are grown and fed to livestock. Insuring jointly crop production risk, and livestock margin risk may reduce the price of an insurance policy, compared to policies that focus just on a crop, or solely on livestock.

In a series of papers, dairy scientists and economists from University of Wisconsin-Madison (Gould, Mitchell and Cabrera, 2008; Cabrera, Gould and Valvekar, 2010) explain and evaluate Livestock Gross Margin for Dairy Cattle (LGM-Dairy). Cabrera and Solis (2010) evaluated the usefulness of climate forecasts in choosing LGM-Diary coverage levels by Wisconsin dairy farms. Valvekar, Cabrera, and Gould (2010) propose a decision tool based on nonlinear programming model that a producer can use to minimize the contract premium, subject to desired level of income over feed costs. Finally, Valvekar, Chavas, Gould and Cabrera (2011) looked at the role of risk aversion and premium subsidies on percent of milk insured via LGM-Dairy. Similar question has been addressed by Thraen (2012) who suggests that the amount of gross margin covered should be chosen with the objective of keeping the ratio of total equity to total assets above the minimum threshold level at which financial health of the dairy becomes a major concern.

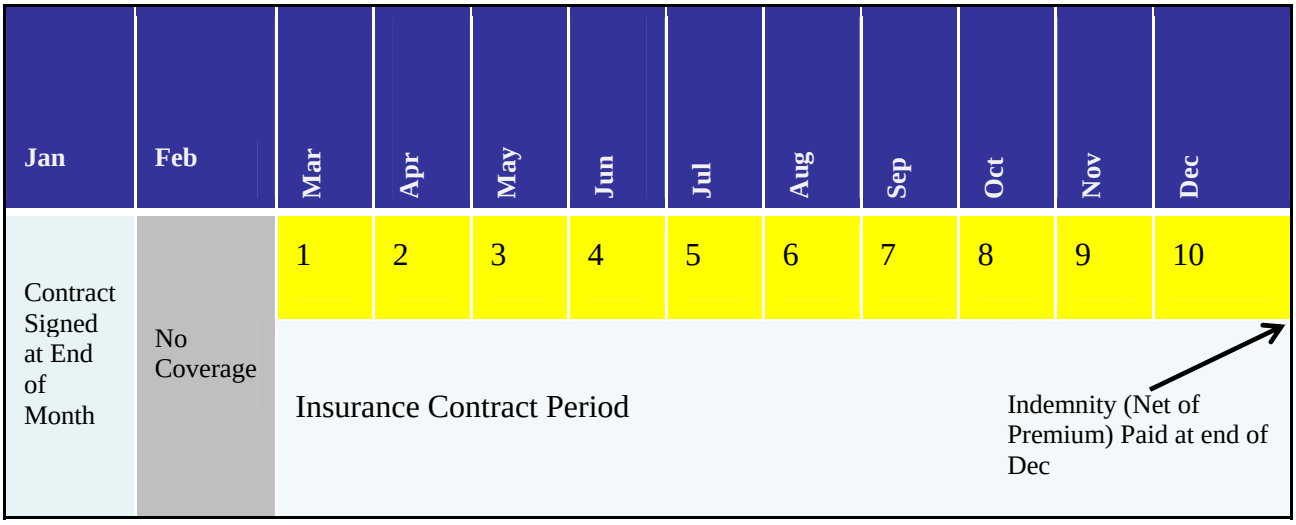
LGM-Dairy Insurance: Product Overview

The need for adequate price and revenue risk management tools for use by dairy producers is widely recognized. Bozic, Kanter and Gould (2011) report that milk price range-based volatility has increased from 16.5% in 1980-1985, and 32.5% in 1990-1995 to 67.3% in 2005-2010 period.[2] In response to increased price risk, various dairy futures and options contracts have been designed since 1993, with cash-settled Class III milk futures and options emerging as the largest and most liquid of six CME dairy products traded (Bozic and Fortenbery, 2011). Traditionally, with stable feed prices, examining dairy farm net revenue volatility could be addressed by focusing on the volatility of farm-milk prices. With recent increases in the level and volatility in feed grain prices the focus of attention in the dairy industry has turned from trying to manage milk ***price*** volatility to one of managing ***gross milk revenue net of feed cost*** volatility. In face of these developments, current federal dairy price support policy, one that is primarily based on a nominal level of $9.90 per cwt. and a supplemental target price deficiency payment program known as Milk Income Loss Contract, or MILC, is seen as an outdated and inadequate safety net.

[2] They define range-based volatility measure as the difference of the highest to lowest monthly all-milk price observed over a particular time period, divided by the average price evaluated over the same period, and expressed as percentage.

The LGM-Dairy insurance product allows dairy farm operators to purchase insurance to protect against decreases in their gross margin, where gross margin is defined as the difference between milk revenue and feed costs (or income over feed cost, IOFC).[3] Under this insurance policy, an indemnity at the end of coverage period is the difference, if positive, between the total contract anticipated gross margin determined at contract purchase and total actual gross margins obtained by the end of the insurance contract. Figure 1 illustrates the insurance contract with the sales event set for January.

Figure 1. Example of a LGM-Dairy contract life-cycle.



Under LGM-Dairy contract, expected milk revenues are based on the three day average Class III futures contract settle prices $\left(f^{M}_{t+i,t}\right)$ prior to contract purchase, and declared milk marketings in each of the 10 insurable months $\left(M_{t+i}\right)$. Feed costs are based on the three day average futures settle prices for corn and soybean meal $\left(f^{C}_{t+i,t}; f^{SBM}_{t+i,t}\right)$ and declared feed amounts for each of the months covered by the contract $\left(C_{t+i}; SBM_{t+i}\right)$. Prices for contract months not traded are defined as weighted average of surrounding months. For example, expected corn price for January is the weighted average of December and March corn futures prices. In addition to monthly milk marketings and feed usage, a farmer must decide on Gross Margin Deductible $\left(D\right)$, i.e. threshold decline in expected gross margin at which LGM-Dairy will begin paying indemnities. For example, if a farmer chose $1.10 deductible and declared 40,000 cwt total milk marketings over the insurance contract period then LGM-Dairy will not pay any indemnities if shortfall in gross margin is less than $44,000. Deductible amounts from $0.00 to $2.00/cwt are allowed. LGM-Dairy contracts can be purchased once each month after the futures markets close on the last business Friday of the month. Only one LGM-Dairy contract can be purchased by a dairy operation per month, and a farmer may insure at most 10 months of revenue under any one

[3] Milk revenue includes only the gross revenue from the sale of milk. On a typical U.S. dairy farm, gross revenue will incorporate additional sources of revenue generated by the sale of livestock for example. This additional revenue is not included in the LGM-Dairy definition.

contract, excluding the first month after the sales date. Maximum insurance coverage is limited to 240,000 cwt. per farm operation. Given the decision on milk marketings, purchased feed usage and deductible level, gross margin guarantee is calculated as:

$$G = \sum_{i=2}^{11}\left(f_{t+i}^{M} - D\right)\times M_{t+i} - \sum_{i=2}^{11} f_{t+i}^{C}\times C_{t+i} - \sum_{i=2}^{11} f_{t+i}^{SBM}\times SBM_{t+i} \quad (1)$$

Realized gross margin is calculated via the same formula, with actual prices defined as three-day average of futures prices prior to futures contract expiration.

As stated before, LGM-Dairy is designed to be actuarially fair, so in order to calculate insurance premium expected indemnity needs to be calculated. LGM-Dairy official rating method uses Monte Carlo simulation that proceeds as follows:

1) Futures prices are taken as unbiased forecasts of terminal futures prices.
2) Marginal distributions of terminal futures prices for each commodity and for each month is assumed to be lognormal, with variance calculated based on average implied volatility of at-the-money call and put options, and time left to actual price determination period.
3) Marginal distributions are coupled into joint distribution function using rank-based correlation coefficients calculated using ranks of historical price deviates (Expected – Actual Price). Period 1978-2005 is used to calculate intra- and inter-commodity Spearman's correlation coefficients for corn and soybean meal, and period 1998-2005 is used to calculate Spearman's correlation coefficients among milk prices for ten insurable months. Authors of the LGM-Dairy rating method indicate that milk-corn and milk-soybean meal correlation matrix is not positive definite and therefore they assume milk-feed correlations to be zero.
4) Based on correlated draws from marginal distributions gross margins are simulated for each insured month in 5000 simulation rounds.
5) For each simulation round, realized gross margin is calculated as the sum of monthly gross margins, and indemnity is calculated as the difference between guaranteed and realized gross margin, if positive.
6) Expected indemnity is the simple average of indemnities over 5000 rounds. Premium, before administrative and overhead fee is set at 1.03 times the expected indemnity.

The United States Department of Agriculture, Risk Management Agency serves as the insurance underwriter for LGM-Dairy. There are no offsetting purchases of futures and options by USDA/RMA. As LGM-Dairy underwriting capacity is limited by federal appropriation, sales of the product can be restricted when this capacity is reached, as happened in 2011 and 2012.

The initial rollout of LGM-Dairy in 2008 was uninspiring as it was only available in a handful of states and dairy farmers were responsible for the full premium. However, in 2010, an array of changes to LGM-Dairy were unveiled. Beginning in July 2010 LGM-Dairy was made available in all contiguous 48 states, and by December 2010 LGM-Dairy included a premium subsidy, a longer sales period, deferred the premium payment to the end of the contract period, and increased the maximum amount of the deductible. These program changes contributed to an upsurge in LGM-Dairy participation. The number of policies sold increased by 1,259 from 2010

to 2011. As of the 2012 reinsurance year dairy producers have purchased nearly 3,000 insurance contracts representing 8.9 billion lbs of milk and $1.5 billion dollars in gross margin liabilities.

Total premiums collected for LGM-Dairy coverage since 2008 are over $45.3 million dollars. Of the $45.3 million dollars collected dairy farmers paid $25.7 million and the USDA has contributed $19.6 million. The USDA/taxpayer contribution represents approximately 43 percent of the LGM-D premium. Of the $45.3 million dollars collected, only $1.1 million dollars has been paid out in indemnities (there remains a possibility that more indemnities will be paid during 2012). The difference between the premiums collected and the indemnities leaves more than $44 million dollars in underwriter gain (this may be used to pay future indemnities).

Given the gap between premiums collected and indemnities paid out, serious concerns as to the usefulness of LGM-Dairy as an insurance product have been raised by some dairy economists, farmer representatives, and policy analysts. In this article we seek to investigate some features of the premium determination methodology that may be producing premiums that are above the actuarially fair level.

Testing LGM-Dairy Rating Method I: Dependency Structure

The current actively utilized LGM-Dairy rating method imposes a restriction of zero correlation between milk futures and corn futures, as well as between milk and soybean meal futures contracts. In the description of the rating method (RMA, 2005), it is stated that authors were unable to obtain positive definite correlation matrix without imposing this restriction. The reason quoted is the short period of time milk futures traded at the time when rating method was being developed. However, even with 14 years of milk futures trading data, the current method would not be able to produce full unrestricted correlation matrix. The impossibility of positive-definiteness is a result stemming from price correlations calculated to be month-specific, i.e. these correlations differ across sales events. In other words, conditional correlation between milk and corn prices for 2nd insurable month for January sales event is different than the correlation coefficient for the same commodities and time-to-maturity horizon for February, March or any other sales event. With 23 marginal distributions in play (10 contracts for milk futures, 6 for corn futures, and 7 for soybean meal futures), 14 years of available data for milk futures is simply not enough to produce a full rank correlation matrix if only one observation per year is used, as is the case with current rating method. To calculate correlations between milk futures with different time-to-maturity, LGM-Dairy developers used a simplification with the core feature that correlation between contracts is not dependent on the month of the sales event. We take a similar approach and simplify LGM-Dairy rating method by stipulating that correlation coefficient is not dependent on the month of the sales event, but only on time-to-maturity horizons for each commodity-insurable month pair. To check if this simplification would introduce bias in calculated LGM-Dairy premiums, we perform a Monte Carlo experiment in which all other restrictions are maintained (i.e. zero milk-feed correlation), but nearby-based instead of sales month-based correlations are used. We then create six different hedging profiles that are used to test for statistical significance in premium changes. Benchmark hedging profiles differ in amounts of declared feeds (minimum, default, maximum) and deductible levels ($0.00 and $1.0/cwt). Each hedging profile was set in such way that 1,600 cwt of milk is insured in each of 10 insurable months, for a total of 16,000 cwt. We assume LGM-Dairy contracts with given

specifications were bought in *each* of the 12 potential sales events in 2011, and focus on the value of the average premium paid over the 12 contracts purchased.

To complete the empirical analysis we run 30 replications of premium calculations under this alternative rating method. Given the extremely computationally intensive nature of the experiment, time needed for each replication was about 15 minutes. All premiums in the tables below exclude any government subsidies and administrative and overhead fees. Standard deviations are shown in parenthesis.

Table 1. Comparing original and nearby-based LGM-Dairy premiums

Feed	**Min.**	**Default**	**Max.**	**Min.**	**Default**	**Max.**
Deductible	$0.00	$0.00	$0.00	$1.00	$1.00	$1.00
Original premium	$14,568	$15,214	$20,350	$7,379	$8,082	$13,307
Nearby-based rating method	$14,739 ($81)	$15,410 ($87)	$20,651 ($124)	$7,514 ($50)	$8,212 ($57)	$13,552 ($100)
Nearby > Original	30/30	30/30	30/30	30/30	30/30	30/30
Nearby/Original -1	1.17%	1.29%	1.48%	1.83%	1.60%	1.35%

We find that while our alternative method seems to be producing premiums that are statistically significantly higher, premiums are not substantially higher, with increases not exceeding 2% of the original rating method.

With recent changes in commodity markets, an assumption of zero conditional correlation between milk and corn, as well as between milk and soybean meal futures prices may no longer be valid. To investigate this we updated the rating method to include price data from the last six years, and allowing for non-zero correlation between milk and feedstuffs. In order to be able to insure positive-definiteness of the full correlation matrix we only use futures prices from those months in which full information is available for the ten milk contracts, six corn contracts and seven soybean meal contracts used in calculation of conditional correlation coefficients. That restricts our data sample to 1998-2010 and a total of 142 time periods used in calculating correlation coefficients.

We find that correlation coefficients are markedly positive, especially for deferred contract months. Table 2 below presents correlation coefficients for milk and corn contracts for nearby and deferred milk contracts and first five nearby corn contracts.

Table 2. Milk-corn conditional rank correlations

		Corn				
		1st nearby	2nd nearby	3rd nearby	4th nearby	5th nearby
Milk	1st nearby	0.07	0.11	0.05	0.06	0.08
	2nd nearby	0.13	0.21	0.17	0.21	0.17
	3rd nearby	0.14	0.23	0.23	0.28	0.24
	8th nearby	0.11	0.22	0.26	0.32	0.35
	9th nearby	0.10	0.23	0.32	0.34	0.40
	10th nearby	0.14	0.21	0.35	0.37	0.45

Using a similar Monte Carlo experiment as before we compare original premiums; premiums using our method maintaining correlation matrix restrictions, but with updated data (1998-2011) used for both milk and feedstuffs; and premiums based on rating method that utilizes full correlation matrix with updated data (1998-2010). The results are shown in Table 3.

Table 3. LGM-Dairy premiums under non-zero milk-feed correlations

Feed	**Min.**	**Default**	**Max.**	**Min.**	**Default**	**Max.**
Deductible	$0.00	$0.00	$0.00	$1.00	$1.00	$1.00
Original premium	$14,568	$15,214	$20,350	$7,379	$8,082	$13,307
Nearby 1998-2010 Restricted corr.	$15,462 ($82)	$16,027 ($90)	$20,556 ($134)	$8,140 ($51)	$8,739 ($57.13)	$13,379 ($101)
Restricted/Original - 1	6.08%	5.61%	1.01%	10.31%	8.12%	1.31%
Nearby 1998-2010 Unrestricted corr.	$14,998 ($60)	$14,344 ($57)	$16,439 ($71)	$7,718 ($43)	$7,164 ($40)	$9,504 ($50)
Unrestricted/Original - 1	2.94%	-5.72%	-19.22%	4.59%	-11.36%	-28.58%
Unrestricted/Restricted - 1	-2.96%	-10.50%	-20.03%	-5.18%	-18.02%	-28.97%

Results reveal a differential impact for premiums charged to farmers that grow all their own feed, i.e., those selecting the minimum feed coverage, and those that purchase all feed, i.e., those selecting maximum feed coverage Compared to current in-use rating methodology, calculated premiums actually increase by 3-5% for minimum-feed hedging profile, depending on the level of deductible chosen. However, insurance for dairy farmers that purchase all their feed, and would seek to declare maximum allowed feed amounts in LGM-Dairy contracts are substantially overpriced by the current official rating method. Our updated rating method shows that

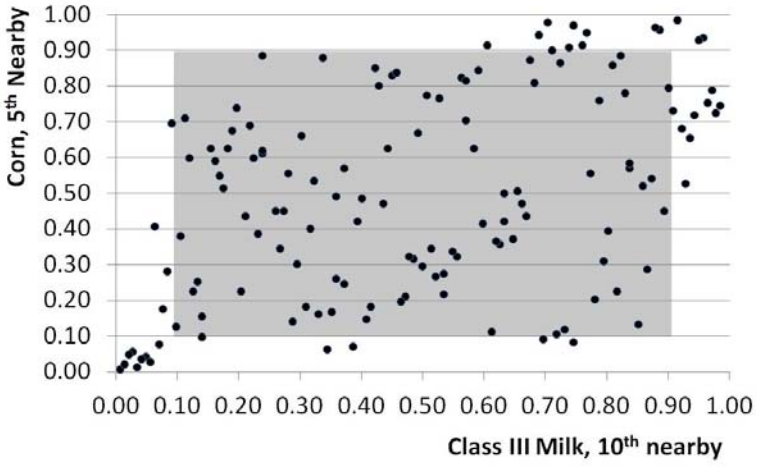
premiums should be between 20% and 29% lower. These premium differences apply to our hedging profiles that insure equally over all insurable months. Should a farmer choose to insure only deferred production months, savings over the original rating method would be expected to be even higher.

Whereas under original rating method and $1.00 deductible scenario farmers buying all their feed (max-feed) pay 80% more for premiums than farmers that grow all their feed (min-feed), under our updated rating method that allows for milk-feed correlations this price differential is only 23%.

Tail Dependence

Although allowing for milk-feed correlations markedly affects insurance premiums, a closer look at the data reveals that measures of linear dependency, such as Spearman's rank correlation coefficient, do not tell the complete story. Some of the most dramatic increases in corn prices were also accompanied by extremely strong increases in milk price. More importantly, dramatic declines in milk prices in the first half of 2009 were accompanied by equally profound declines in corn futures prices. What these examples suggest is that milk and feed commodities may demonstrate stronger co-movements in extreme business environments than in normal economic times. In other words, joint milk-feed distribution may exhibit tail dependence. A simple graph will help illustrate this effect. In Figure 2. we have plotted percentiles of unexpected price deviates for both 10^{th} nearby Class III milk futures and 5^{th} nearby corn futures prices. The central part of this graph shades 10^{th}-90^{th} percentile for both commodities. Within the shaded area there appears to be very little evidence of co-movements between milk and corn futures price. In the tails of both distributions, however, co-movement is remarkably strong. Looking at Figure 2, we can see that the highest unexpected increases in milk prices (90^{th}+ percentile) have been accompanied by corn price deviates with high percentile scores, and vice versa. This relationship is even stronger for extreme unexpected decreases in milk prices (0-10^{th} percentile).

Figure 2. Percentiles of Unexpected Price Deviates for Corn and Class III milk Futures.



To model such non-linear dependence between corn and milk prices we need to abandon rank correlation measures and utilize copulas. Standard parametric copulas such Clayton, Gumbel, or t-copula are not flexible enough to reflect this nonlinear structure. Furthermore, as the full joint

distribution has between 22 and 24 marginal distributions (depending on the month of the sales event), high dimensionality renders the parametric approach entirely too complex. To capture what is evident in the data, we utilize the empirical copula approach and model dependencies in a completely non-parametric fashion which allows the data to inform the tail dependence.

An empirical copula approach is implemented using the following protocol:

1) Unexpected price deviates are calculated for each commodity, each month for period 1998-2011.
2) Each obtained observation is ranked such as to create ranked-deviates matrix with 140 rows.
3) Using van der Waerden scores 5000 points on each marginal implied probability distribution are identified, corresponding to percentiles of the distribution (F(1/5000), F(2/5000), etc.). These van der Waerden scores are split in 140 blocks, such that the first block contains percentiles 1/5000-140/5000, second contains 141/5000-280/5000, etc.
4) Next, 5000 rounds of simulation are performed. In each round, a single row in the ranked-deviates matrix is randomly chosen. Ranks in that row determine blocks of van der Waerden scores of each commodity and contract month to be used for drawing in the next stage.
5) Data is drawn randomly from each previously described block, with draws from blocks of different commodities/months being independent.

Table 4. compares original, correlation-based on empirical copula-based premiums for the six hedging profiles previously described. We find that empirical copula reinforces the effect already present in our ranked correlation based rating method: premiums for minimum feed hedging profiles increase further, and premiums for default and maximum feed hedging profile decrease even more.

Table 4. Empirical copula based LGM-Dairy rating method: Effect on insurance premiums

Feed	**Min.**	**Default**	**Max.**	**Min.**	**Default**	**Max.**
Deductible	$0.00	$0.00	$0.00	$1.00	$1.00	$1.00
Original premium	$14,568	$15,214	$20,350	$7,379	$8,082	$13,307
Nearby 1998-2010 Rank-correlation	$14,998 ($60)	$14,344 ($57)	$16,439 ($71)	$7,718 ($43)	$7,164 ($40)	$9,504 ($50)
Nearby 1998-2010 Empirical Copula	$15,286 ($98)	$14,104 ($101)	$15,550 ($131)	$8,028 ($72)	$6,759 ($65)	$8,335 ($82)
Copula/Original - 1	4.92%	-7.30%	-23.58%	8.79%	-16.38%	-37.37%
Copula/Correlation - 1	1.92%	-1.68%	-5.41%	4.00%	-5.66%	-12.30%

After accounting for tail dependence, we find that premiums for minimum feed hedging profiles increase by close to 2% compared to premiums under the correlation-based rating method. An intuitive explanation would be that the odds of extreme unexpected movements in milk futures to be accompanied by extreme changes in milk futures for a month immediately following are higher than would be implied by rank correlation coefficients. Not surprisingly, the effect on premiums is stronger for policies with high deductible levels designed to protect against catastrophic risks.

The most dramatic impact is again seen for maximum-feed hedging profiles. The empirical copula based method reduces premiums compared to premiums under correlation-based rating method by 5.41% in zero deductible case, and as much as 12.30% for $1.00 deductible. In total, the calculated premium for a "catastrophic risk" (i.e. $1.00 deductible) insurance policy for max-feed hedging profile is lower by a very substantial 37.37% compared to the premium obtained under current official rating method. Furthermore, we observe that cost-advantage that the min-feed hedging profile enjoyed under the original rating method is all but vanished under $1.00 deductible scenario and empirical copula based rating method.

A more general point can be stated as well. In managing any portfolio, and setting a price of a risk insurance product whose payoff depends on multiple securities, it is essential to properly account for the joint distribution of returns of the included securities. In doing so, treating correlation coefficients as sufficient statistics of dependence may not be prudent, as "the devil is in the tails" as noted by Donnely and Embrechts (2010). For example, use of Gaussian copula to model dependence of corporate bonds in a portfolio cannot appropriately capture the phenomena of defaults clustering, i.e. the fact that in time of crisis, when one company defaults, others are more likely to default. In such a case, proper accounting of extremal dependence would *increase* the price of risky derivatives, i.e. the cost of risk transfer would be higher. Similarly, tail dependent flood risks could increase the price of house insurance (Kousky and Cooke, 2009). Unlike other portfolio situations, however, our paper suggests that agricultural gross margin insurance may present a curious case where proper representation of tail dependence actually *reduces* the cost of insurance,i.e., the premium.

Furthermore, as new generation of dairy policy tools are designed, models used to score the fiscal implications of different proposals would do well to incorporate richer structure of inter-commodity dependence than simple measures of linear correlation.

Testing LGM-Dairy Rating Method II: Marginal Distributions

LGM-Dairy premiums are established using Monte Carlo simulations which assume that conditional on present information, distribution of terminal futures price for each commodity and for any month are distributed lognormally. This is consistent with Black's model for pricing options on futures contracts. Implied volatility and time to maturity are used to calculate the variance of terminal log-prices. This assumption was likely introduced primarily to simplify the premium determination process. Under lognormality, it suffices to utilize at-the-money options in calculating variance of terminal prices. Allowing for more flexible distributions would necessitate utilizing options for all traded strikes, more complex option pricing formula, and possibly high-frequency data as well.

Williams and Wright (1991) suggest that distribution of future cash prices for storable commodities should be skewed, and that, other things equal, degree of skewness should be higher when commodity stocks are lower. Bozic and Fortenbery (2011) demonstrated the existence of such effect in options on corn futures, finding that implied skewness for corn options is systematically higher than what would be consistent with lognormal distribution. The previous section of this paper addressed tail dependence, and now the focus of this section is on heavy tails, and skewness that may be expected to be higher than is consistent with lognormal distribution.

In order to test whether allowing for flexible skewness and kurtosis in marginal distributions of milk, corn and soybean meal prices affects the LGM-Dairy premium we replaced the lognormal with a four-parameter generalized lambda distribution (GLD) (Bozic and Fortenbery, 2011).

As a first test, we calculated higher moments of generalized lambda distribution such that they match skewness and kurtosis of lognormal distribution. We found that under these restrictions, simulations based on GLD produce premiums that are on average less than 1.5% different than premiums obtained using lognormal distribution. It follows that GLD-based LGM-Dairy rating method can be treated as generalization of the rating method founded on lognormal marginal distributions.

We then let data determine higher moments of marginal distribution using high-frequency futures and options data for 2011, following procedure described in Bozic and Fortenbery (2011). In order to test if premiums are statistically significantly different we used the same Monte Carlo procedure as in the previous section.

Table 5 presents LGM-Dairy premiums for the six hedging profiles previously described under the original and five alternative rating methods. Except for the original premium method, all of our alternative methods use data for the period 1998-2011. Marginal distributions and dependence structure were as follows:

1) Spearman's rank correlation matrix (full), lognormal marginal distributions
2) Spearman's rank correlation matrix (full), GLD marginal distributions (matching higher moments of lognormal distribution)
3) Spearman's rank correlation matrix (full), GLD marginal distributions, flexible moments
4) Empirical copula, lognormal marginal distributions
5) Empirical copula, GLD marginal distributions, flexible moments

Table 5. Effects of flexible higher moments of marginal distributions on LGM-Dairy premiums

Feed	**Min.**	**Default**	**Max.**	**Min.**	**Default**	**Max.**
Deductible	$0.00	$0.00	$0.00	$1.00	$1.00	$1.00
Original premium	$14,568	$15,214	$20,350	$7,379	$8,082	$13,307
Rank-correlation Lognormal	$14,998 ($60)	$14,344 ($57)	$16,439 ($71)	$7,718 ($43)	$7,164 ($40)	$9,504 ($50)
Rank-correlation GLD lognormal moments	$14,936 ($79)	$14,288 ($67)	$16,386 ($92)	$7,615 ($50)	$7,076 ($49)	$9,453 ($62)
Rank-correlation GLD flexible moments	$15,156 ($79)	$14,495 ($74)	$16,308 ($85)	$7,895 ($50)	$7,336 ($50)	$9,403 ($59)
Empirical copula Lognormal	$15,286 ($98)	$14,104 ($101)	$15,550 ($131)	$8,028 ($72)	$6,759 ($65)	$8,335 ($82)
Empirical copula, GLD flexible moments	$15,445 ($102)	$14,239 ($105)	$15,477 ($125)	$8,218 ($76)	$6,959 ($69)	$8,245 ($83)

Upon examining the results of our rating method simulations, we conclude that flexible higher moments of marginal distributions do not substantially alter LGM-Dairy premiums, irrespective of the feed amounts, deductible levels, or method for modeling milk-feed dependence in joint

distribution function. A further look at the simulation results reveals that allowing for more flexible higher moments in marginal distribution affects the *distribution* of indemnities (obtained from 5000 simulation rounds), especially the skewness and kurtosis. However, the empirical mean of the indemnity distribution is not very sensitive to the shape of the marginal distributions, at least within the range of higher moments as implied from the options traded in the year 2011. Therefore, we conclude that assumption of lognormality as currently used in LGM-dairy rating method may be considered as an appropriate simplification that does not violate the basic principle of actuarially fair rating method for calculating the insurance premium.

Conclusions

Examining the rating method assumptions of the LGM-Dairy we found that:

- Assumption of lognormality of marginal distributions for milk, corn and soybean meal captures salient aspects of information embedded in the option premiums, and can be considered as useful simplification that is appropriate for the task at hand.
- Official rating method, based on historical data up to 2005 does not capture well the structural changes in milk and feed markets that have occurred since 2005. In particular, milk and feed prices should not be treated as independent variables. Allowing non-zero milk-corn and milk-soybean meal correlations reduces premiums substantially for hedging profiles that declare more than minimum amounts of feeds.
- Coupling marginal distributions using linear rank correlation coefficients does not capture well the non-linear nature of milk and feed dependence. In particular, milk and feed markets seem to exhibit tail dependence at longer time-to-maturity horizons. The premium determination method based on empirical copula approach suggests tail dependence is an important feature of data structure that substantially affects insurance premiums, increasing them for minimum-feed hedging profile, but considerably decreasing them for insurance policies with maximum feed amounts declared.

Based on our findings we recommend a revision of LGM rating method that addresses issues of milk-corn and milk-soybean meal dependence. Although our analysis suggests that certain hedging profiles are charged severely upward biased insurance premiums, we wish to emphasize that the ratio of indemnities to premiums as observed over the past three and a half years for which the LGM-Dairy product has been offered is likely *not* significantly caused by the stated shortcomings in the rating method. In particular, a predominant majority of the LGM-Dairy policies sold have been specified with minimum, or very low feed amounts declared. Neither analysis of dependence between milk and feed prices, nor investigation of the shapes of marginal distributions find rating method producing upward biased premiums for the type of insurance policies that have been mostly sold, i.e. contracts with minimal feed amounts declared. There does remain one aspect of the rating method that still needs to be investigated. Class III milk options are a thin market, especially for options with high time-to-maturity. If options premiums embed a liquidity premium in addition to pricing expected volatility, then methods used to obtain implied volatility would lead to a variance of marginal distributions of milk futures prices that is higher than warranted. If this was indeed the case, LGM-Dairy premiums could be found to be too high, even for minimum-feed hedging profiles. Further research is planned to investigate this issue.

Finally, we wish to briefly discuss the rational for government involvement in offering a risk management tool such as LGM-Dairy. The LGM-Dairy design creates an insurance product that is non-redundant, i.e. whose payoff structure cannot be replicated using a combination of existing market instruments. As such several benefits may be achieved:

- Insurance products that can be customized to fit the size of the producer eliminate the over- or underinsuring that may occur when fixed size contracts traded at the exchange are used to manage risk.
- By creating an insurance product whose payout structure depends solely on *relative* movements of milk and feed prices, considerable savings can be obtained compared to bundled option strategy.
- By allowing insured margin to encompass more than one period, savings can be achieved compared to buying a pack of options on futures for several consecutive delivery months, due to averaging effect similar to those seen in Asian options that settle against an average price of the underlying asset.
- Because liabilities that an underwriter assumes by issuing a gross margin insurance policy are not offset by taking positions at the exchange, the liquidity constraints of the thinly traded CME products are avoided, as option premiums will need not change as number of insurance policies issued increases.

While above reasons justify existence of a gross margin insurance product even in an economy with vibrant private risk management tools, they do not by themselves suffice to explain why this product should involve government intervention, rather than just be left to market to create as just another over-the-counter financial derivative. We see three reasons that justify government involvement:

1) Complex market regulation of the dairy sector together with extensive knowledge needed to map realities of dairy cattle nutrition to a small set of traded grains and feedstuffs futures imply that incentives to create an attractive and relatively simple margin contract may be lacking, if not subsidized by the federal government.
2) Wide adoption of such a gross margin product would expose private underwriters to substantial liability risk, as, unlike for standard insurance products, aggregate portfolio risk does not decline with increase in number of policies sold. As such, reinsurance of gross margin contracts through Federal Crop Insurance Corporation seems to be necessary condition for existence of LGM-Dairy.
3) Finally, federal policy costs of providing safety net are made more predictable through subsidizing insurance premiums, rather than mitigating incurred losses.

While our hope is that analysis presented in this paper will find a practical application in design of risk management tools in the U.S. dairy sector, we wish to close this paper by reiterating what seems to us as the most curious finding of this research endeavor. Across the field of finance, there is a burgeoning movement to appropriately model tail risk, and tail dependence in an insurance portfolio. The recent worldwide economic recession is at least partially caused by inappropriate models of cascading security defaults, which can be understood as tail dependence among securities that constitute an investment portfolio. Copula models that account for tail dependence therefore find *higher* insurance risk, compared to models that represent dependence using linear correlations. We find that agricultural gross margin insurance may represent case

where tail dependence *decreases* insurance risk – a rare, and curious departure from the conventional insurance portfolio situation.

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