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by

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Volatility Models for Commodity Markets

Paul L. Fackler and Yanjun Tian*

The time structure of volatility in futures prices and implied volatility implicit in option premia is derived from an underlying model of spot price behavior. The model suggests a number of characteristic features that should be present in observed market prices. These features are found in soybean futures and options on soybean futures.

Introduction

Futures and options are derivative assets whose value depends on the behavior of other, underlying, assets. The derivative nature of futures and options, however, is often ignored in empirical analysis. For example, it is common to treat futures prices as if they were an ordinary time series of observations, despite the obvious problem that futures contracts mature. To address this problem, it is common to artificially create a continuous sequence of prices by “rolling over” in the next nearby contract when one reaches its expiration month.

Unfortunately, using off-the-shelf times series approaches to model derivatives can result in misrepresentation of their dynamic behavior. The alternative is to treat the behavior of the underlying assets as primitive and exploit the implications of the derivative nature of futures and options in modeling their time series behavior.

In this paper, we will first discuss this idea using a simple, one factor, model of a spot commodity price that exhibits mean reversion and seasonally varying volatility. The model is used to illustrate the implications of arbitrage relationships for the behavior of futures price volatility and the implied volatility associated with options on futures. These implications are examined using price data on soybean futures and options on soybean futures. Although the simple model fails to predict some interesting aspects of price volatility, it illustrates that the seemingly complex seasonal volatility patterns in grain futures and options can be largely accounted for using a simple model, thereby highlighting the importance of accounting for the timing and maturity structure of futures and options contracts. Some suggestions for further analysis are included in the final section of the paper.

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Volatility in Futures and Options on Futures

The main message of this paper is that futures price volatility and option-based implied volatility depends on the volatility of underlying fundamental factors, especially that of the spot price. To illustrate this idea, we use a simple model for the log of the spot price, p , described by the stochastic differential equation

$$dp = \kappa(\alpha(t) - p)dt + \sigma(t)dz,$$

where the functions α and σ are seasonal functions of time.¹ This process exhibits mean reversion to a seasonally varying mean and exhibits seasonal variation in volatility. It can be shown that this continuous process exhibits changes (in logs) that are normally distributed with conditional mean

$$E_t[p(t+h)] = \mu(t+h) + e^{-\kappa h}(p(t) - \mu(t)),$$

and variance

$$\text{Var}_t[p(t+h)] = \nu(t+h) - e^{-2\kappa h}\nu(t),$$

where μ and ν are seasonal functions that can be derived from α and σ .² To provide a rough benchmark for the models of the next section, this spot price model was estimated for central Illinois elevator cash bids for soybeans. The seasonal squared volatility was modeled as a Fourier series with seven terms; the resulting volatility function, $\sigma(t)$, is plotted in Figure 1.

For simplicity, we assume that the futures price for a contract expiring at time T is a function of the spot price alone (a one-factor model). If the spot process describes the so-called risk-neutral process or if the spot price exhibits no risk premia, then the futures price is a zero drift process (Hull):

$$dF = FV_T(t)dz.$$

Using Ito's Lemma we can write

$$dF = \sigma(t)PF_P dz$$

so the volatility of the futures price process is

$$V_T(t) = \sigma(t)F_P \frac{P}{F} dz,$$

¹The model, with no seasonality, was used by Brennan and Schwartz and by Schwartz to model commodity spot prices.

²Specifically, they can be shown to satisfy

$$\alpha(t) = \mu(t) + \mu'(t)/\kappa$$

and

$$\sigma^2(t) = 2\kappa\nu(t) + \nu'(t).$$

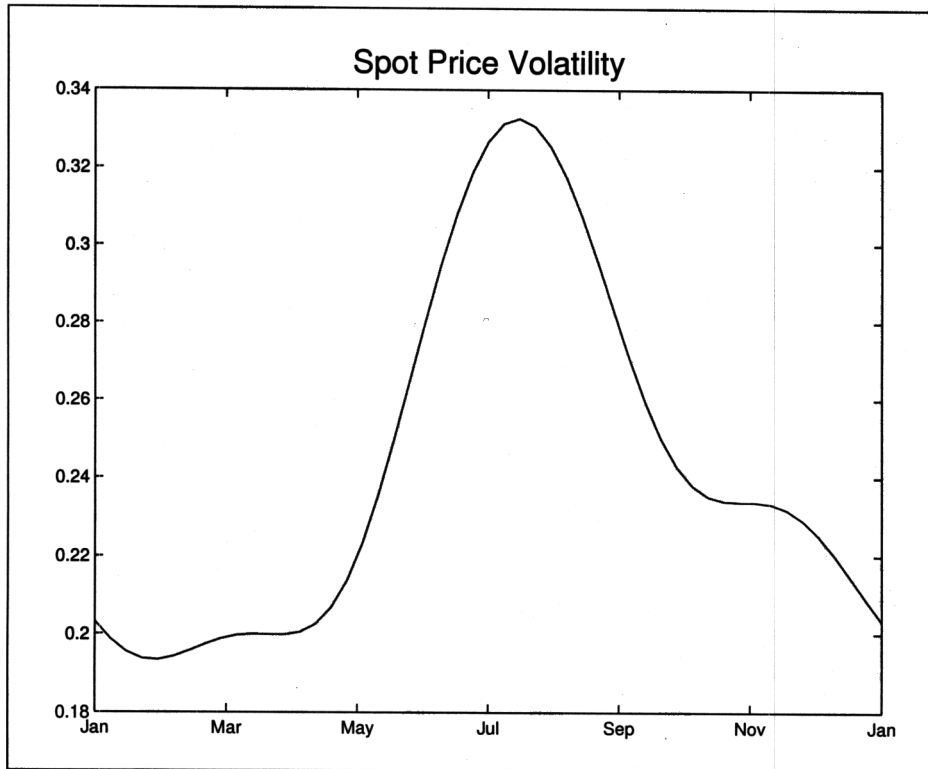


Figure 1

i.e., the volatility of the futures price process equals the volatility of the spot times the elasticity of the response of futures to spot price changes. The zero drift condition is ensured by setting the futures price equal to the expectation of the spot. As the spot price is lognormally distributed, its expectation is equal to

$$E_t[P(T)] = \exp \left(E_t(p(T)) + \frac{1}{2} \text{Var}_t(p(T)) \right).$$

Thus

$$F(P, t; T) = \exp \left(\mu(T) + e^{-\kappa(T-t)} (\ln(P) - \mu(t)) + \frac{1}{2} (\nu(T) - e^{-2\kappa(T-t)} \nu(t)) \right).$$

Armed with an explicit expression for the futures price in terms of P and t , we see that

$$F_P = \frac{F}{P} e^{-\kappa(T-t)}$$

and the volatility of the futures price is

$$V_T(t) = e^{-\kappa(T-t)} \sigma(t).$$

Thus the futures price exhibits a damped seasonality in its volatility: it reflects the seasonal nature of the volatility of the spot price, but with a damping factor that causes the volatility to decrease as the time to maturity increases:

$$V_{T_1}(t) > V_{T_2}(t) \text{ for } T_1 < T_2.$$

The damping term is a specific manifestation of a phenomenon known as the Samuelson hypothesis. Samuelson argued that futures prices should exhibit increased volatility as they approach their maturity date. An alternative, although not competing, hypothesis is that volatility is related to information flows (associated commonly with Anderson and Danthine). In this simple model, the idea of variation in the rate of information flow is modeled as seasonality in spot prices. In seasonally produced crops, information flows tend to be greatest during the growing season of the crop and hence, for fall harvested crops, spot price volatility tends to be highest in the summer.

The effect of the time-to-maturity term is illustrated in Figure 2, using an estimated speed of mean reversion parameter, $\kappa = 0.82$, and the spot price volatility function from Figure 1. Soybean futures contracts mature every two months starting in January (there is also an August contract that we ignore). The volatility functions for the six contracts clearly display the Samuelson effect: at any given time, volatility declines with time-to-maturity. Furthermore, in spite of the strong summer spot price volatility, it is possible for volatility of a given contract to be higher at other times. The May contract volatility function, for example, peaks at expiration. Furthermore, nearness-to-expiration at the peak of spot price volatility makes the July contract especially volatile at expiration.

The same approach can be applied to the seasonal pattern of implied volatility exhibited by the premia for options on futures. The simple model of spot prices results in lognormally distributed futures prices. The Black-Scholes model with deterministically time varying volatility is the appropriate option pricing model for this process. It is well known that the volatility applicable in this version of the Black-Scholes model is to use the average variance over the remaining life of the option (Cox and Rubenstein, p. 212). For simplicity, assume that the option expires at the same date, T , as the futures contract matures. Then the implied volatility of the option is given by

$$I_T(t) = \sqrt{\frac{\int_t^T V_T^2(\tau) d\tau}{T-t}} = \sqrt{\frac{\int_t^T e^{-2\kappa(T-\tau)} \sigma^2(\tau) d\tau}{T-t}}$$

The effect on options implied volatilities is far more complicated due to the smoothing effect of time integration, as is illustrated in Figure 3. The January and May contracts display little variation in implied volatility over the season. This is because the summer period high volatility is averaged together with lower volatility periods latter in the life of the contracts. July and September, on the other hand, display strong upward movements in implied volatility because the increasingly short time periods over which the average is taken are also increasingly volatile.

The interaction of the seasonal and contract maturity effects in implied volatility is quite complicated, in spite of the fact that it is based on a rather simple behavioral model. It is

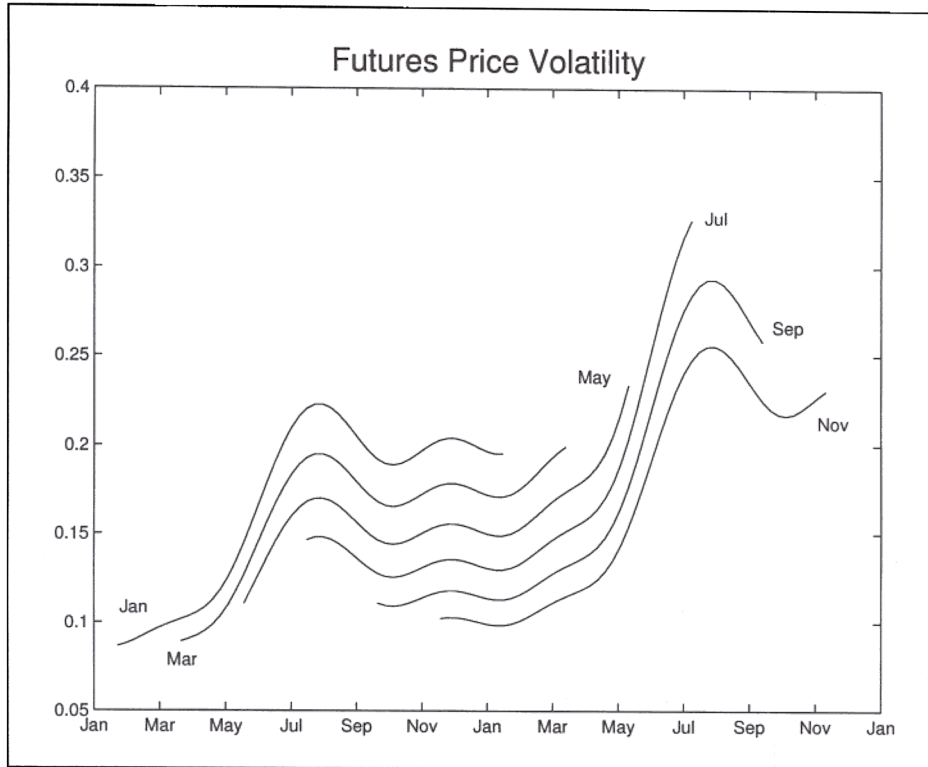


Figure 2

important to remember that these patterns are all derived from the parameters of the spot price volatility function of Figure 1 and the speed of mean reversion parameter, κ .

Empirical Results

The previous section suggested several empirical hypotheses. First, it suggests that the volatility of futures prices at a specified time of year will decline as the time to maturity increases. Second, futures contracts expiring early in the crop year (November through March) are expected to exhibit volatility that increases rapidly during the early life of the contract and then levels off or even declines. The contracts that expire late in the crop year (May through September) have relatively flat volatility early in their life and that rises sharply as the contract matures.

The implied volatility on options is more difficult to characterize. Ignoring the July contract, the implied volatility curve for each new contract lies below that of its predecessor, starting with the September contract. The curves also exhibit a progressive flattening out going from September to May. The July contract's behavior is quite different, beginning its life at a quite low level and ending at the highest level of any contract.

To examine whether these characterizations hold in actual markets, we first examine futures price volatility using a sample of weekly soybean futures prices for the period November,

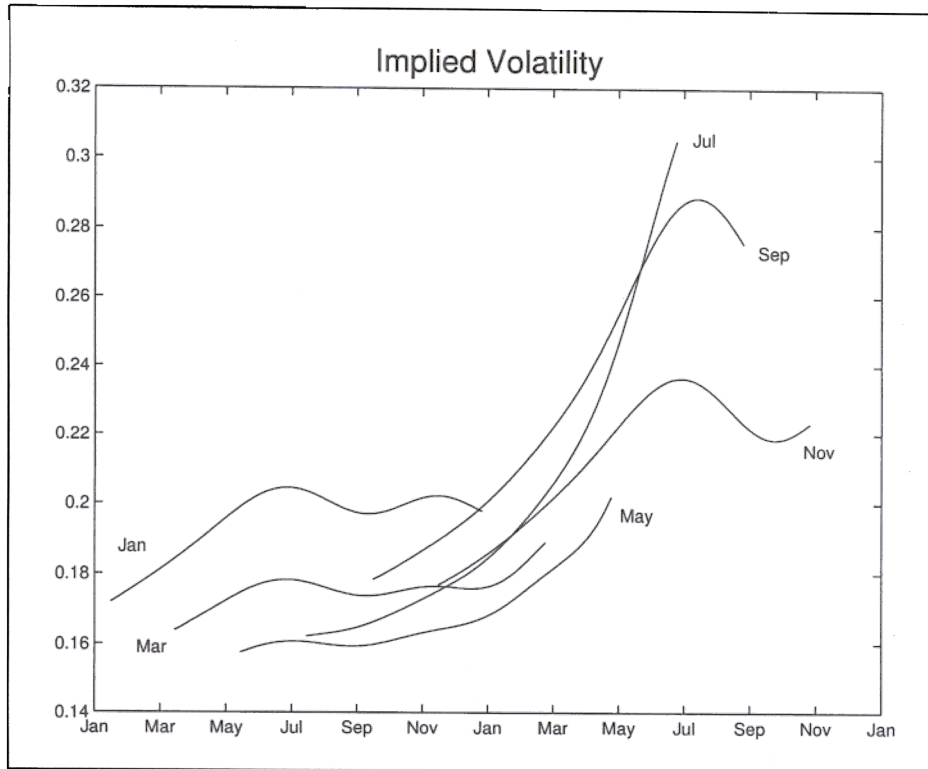


Figure 3

1974 to March, 1998. A separate function is estimated for each of the six contract months (the August contract is again ignored), with the time pattern of volatility represented by a cubic spline function of the time to maturity with eight evenly spaced breakpoints on the interval $[0,1.25]$.³ Thus the absolute weekly changes in the logs of futures prices were regressed on the time to maturity using a cubic spline with eight evenly spaced breakpoints. The estimated functions, $V_T(t)$, are plotted in Figure 4.

The figure, although clearly messier than Figure 2, displays similar features. The Samuelson effect is present in the sense that the nearest to mature contract is generally the most volatile at any specific point in time. Furthermore, the magnitudes of the volatilities are roughly similar, even though computed using very different methods and data.⁴ Perhaps the most striking difference between the constructed and the estimated volatility curves occurs near the maturity of the contract. With the exception of July, all of the contracts exhibits a marked decline in volatility in the month prior to delivery. Although some market microstructure explanation relating to liquidity effects may explain this, we leave it as an

³The September contract consistently traded for a shorter period and the spline endpoints were adjusted accordingly.

⁴The use of absolute changes rather than squared changes led to a slight downward estimated relative to the spot price based model.

interesting puzzle.

If this end of contract effect is discounted, the basic shapes of the curves are as suggested above. The Samuelson effect, however, appears to be quite a bit weaker than the simple model would suggest. Furthermore, there appears in the curves an indication that there are old crop/new crop effects that the simple model does not account for. The summer months are volatile primarily because of weather news that influences production projections for the crop to be harvested in the fall. This suggests that new crop contracts would be most effected by the weather shocks and that old crop (July and September) contracts would be affected to the extent that current storage decisions are altered in response to the shocks.

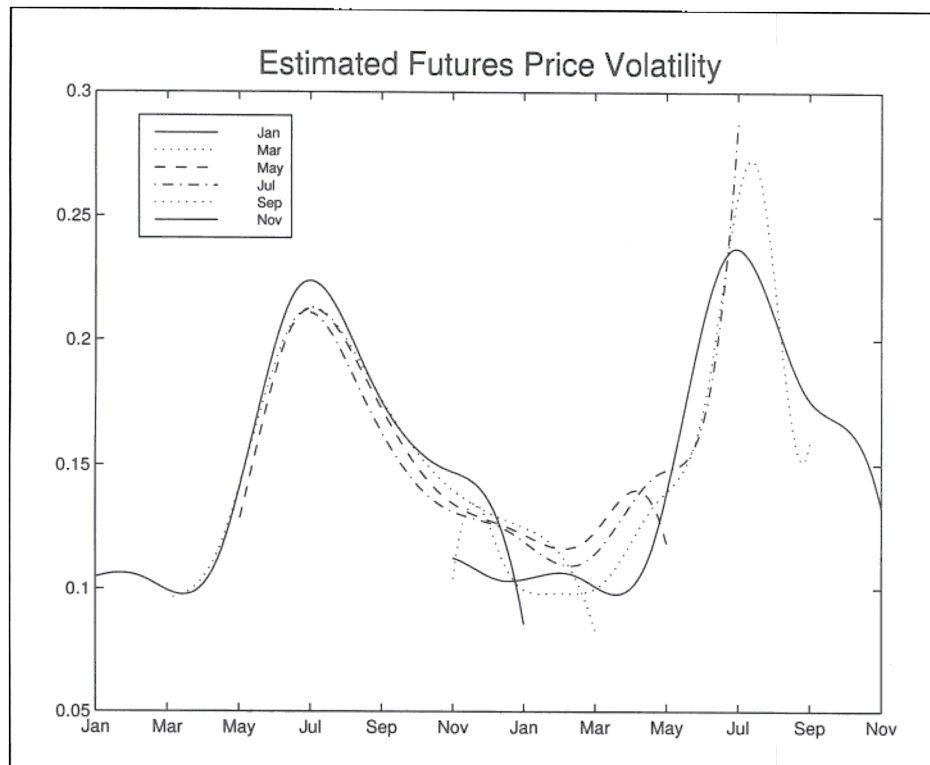


Figure 4

To examine the hypothesis concerning option-implied volatilities, we used a sample of weekly option premia for the period 1985 through 1998. We first computed the at-the-money implied volatility for call options by linearly interpolating the implied volatilities for the two strike prices that bracket the current futures price. These are regressed on the time to maturity for the option using a cubic spline with four evenly-spaced breakpoints. Estimates for each of the six contract months are displayed in Figure 5.

Here the patterns are quite consistent with those of Figure 3. The overall decline and flattening of the curves from the September contract to the May contract is present. This conclusion is especially valid if the end-points of the curves are discounted. The behavior

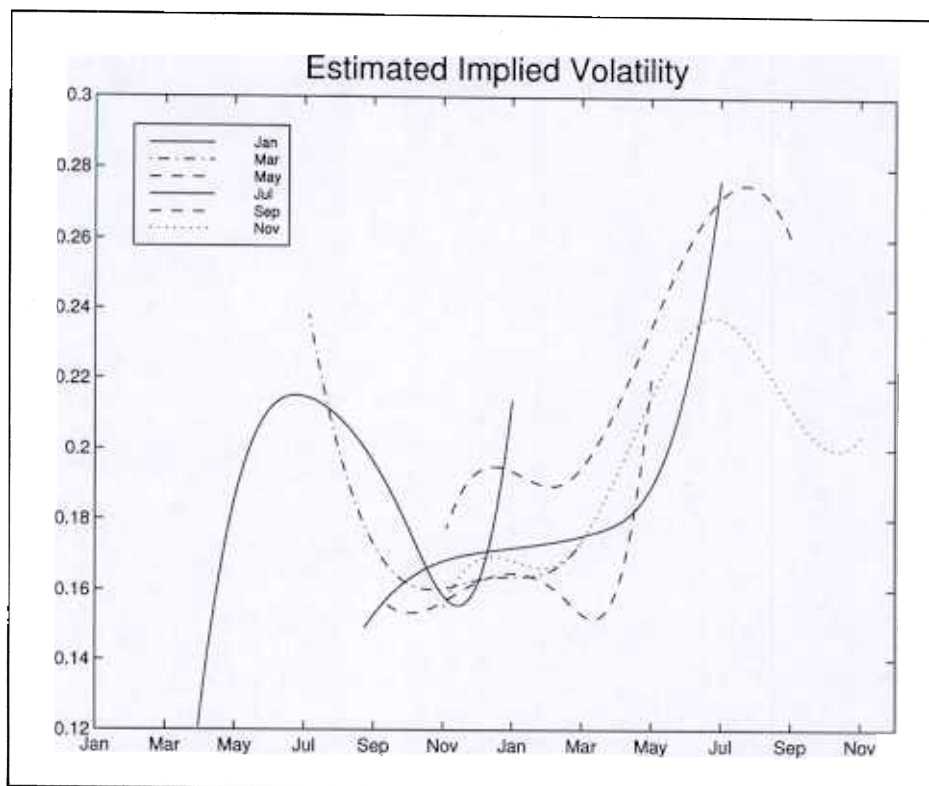


Figure 5

at the end point seems to be highly sensitive to exact sample used and to the order of the approximation. It is likely that liquidity is very thin during these periods and hence the implied volatilities are likely to be noisier than during their most actively traded periods.

The behavior of the July contract is especially noteworthy. It behaves in precisely the curious fashion predicted, starting quite low and rising steeply as it nears expiration. Given the simplicity of the spot volatility based model, it is surprising and gratifying that it is supported so well by the data.

Discussion and Conclusions

Futures and options are derivative assets that reflect information about the dynamic behavior of the fundamental factors from which they derive their value. It is important, however, to recognize that the maturity structure of futures and options has important implications for how that information is reflected in price behavior. For example, a fairly common practice in futures price analysis is to create a continuous series of prices by rolling into the next contract month at each delivery date. Standard time series methods are then applied to the constructed series. Even our simple model suggests, however, that the correlation of the spot and futures price is increasing in the time to maturity. An important

implication is that estimates of optimal hedge ratios, which depend on the variance of the futures price and the covariance of the futures with a specific local cash price, should account for when the hedge is placed and when it is lifted in relation to the contract maturity date.

The empirical results presented clearly illustrate that the effects of seasonality and contract maturity are important elements in understanding commodity price volatility. Futures price exhibit the Samuelson effect that, *ceteris paribus*, volatility tends to decrease with time-to-maturity, although the effect is not particularly strong in the soybean market. The effect of time averaging on implied volatility is also illustrated clearly in the soybean market and provides an explanation for the complicated seasonal patterns exhibited by implied volatility.

The simple one-factor model, however, is clearly fails to capture important aspects of volatility in the soybean market. Some fairly straightforward remedies suggest themselves. First, the model of the underlying factors influencing futures prices is too simple. Schwartz demonstrated that a one factor model did not capture the behavior of futures price term structure in the oil and copper markets nearly as well as two and three factor models which included not only the spot price but also stochastic convenience yields and stochastic interest rates as additional factors. The Schwartz model is still a Gaussian model and it would be reasonably straightforward to extend the current analysis to such a model.

Several facts, however, suggest that the Schwartz model will be incapable of modeling important features of futures and options prices. First, there is considerable empirical evidence for ARCH/stochastic volatility effects in futures prices. Second, implied volatility exhibits considerable variation that appears to be neither seasonal nor related to time-to-maturity. Third, implied volatilities in options on commodity futures exhibit a curved (generally upward sloping) relationship as a function of their strike price. A Gaussian model always produces the flat smile of the Black-Scholes model and hence cannot capture the smile effect.

These facts suggest that the use of stochastic volatility models, which are known to produce smile effects, should be examined. These model are non-Gaussian and complicate the analysis because futures volatility becomes dependent on stochastic state variables. Of course, it is precisely this dependence that provides the increased richness of these model that makes them attractive.

It may also be fruitful to take the spot price as endogenous and make the primitives of the model be economic fundamentals such as shocks to demand and to the expected harvest. In this way the recognition that the relationship between spot and futures prices depends on whether a harvest occurs before the futures contract matures could be explicitly accounted for. This would formalize a link to the rich literature on the price behavior of storable commodities (e.g., Williams and Wright, Ng and Pirrong).

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