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EVALUATING POTENTIAL CHANGES IN PRICE REPORTING ACCURACY

Clement E. Ward and Seung-Churl Choi¹

Non-cash-market transactions for fed cattle have increased. Price discovery depends in part on the accuracy of reported cash market prices. Cattlemen and others have expressed concern that as non-cash-market transactions increase, reported cash market prices may no longer accurately reflect supply-demand conditions. Equations based on Chebyshev's inequality are used in conjunction with experimental market data from the *Fed Cattle Market Simulator* to explore relationships related to price reporting accuracy for several subpopulations of prices versus the known population. Price means and variances and distribution of prices were invariant to number of transaction prices. Mean prices and variance of prices also were invariant to number of observations. Only when the reduction in prices reached 80 % was there a significant relationship between number of observations and two pairs of variables, i.e., reported price precision and confidence of a given level of precision. With the exception of the smallest reduction in transactions, no differences were found between the subpopulations and population for reported price precision versus probability of a given level of precision, for reported price precision versus estimated number of observations with a given degree of confidence, and for probability of a given level of precision versus estimated number of observations with a given level of precision. Results suggest the *possibility* that number of non-reported fed cattle transaction prices could increase significantly before the industry faces serious concerns regarding the accuracy of reported prices *ceteris paribus*.

INTRODUCTION

Thin markets can take many forms which are not necessarily mutually exclusive (Hayenga 1980). One form is few, large transactions, such as has characterized the wholesale cheese market for many years. Another form is few, large buyers (sellers), such as the concentrated wholesale boxed beef market. A third form, and the one considered in this work, is few reportable or reported transactions among all transactions between buyers and sellers in a given time period and geographic area.

In the fed cattle and wholesale beef markets, the latter form of a thin market is becoming an increasing concern relative to historical periods. Data compiled by the Grain Inspection, Packers and Stockyards Administration (GIPSA) (1997) of the U.S. Department of Agriculture (USDA) beginning in 1988 indicate that on an annual average basis, non-cash-market transactions for fed cattle procured by the four largest firms ranged from 17.5 % in 1993 to 24.9 % in 1989. Non-cash-market transactions for fed cattle include packer feeding, marketing agreements and formula pricing, and basis and fixed price forward contracts. Transaction prices in these

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arrangements are not publicly available. Some are internal to the firm (such as in packer feeding) and some are privately shared between buyer and seller. Some occur well before cattle are slaughtered (such as forward contracts) and some occur after slaughter when carcass weight and quality are known.

Packers and feeders interviewed in a study by Schroeder et al. (1997) indicated they expect the proportion of cattle traded by non-cash-price methods to increase above that reported by GIPSA. Research shows that pricing accuracy, i.e., how closely fed cattle prices reflect actual wholesale values, increases when fed cattle are priced on a carcass weight versus live weight basis (Feuz, Fausti, and Wagner 1993). Most marketing agreements and strategic alliances in the beef industry involve formula pricing (Schroeder et al. 1997), which consists of a base price with specified premiums and discounts for carcasses above and below standard quality specifications.

Formula pricing on a carcass weight basis, while believed by many to be a positive trend toward value based marketing, is not without criticism. Cattle marketed by these methods bypass the traditional method of market price reporting. Price is not discovered until after cattle are slaughtered, and no reported price is available for subsequent price discovery. Some cattle feeders are concerned that formula-priced cattle will erode the current price reporting base and adversely affect the accuracy of reported cash cattle prices. They raised the concern that reported prices may not accurately reflect actual supply and demand conditions (Schroeder et al. 1997). This concern is in essence a thin market issue. As the number of cash market transactions decline, and concomitantly as the proportion of non-cash-market transactions increase, at what point do the remaining cash market reported prices no longer provide an accurate reflection of supply and demand conditions and cease to be useful for subsequent price discovery?

Information is costly to obtain and market prices cannot reflect all available information according to Grossman and Stiglitz (1980). Therefore, the informational content of market prices or quality of information contained in market prices is inversely related to the resources devoted to collecting and reporting market prices. Quality of information as used here refers to the accuracy with which reported prices reflect market supply and demand conditions. Stigler (1961) linked the informational content of prices with pricing efficiency. He equated price dispersion or price variation with ignorance in the market and further linked price dispersion with search costs of collecting and reporting prices. Anderson et al. (1998) found that price dispersion (i.e. price variance) increased with declining amounts of market information available to participants in an experimental market for fed cattle. Their results support the concern raised regarding fed cattle and wholesale beef prices as structural and behavioral changes occur in those markets.

Determining the accuracy of reported prices is difficult due to data limitations. In one of the rare studies using industry data, Hudson, Ethridge, and Brown (1996) used hedonic pricing models to assess the accuracy of reported cotton prices. However, for reported fed cattle prices, insufficient information is available for both cattle and sale lot attributes to estimate hedonic models with publicly available data. Such detail is available under special data collection procedures, such as those in Jones et al. (1992), but then comparable reported data are not available at the same level of detail.

This study used unique data from the *Fed Cattle Market Simulator (FCMS)* (Ward et al. 1996) to examine the relative accuracy of reported prices as the availability of reported prices declined under various scenarios. The unique data enabled examining trade-offs in pricing accuracy as the availability of reported prices decline. This study was inspired by Tomek's (1980) award-winning article, and like his work is "... essentially a search for hypotheses and an

exploration of methods ...” (p.434) rather than presenting conclusive findings. Research reported here is *one* step towards addressing industry concerns about price reporting accuracy. Tomek states, “... the trade-off between precise prices and the number of transactions ... is an area for fruitful research ...” (p.436). Results using experimental market data have clear limitations when drawing inferences from the experimental market to the real-world fed cattle market. But despite this caveat, work reported here is unique both in terms of what it attempts to do and in the data used, being the first study to explore trade-offs in parameters related to price reporting accuracy for various samples of reported prices compared with a known population of prices.

THEORY

Tomek (1980) links statistical theory with economics to define a thin market. He uses Chebyshev’s inequality as stated in (1) (Hoel, Port, and Stone 1971)

$$(1) P(|X_n - \mu| \geq c) \leq \frac{\sigma_n^2}{c^2},$$

where P is the probability, c is an arbitrary value, X_n is a random variable with mean μ and variance σ_n^2 .

Chebyshev’s inequality can be used to ask three questions. Given the variance of prices (σ_n^2), what are the number of transactions necessary (n) to ensure a given level of pricing precision (c) with a given level of confidence (P)?

$$(2) n = \frac{\sigma_n^2}{(1 - P)c^2}$$

Given the variance of prices (σ_n^2) and the number of transactions (n), what is the probability (P) reported prices are within a given level of pricing precision (c)?

$$(3) P = 1 - \frac{\sigma_n^2}{nc^2}$$

And, given the variance of prices (σ_n^2) and the number of transactions (n), what is the level of pricing precision (c) with a given level of confidence (P)?

$$(4) c = \frac{\sigma_n}{\sqrt{(1 - P)n}}$$

Let k , the variance of the mean, be a constant. Then from Chebyshev’s inequality, as k increases, P declines, and a trade-off occurs between variance (σ_n^2) and level of confidence or probability (P) of a given level of pricing precision. As k increases, c also increases, suggesting

that as variance (σ_n^2) increases, so does pricing precision (c). Together from the above, for a given variance (σ_n^2), c and P are inversely related, signifying a trade-off between pricing precision (c) and a level of confidence or probability (P) in the accuracy of reported prices.

In this research, we explore these relationships from Chebyshev's inequality for a known population of prices and for a series of subpopulation samples.

DATA

The *FCMS* has been described elsewhere (Ward et al. 1996, Anderson et al. 1998) so will be described only briefly here. The emphasis will be on the types of information generated or reported in the experimental market. The *FCMS* creates a market for fed cattle. Teams of participants role play as one of eight feedlot marketing managers or one of four meatpacking procurement managers. Feedlot marketing managers attempt to market fed cattle from their feedlot at a profit when cattle reach acceptable finish weights; and meatpacking procurement managers attempt to purchase fed cattle at a profit for processing into boxed beef.

During trading periods of about seven minutes, feedlots and meatpackers negotiate prices and finalize trades, i.e., about 40 transactions per trading period in market equilibrium. Each feedlot has a visible array of paper pens of cattle, each sheet of paper representing 100 steers on a show list, i.e., cattle available for sale during a five-week market window. Prices are negotiated and sales occur for the range of available weights of show-list cattle, from 1100 to 1200 pounds in 25-pound increments. Completed transaction sheets are scanned into a computer for record keeping and analysis.

Continuous market information is provided during the trading period on two digital display bars, one which scrolls cash market information (trading volume and high-low prices) and the other which scrolls futures market information (trading volume and current prices for three futures market contracts). Current market information parallels within-week or within-day market information available to fed cattle buyers and sellers from the Agricultural Marketing Service (AMS)-USDA and the Chicago Mercantile Exchange (CME). The focus for this study is on the weekly average price for 1150-lb cattle.

Data for this study were the same as for Ward et al. (1996). Data were collected from an agricultural economics course that met weekly for 90 minutes during the spring 1994 semester at Oklahoma State University. Students, primarily juniors and seniors majoring in agricultural economics, animal science, and agricultural education, participated in the *FCMS* from trading weeks 21 to 103. Data collected for this research included trading weeks 30 to 101, i.e., 72 trading weeks or just under one and one-half years of simulated market activity.

Each data record was one transaction, i.e., sale/purchase of one pen of 100 steers between one feedlot and one meatpacker, and the data set consisted of 35 cash market transactions per trading period on average for the 72-week period, a total of 2,515 observations.² Data for each transaction included: week traded, meatpacker purchasing cattle, feedlot selling cattle, weight of cattle traded, transaction price, and type of transaction (cash or forward contract). Other data for

² Contract transactions, totaling 167, were omitted for this study unlike the Ward et al. (1996) work.

each week of trading in the simulated market included: break-even price for 1150-pound cattle for each feedlot and the largest meatpacker, boxed beef price at which meat would be sold that week, closing nearby futures market price for the preceding week, fed cattle marketings for the previous week, and number of pens of cattle on the show list at the beginning of each trading week.

PROCEDURE

Population parameters for prices in the experimental market were known. Population parameters were examined along with those from Chebyshev's inequality (equations 2-4). Samples from the known populations were drawn in alternative ways to create subpopulations which were then examined in the same manner as population parameters.

Sampling scenarios were intended to reflect industry conditions or possible conditions. First, about 20 % of fed cattle have been procured by captive supply methods on average over the 1988-97 period (Grain Inspection, Packers and Stockyards Administration 1997), reducing the number of reportable transactions by a like percentage. Thus, we initially reduced the number of reportable prices from the population by randomly deleting 20 % of the transactions. Then, in accordance with industry perception, i.e., anticipating an increase in non-cash-market transactions, we reduced the population by randomly deleting 40, 60, and 80 % of the transactions.

Second, fed cattle price reporting currently is voluntary, both for cattle feeders and meatpackers. We assumed the largest packer ceased to cooperate with AMS-USDA in reporting prices. Thus, in the *FCMS*, we deleted all transactions involving the largest packer (packer 4), a reduction of 31.8 %. Wondering what would occur if a second large packer followed the largest firm, we also reduced the population by deleting all transactions for the two largest packers (packers 3 and 4), a reduction of 57.8 %.

Third, feeders and packers admit that some transactions above current market prices are made with the stipulation that they not be reported. Therefore, we deleted all transactions each week with the highest transaction price, a reduction of 14.4 %.

For the population of prices and for each sample, we calculated the weekly mean, variance, and number of transactions. We also calculated c , P , and the estimated n for given levels of c and P . For subsequent results reported here, we assumed industry participants would be satisfied to have reported prices for weekly average prices be within \$0.25/cwt (c) of the population mean 90 percent of the time (P). While this was judged to be a sufficient level of price reporting accuracy for price discovery, we recognize the arbitrariness of the assumption.

We tested whether or not means and variances of subpopulations differed from the population; and tested whether or not subpopulations were distributed differently than the population. Then, Ordinary Least Squares (OLS) regressions of the form

$$(5) Y_{ij} = \alpha + \beta_i X + e_{ij}$$

were estimated for combinations of the population parameters (i.e., n , σ_n^2 , $\bar{X}_n$) and those computed from Chebyshev's inequality (i.e., $\hat{c}$, $\hat{P}$, and $\hat{n}$). A Chow test was used to test for differences between regression coefficients for subpopulations versus those for the population.

RESULTS

Summary statistics for the population and subpopulations are shown in Table 1. Using a t test, no significant difference was found between subpopulation and population means at a 0.01 significance level. No significant difference was found between subpopulation and population variances using a χ^2 test at the 0.01 significance level.

An Omnibus χ^2 test at the 0.01 level found that the distribution of weekly mean prices was normally distributed. The distribution of mean prices in each subpopulation was also normally distributed based on the Omnibus χ^2 test. Thus, reducing the number of reported prices in the experimental market did not alter the distribution of weekly mean prices.

Figures 1 and 2 are difficult to decipher in detail but provide an indication of how means and variances did not and did vary, respectively, in the subpopulations compared with the population of weekly average prices. Figure 1 shows relatively small differences among the subpopulation means relative to the population means. From Figure 1, the subpopulations for which the means appear most variant from others are for the scenarios in which transactions were reduced 80 % and when the highest transaction prices each week were deleted. Figure 2, while messy, show larger differences in price variances for some weeks for the subpopulations compared with the population parameter. A cursory examination shows no consistency of highest variances. However, smallest variances appear to occur most often in the subpopulation excluding packers 3 and 4, and suggest prices paid by packers 1 and 2 some weeks were nearly identical and resulted in near-zero price variance.

Table 2 shows slope coefficients from regressions between pairs of variables, both for the population and subpopulations. No significant relationship or trade-off was found between several pairs of population and subpopulation parameters; e.g., mean versus number of observations, variance versus number of observations, mean versus variance, and number of observations versus estimated number of observations from Chebyshev's inequality. No relationship or trade-off was found between precision versus number of observations, and confidence level versus number of observations except for the random 80 % reduction. The significant relationship was negative for precision versus number of observations and positive for confidence level versus number of observations. Strong, significant relationships were found between three pairs of parameters; precision versus confidence level, precision and estimated number of observations, and confidence level versus estimated number of observations.

Table 3 shows Chow test results for structural changes between slope coefficients for each subpopulation versus the population. Significant structural changes paralleled slope coefficient significance. The subpopulation with a random 80 % reduction in observations was significantly different than the population for precision versus number of observations and for confidence level versus number of observations.

Only for the subpopulation without maximum prices, was there no significant difference between the subpopulation and population for precision versus confidence level, precision and estimated number of observations, and confidence level versus estimated number of observations. All other subpopulations differed from the population for these three pairs of variables.

IMPLICATIONS AND CONCLUSIONS

As noted earlier, results here have distinct limitations when drawing inferences from the experimental market for the real-world market. Three limitations are noted here. First, cattle characteristics in the experimental market vary among weight groups but are fixed within weight groups. Second, the experimental market has no geographic market component. Third, ex post sampling from a known population assumes implicitly that price discovery dynamics remain fixed in the absence of non-reportable or non-reported transactions.

Given the above limitations, what was found for the experimental market? What relevance does it have for price reporting accuracy and price discovery as real-world fed cattle markets becoming increasingly thin over time?

While not planned per se, the subpopulations represent a continuum of reduced numbers of transactions.

Subpopulation	Reduction in observations (%)
Scenario III: w/o maximum prices	14.4
Scenario I: random 20 %	20.0
Scenario II: w/o packer 4	31.2
Scenario I: random 40 %	40.0
Scenario II: w/o packers 3-4	57.8
Scenario I: random 60 %	60.0
Scenario I: random 80 %	80.0

Some relationships were invariant to number of observations and method of reducing transactions. The reported mean, variance, and distribution of mean prices were not significantly affected by any reduction of reported number of transaction prices. And the subpopulation relationships for these variables did not differ from the population relationships. Note that with the population of prices, number of transactions per week averaged 35, while with a random 80 % reduction in prices, number of transactions per week averaged just 7. Reduced number of transactions had no significant effect on the relationship between:

- Mean price and number of observations
- Variance of prices and number of observations
- Mean price and variance of prices
- Number of observations and estimated number of observations for a given level of pricing accuracy with a given probability or level of confidence.

Only when the reduction in prices reached 80 % was there a significant relationship between two pairs of variables, indicating at that point only did the subpopulation differ from the population. They were:

- Reported price precision and number of observations
- Probability of a given level of precision and number of observations.

Three pairs of variables had strong significant relationships. For each pair, only with the exception of the smallest reduction in transactions, was there no difference between the subpopulation and population. These were:

- Reported price precision and probability of a given level of precision
- Reported price precision and estimated number of observations for a given level of precision with a given probability

- Probability of a given level of precision and estimated number of observations with a given level of pricing precision.

Results suggest the *possibility* that number of non-reported fed cattle transaction prices could increase significantly before the industry faces significant concerns regarding the accuracy of reported prices *ceteris paribus*. However, as number of observations declined, the theoretically expected trade-off between pricing accuracy and the confidence in a given level of pricing precision occurred. That trade-off increased as the percentage reduction in transactions increased from 20 to 80 percent, regardless of two sampling method employed across that range.

These conclusions must be interpreted in the context of limitations inherent in the experimental market data. *If* non-reported fed cattle transaction prices increased disproportionately for specific qualities of fed cattle, i.e., specific grades, yield grades, breeds, etc., or in specific geographic areas, then conclusions drawn from experimental market data may be less valid. Research is needed to determine how consistently reductions in price reporting occur over cattle qualities or types and over geographic regions. Research is also needed to apply the methodology in this paper to transaction data collected specifically for that purpose, to corroborate or refute findings here. Lastly, research of a similar type is needed at the wholesale boxed beef level as the wholesale market increasingly becomes the focal point for fed cattle price discovery with increases in formula pricing of fed cattle on a carcass weight basis.

REFERENCES

- Anderson, J.D., C.E. Ward, S.R. Koontz, D.S. Peel, and J.N. Trapp. "Experimental Simulation of Public Information Impacts on Price Discovery and Marketing Efficiency in the Fed Cattle Market." *Journal of Agricultural and Resource Economics* 23(1998):1 forthcoming.
- Feuz, D.M., S.W. Fausti, and J.J. Wagner. "An Analysis of the Efficiency of Four Marketing Methods for Slaughter Cattle." *Agribusiness: An International Journal* 9(1993):6 453-63.
- Grain Inspection, Packers and Stockyards Program. *Packers and Stockyards Statistical Report: 1995 Reporting Year*. U.S. Department of Agriculture, GIPSA SR-97-1, September 1997.
- Grossman, S. and J.E. Stiglitz. "On the Impossibility of Informationally Efficient Markets." *American Economic Review* 70(1980):393-408.
- Hayenga, M.L. (ed.) *Pricing Problems in the Food Industry (with Emphasis on Thin Markets)*. University of Wisconsin, NC117 Monograph 7, 1979.
- Hoel, P.G., S.C. Port, and C.J. Stone. *Introduction to Probability Theory*. Houghton-Mifflin.
- Hudson, D., D. Ethridge, and J. Brown. "Producer Prices in Cotton Markets: Evaluation of Reported Price Information Accuracy." *Agribusiness: An International Journal* 12(1996):4 353-62.
- Jones, R., T. Schroeder, J. Mintert, and F. Brazle. "The Impacts of Quality on Cash Fed Cattle Prices." *Southern Journal of Agricultural Economics* 24(1992):149-62.
- Schroeder, T.C., C.E. Ward, J. Mintert, and D.S. Peel. "Beef Industry Price Discovery: A Look Ahead." *Price Discovery in Concentrated Livestock Markets: Issues, Answers, Future Directions*. W.D. Purcell, ed. Virginia Tech, Research Institute on Livestock Pricing. February 1997.
- Stigler, G.J. "The Economics of Information." *Journal of Political Economy* 69(1961):213-25.

Tomek, W.G. "Price Behavior on a Declining Terminal Market." *American Journal of Agricultural Economics* 61(1980:433-44).

Ward, C.E., S.R. Koontz, D.S. Peel, and J.N. Trapp. "Price Discovery in an Experimental Market." *Review of Agricultural Economics* 18(1996):449-66.

Table 1. Population and Subpopulation Summary Statistics

Data Sets	Total Observations	Mean Price	Standard Deviation	Minimum Price	Maximum Price
Cash Transactions	2,515		3.84		
Scenario I:					
<i>Random Reduction by</i>					
20%	2,011	78.66	3.84	70.00	86.05
40%	1,508	78.57	3.84	70.00	86.05
60%	1,004	78.47	3.79	70.25	86.05
80%	502	78.55	3.88	71.25	86.05
Scenario II:					
<i>Without Packer 4</i>	1,716	78.46	3.83	70.00	86.05
<i>Without Packers 3-4</i>	1,060	78.56	3.84	70.00	86.05
Scenario III:					
<i>Without Maximum Prices</i>	2,154	78.29	3.77	70.00	86.00

Table2. Slope Coefficients

Regression	Base	Scenario I				Scenario II		Scenario III
		20%	40%	60%	80%	W/o Packer 4	W/o Packers 3-4	W/o Max Prices
$\bar{X}$ on n	-0.026	-0.006	-0.065	-0.142	-0.046	-0.095	0.002	-0.105
t-ratio	-0.415	-0.081	-0.727	-1.346	-0.317	-1.361	0.018	-1.784
σ^2 on n	0.012	0.012	0.034	0.038	-0.008	0.011	0.020	0.010
t-ratio	1.167	0.996	2.360	2.058	-0.272	0.922	1.242	1.172
$\bar{X}$ on σ^2	-0.167	-0.216	-0.229	-0.136	0.273	-0.506	-0.160	-0.453
t-ratio	-0.227	-0.289	-0.322	-0.203	0.454	-0.707	-0.221	-0.565
c on n	-0.003	-0.005	-0.003	-0.006	-0.057	-0.004	-0.001	-0.004
t-ratio	-1.119	-1.297	-0.513	-0.627	-2.812	-0.743	-0.166	-1.297
P on n	0.003	0.005	-0.002	0.013	0.191	0.008	0.015	0.004
t-ratio	0.622	0.787	-0.174	0.656	2.456	0.995	0.816	0.924
$\hat{n}$ on n	1.869	1.861	5.473	6.059	-1.255	1.722	3.145	1.669
t-ratio	1.167	0.996	2.360	2.058	-0.272	0.922	1.241	1.173
P on c	-1.366	-1.520	-1.718	-2.257	-3.612	-1.610	-2.038	-1.347
t-ratio	-35.330	-35.100	-34.700	-32.490	-27.170	-28.980	-22.310	-32.050
c on $\hat{n}$	0.002	0.002	0.002	0.003	0.004	0.002	0.003	0.002
t-ratio	19.850	20.010	17.090	15.080	16.32	18.030	15.990	17.170
P on $\hat{n}$	-0.003	-0.003	-0.004	-0.006	-0.016	-0.004	-0.007	-0.003
t-ratio	-30.170	-29.720	-25.840	-18.110	-19.500	-22.730	-18.150	-24.080

Table 3. Chow Test for Structural Change

Regression	<u>Scenario I</u>				<u>Scenario II</u>		<u>Scenario III</u>
	20%	40%	60%	80%	W/o Packer 4	W/o Packers 3 - 4	W/o Max Prices
$\bar{X}$ on n	0.033	0.228	0.852	0.129	0.601	0.095	0.726
σ^2 on n	0.228	2.254	2.456	0.553	0.127	0.422	0.265
$\bar{X}$ on σ^2	0.002	0.002	0.001	0.108	0.058	0.016	0.089
c on n	0.132	0.577	1.244	5.585	0.003	0.163	0.845
P on n	0.344	1.649	1.665	5.128	0.332	0.555	0.325
$\hat{n}$ on n	0.228	2.254	2.456	0.553	0.127	0.422	0.265
P on c	3.548	14.268	39.456	33.050	7.611	14.610	0.844
c on $\hat{n}$	7.755	27.163	69.571	146.809	14.304	43.544	2.405
P on $\hat{n}$	26.344	80.652	117.279	201.162	48.335	101.320	7.498